5.4 Elastic Stress-strain Relations

▶ Assumptions in this section

i) We shall generalize the elastic behavior in the tension test to arrive at relations which connect all six components of stress with all six components of elastic strain.

ii) We shall restrict ourselves to materials which are linearly elastic. (linear elasticity)

iii) We also restrict ourselves to strains small compared to unity. (small strain)

iv) We shall consider the materials that are independent of orientation which is assumed to be isotropic. (isotropic)

▶ Definitions

$$\sigma_x = E\epsilon_x, \qquad \epsilon_x = \frac{\sigma_x}{E}$$

1. Young's modulus (or modulus of elasticity)

i) The modulus of elasticity E is numerically equal to the slope of the linear-elastic region in stress-strain curve and it is the material property.

ii) The modulus of elasticity at compression and extension is same.

iii) Unit: Because ϵ is a dimensionless number, it is homogeneous to stress σ.

$$\tau_{xy} = G\gamma_{xy}, \qquad \gamma_{xy} = \tau_{xy}/G$$

2. Shear modulus of elasticity G

i) Unit: $[G] = [E] = [\sigma] = [\tau]$

ii) The relation between G and E

$$G = \frac{E}{2(1+\nu)} \tag{5.3}$$

→ $E, G,$ and ν are dependent each other.

→ In common materials, $0 < \nu < 0.5$, so $\frac{E}{3} < G < \frac{E}{2}$.

3. Poisson's ratio

→ Tests in uniaxial compression show a lateral extensional strain which is this same fixed fraction of the longitudinal compressive strain.

$$\nu = -\frac{Lateral \quad Strain}{Axial \quad Strain}$$

i) Poisson's ratio is the example of non-stress strain and thermal strain.

ii) For isotropic, linear-elastic material

$$\epsilon_y = \epsilon_z = -\nu\epsilon_x = -\nu\sigma_x/E$$

cf. The conditions that lateral strain in proportional to axial strain in linear-elastic region

① Material has the same components in all regions.

→ Homogeneous

② Material properties are independent of orientation.

→ Isotropic

Meanwhile, the lumbers are not isotropic but homogeneous.

In general, the structural materials (i.e., steel) is satisfied with the above requirements.

▶ The conclusions obtained under the assumption that the material is isotropic

i) No shear-strain due to normal stress components.

ii) The principal axes of strain at a point of a stressed body coincide with the principal axes of stress at that point.

iii) Each shear stress component produces only its corresponding shear-strain component.

iv) No strain components other than γ_{zx}, can exist, singly or in combination, as a result of the shear-stress component τ_{zx}.

v) The thermal strain cannot produce the shear strain.

▶ The stress-strain relations of a linear-elastic isotropic material with all components of stress present

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad (5.2)$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

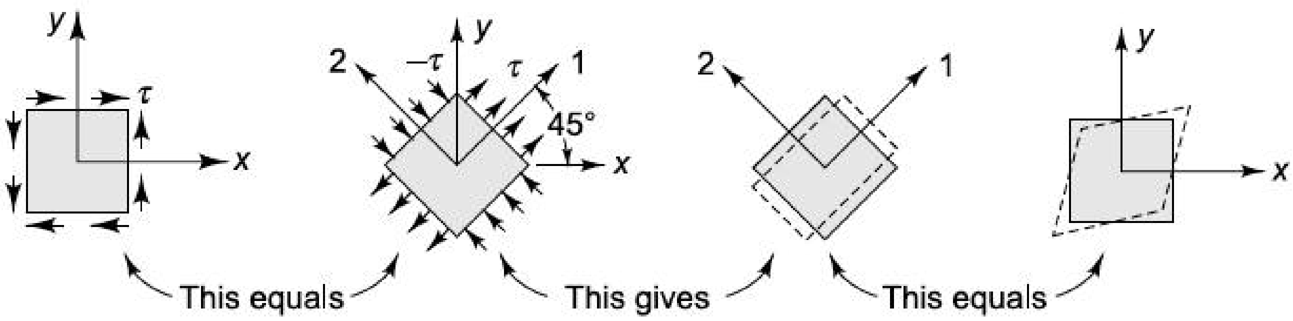


Fig. 5.16 *Equivalent states of stress and strain*

→ From Fig. 5.16,

$$\epsilon_1 = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E} = \frac{\tau(1+\nu)}{E}, \qquad \epsilon_2 = \frac{\sigma_2}{E} - \nu\frac{\sigma_1}{E} = -\frac{\tau(1+\nu)}{E}$$

Meanwhile, upon use of the strain transformation formulas

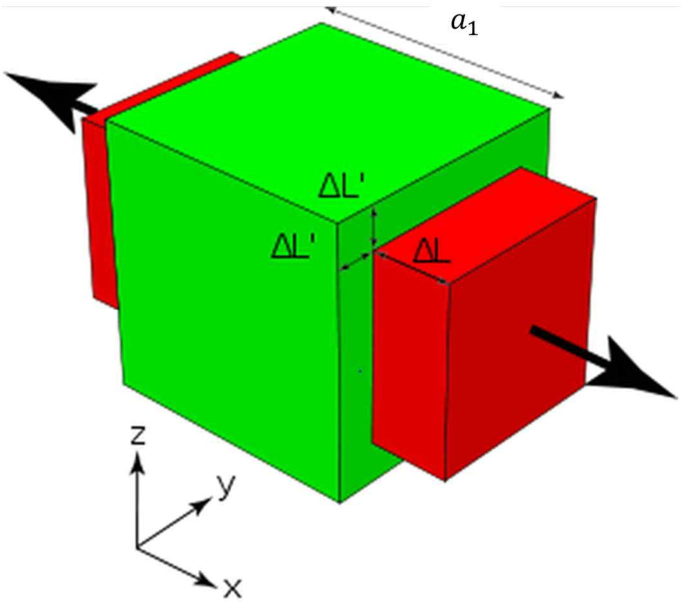
$$\gamma_{xy} = \epsilon_1 - \epsilon_2 = \frac{2(1+\nu)}{E}\tau$$

This equation and $\gamma_{xy} = \frac{\tau}{G}$ must be equal, so

$$G = \frac{E}{2(1+\nu)} \tag{5.3}$$

→ It is true, although it will not be proved here, that no other choice of coordinate axes gives any added information about the elastic constants, and thus for an isotropic material there are just two independent elastic constants.

▶ Volume change of the isotropic, linear-elastic material at extension



$$\Delta L = a_1\epsilon$$

$$\Delta L' = b_1\nu\epsilon = c_1\nu\epsilon$$

The length of each side after deformation is

$$\begin{cases} a_1(1+\epsilon) \\ b_1(1-\nu\epsilon) \\ c_1(1-\nu\epsilon) \end{cases}$$

$$\therefore\ v_f = a_1 b_1 C_1 (1+\epsilon)(1-\nu\epsilon)^2$$

$$= a_1 b_1 c_1 (1 - 2\nu\epsilon + \nu^2\epsilon^2 + \epsilon - 2\nu\epsilon^2 + \nu^2\epsilon^3)$$

$$v_f = a_1 b_1 c_1 (1 + \epsilon - 2\nu\epsilon)$$

$$\therefore\ e = \frac{\Delta V}{V_0} = \frac{V_f - V_0}{V_0} = \frac{a_1 b_1 c_1(\epsilon - 2\nu\epsilon)}{a_1 b_1 c_1}$$

$$= \epsilon(1-2\nu) = \frac{\sigma}{E}(1-2\nu)$$

→ Volume increases of slender members' tensile test can be obtained when ϵ, ν are known.

cf. If $\nu > 0.5$, there is a contradiction that volume decreases when material is extended, so $\nu_{max} = 0.5$.

i) In linear-elastic region: $\frac{1}{4} \sim \frac{1}{3}$ → $\therefore 0.3\epsilon < e < 0.5\epsilon$

ii) In plastic region: in general, $\Delta V = 0$, so it is fine that $\nu = 0.5$.

▶ Unit volume change in three-axial stresses

→ Having unit length and $V_0 = 1$,

$$V_f = (1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z)$$

$$e = \frac{\Delta V}{V_0} = \frac{V_f - V_0}{V_0} = \frac{V_f}{V_0} - 1 \fallingdotseq \epsilon_x + \epsilon_y + \epsilon_z$$
$$= \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

cf. The shear-stress components cannot have an effect on the volume change.

$$e = -\frac{3(1-2\nu)}{E}p \qquad \frac{1}{k} = \frac{E}{3(1-2\nu)}$$

κ: bulk module or modules of compression

5.5 Thermal strain

▶ In the elastic region the effect of temperature on strain appears in two ways.

i) By causing a modification in the values of the elastic constants

ii) By directly producing a strain even in the absence of stress

cf. For an isotropic material, symmetry arguments show that the thermal strain must be a pure expansion or contraction with no shear-strain components referred to any set of axes.

$$\begin{cases} \epsilon_x^t = \epsilon_y^t = \epsilon_z^t = \alpha(T - T_0) \\ \quad \gamma_{xy}^t = \gamma_{yz}^t = \gamma_{zx}^t = 0 \end{cases} \tag{5.4}$$

▶ Total strain ϵ

$$\epsilon = \epsilon^t + \epsilon^e \tag{5.5}$$

5.6 Complete equations of elasticity

→ The problem was outlined previously in broad generality by the three steps given in (2.1). For convenience we summarize below, under the three steps of (2.1), explicit equations which must be satisfied at each point of a nonaccelerating, isotropic, linear-elastic body subject to small strains.

▶ Equilibrium (3 equations; 6 unknowns)

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + X = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + Y = 0 \tag{5.6}$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + Z = 0$$

▶ Geometry (6 equations and 9 unknowns)

$$\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} & \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \epsilon_y &= \frac{\partial v}{\partial y} & \gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial y} \\ \epsilon_z &= \frac{\partial w}{\partial z} & \gamma_{zx} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{aligned} \tag{5.7}$$

▶ Stress-strain-temperature relation (6 equations)

$$\begin{aligned} \epsilon_x &= \tfrac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha(T - T_0) & \gamma_{xy} &= \tfrac{\tau_{xy}}{G} \\ \epsilon_y &= \tfrac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] + \alpha(T - T_0) & \gamma_{yz} &= \tfrac{\tau_{yz}}{G} \\ \epsilon_z &= \tfrac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha(T - T_0) & \gamma_{zx} &= \tfrac{\tau_{zx}}{G} \end{aligned} \tag{5.8}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_x + \nu\left(\epsilon_y + \epsilon_z\right)\right]$$

→ The equilibrium equations (5.6), the strain-displacement equations (5.7), and the strain-stress-temperature relations (5.8) provide 15 equations for the six components of stress, the six components of strain, and the three components of displacement.

cf. The complete equations (5.6), (5.7), and (5.8) apply to deformations of isotropic, linearly elastic solids which involve small strains and for which it is acceptable to apply the equilibrium requirements in the undeformed configuration.

cf. We shall be primarily concerned with the three steps of (2.1), expressed not in the infinitesimal formulation of (5.6), (5.7), and

(5.8) but expressed, instead, on a macroscopic level in terms of rods, shafts, and beams.

▶ Example 5.2 A long, thin plate of width b, thickness t, and length L is placed between two rigid walls a distance b apart and is acted on by an axial force P, as shown in Fig. 5.17 (a). We wish to find the deflection of the plate parallel to the force P. We idealize the situation in Fig 5.17 (b).

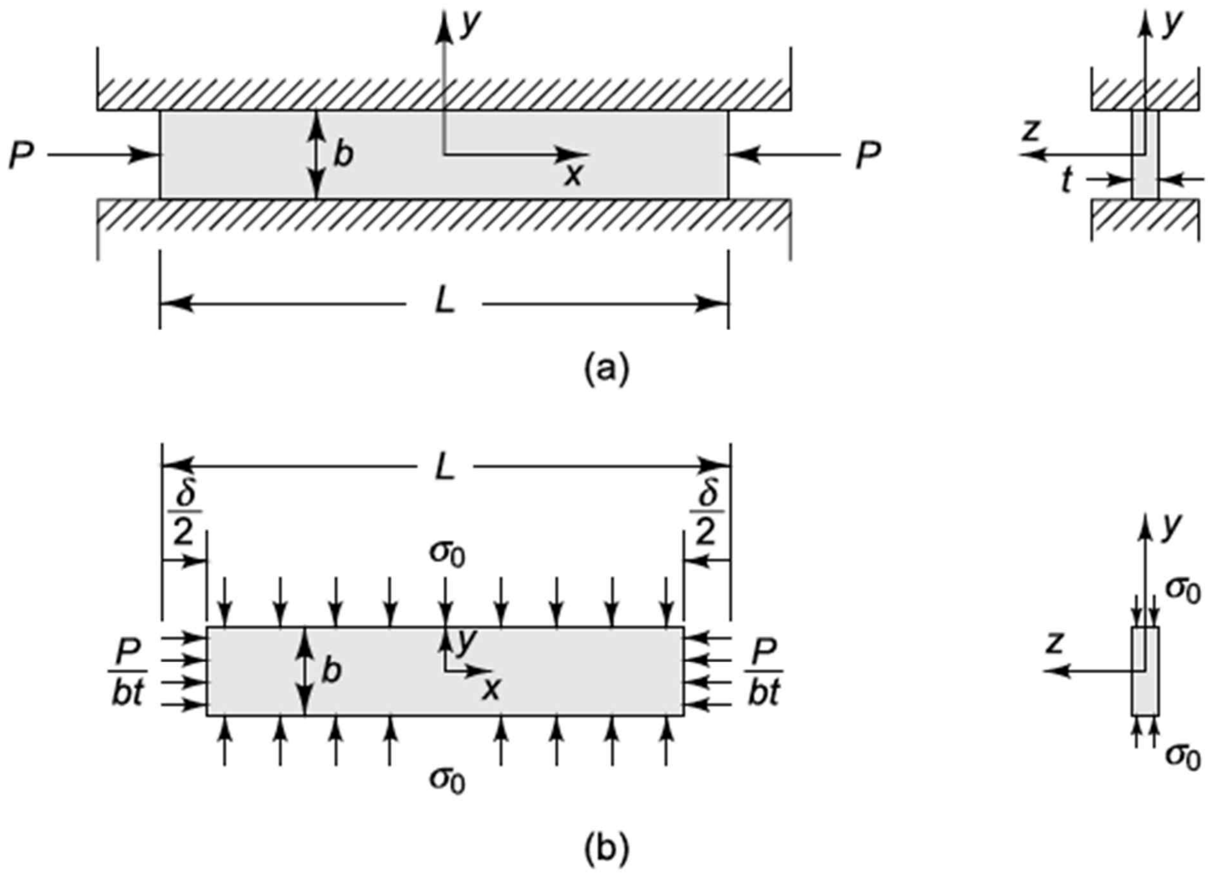


Fig. 5.17 *Example 5.2. (a) Actual problem; (b) idealized model*

▷ Assumptions

i) The axial force P results in an axial normal stress uniformly distributed over the plate area, including the end areas.

ii) There is no normal stress in the thin direction. (Note that this implies a case of plane stress in the xy plane.)

iii) There is no deformation in the y direction, that is, $\epsilon_y = 0$. (Note that this implies a case of plane strain in the xz plane.)

iv) There is no friction force at the walls (or, alternatively, it is small enough to be negligible).

v) The normal stress of contact between the plate and wall is uniform over the length and width of the plate. We now satisfy the requirements (2.1) for the idealized model of Fig. 5.17 (b).

▷ Equilibrium

$$\sigma_x = -\frac{P}{bt}, \quad \sigma_y = -\sigma_0, \quad \sigma_z = 0 \tag{a}$$

$$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$$

These stresses also satisfy the equilibrium equations (5.6).

▷ Geometry

$$\epsilon_y = 0 \tag{b}$$

$$\epsilon_x = -\frac{\delta}{L} \tag{c}$$

▷ Stress-strain relation

→ eq. (5.8) is

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y), \quad \epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x), \quad \epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0 \tag{d}$$

→ Solving the system of equations (a), (b), (c), and (d)

$$\sigma_y = \nu\sigma_x = -\frac{\nu P}{bt}, \quad \delta = \frac{(1-\nu^2)PL}{Ebt}$$

$$\epsilon_z = \frac{\nu(1+\nu)P}{Ebt} = \frac{\nu}{1-\nu}\frac{\delta}{L}$$

cf. We note that the presence of the rigid walls reduces the axial deflection of the plate by the factor $(1-\nu^2)$.

▷ Strain-displacement relation

$$u = -\frac{\delta}{L}x\,, \quad v = 0\,, \quad w = \frac{\nu}{1-\nu}\frac{\delta}{L}z$$

cf. It is relatively easy to get an exact or nearly exact solution to an idealized approximation of the real problem.

5.7 Complete Elastic Solution for a Thick-walled Cylinder

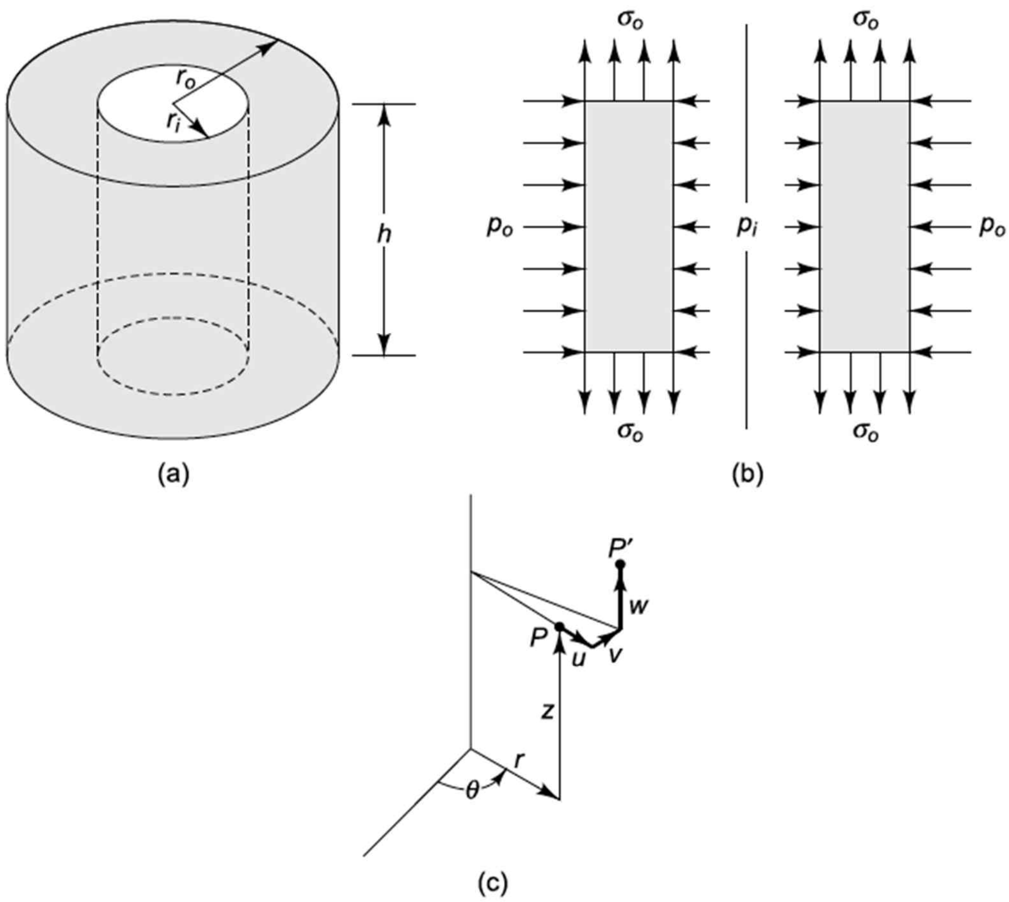


Fig. 5.18 *Thick-walled cylinder (a) subjected to inner and outer pressures and axial tension (b). Cylindrical coordinates and displacement components (c).*

→ There is uniform inner pressure p_i, uniform outer pressure p_o, and uniform axial tensile stress σ_o.

▶ Boundary condition

i) for $r = r_i$

$$\sigma_r = -P_i\,, \qquad \tau_{rz} = 0\,, \quad \tau_{r\theta} = 0 \tag{a}$$

ii) for $r = r_0$

$$\sigma_r = -P_0\,, \qquad \tau_{rz} = 0\,, \quad \tau_{r\theta} = 0 \tag{b}$$

iii) for $z = 0$ & $z = h$

$$\sigma_z = \sigma_0, \qquad \tau_{rz} = 0\,, \qquad \tau_{\theta z} = 0 \tag{c}$$

▶ Geometry

→ Based on the uniformity of the axial loading, $\sigma_z = \sigma_0$ throughout the interior and that all stresses and strains are independent of z.

→ Based on symmetry, we shall look for a solution in which ν, the θ component of displacement, vanishes everywhere and in which all stresses, strains, and displacements are independent of θ.

→ The shear stresses $\tau_{r\theta}, \tau_{\theta z}, \tau_{rz}$ and the corresponding strains $\gamma_{r\theta}, \gamma_{\theta z}, \gamma_{rz}$ vanish everywhere.

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{rz} = \gamma_{r\theta} = \gamma_{\theta z} = \gamma_{rz} = 0$$

▶ Equilibrium equation for cylindrical coordinate system

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{zr}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + 2\frac{\tau_{r\theta}}{r} = 0$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{zr}}{r} = 0$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{d}$$

▶ Strain-displacement equations for cylindrical coordinate system

$$\epsilon_r = \frac{\partial u}{\partial r} \qquad \epsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r} \qquad \epsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{r\theta} = \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r} \qquad \gamma_{\theta z} = \frac{1}{r}\frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial z} \qquad \gamma_{zr} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}$$

$$\epsilon_r = \frac{du}{dr}, \qquad \epsilon_\theta = \frac{u}{r}, \qquad \epsilon_z = \frac{dw}{dz} \tag{e}$$

<proof>

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\epsilon_r = \frac{du}{dr} \qquad \epsilon_\theta = \frac{u}{r} \qquad \epsilon_z = \frac{dw}{dz}$$

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_r + \nu(\epsilon_\theta + \epsilon_z)$$

$$\sigma_\theta = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_\theta + \nu(\epsilon_r + \epsilon_z)$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_z + \nu(\epsilon_r + \epsilon_\theta)$$

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{du}{dr} + \nu\left(\frac{u}{r} + \epsilon_z\right)]$$

$$\sigma_\theta = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{u}{r} + \nu\left(\frac{du}{dr} + \epsilon_z\right)]$$

$$k = \frac{E}{(1+\nu)(1-2\nu)}$$

$$\frac{d\sigma_r}{dr} = k\left[\frac{(1-\nu)\,d^2u}{dr^2} + \nu\left(\frac{1}{r}\frac{du}{dr} - u\frac{1}{r^2}\right)\right]$$

$$\frac{\sigma_r - \sigma_\theta}{r} = k[(1-2\nu)\frac{1}{r}\frac{du}{dr} + 2\nu\frac{u}{r^2} - \frac{u}{r^2}]$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = k(1-\nu)\left[\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2}\right] = 0 \quad \rightarrow \text{Cauchy equation}$$

$$\therefore\ u = Ar + \frac{B}{r} \qquad \epsilon_r = \frac{du}{dr} = A - \frac{B}{r^2} \qquad \epsilon_\theta = A + \frac{B}{r^2}$$

$$\therefore\ \sigma_r = k\left[A + \nu\epsilon_z - (1-2\nu)\frac{B}{r^2}\right]$$

$$\sigma_\theta = k\left[A + \nu\epsilon_z + (1-2\nu)\frac{B}{r^2}\right]$$

▶ <u>Boundary condition</u>

$$\sigma_{r_i} = k\left[A + \nu\epsilon_z - (1-2\nu)\frac{B}{r_i^2}\right] = -P_i$$

$$\sigma_{r_0} = k\left[A + \nu\epsilon_z - (1-2\nu)\frac{B}{r_0^2}\right] = -P_0$$

$$\begin{bmatrix} -\frac{P_i}{k} - \nu\epsilon_z \\ -\frac{P_0}{k} - \nu\epsilon_z \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1-2\nu}{r_i^2} \\ 1 & -\frac{1-2\nu}{r_0^2} \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \mathrm{X}\begin{bmatrix} A \\ B \end{bmatrix}$$

$$det\ (X) = \frac{(1-2\nu)(r_0^2 - r_i^2)}{(r_0 r_i)^2}$$

$$X^{-1} = \frac{1}{dex\,(X)}\begin{bmatrix} -\frac{1-2\nu}{r_0^2} & \frac{1-2\nu}{r_i^2} \\ -1 & 1 \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} A \\ B \end{bmatrix} = \frac{(r_0 r_i)^2}{(1-2\nu)(r_0^2 - r_i^2)}\begin{bmatrix} -\frac{1-2\nu}{r_0^2} & \frac{1-2\nu}{r_i^2} \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -\frac{P_i}{k} - \nu\epsilon_z \\ -\frac{P_0}{k} - \nu\epsilon_z \end{bmatrix}$$

$$\therefore\ A = \frac{1}{k}\frac{-P_0 r_0^2 + P_i r_i^2}{r_0^2 - r_i^2} - \nu\epsilon_z$$

$$B = \frac{1}{(1-2\nu)k}(-P_0 + P_i)\frac{(r_0 r_i)^2}{r_0^2 - r_i^2}$$

$$\therefore\ \sigma_r = k\left[\frac{1}{k}\frac{-P_0 r_0^2 + P_i r_i^2}{(r_0^2 - r_i^2)} - (1-2\nu)\frac{1}{(1-2\nu)k}(-P_0 + P_i)\frac{\left(\frac{r_0 r_i}{r}\right)^2}{r_0^2 - r_i^2}\right]$$

$$= -\frac{P_i\left[(r_0/r)^2 - 1\right] + P_0\left[(r_0/r_i)^2 - (r_0/r)^2\right]}{(r_0/r_i)^2 - 1}$$

$$\sigma_\theta = k\left[\frac{1}{k}\frac{-P_0r_0^2+P_ir_i^2}{(r_0^2-r_i^2)} + (1-2\nu)\frac{1}{(1-2\nu)k}(-P_0+P_i)\frac{\left(\frac{r_0r_i}{r}\right)^2}{r_0^2-r_i^2}\right]$$

$$= \frac{P_i[(r_0/r)^2+1]-P_0[(r_0/r_i)^2+(r_0/r)^2]}{(r_0/r_i)^2-1}$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_r+\sigma_\theta)]$$

$$= \frac{\sigma_0}{E} - \frac{2\nu}{E}\frac{P_ir_i^2-P_0r_0^2}{r_0^2-r_i^2}$$

→ Note that ϵ_z is independent of position within the cylinder.

▶ Stress-strain equations

From generalized Hooke's law

$$\epsilon_r = \frac{1}{E}[\sigma_r - \nu(\sigma_\theta+\sigma_z)]$$

$$\epsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_z+\sigma_r)] \qquad \text{(f)}$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_r+\sigma_\theta)]$$

i) From the first two equations of (f) we solve for the transverse stresses σ_r and σ_θ, in terms of ϵ_r and ϵ_θ and thus obtain the stresses also as functions of u.

ii) Finally, substituting the stresses into (d) leads to the following differential equation for $u(r)$

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0 \qquad \text{(h)}$$

$$\therefore\ r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u = 0$$

$u = Ar + \frac{B}{r}$ → general solution (i)

$$r^2\frac{d^2u}{dr^2}+r\frac{du}{dr}-u=r^2m(m-1)r^{m-2}+m\,r^{m-1}-r^m$$

$$\therefore\ (m(m-1)+m-1)r^m=0$$

$$\therefore\ v=C_1r+c_2r^{-1}$$

Apply the boundary conditions

$$\begin{cases}\sigma_r=-\dfrac{P_i[(r_0/r)^2-1]+P_0[(r_0/r_i)^2-(r_0/r)^2]}{(r_0/r_i)^2-1}\\ \sigma_\theta=\dfrac{P_i[(r_0/r)^2+1]-P_0[(r_0/r_i)^2+(r_0/r)^2]}{(r_0/r_i)^2-1}\end{cases}\qquad(5.9)$$

→ The axial strain is obtained by substituting these stresses together with $\sigma_z=\sigma_o$ into the third equation of (f).

$$\therefore\ \epsilon_z=\frac{\sigma_0}{E}-\frac{2\nu}{E}\frac{P_ir_i^2-P_0r_0^2}{r_0^2-r_i^2}\qquad(5.10)$$

→ Note that ϵ_z is independent of position within the cylinder.

▶ Analysis

i) The axial displacement w thus varies linearly with z.

ii) The transvers stresses (σ_r,σ_θ) are independent of σ_o. ϵ_z depends on the axial loading σ_o.

iii) When the inner and outer pressures are both equal (that is, $p_i=p_o=p$), we find that $\sigma_r=\sigma_\theta=-p$ throughout the interior.

iv) When the outer pressure is absent ($p_o=0$), we note that an inner pressure p_i results in a compressive radial stress which varies from $\sigma_r=-p_i$ at the inner wall to $\sigma_r=0$ at the outer wall.

v) Note that the numerically greatest stress in both Fig. 5.19 (a) and Fig. 5.19 (b) is the tangential stress σ_θ at the inner wall of the cylinder.

vi) When the cylinder wall-thickness $t = r_o - r_i$ becomes small in comparison with r_i, the solution (5.9) approaches the thin-walled-tube approximation of Prob. 4.10 (see Prob. 5.47).

vii) When the axial stress vanishes $(\sigma_o = 0)$, the cylinder is said to be subject to a plane stress distribution. In this case the axial strain ϵ_z is generally not zero. ($\because$ plane stress distribution $\neq$ plane strain distribution)

viii) We can use the exact result (5.9) to illustrate the concept of stress concentration.

cf. A characteristic of the solution (5.9) is that, although it depends on the material's being homogeneous, isotropic, and linearly elastic, the stresses are independent of the actual magnitudes of the elastic parameters E and ν.

cf. Note that the results (5.9) and (5.10) involve the quite significance in engineering.

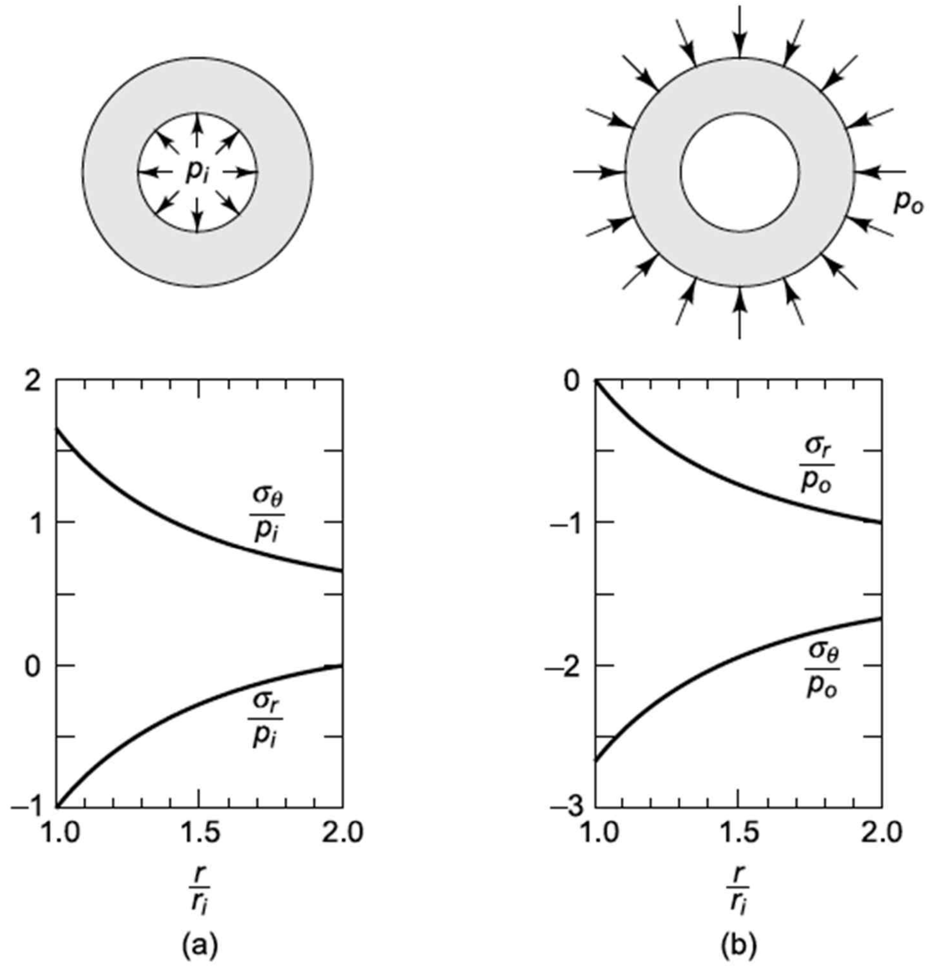


Fig. 5.19 *Distribution of radial stress $\sigma_r(r)$ and tangential stress $\sigma_\theta(r)$ in cylinder with $r_o = 2r_i$ due to (a) inner pressure p_i and (b) outer pressure p_o*