Build a Flow Equation

2019년 7월 19일 금요일 오후 7:07

① Mass conservation

$$\nabla \cdot (\rho v) + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x \partial y \partial z} \frac{\partial(\rho V_{ext})}{\partial t}$$

② Darcy law

$$v = -\frac{k}{\mu} \nabla p$$

③ Constitutive equations

$$C_R = \frac{1}{\phi}\frac{\partial \phi}{\partial p}, \quad C_f = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$$

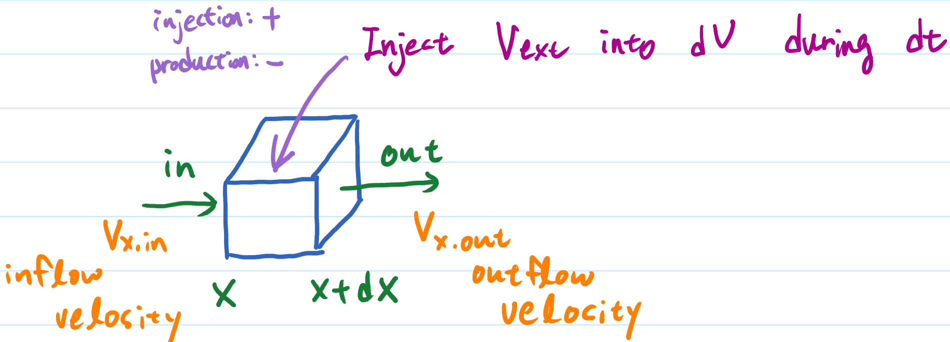
④ BC & IC

⑤ Single or Multiphase

Mass Conservation

2019년 7월 19일 금요일 오후 7:08

mass balance over $\underset{\text{small volume}}{dV}$ and $\underset{\text{small time interval}}{dt}$



First only x-direction is considered.

Accumulation in dV during dt is calculated in two different ways, and they equal.

i) in-flow mass : $[A v_x dt \rho]_x$

out-flow mass : $[A v_x dt \rho]_{x+dx}$

accumulation in dV : $[A v_x dt \rho]_x - [A v_x dt \rho]_{x+dx} + [\rho V_{ext}]_{t+dt} - [\rho V_{ext}]_t$

ii) accumulation during dt :

$[\phi \, dV \rho]_{t+dt} - [\phi \, dV \rho]_t$

i) = ii)

$$[\underbrace{A}_{dydz} v_x dt \rho]_x - [\underbrace{A}_{dydz} v_x dt \rho]_{x+dx} + [\rho V_{ext}]_{t+dt} - [\rho V_{ext}]_t = [\phi \underbrace{dV}_{dxdydz} \rho]_{t+dt} - [\phi \underbrace{dV}_{dxdydz} \rho]_t$$

divided by dx dy dz dt

$$-\frac{[\rho v_x]_{x+dx} - [\rho v_x]_x}{dx} + \frac{[\rho V_{ext}]_{t+dt} - [\rho V_{ext}]_t}{dx\,dy\,dz\,dt} = \frac{[\phi\rho]_{t+dt} - [\phi\rho]_t}{dt}$$

$$\frac{d(\rho v_x)}{dx} + \frac{d(\phi\rho)}{dt} = \frac{1}{dxdydz}\frac{d(\rho V_{ext})}{dt}$$

x (space) and t (time) are independent

$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x \partial y \partial z}\frac{\partial(\rho V_{ext})}{\partial t}$$

Consider y-, z- directions

$$\underbrace{\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}} + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x \partial y \partial z}\frac{\partial(\rho V_{ext})}{\partial t}$$

$\nabla \cdot (\rho v)$: sum of divergence (out flow) in x, y, z

multiply $\partial t \partial x \partial y \partial z$

$\partial t \partial y \partial z \, \partial(\rho V_x) + \partial t \partial z \partial x \, \partial(\rho V_y)$

multiply $\partial t \partial x \partial y \partial z$

$$\partial t \partial y \partial z \, \partial(\rho V_x) + \partial t \partial z \partial x \, \partial(\rho V_y)$$

$$+ \partial t \partial x \partial y \, \partial(\rho V_z) + \partial x \partial y \partial z \, \partial(\phi \rho) = \partial(\rho V_{ext})$$

$$\partial(\rho V_{ext}) - [\partial t \partial y \partial z \, \partial(\rho V_x) + \partial t \partial z \partial x \, \partial(\rho V_y) + \partial t \partial x \partial y \, \partial(\rho V_z)] = \partial x \partial y \partial z \, \partial(\phi \rho)$$

Injected mass Outflow mass Accumulation

For an infinitesimal volume in porous media, the sum of mass change in x-, y-, z-directions and total volume is same to the externally injected mass.

$$\nabla \cdot (\rho v) + \frac{\partial(\phi \rho)}{\partial t} = \frac{1}{\partial x \partial y \partial z} \frac{\partial(\rho V_{ext})}{\partial t}$$

1D Flow Equation

2019년 7월 19일 금요일 오후 7:47

① Mass conservation

$$\nabla\cdot(\rho v) + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x\partial y\partial z}\frac{\partial(\rho V_{ext})}{\partial t}$$

② Darcy law

$$v = -\frac{k}{\mu}\nabla p$$

③ Constitutive equations

$$C_R = \frac{1}{\phi}\frac{\partial\phi}{\partial p}, \quad C_f = \frac{1}{\rho}\frac{\partial\rho}{\partial p}$$

④ BC & IC ⑤ Single phase

$$\nabla\cdot(\rho v) + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x\partial y\partial z}\frac{\partial(\rho V_{ext})}{\partial t}$$

$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x\partial y\partial z}\frac{\partial(\rho V_{ext})}{\partial t}$$

$$v = -\frac{k}{\mu}\nabla p$$

$$v_x = -\frac{k}{\mu}\frac{\partial p}{\partial x}$$

$$C_R = \frac{1}{\phi}\frac{\partial\phi}{\partial p}, \quad C_f = \frac{1}{\rho}\frac{\partial\rho}{\partial p}$$

$$\frac{\partial}{\partial x}\left(-\frac{k\rho}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x\partial y\partial z}\frac{\partial(\rho V_{ext})}{\partial t}$$

$$\frac{\partial}{\partial x}\left(\frac{k\rho}{\mu}\frac{\partial p}{\partial x}\right) + \frac{1}{\partial x\partial y\partial z}\frac{\partial(\rho V_{ext})}{\partial t} = \frac{\partial(\phi\rho)}{\partial t}$$

$$\rho = \frac{\text{constant}}{B}$$ fluid formation volume factor

See another sheet

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu}\frac{\cancel{\text{constant}}}{B}\frac{\partial p}{\partial x}\right) + \frac{1}{\partial x\partial y\partial z}\frac{\partial\left(\cancel{\text{constant}}\,\frac{V_{ext}}{B}\right)}{\partial t} = \frac{\partial}{\partial t}\left(\phi\frac{\cancel{\text{constant}}}{B}\right)$$

$\partial y\,\partial z = A$

$\frac{V_{ext}}{B}$ = Vext at surface

– surface flow rate

$$\frac{V_{ext}}{B} = V_{ext} \text{ at surface}$$

$$\frac{\partial V_{ext} \text{ at surface}}{\partial t} = Q \leftarrow$$ surface flow rate (inj: +, prod: −)

$$\frac{\partial}{\partial x}\left(\frac{K}{\mu B}\frac{\partial P}{\partial x}\right) + \frac{Q}{A\partial x} = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)$$

ρ to B

2018년 8월 13일 월요일 오후 3:29

B = formation volume factor = $\frac{rbbl}{STB}$ or $\frac{rcf}{scf}$

ex) $B_g = 0.001$

Gas
1 scf → $\rho_{g,surface}$ lb

surface

why shrink?
$P \uparrow$

reservoir

0.001 rcf → $\rho_g = \frac{\rho_{g,surface}}{B_g}$

Mostly $B_w \approx 1$ → That's why water is called incompressible fluid

Higher Bw is more incompressible?

$B_o = 2$

Oil
1 STB + Dissolution gas R_s scf/STB

Surface

why expand?
dissolution gas

Reservoir

2 rbbl

ppg = pound per gallon, 1 bbl = 42 gallon

$\rho_{o,surface}$, $\rho_{g,surface}$ (lb/ft³), R_s, B_o are given

What is ρ_o in reservoir conditions?

Oil 1 STB = 42 $\rho_{o,s}$ lb

Gas R_s scf = $R_s \rho_{g,s}$ lb

surface

surface

Mass doesn't change!

Reservoir $\quad B_o$ (rbbl)

$$\rho_o = \frac{42\,\rho_{o,s} + R_s\,\rho_{g,s}}{42\,B_o}$$

These are fluid properties at surface conditions

→ Constant!

Compressibility

2019년 9월 17일 화요일 오전 11:05

③ Constitutive equations

$$C_R = \frac{1}{\phi}\frac{\partial \phi}{\partial p}, \quad C_f = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$$