457.204 Elementary Fluid Mechanics and Lab.
Elementary Test

Hydrostatic Pressure

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Objectives

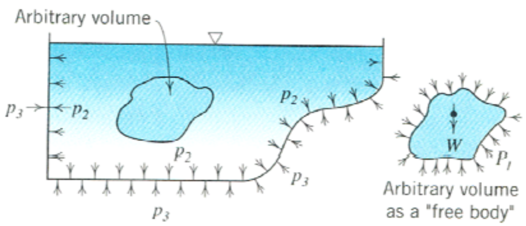
Chapter 1. Hydrostatic Pressure

- To determine the center position of pressure of a plane surface immersed in water
- To compare the experimental position with the theoretical results

Background theory

What is fluid statics…?

- Fluid pressure ?
  The pressure at some point within a fluid
- Fluid statics ?
  - No relative motion between fluid elements
  - No shear stress

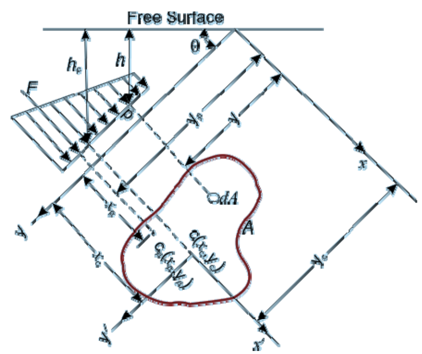


Can't support shearing stresses

Pressure must be acting on normal to surface

Background theory

- Hydrostatic pressure ?
  The pressure at any given point of a static fluid is called the hydrostatic pressure

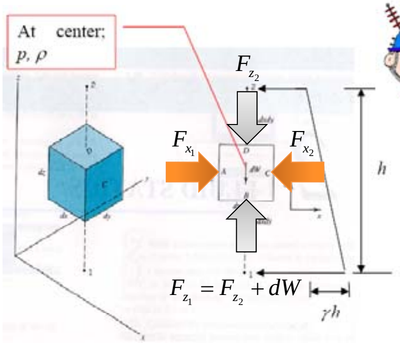


Background theory

- How to calculate hydrostatic pressure ?

$p = \gamma h$ (γ : specific gravity, h : depth)

Why?



- No shear stress
  & Acting normal to surface
- Force Equilibrium

$$\sum F_x = \left[\left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right) - \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right)\right] dz = 0$$

$$\sum F_z = \left[\left(p - \frac{\partial p}{\partial z}\frac{dz}{2}\right) - \left(p + \frac{\partial p}{\partial z}\frac{dz}{2}\right)\right] dx - dW = 0$$

$$\vdots$$

Background theory

- How to calculate hydrostatic pressure ?

$p = \gamma h$ (γ : specific gravity, h : depth)

Eventually!

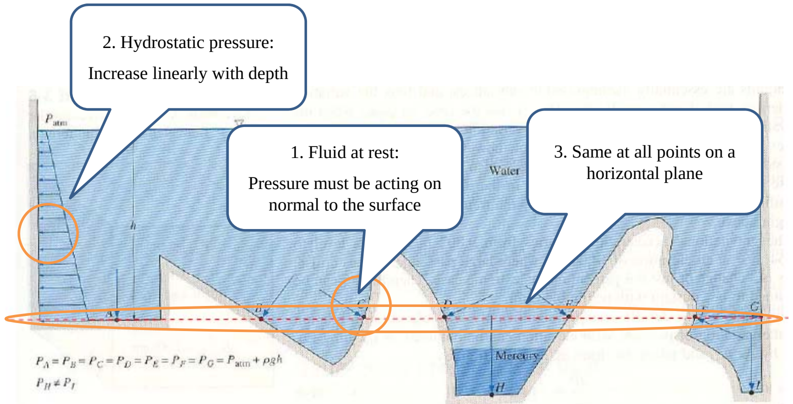
$\sum F_x : \frac{\partial p}{\partial x} = 0$ → Pressure is constant in a horizontal plane in a static fluid

$\sum F_z : \frac{\partial p}{\partial z} = \rho g = \gamma$ → Increase of pressure with depth in a fluid of constant density – linearly increase

$$\Delta p = \gamma \Delta z$$

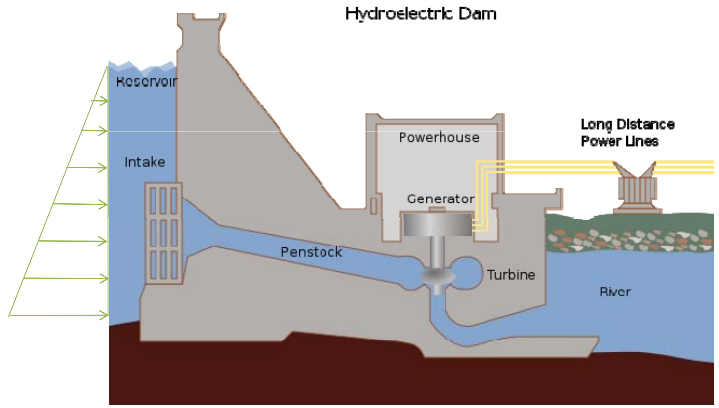
Background theory

- Summary



Example of hydrostatic pressure

- Hydroelectric dam
  From pressure difference with depth, water gets energy to operate turbine.



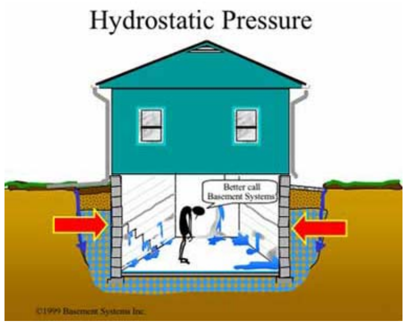
Example of hydrostatic pressure

- Basement leaks & hydrostatic pressure

Backfill becomes saturated

Hydrostatic pressure makes a crack at the basement wall

Water infiltrates the basement wall



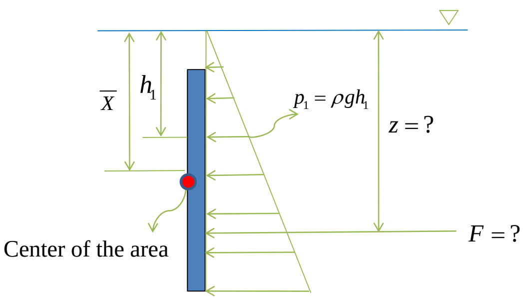
Example of hydrostatic pressure

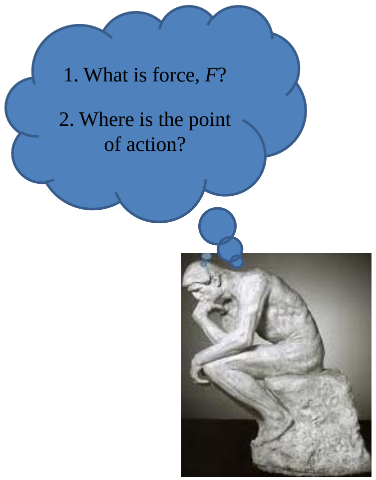
- There are many kinds of problems….
  - Which point should we choose to intake water from dam?
  - Where should we release water to get sufficient energy?
  - How can we construct safe dam?
  - How can we protect water leak problem?

Basic solutions of these problems come from understanding hydrostatic pressure

Hydrostatic pressure on submerged plane

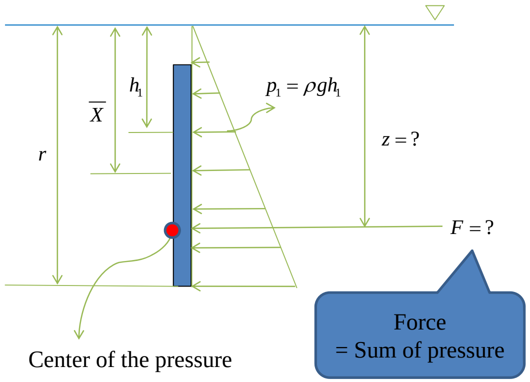
- Pressure distribution diagram
  Pressure is linearly increase from water surface to bottom





Hydrostatic pressure on submerged plane

- Total force acting on surface



- Pressure at h

$$p = \rho g h$$

- Force acting on surface

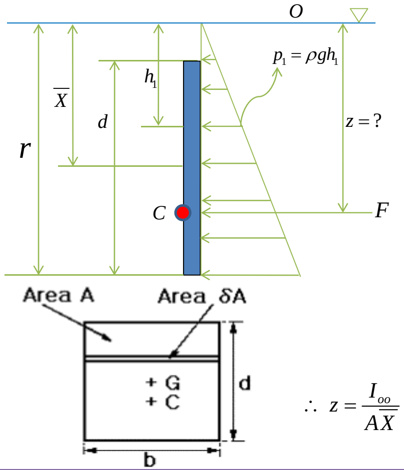
$$F = \int p\,dA = \int \rho g X\,dA$$
$$= \rho g \overline{X} A$$

- First moments of areas

$$\int X\,dA = \overline{X} A$$

Hydrostatic pressure on submerged plane

- Position of the total force



- Moment at O

$$M = \int X^2 \rho g \, dA = \rho g I_{oo} = Fz$$

(Second moment of area: $\int X^2 dA = I_{oo}$)

$$\therefore \ z = \frac{M}{F} = \frac{\rho g I_{oo}}{\rho g A\overline{X}} = \frac{I_{oo}}{A\overline{X}}$$

- From parallel axis theorem

$$I_{oo} = I_{gg} + A\overline{X}^2 \quad \text{(Rectangular : } I_{gg} = \frac{bd^3}{12} \text{)}$$

$$\therefore \ z = \frac{I_{oo}}{A\overline{X}} = \frac{I_{gg} + A\overline{X}^2}{A\overline{X}} = \frac{I_{gg}}{A\overline{X}} + \overline{X} = \frac{bd^3/12}{bd \times (r - d/2)} + \left(r - \frac{d}{2}\right)$$

When t=0, Center of pressure is located on 2/3 down
from water surface to bottom of submerged plate

Observation for Total Immersion

- In case of rectangular plane

$$\overline{X} = r - \frac{d}{2}$$

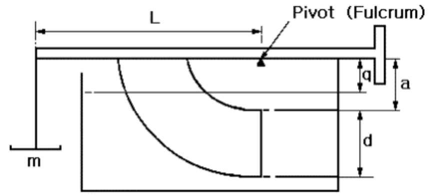
- Resultant Force

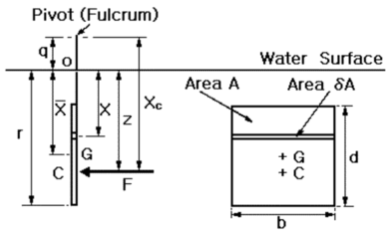
$$F = \rho g \overline{X} A$$

- Center point of pressure

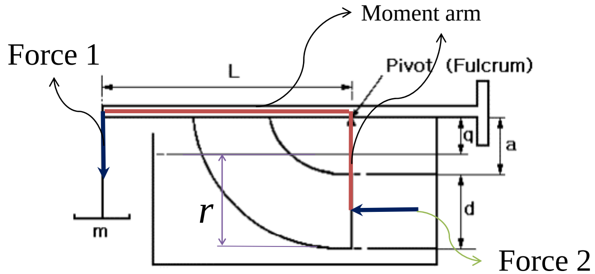
$$z = \frac{bd^3/12}{bd \times (r - d/2)} + \left(r - \frac{d}{2}\right) \quad \text{from water surface}$$

$$X_c = \frac{bd^3/12}{bd \times (r - d/2)} + \left(r - \frac{d}{2}\right) + q \quad \text{from pivot (Fulcrum)}$$



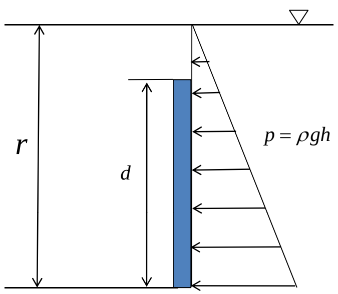


Theoretical Value for Total Immersion

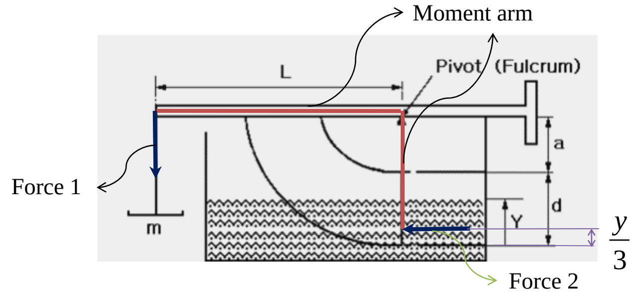


$$M_1 = M_2 \ \rightarrow \ F_1 L = F_2 X_c \ \rightarrow \ X_c = \frac{F_1 L}{F_2}$$

$$\begin{cases} F_1 = mg \\ F_2 = \frac{1}{2} \times \rho g r \times r \times b - \frac{1}{2} \times \rho g (r - d) \times (r - d) \times b \end{cases}$$



Observation for Partial Immersion

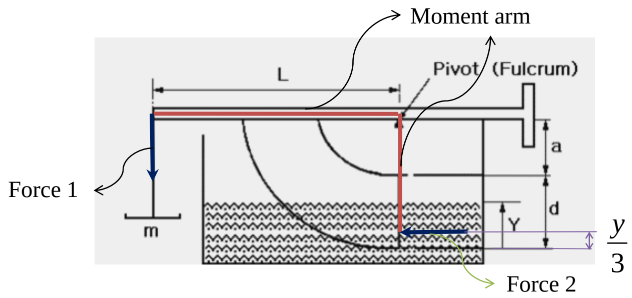


- Center point of pressure

$$z = \frac{2}{3} y \qquad \qquad X_c = a + d - \frac{y}{3}$$

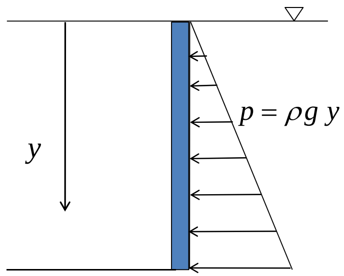
from water surface — from pivot (Fulcrum)

Theoretical Value for Partial Immersion

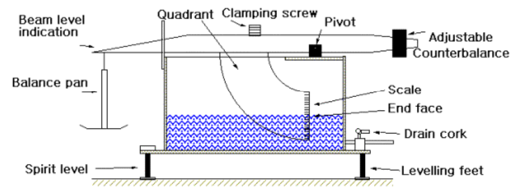


$$M_1 = M_2 \;\rightarrow\; F_1 L = F_2 X_c \;\rightarrow\; X_c = \frac{F_1 L}{F_2}$$

$$\begin{cases} F_1 = mg \\ F_2 = \rho g \overline{X} A = \rho g \dfrac{by^2}{2} \end{cases}$$

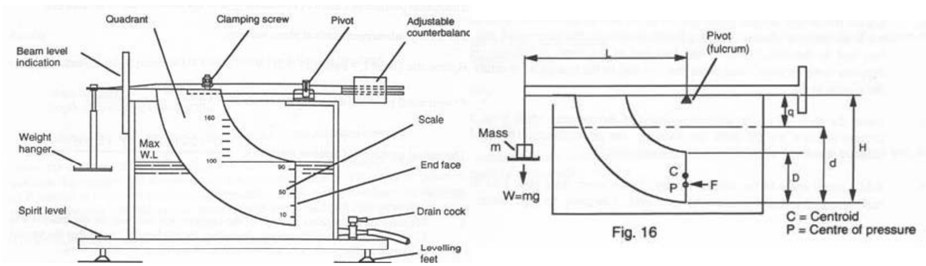


Principle of Experiment



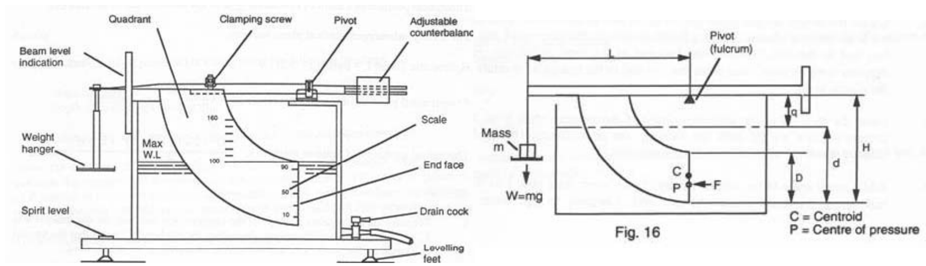
- Place a weight until the balance arm is horizontal.
- Record the water level on the quadrant and the weight on the balance pan.
- Repeat this procedure until the water level reaches the top of the quadrant.

Procedure



① Place the quadrant on the two dowel pins and using the clamping screw, fasten to the balance arm.
② Measure values of a, L, depth d and width b of the quadrant end face.
③ With the Perspex tank on the bench, position the balance arm on the knife edges (pivot).
④ Hang the balance pan from the end of the balance arm.
⑤ Connect a hose from the drain cock to the sump and another from the bench feed to the triangular aperture on the top of the Perspex tank.
⑥ Level the tank using the adjustable feet and spirit level.

Procedure



⑦ Move the counter balance weight until the balance arm is horizontal.
⑧ Close the drain cock and admit water until the level reaches the bottom edge of the quadrant.
⑨ Place a weight on the balance pan, slowly adding water into the tank until the balance arm is horizontal.
⑩ Record the water level on the quadrant and the weight on the balance pan.
⑪ Repeat the above for each increment of weight until the water surface level reaches the top of the quadrant end face. Then remove each increment of weight noting weights and water levels until the weights have been removed.

Results

a (m)	Arm L (m)	depth d (m)	Width b (m)	Mass m (kg)	Measured length X_{cm}	Theoretical Value X_{ct}

Discussion

1. Confirm that the center of pressure is below the center of area, and discuss why.

2. Compare the theoretical value with measured length, discuss why does the difference occurs.

3. Discuss why we do not consider the pressure at the surface of arc. Why we just calculate the pressure of the rectangular section?