Ship Stability

Ch. 9 Numerical Integration Method in Naval Architecture

Spring 2018

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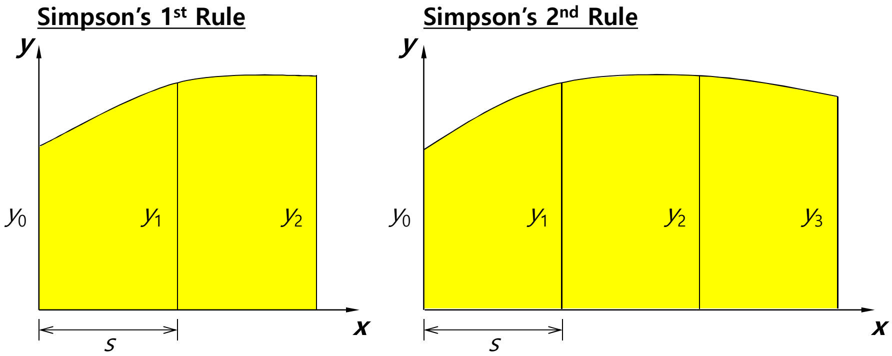
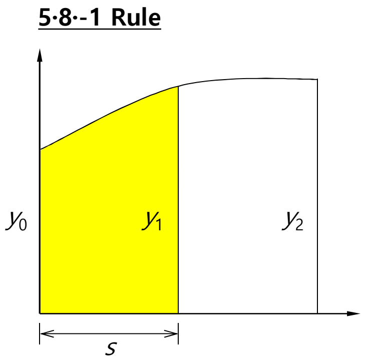
1. Simpson's Rule

Simpson's 1st and 2nd Rules

Simpson's 1st and 2nd Rules



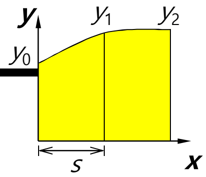
5·8·-1, 3·10·-1, and 7·36·-3 Rules



3·10·-1 Rule

$$M_y = \frac{1}{24} s^2 (3y_0 + 10y_1 - 1y_2)$$

Derivation of Simpson's 1st Rule (1/4)



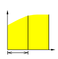
st Rule:
te the function y by a parabola (polynomial curve) whose
s the form

$$y = a_0 + a_1 x + a_2 x^2$$

ıla is represented by three points defining this curve.
ɔoints (y_0, y_1, y_2) are obtained by dividing the given interval into equal
"s".

Parabola :

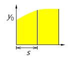
②
③

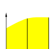


Derivation of Simpson's 1st Rule (3/4)

$$a_0 + a_1 x + a_2 x^2$$

Integrate the area A from 0 to 2s. (Definite Integral)





Derivation of Simpson's 2nd Rule (1/4)

Simpson's 2nd rule :

:e the function by a polynomial curve whose equation has the

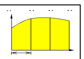
$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

›olynomial curve is represented by four points defining this curve.

›ints (y_0, y_1, y_2, y_3) are obtained by dividing the given interval into equal "s".

Cubic polynomial curve:

The relation between the coefficients a_0, a_1, a_2, a_3 ("Find") and y_0, y_1, y_2, and y_3 are

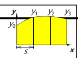


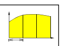
The unknown coefficients, a_0, a_1, a_2, and a_3 lead to

Derivation of Simpson's 2nd Rule (3/4)

$$+a_1 x + a_2 x^2 + a_3 x^3$$

Integrate the area A from 0 to 3s.





Derivation of 5·8·-1 Rule (1/4)

te the function y by a parabola whose equation has the form

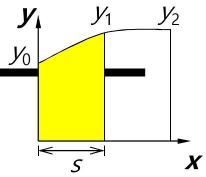
$$y = a_0 + a_1 x + a_2 x^2$$

la is represented by three points defining this curve.
oints (y_0, y_1, y_2) are obtained by dividing the given interval into equal
"*s*".

Parabola :

②
③

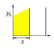
Derivation of 5·8·-1 Rule (3/4)



$$a_0 + a_1 x + a_2 x^2$$

Integrate the area A from 0 to s.

Derivation of 3·10·-1 and 7·36·-3 Rules

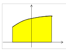


$$M_y = M_L = \int_0^s x\,dA = \int_0^s xy\,dx = \int_0^s a_0 x + a_1 x^2 + a_2 x^3\,dx$$

$$= \frac{1}{24} s^2 (3y_0 + 10y_1 - y_2)$$

3·10·-1 Rule: The first moment of area about y axis

7·36·-3 Rule: The second moment of area about y axis



Given: Function $f(t)$

Find: Integration of $f(t)$ at a given interval [-1, 1]

Gaussian quadrature:

Calculation of Area by Using Gaussian Quadrature

n	**Node** t_j
3	t_1 = -0.7745966692 t_2 = 0 t_3 = 0.7745966692

Coefficients

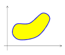
A_1 = 0.5555555556

A_2 = 0.8888888889

A_3 = 0.5555555556

Calculation of Area by Using Green's Theorem

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy = \oint_C \left(Mdx + Ndy \right)$$



Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, $\partial M/\partial y$, and $\partial N/\partial x$ are continuous on R.

Green's theorem gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.

※ The region should be the left-hand of the curve.

If

L.H.S

(A: Area)

※ The region should be the left-hand of the curve.

Surface Integral Line Integral

M, N: The functions of x and y. And M, N, ∂M/∂y, and ∂N/∂x are continuous on R.

If

Calculation of First Moment of Area by Using Green's Theorem (2/2)

※ The region should be the left-hand of the curve.

Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

LHS =

$$\therefore 2M_{A,x} = \oint_C \left(xydy - \frac{y^2}{2}dx \right)$$

RHS =

※ The region should be the left-hand of the curve.

Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓Second moment of area about the y-axis in x direction

Calculation of Second Moment of Area by Using Green's Theorem (2/2)

Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

※ The region should be the left-hand of the curve.

✓Second moment of area about the x-axis in y direction

L.H.S =

$$I_{A,x} = \frac{1}{2}\oint\left(xy^2dy - \frac{y^3}{3}dx\right)$$

R.H.S =

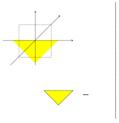
Segment ①:

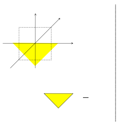
Using the chain rule, convert the line integral for y and z into the integral for only one parameter "*t*".

Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

Green's theorem

"Water pla





[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (2/10)

Segment ②:

Segment ①:

$$\frac{1}{2}\int_{②} ydz - zdy = \frac{1}{2}\int_{0}^{\sqrt{2}}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right)dt$$

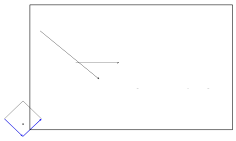
✓Area A

③

①

②

Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;



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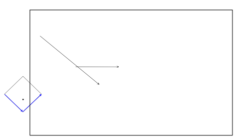
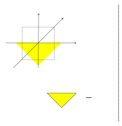
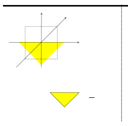
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (3/10)

✓Area A

Segment ③:







[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (4/10)

Green's theorem

$$= \frac{1}{2}\oint_C \frac{y^2}{2} dz - yzdy$$

Segment ①

✓First moment of area about the z-axis in y direction $\boldsymbol{M}_{A,z}$

③

①

②

Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

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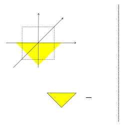
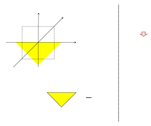
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (5/10)





[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (6/10)

$$\frac{1}{2}\int_{①}\frac{y^2}{2}dz - yzdy = -\frac{\sqrt{2}}{3}$$

✓First moment of area about the z-axis in y direction $M_{A,z}$

Segment ①:

Segment ②:

Segment ③:

Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

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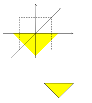
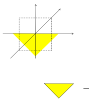
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (7/10)

Segment ①:





[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (8/10)

$$= \frac{1}{2}\int_0^{\sqrt{2}} \left(t(t-\sqrt{2})\cdot 1 - \frac{(t-\sqrt{2})^2}{2}\cdot 1 \right) dt$$

✓First moment of area about the y-axis in z direction $M_{A,y}$

Segment ①:

Segment ②:

③

①

②

Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

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[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (9/10)

[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (10/10)

$$M_{A,z} = \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy = 0$$

✓Area A

✓First moment of area about the z-axis in y direction $M_{A,z}$

✓First moment of area about the y-axis in z direction $M_{A,y}$

✓Centroid

③

①

②

Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

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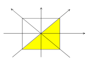
Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

2:1

[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (2/10)

$$= \frac{1}{2}\int_{-1}^{1}(1\cdot 1 - t\cdot 0)dt$$



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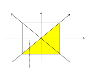
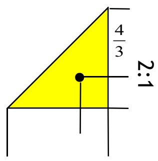


[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (4/10)



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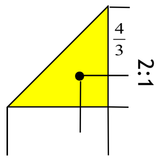
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (6/10)



① ② ③

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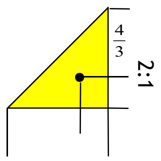
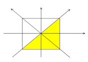
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (8/10)





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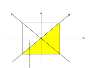
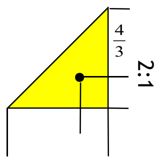


[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (10/10)



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4. Calculation of Hydrostatic Values by Using Simpson's Rule

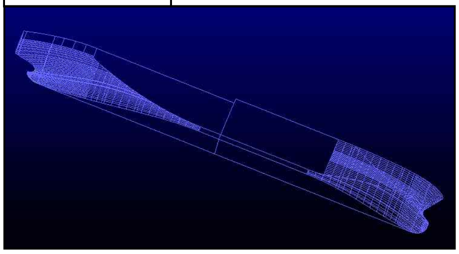
What is a "Hull form"?

☑ **Hull form**
- **Outer shape of the hull** that is streamlined in order to satisfy requirements of a ship owner such as a deadweight, ship speed, and so on
- **Like a skin of human**

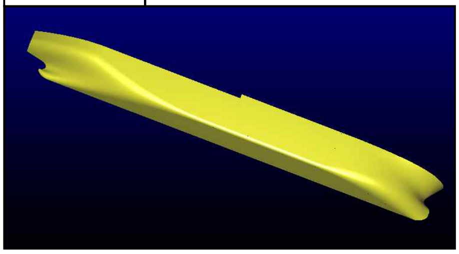
☑ **Hull form design**
- **Design task that designs the hull form**

Hull form of the VLCC(Very Large Crude oil Carrier)

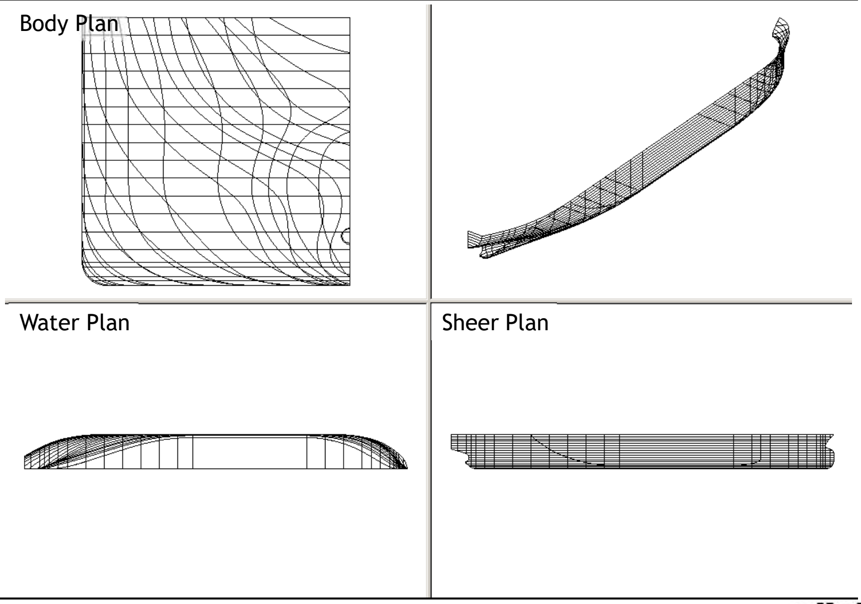
Wireframe model



Surface model



Lines of a 320,000ton VLCC



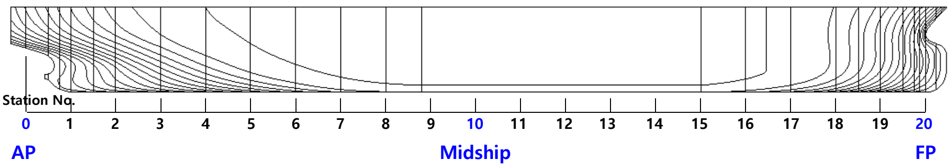
Station

☑ **Stations are ship hull cross sections at a spacing of L_{BP}/20.**

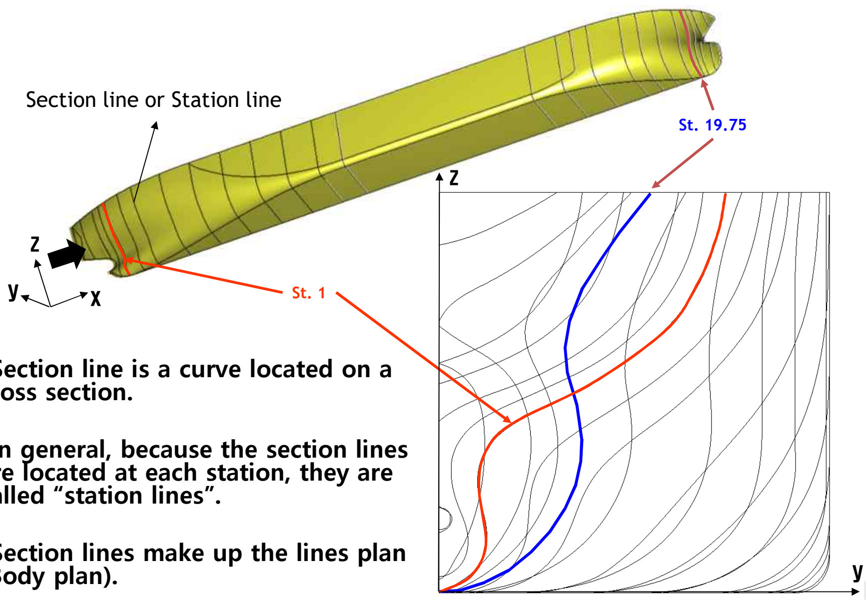
☑ **The station 0 is located at the aft perpendicular and the station 20 is at the forward perpendicular. And the station 10 therefore represents the midship section.**

- Station spacing = **L_{BP} / 20**
- X position of the Station "A" = **Station No. of "A"** × **Station spacing**

Sheer Plan (Elevation View)



Section Line and Body Plan



☑ Section line is a curve located on a cross section.

☑ In general, because the section lines are located at each station, they are called “station lines”.

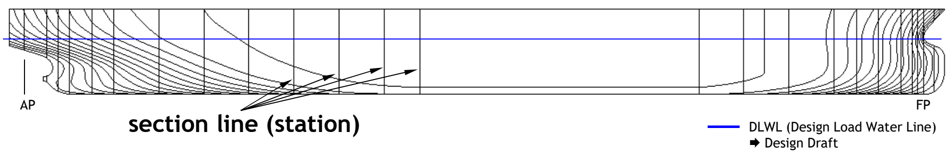
☑ Section lines make up the lines plan (Body plan).

Buttock Line and Sheer Plan (Buttock Plan)

☑ Buttock line is a curve located on a profile (lateral) section (x-z plane).

☑ Buttock lines make up the **sheer plan** or **buttock plan** of lines.

Sheer Plan (Elevation View)

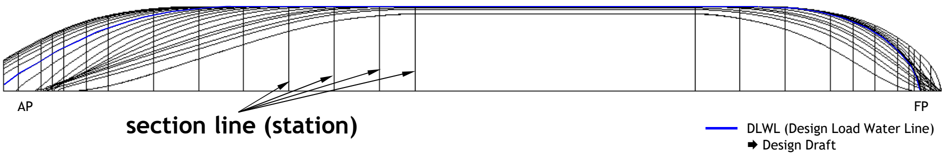


Example of water line of a 320K VLCC

Water Line and Water Plan (Half-Breadth Plan)

- ☑ Water line is a curve located on a water plane (vertical) section (x-y plane).
- ☑ Water lines make up the **water plan** or **half-breadth plan** of lines.

Water Plan (Plan View)



Example of water line of a 320K VLCC

Example of Offsets Table of a 6,300TEU Container Ship

→ Waterline

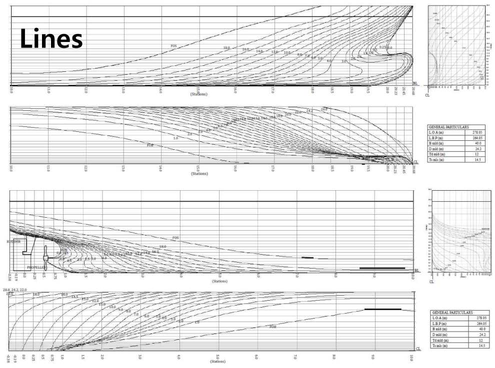
* Unit: mm

↓ Stations

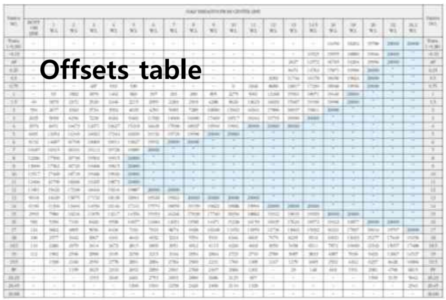
Half-Breadth

Station NO.	HALF BREADTH FROM CENTER LINE																				Station NO.
	BOTTOM LINE	1 W.L.	2 W.L.	3 W.L.	4 W.L.	5 W.L.	6 W.L.	7 W.L.	8 W.L.	9 W.L.	10 W.L.	11 W.L.	12 W.L.	13 W.L.	14.5 W.L.	16 W.L.	18 W.L.	20 W.L.	22 W.L.	24.2 W.L.	
Trans. (-0.38)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	14450	18262	19780	20000	20000	Trans. (-0.38)
-0.19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10525	15955	18890	19944	20000	*	-0.19
AP	-	-	-	-	-	-	-	-	-	-	-	-	-	2627	12572	16765	19204	19991	20000	*	AP
0.25	-	-	-	-	-	-	-	-	-	-	-	-	-	8474	14763	17871	19596	20000	*	*	0.25
0.5	-	-	-	-	-	-	-	-	-	-	-	-	3283	11746	16178	18456	19824	20000	*	*	0.5
0.75	-	-	-	487	933	530	-	-	-	-	0	1846	8680	13817	17230	18948	19956	20000	*	*	0.75
1	-	93	1802	1870	1462	863	397	183	280	895	2275	5061	12168	15561	18071	19440	20000	*	*	*	1
1.5	49	1879	2372	2520	2446	2215	2059	2283	2919	4288	9026	13623	16033	17687	19196	19906	20000	*	*	*	1.5
2	534	2677	3363	3734	3932	4029	4250	5085	7289	10680	13943	16341	17896	18937	19811	20000	*	*	*	*	2
3	2025	5058	6294	7228	8182	9483	11585	14000	16000	17469	18517	19244	19735	19990	20000	*	*	*	*	*	3
4	3974	8451	10473	12071	13627	15218	16635	17938	18937	19594	19941	20000	20000	20000	*	*	*	*	*	*	4
5	6091	12054	14349	16032	17344	18359	19152	19729	19996	20000	20000	*	*	*	*	*	*	*	*	*	5
6	8152	14697	16708	18069	19011	19627	19952	20000	20000	*	*	*	*	*	*	*	*	*	*	*	6
7	10187	16515	18101	19113	19728	19985	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	7
8	12286	17500	18738	19502	19915	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	*	8
9	13900	17562	18720	19408	19815	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	*	9
10	13517	17469	18718	19466	19926	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	*	10
11	12406	16799	18506	19265	19873	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	*	11
12	11001	15632	17338	18464	19316	19887	20000	20000	*	*	*	*	*	*	*	*	*	*	*	*	12
13	9018	14029	15875	17152	18138	18941	19528	19922	20000	20000	20000	20000	*	*	*	*	*	*	*	*	13
14	6196	11304	13404	14934	16146	17141	17974	18650	19199	19622	19886	19994	20000	20000	20000	*	*	*	*	*	14
15	2993	7980	10216	11870	13217	14356	15353	16246	17038	17740	18354	18882	19312	19633	19929	20000	20000	*	*	*	15
16	583	5356	7103	8420	9598	10677	11684	12651	13581	14471	15328	16159	16935	17624	18572	19322	19877	20000	20000	*	16
17	124	3602	4805	5656	6434	7181	7919	8674	9438	10248	11052	11859	12734	13663	15032	16321	17837	19014	19797	20000	17
18	100	2577	3442	3967	4341	4643	4932	5224	5554	5931	6346	6845	7479	8235	9516	10921	13033	15277	17449	19250	18
18.5	110	2286	2979	3414	3673	3815	3893	3951	4012	4115	4320	4603	4959	5458	6511	7872	10049	12543	15057	17488	18.5
19	112	1982	2596	2988	3195	3258	3215	3104	2954	2804	2723	2710	2780	3087	3833	4987	7036	9433	11867	14527	19
19.5	-	1538	2160	2550	2778	2891	2894	2784	2569	2231	1760	1385	1247	1279	1685	2532	4262	6257	8428	10884	19.5
FP	-	-	1195	1825	2310	2652	2859	2901	2768	2497	2060	1301	-	29	148	603	1551	2981	4700	6815	FP
20.23	-	-	-	1353	2045	2481	2753	2893	2890	2686	2125	697	-	-	-	-	-	1590	3135	5042	20.23
20.45	-	-	-	-	-	1300	1910	2258	2420	2400	2110	1320	-	-	-	-	-	-	-	2343	20.45
20.68	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	20.68

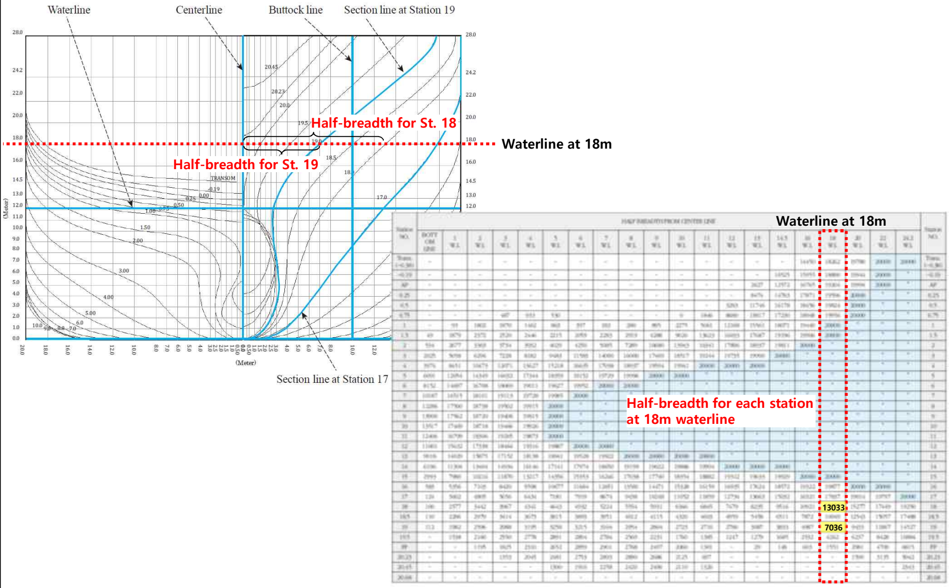
Relationship Between Lines and Offsets Table (1/2)

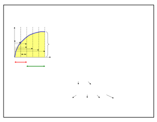


Generation of offsets table from the lines

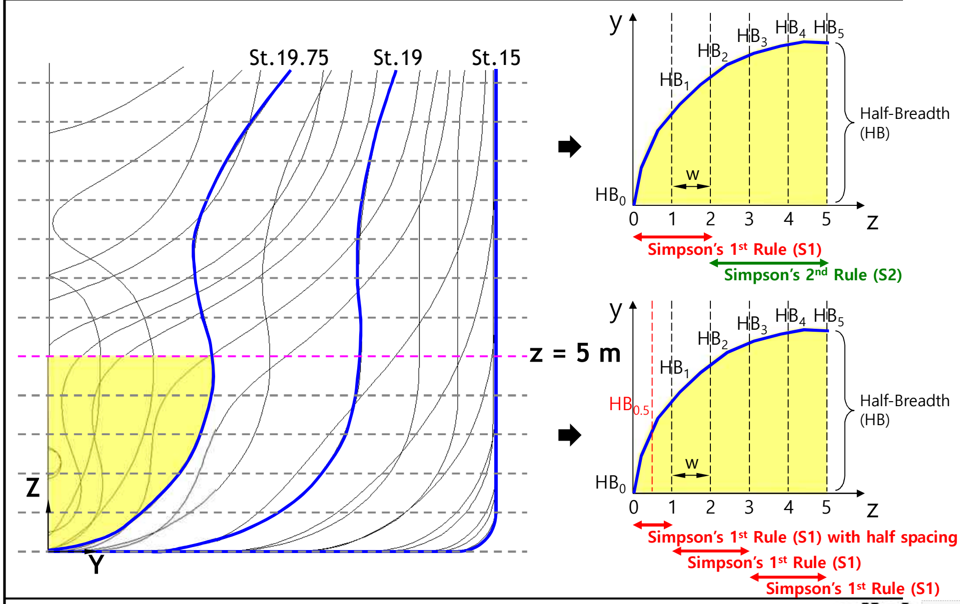


Relationship Between Lines and Offsets Table (2/2)





Calculation of Sectional Area

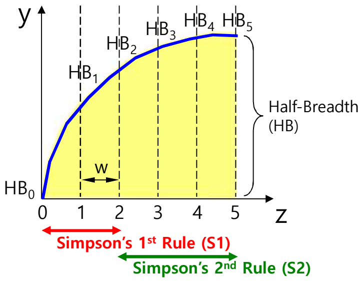


Calculation of the First Moment of Sectional Area

$$Area_1 = \int dA = \frac{1}{3}s(y_0 + 4y_1 + y_2) = \frac{1}{3}w(HB_0 + 4HB_1 + HB_2)$$

Calculation of the First Moment of Sectional Area

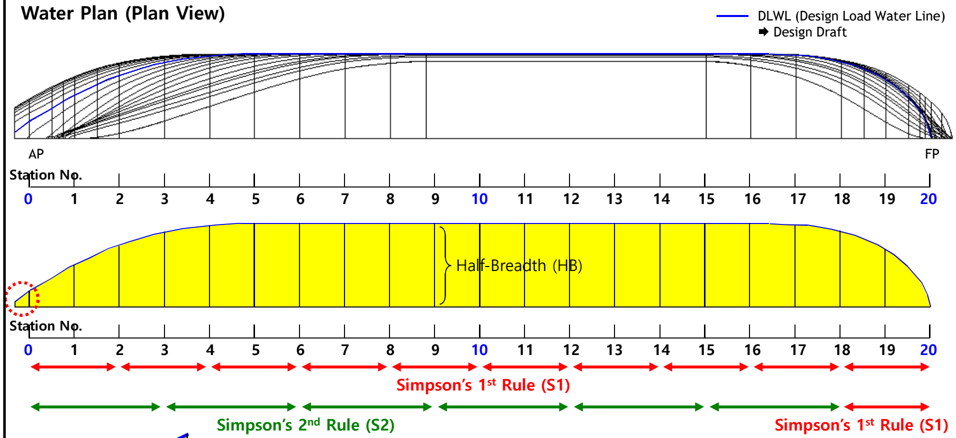
Calculation of the First Moment of Sectional Area (about z axis)

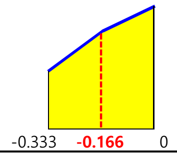


$$M_{z,1} = \int z dA = \frac{1}{3} s (Y_0 + 4Y_1 + Y_2)$$
$$= \frac{1}{3} s \left(1 \cdot ((y_0/2) \cdot y_0) + 4 \cdot ((y_1/2) \cdot y_1) + 1 \cdot ((y_2/2) \cdot y_2)\right)$$
$$= \frac{1}{3} w \left(1 \cdot ((HB_0/2) \cdot HB_0) + 4 \cdot ((HB_1/2) \cdot HB_1) + 1 \cdot ((HB_2/2) \cdot HB_2)\right)$$

Calculation of Water Plane Area

Water Plan (Plan View)

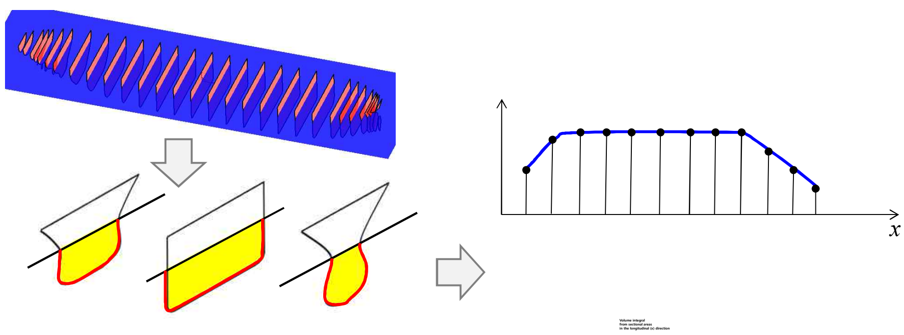




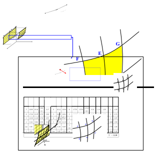
1. Generate a temporary section (e.g., -0.166)
2. Perform Simpson's 1st Rule.

Calculation of Displacement Volume

☑ The displacement volume (underwater volume) at a certain draft can be calculated **by integrating sectional areas in the longitudinal direction**.



☑ In addition, the volume can be calculated **by integrating water plane areas in the vertical direction**. There can be a difference between two volumes due to approximation.



Calculation for Wetted Surface Area

- ☑ **The wetted surface area means ship's area which contacts with water.**
- ☑ **This area can be calculated with the following approximate formula.**

$$S = \delta z \int_{Sta.\,4}^{Sta.\,6} \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dx$$

$$\delta z = (6-3) = 3\,m$$

1)

In the table,
[(1.2) − (1.1)]

Example of Calculation for Wetted Surface Area (2/7)

Example of Calculation for Wetted Surface Area (3/7)

2)

HB: Half breadth for waterline
HB_A: Half breadth afterward
HB_F: Half breadth forward
S: Wetted surface area of the ship

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19,66	18,41	0,42 (1)	0,17	6	20,12	19,84	4	17,56	15,56	−0,12 (2)	0,01	1,18	1,09	1	1,09
4	17,56	15,47	0,70	0,49	5	19,66	18,41	3	13,38	11,16	−0,24	0,06	1,55	1,24	3	3,72
3	13,38	11,16	0,74	0,55	4	17,56	15,47	2	8,14	6,64	−0,33	0,11	1,66	1,29	3	3,87
2	8,14	6,64	0,50	0,25	3	13,38	11,16	1	2,62	2,16	−0,35	0,13	1,38	1,17	1,444	1,69
1 1/2	5,43	4,39	0,35	0,12	2	8,14	6,64	1 (3)	2,62	2,16	−0,36	0,13	1,25	1,12	1,778	1,99
1	2,62	2,16	0,15	0,02	1 1/2	5,43	4,39	1/2	−0,22*	−0,28*	−0,37	0,14	1,16	1,08	0,444	0,48
																Σ = 12,84

2. Substituting 1) and 2) into the formula.

$$S \approx \delta z \int_{Sta.1}^{Sta.5} \sqrt{1+\left(\frac{\delta y}{\delta x}\right)^2+\left(\frac{\delta y}{\delta z}\right)^2}\,dx$$

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(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19,66	18,41	0,42 (1)	0,17	6	20,12	19,84	4	17,56	15,56	-0,12 (2)	0,01	1,18	1,09	1	1,09
4	17,56	15,47	0,70	0,49	5	19,66	18,41	3	13,38	11,16	-0,24	0,06	1,55	1,24	3	3,72
3	13,38	11,16	0,74	0,55	4	17,56	15,47	2	8,14	6,64	-0,33	0,11	1,66	1,29	3	3,87
2	8,14	6,64	0,50	0,25	3	13,38	11,16	1	2,62	2,16	-0,35	0,13	1,38	1,17	1,444	1,69
1 1/2	5,43	4,39	0,35	0,12	2	8,14	6,64	1 (3)	2,62	2,16	-0,36	0,13	1,25	1,12	1,778	1,99
1	2,62	2,16	0,15	0,02	1 1/2	5,43	4,39	1/2	-0,22*	-0,28*	-0,37	0,14	1,16	1,08	0,444	0,48
															Σ	= 12,84

$$S \approx \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{1}{2} \left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2 \cdot \delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2 \cdot \delta x} \right) \right)^2 + \left(\frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z} \right)^2 } \, dx$$

Example of Calculation for Wetted Surface Area (6/7)

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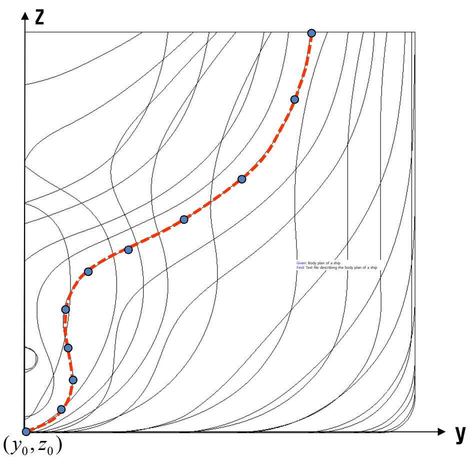
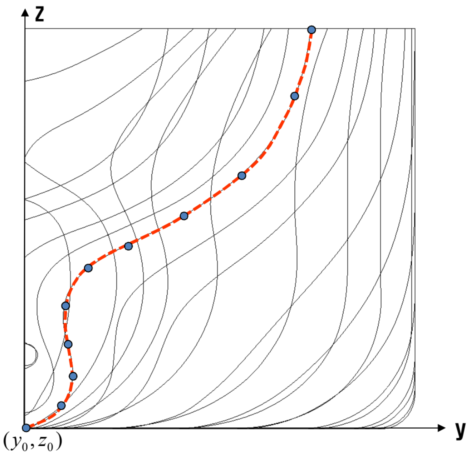
5. Calculation of Hydrostatic Values by Using Gaussian Quadrature and Green's Theorem

Description of Section Lines (1/2)



Example of text file for describing the body plan of a ship

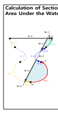
300.0 50.0 27.0 18.0	// *LBP*, B_{mld}, D_{mld}, *T*
27	// Section Line Num.
…	
1.0 11	// Station, Point Num.
y_0 z_0	// Y coord., Z coord.
y_1 z_1	
y_2 z_2	
…	
y_{10} z_{10}	
1.5 10	
…	


z
y
(y_0, z_0)

Given: B-spline curve, the intersection points between the B-spline curve and water plane, and B-spline parameter "u" at each end point of the line segments
Find: Sectional area and 1st moment of section

Green's Theorem

Calculation of Sectional Area and 1st Moment of Sectional Area Under the Water Plane (2/4)



$$u = \frac{(t+1)(u_{\max} - u_{\min})}{2} + u_{\min}$$

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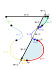

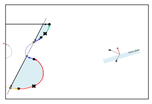

※ Method to check whether the line segments are located under the water plane or not

Pointₖ

: Origin

Pointₖ

✓Perform only line integration for the segments which are on or below the water plane.

The line segment : ➡ No integration

$$\mathbf{n}\cdot(\mathbf{X}-\mathbf{O}) > 0$$

- To calculate the sectional area under the water plane, it is required to check whether the points on the line segments are located under the water plane or not.

✓Check the location of the point by using the sign of dot product of normal vector of the water plane and position vector of the point

: The point is above the water plane.

: The point is on or below the water plane.

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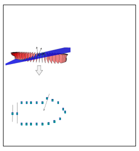
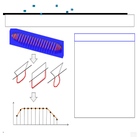




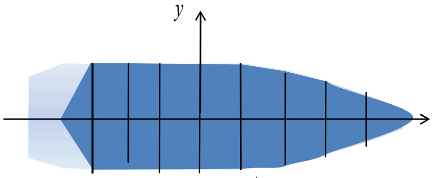
Calculation of Water Plane Area, 1st and 2nd Moment of Water Plane Area

Given: Intersection points between the water plane and the section lines
Find: Water plane area, 1st moment and 2nd moment of the water plane area

✓ Transform the intersection points decomposed in body fixed frame into the points decomposed in water plane fixed frame (inertial frame).

✓ Generate the curve which interpolates the intersection points. If a section 'x' has no intersection point, input the point as (x, 0, 0).

✓ Calculate the area, 1st moment and 2nd moment of area using Green's theorem or Gaussian quadrature.



Calculation procedure

Intersection point between the water plane area and the section lines

5) Integrate the area along ship's length using Green's theorem or Gaussian quadrature.

➡ Wetted surface area is calculated.