Field and Wave Electromagnetic

Chapter10

Waveguides and Cavity Resonators

Introduction (1)

* Waveguide

- TEM waves are not the only mode of guided waves
- The three types of transmission lines (parallel-plate, two-wire, and coaxial) are not the only possible wave-guiding structure.
- Attenuation constant α for loss line

$$\alpha \cong \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) \propto R \quad (G\sqrt{L/C} \to 0)$$

$$R = 2\left(\frac{R_s}{w}\right) = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} \propto \sqrt{f}$$

$\therefore$ Attenuation of TEM waves tends to increase monotonically with frequency $\Rightarrow$ prohibitively high in the microwave range.

Introduction (2)

- TEM waves : $E_z = H_z = 0$
- TM waves : transverse magnetic waves ($H_z = 0, E_z \neq 0$)
- TE waves : transverse electric(TE) waves
- single conductor wave guide
  : rectangular and cylindrical wave guide.
- dielectric-slab waveguide : surface waves

General Wave Behaviors along Uniform Guiding Structures (1)

* General wave behaviors along uniform g uiding structures

- straight guiding structures with a uniform cross section.

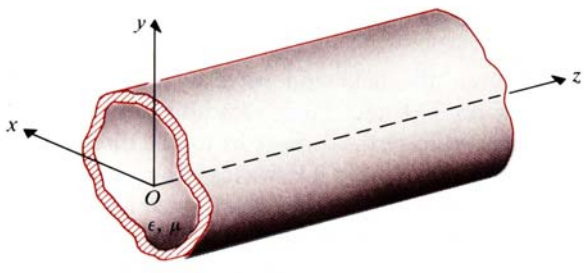

FIGURE 10–1
A uniform waveguide with an arbitrary cross section.

- Assume that the waves propagate in the +z direction with a propagation constant $\gamma = \alpha + j\beta$
- For harmonic time dependence on z and t for all field components : $e^{-\gamma z} e^{j\omega t} = e^{(j\omega t - \gamma z)} = e^{-\alpha z} e^{j(\omega t - \beta z)}$

General Wave Behaviors along Uniform Guiding Structures (2)

- For a cosine reference

$$\bar{E}(x,y,z;t) = \Re e[\bar{E}^0(x,y)e^{(j\omega t-\gamma z)}]$$

where $\bar{E}^0(x,y)$: two-dimensional vector phasor

- $\frac{\partial}{\partial t} \to jw,\ \frac{\partial}{\partial z} \to -\gamma,$ in using a phasor representation
- In the charge-free dielectric region inside, Helmholtz's equations should be satisfied

$$\nabla^2 \bar{E} + k^2 \bar{E} = 0$$

$$\nabla^2 \bar{H} + k^2 \bar{H} = 0, \quad \text{where } k = \omega\sqrt{\mu\varepsilon}$$

- In Cartesian coordinates, rectangular wave guide

$$\nabla^2 \bar{E} = (\nabla_{xy}^2 + \nabla_z^2)\bar{E} = (\nabla_{xy}^2 + \frac{\partial^2}{\partial z^2})\bar{E} = \nabla_{xy}^2 \bar{E} + \gamma^2 \bar{E}$$

General Wave Behaviors along Uniform Guiding Structures (3)

$$\therefore\ \nabla_{xy}^2 \bar{E} + (\gamma^2 + k^2)\bar{E} = 0$$

$$\nabla_{xy}^2 \bar{H} + (\gamma^2 + k^2)\bar{H} = 0$$

cf) $\nabla_{xy}^2 \bar{E} + (\gamma^2 + k^2)\bar{E} = 0$

$$\to \nabla_{xy}^2(\hat{x}E_x + \hat{y}E_y + \hat{z}E_z) + (\gamma^2 + k^2)(\hat{x}E_x + \hat{y}E_y + \hat{z}E_z) = 0$$

i.e $\nabla_{xy}^2 E_x + (\gamma^2 + k^2)E_x = 0,\ \nabla_{xy}^2 E_y + (\gamma^2 + k^2)E_y = 0$

$$\nabla_{xy}^2 E_z + (\gamma^2 + k^2)E_z = 0$$

The solution of above equations depends on the cross-sectional geometry and the boundary conditions

cf) $\nabla_{r\phi}^2$ instead of ∇_{xy}^2 for waveguides with a circular cross section

General Wave Behaviors along Uniform Guiding Structures (4)

- Interrelationships among the six components in Cartesian coordinates

$\nabla \times \bar{E} = -j\omega\mu\bar{H}$ $\qquad$ $\nabla \times \bar{H} = j\omega\varepsilon\bar{E}$

1 $\dfrac{\partial E_z^0}{\partial y} + \gamma E_y^0 = -j\omega\mu H_x^0$ $\qquad$ 4 $\dfrac{\partial H_z^0}{\partial y} + \gamma H_y^0 = j\omega\varepsilon E_x^0$

2 $-\gamma E_x^0 - \dfrac{\partial E_z^0}{\partial x} = -j\omega\mu H_y^0$ $\qquad$ 5 $-\gamma H_x^0 - \dfrac{\partial H_z^0}{\partial x} = j\omega\varepsilon E_y^0$

3 $\dfrac{\partial E_y^0}{\partial x} - \dfrac{\partial E_x^0}{\partial y} = -j\omega\mu H_z^0$ $\qquad$ 6 $\dfrac{\partial H_y^0}{\partial x} - \dfrac{\partial H_x^0}{\partial y} = j\omega\varepsilon E_z^0$

cf) $\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\bar{H}$ $\qquad$ $\begin{pmatrix} \dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} \\ \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} \end{pmatrix} = \begin{pmatrix} -j\omega\mu H_x \\ -j\omega\mu H_y \\ -j\omega\mu H_z \end{pmatrix}$

where $\dfrac{\partial}{\partial z} \to -\gamma$ and $e^{-\gamma z}$ is surpressed

General Wave Behaviors along Uniform Guiding Structures (5)

- Transverse component can be expressed in terms of longitudinal components.

Ex) Combining 1 and 5

1 : $\dfrac{\partial E_z^0}{\partial y} + \gamma E_y^0 = -j\omega\mu H_x^0$

5 : $-\gamma H_x^0 - \dfrac{\partial H_z^0}{\partial x} = j\omega\varepsilon E_y^0$

eliminating E_y^0 from 1,5

1' : $j\omega\varepsilon\dfrac{\partial E_z^0}{\partial y} + \gamma\, j\omega\varepsilon E_y^0 = \omega^2\mu\varepsilon H_x^0$

5' : $-\gamma^2 H_x^0 - \gamma\dfrac{\partial H_z^0}{\partial x} = \gamma\, j\omega\varepsilon E_y^0$

$(k^2 + \gamma^2)H_x^0 = j\omega\varepsilon\dfrac{\partial E_z^0}{\partial y} - \gamma\dfrac{\partial H_z^{\ 0}}{\partial x}$

$\therefore\ H_x^0 = -\dfrac{1}{h^2}\left(-j\omega\varepsilon\dfrac{\partial E_z^0}{\partial y} + \gamma\dfrac{\partial H_z^0}{\partial x}\right)$ where $h^2 = k^2 + \gamma^2$

General Wave Behaviors along Uniform Guiding Structures (6)

- i.e

$$H_x^0 = -\frac{1}{h^2}(-j\omega\varepsilon\frac{\partial E_z^0}{\partial y} + \gamma\frac{\partial H_z^0}{\partial x})$$

$$H_y^0 = -\frac{1}{h^2}(+j\omega\varepsilon\frac{\partial E_z^0}{\partial x} + \gamma\frac{\partial H_z^0}{\partial y})$$

$$E_x^0 = -\frac{1}{h^2}(j\omega\mu\frac{\partial H_z^0}{\partial y} + \gamma\frac{\partial E_z^0}{\partial x})$$

$$E_y^0 = -\frac{1}{h^2}(-j\omega\mu\frac{\partial H_z^0}{\partial x} + \gamma\frac{\partial E_z^0}{\partial y})$$

where $h^2 = k^2 + \gamma^2$

note First, solve $\nabla_{xy}^2 \bar{E} + h^2 \bar{E} = 0$

and $\nabla_{xy}^2 \bar{H} + h^2 \bar{H} = 0$ for longitudinal components

then find H_x, H_y, E_x, E_y using above equation

TEM Waves (1)

* TEM wave

- $E_z = 0,\ H_z = 0$ for TEM waves

$\therefore\ E_x = E_y = H_x = H_y = 0$ unless $h^2 = 0$

- TEM waves exist only

where $\gamma_{TEM}^2 + k^2 = 0$

or $\gamma_{TEM}^2 = jk = j\omega\sqrt{\mu\varepsilon}$: propagation constant of a uniform plane wave on a lossless transmission line.

- Phase velocity $\mu_{p(TEM)} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$

TEM Waves (2)

- Wave impedance $Z_{TEM} = \frac{E_x^0}{H_y^0} = \frac{j\omega\mu}{\gamma_{\text{TEM}}} = \frac{\gamma_{\text{TEM}}}{j\omega\varepsilon}$ from 2,4

$$= \sqrt{\frac{\mu}{\varepsilon}} = \eta$$

note Z_{TEM} is the same as the intrinsic impedance of the dielectric medium

$$Z_{TEM} = -\frac{E_y^0}{H_x^0} = -\sqrt{\frac{\mu}{\varepsilon}}$$

$$\therefore \boxed{\overline{H} = \frac{1}{Z_{TEM}} \hat{z} \times \overline{E}}$$

TEM Waves (3)

* Single-conductor waveguides cannot support TEM waves.

Why?

1. $\overline{B}$ lines always close upon themselves

2. For TEM waves to exist, $\overline{B}$ and $\overline{H}$ lines would form closed loops in a transverse plane.

3. By the Ampere's circuital law.

$\oint \overline{H} \bullet d\overline{l} = I_c + I_d$ transverse plane

I_c : conductor current

I_d : displacement current

4. Without an inner conductor

$I_c = 0$

TEM Waves (4)

5. For TEM wave, $E_z = 0 \rightarrow$ no longitudinal displacement current

   cf) $I_d = \int_s \frac{\partial \bar{D}}{\partial t} \bullet ds = 0$ in the z direction

   $\because E_z = 0$

6. Therefore there can be no closed loops of magnetic field lines in any transverse plane

7. Assuming perfect conductors, a coaxial transmission line having an inner conductor can support TEM waves

8. When the conductors have losses, no longer TEM waves

TM / TE Waves (1)

For TM waves $H_z = 0$	For TE waves $E_z = 0$
solving	solving
$\nabla_{xy}^2 E_z^0 + h^2 E_z^0 = 0$	$\nabla_{xy}^2 H_z + h^2 H_z = 0$
for E_z. with proper boundary conditions	*for* H_z. with proper boundary conditions
$H_x^0 = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z^0}{\partial y}$: ①	$H_x^0 = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial x}$: ⑤
$H_y^0 = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z^0}{\partial x}$: ②	$H_y^0 = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial y}$: ⑥
$E_x^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial x}$: ③	$E_x^0 = -\frac{j\omega\mu}{h^2}\frac{\partial H_z^0}{\partial y}$: ⑦
$E_y^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial y}$: ④	$E_y^0 = \frac{j\omega\mu}{h^2}\frac{\partial H_z^0}{\partial x}$: ⑧

TM / TE Waves (2)

$(E_T^0)_{TM} = \hat{x}E_x^0 + \hat{y}E_y^0 = -\frac{\gamma}{h^2}\nabla_T E_z^0$

E_x^0, E_y^0 are given, H_x^0, H_y^0 can be determined from the wave impedance for the TM mode from ②,③ and ①,④

$$Z_{TM} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{\gamma}{j\omega\varepsilon}$$

$cf)\ Z_{TM} \neq \frac{j\omega\mu}{\gamma}$

∵ γ for TM is not equal to $j\omega\sqrt{\mu\varepsilon}$, which is γ_{TEM}

$$\therefore\ \bar{H} = \frac{1}{Z_{TM}}(\hat{z}\times\bar{E})$$

$(H_T^0)_{TE} = \hat{x}H_x^0 + \hat{y}H_y^0 = -\frac{\gamma}{h^2}\nabla_T H_z^0$

similar way, E_x^0, E_y^0 can be obtained from H_x^0, H_y^0

from ⑥,⑦ and ⑤,⑧

$$Z_{TE} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{j\omega\mu}{\gamma}$$

$cf)\ Z_{TE} \neq Z_{TM} = \frac{\gamma}{j\omega\varepsilon}$

$$\therefore\ \bar{E} = -Z_{TE}(\hat{z}\times\bar{H}) = Z_{TE}(\bar{H}\times\hat{z})$$

TM / TE Waves (3)

solution of

$\nabla_{xy}^2 E_z^0 + h^2 E_z^0 = 0$

for a given boundary condition are possible only for discrete values of h

⇒ infinity of h's

⇒ but solutions are not possible for all values of h

⇒ eigenvalues or characteristic values

$h^2 = \gamma^2 + k^2$

$\gamma = \sqrt{h^2 - k^2}$

$= \sqrt{h^2 - \omega^2\mu\varepsilon}$

$\nabla_{xy}^2 H_z^0 + h^2 H_z^0 = 0$

eigen values

TM / TE Waves (4)

<table>
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$for\ \gamma = 0,$

$\omega^2 \mu\varepsilon = h^2$

$f_c = \dfrac{h}{2\pi\sqrt{\mu\varepsilon}}$: cutoff frequency

cf) The value of f_c for a particular mode in a waveguide depends on the eigenvalue of this mode

$\gamma = h\sqrt{1-(\frac{f}{f_c})^2}$

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$\nabla_{xy}^2 H_z^0 + h^2 H_z^0 = 0$

eigen values

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TM / TE Waves (5)

$$\gamma = h\sqrt{1-(\frac{f}{f_c})^2}$$

(a) $(\frac{f}{f_c})^2 > 1$ or $f > f_c$

in this range, $\Rightarrow \omega^2\mu\varepsilon > h^2$ and γ is imaginary

$$\gamma = j\beta = jk\sqrt{1-(\frac{h}{k})^2}$$

$$= jk\sqrt{1-(\frac{f_c}{f})^2}$$

$\Rightarrow$propagation mode with a phase constant β

$$\beta = k\sqrt{1-(\frac{f_c}{f})^2} \quad \text{(rad/m)}$$

TM / TE Waves (6)

- Guided wavelength

$$\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda}{\sqrt{1-(\frac{f_c}{f})^2}} > \lambda \quad \text{where } \lambda = \frac{2\pi}{k} = \frac{1}{f\sqrt{\mu\varepsilon}} = \frac{u}{f}$$

let cutoff wavelength, $\lambda_c = \frac{u}{f_c}$

then $\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2}(1-(\frac{f_c}{f})^2)$

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{f^2}{u^2}(\frac{f_c}{f})^2 = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

$$\therefore \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} = \frac{1}{\lambda^2}$$

TM / TE Waves (7)

- Phase velocity

$$u_p = \frac{\omega}{\beta} = \frac{u}{\sqrt{1-(\frac{f_c}{f})^2}} = \frac{\lambda_g}{\lambda} u \quad > u$$

cf) 1. Phase velocity of guided wave is faster than that of unbounded medium.

2. Phase velocity depends on frequency so that single conductor waveguides are dispersive transmission systems

TM / TE Waves (8)

- Group velocity

$$u_g = \frac{1}{d\beta/d\omega} = u\sqrt{1-(\frac{f_c}{f})^2} = \frac{\lambda}{\lambda_g}u \quad < u$$

$$\therefore \; u_g u_p = u^2$$

cf) $$\frac{d\beta}{d\omega} = \frac{d(k\sqrt{1-(\frac{f_c}{f})^2}}{d\omega} = \frac{d[2\pi f\sqrt{\mu\varepsilon}\sqrt{1-(\frac{f_c}{f})^2}\,]}{d(2\pi f)}$$

$$= \frac{d}{df}\frac{f}{u}\sqrt{1-(\frac{f_c}{f})^2}$$

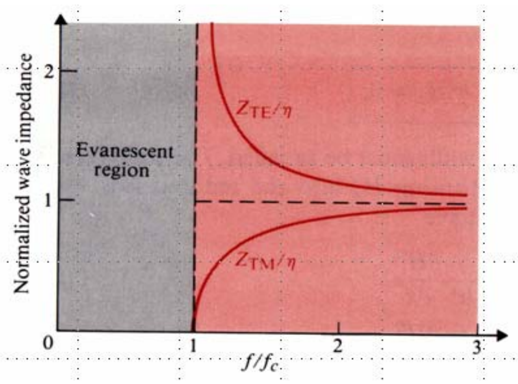
$$= \frac{1}{u}\frac{1}{\sqrt{1-(\frac{f_c}{f})^2}}$$

TM / TE Waves (9)

- $$Z_{TM} = \frac{\gamma}{j\omega\varepsilon} = \frac{jk\sqrt{1-(\frac{f_c}{f})^2}}{j\omega\varepsilon}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-(\frac{f_c}{f})^2} = \eta\sqrt{1-(\frac{f_c}{f})^2}$$

; purely resistive and less than the intrinsic impedance of the dielectric medium



- $$Z_{TE} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{jk\sqrt{1-(\frac{f_c}{f})^2}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1-(\frac{f_c}{f})^2}} = \frac{\eta}{\sqrt{1-(\frac{f_c}{f})^2}}$$

; purely resistive larger than the intrinsic impedance of the dielectric medium

TM / TE Waves (10)

(b) $(\frac{f}{f_c})^2 < 1$ or $f < f_c$

$$\gamma = \alpha = h\sqrt{1-(\frac{f}{f_c})^2}$$: real number

$\therefore\ e^{-\gamma z} = e^{-\alpha z}$ $\Rightarrow$ wave diminishes rapidly with z and is said to be evanescent

$\Rightarrow$ waveguide : high-pass filter

$$Z_{TM} = \frac{\gamma}{j\omega\varepsilon} = \frac{h\sqrt{1-(\frac{f}{f_c})^2}}{j\omega\varepsilon} = -j\frac{h}{\omega\varepsilon}\sqrt{1-(\frac{f}{f_c})^2},\quad f < f_c$$

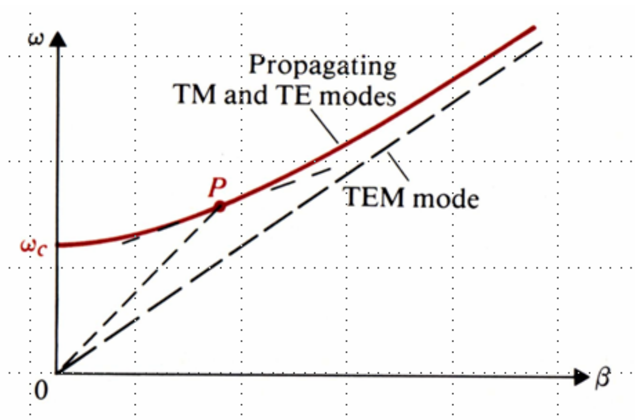
$\Rightarrow$ purely reactive $\Rightarrow$ no power flow associated with evanescent mode

$$Z_{TE} = \frac{j\omega\mu}{\gamma} = j\frac{\omega\mu}{h\sqrt{1-(\frac{f}{f_c})^2}}$$: purely reactive. no power flow.

TM / TE Waves (11)

- $\omega - \beta$ diagram

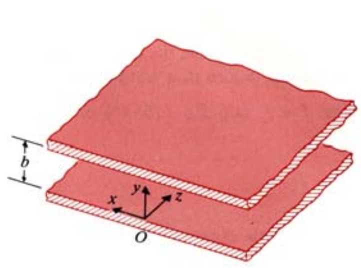
$$\beta = \frac{\omega}{u}\sqrt{1-(\frac{\omega_c}{\omega})^2}$$



Parallel-Plate Waveguide (1)

- Parallel plate waveguide can support TM and TE waves as well as TEM waves

* TM waves between parallel plates



1. Assuming ε and μ
2. Infinite in extent in the x-direction
3. TM waves $(H_z = 0)$
4. $e^{(j\omega t - \gamma z)}$

FIGURE 10–5
An infinite parallel-plate waveguide.

Parallel-Plate Waveguide (2)

$E_z(y,z) = E_z^0(y)e^{-\gamma z}$ (no variation along x-direction)

$$\frac{d^2 E_z^0(y)}{dy^2} + h^2 E_z^0(y) = 0$$

where $h^2 = \gamma^2 + k^2$

B.C.

$E_z^0(y) = 0$ at $y = 0$ and $y = b$

$\therefore\ E_z^0(y) = A_n \sin(\frac{n\pi y}{b})$ from $h = \frac{n\pi}{b}$

where A_n depends on the strength of excitation of the particular TM wave

Parallel-Plate Waveguide (3)

$$\therefore H_x^0(y) = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z^{\ 0}}{\partial y} = \frac{j\omega\varepsilon}{h} A_n \cos(\frac{n\pi y}{b})$$

$$H_y^0(y) = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z^{\ 0}}{\partial x} = 0$$

$$E_x^0(y) = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial x} = 0$$

$$E_y^0(y) = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial y} = -\frac{\gamma}{h} A_n \cos(\frac{n\pi y}{b})$$

$$\gamma = \sqrt{(\frac{n\pi}{b})^2 - \omega^2\mu\varepsilon}$$

Cutoff frequency that makes $\gamma = 0$ $\quad \therefore f_c = \frac{n}{2b\sqrt{\mu\varepsilon}}$

Parallel-Plate Waveguide (4)

cf) $f_{c1} = \frac{1}{2b\sqrt{\mu\varepsilon}}$ for TM_1 mode with n=1

$f_{c2} = \frac{2}{2b\sqrt{\mu\varepsilon}}$ for TM_2 mode with n=2

cf) TM_0 mode is the TEM mode with $f_c = 0$

$\because E_z = 0$

- Dominant mode of the waveguide = the mode having the lowest cutoff frequency
- For parallel plate waveguides, the dominant mode is the TEM mode

Ex. 10-3(1)

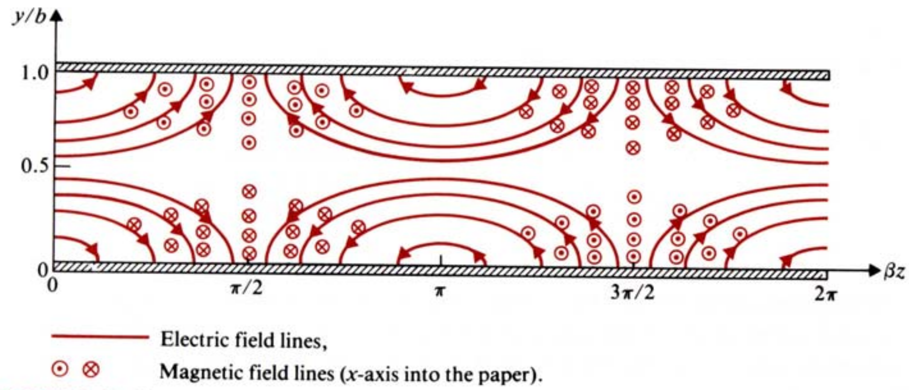


FIGURE 10–6
Field lines for TM_1 mode in parallel-plate waveguide.

cf) Field line : the direction of the field in space

i.e. $d\bar{l} = \hat{x}dx + \hat{y}dy + \hat{z}dz = k\bar{E} = k(\hat{x}E_x + \hat{y}E_y + \hat{z}E_z)$

$$\frac{dx}{E_x} = \frac{dy}{E_y} = \frac{dz}{E_z} = k \Rightarrow$$ field line

Ex. 10-3(2)

∴ in the $y-z$ plane $\quad \frac{dy}{dz} = \frac{E_y(y,z;t=0)}{E_z(y,x;t=0)}$

– For TM_1 mode at t=0,

$$H_x(y,z;0) = \frac{\omega\varepsilon b}{\pi} A_1 \cos(\frac{\pi y}{b}) \sin \beta z$$

– At $y=0$ and $y=b$

- There are surface currents because of a discontinuity in the tangential magnetic field.
- There are surface charges because of the presence of a normal electric field

Ex. 10-4 (1)

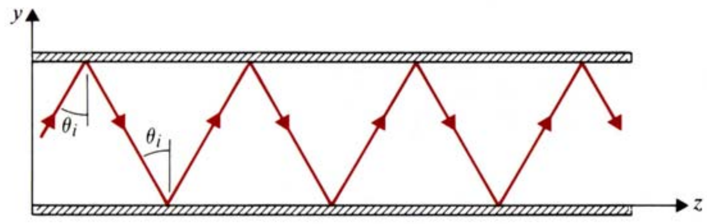


FIGURE 10–7
Propagating wave in parallel-plate waveguide as superposition of two plane waves.

(a) A propagating TM_1 wave = the superposition of two plane waves bouncing back and forth obliquely between the two conducting plates

proof>

$$E_z(y,z) = A_1 \sin(\frac{\pi y}{b}) e^{-j\beta z} = \frac{A_1}{2j}(e^{j\pi y/b} - e^{-j\pi y/b}) e^{-j\beta z}$$

$$= \frac{A_1}{2j}[\underbrace{e^{-j(\beta z - \pi y/b)}}_{1} - \underbrace{e^{-j(\beta z + \pi y/b)}}_{2}]$$

Ex. 10-4 (2)

1 Term : A plane wave propagating obliquely in the $+z$ and $-y$ directions with phase constants β and $\frac{\pi}{b}$

2 Term : A plane wave propagating obliquely in the $+z$ and $+y$ directions with the same phase constants

$$\overline{H} = -\hat{x}H_x$$

$$\overline{E}_i = \hat{y}E_{i0}\sin\theta_i - \hat{z}E_{i0}\cos\theta_i \qquad \overline{E}_r = -\hat{y}E_{r0}\sin\theta_i - \hat{z}E_{r0}\cos\theta_i$$

$$= \hat{y}E_{i0}\sin\theta_i + \hat{z}E_{i0}\cos\theta_i$$

$$\overline{\beta}_i = \hat{y}\beta_1\cos\theta_i + \hat{z}\beta_1\sin\theta_i \qquad \overline{\beta}_r = -\hat{y}\beta_1\cos\theta_i + \hat{z}\beta_1\sin\theta_i$$

Ex. 10-4 (3)

$$E_z(y,z) = E_{i0}\cos\theta_i (e^{j\beta_1 y\cos\theta_i} - e^{-j\beta_1 y\cos\theta_i})e^{-j\beta_1 z\sin\theta_i}$$

$$\therefore\ \beta_1\sin\theta_i = \beta, \qquad \beta_1\cos\theta_i = \frac{\pi}{b}$$

$$\beta = \sqrt{\beta_1^2 - (\frac{\pi}{b})^2} = \sqrt{\omega^2\mu\varepsilon - (\frac{\pi}{b})^2}$$

$\cos\theta_i = \frac{\pi}{\beta_1 b} = \frac{\lambda}{2b}$ $\Rightarrow$ solution exists only for $\lambda \le 2b$

at $\frac{\lambda}{2b} = 1,\ f = \frac{u}{\lambda} = \frac{1}{2b\sqrt{\mu\varepsilon}}$ $\Rightarrow$ cutoff frequency

then $\theta_i = 0$ $\Rightarrow$ waves bounce back and forth in the y-direction and no propagation in the z-direction

$\Rightarrow$ TM_1 mode propagates only when $\lambda < \lambda_c = 2b$ or $f > f_c$.

$$\cos\theta_i = \frac{\lambda}{\lambda_c} = \frac{f_c}{f} \qquad \sin\theta_i = \frac{\lambda}{\lambda_g} = \frac{u}{u_p} = \sqrt{1-(\frac{f_c}{f})^2}$$

TE Waves between Parallel Plates (1)

* TE waves

$$E_z = 0, \quad \frac{\partial}{\partial x} = 0$$

$$\therefore \frac{d^2H_z^0(y)}{dy^2} + h^2H_z^0(y) = 0$$

We note that $H_z(y,z) = H_z^0(y)e^{-\gamma z}$

−B.C. $E_x^0 = -\frac{j\omega\mu}{h^2}\frac{\partial H_z^0}{\partial y} = 0$

i.e $\frac{dH_z^0(y)}{dy} = 0$ at $y = 0$ and $y = b$

$$\therefore H_z^0(y) = B_n\cos(\frac{n\pi y}{b})$$

TE Waves between Parallel Plates (2)

$$\therefore H_x^0(y) = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial x} = 0 \ (\because \frac{\partial H_Z}{\partial x} = 0)$$

$$H_y^0(y) = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial y} = \frac{\gamma}{h} B_n \sin(\frac{n\pi y}{b})$$

$$E_x^0(y) = -\frac{j\omega\mu}{h^2}\frac{\partial H_z^0}{\partial y} = \frac{j\omega\mu}{h} B_n \sin(\frac{n\pi y}{b})$$

$$E_y^0(y) = \frac{j\omega\mu}{h^2}\frac{\partial H_z^0}{\partial x} = 0 (\because \frac{\partial H_Z}{\partial x} = 0)$$

$\Rightarrow \gamma = \sqrt{h^2 - k^2} = \sqrt{(\frac{n\pi}{b})^2 - \omega^2\mu\varepsilon} \Rightarrow$ the same as that for TM waves

$\Rightarrow$ The cutoff frequency is the same

$\Rightarrow$ For $n = 0$, $H_y = 0$ and $E_x = 0$

TE Waves between Parallel Plates (3)

i.e, TE_0 mode doesn't exist

$cf)\ TM_0 = TEM$

$cf)\ TM_{01}$ or TM_{10} does not exist

$$E_z^0(x, y) = E_0 \sin(\frac{m\pi x}{a})\sin(\frac{m\pi y}{b})$$

$cf)\ TM_{01}$ or TM_{10} does exist

for the rectangular waveguide

$$H_z^0(x, y) = H_0 \cos(\frac{m\pi x}{a})\cos(\frac{m\pi y}{b})$$

Energy-transport Velocity (1)

* Energy-transport velocity
  - Wave guide ⇒ high pass filter
  - Broadband signal ⇒ 1. low frequency components may be below cutoff
    2. high frequency components will travel widely different velocity
  - Energy-transport velocity : velocity at which energy propagates along a waveguide

$$u_{en} = \frac{(P_z)_{av}}{W'_{av}} \quad (\text{m/s})$$

$$(P_z)_{av} = \int_s \bar{P}_{av} \bullet d\bar{s}$$: the time average power

Energy-transport Velocity (2)

$$W'_{av} = \int_s [(w_e)_{av} + (w_m)_{av}] ds$$: the time average stored energy per unit length

$$u_{en} = u\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

[H.W] prove that

$$(w_e)_{av} = \frac{\varepsilon}{4}\Re e(\bar{E} \bullet \bar{E}^*)$$

$$(w_m)_{av} = \frac{\mu}{4}\Re e(\bar{H} \bullet \bar{H}^*)$$

Energy-transport Velocity (3)

$$(w_e)_{av} = \frac{\varepsilon}{4} A_n^2 [\sin^2(\frac{n\pi y}{b}) + \frac{\beta^2}{h^2}\cos^2(\frac{n\pi y}{b})]$$

cf) $(\bar{E}\bullet\bar{E}^*) \Rightarrow j\beta\bullet(-j\beta) = \beta^2$

$$\int_0^b (w_e)_{av}\, dy = \frac{\varepsilon b}{8} A_n^2 [1 + \frac{\beta^2}{h^2}] = \frac{\varepsilon b}{8h^2} k^2 A_n^2$$

$$(w_m)_{av} = \frac{\mu}{4}(\frac{\omega^2\varepsilon^2}{h^2}) A_n^2 \cos^2(\frac{n\pi y}{b})$$

$$\int_0^b (w_m)_{av}\, dy = \frac{\mu b}{8h^2}(\omega^2\varepsilon^2) A_n^2 = \frac{\varepsilon b}{8h^2} k^2 A_n^2$$

$$(P_z)_{av} = \int_0^b \bar{P}_{av}\bullet\hat{z}\, dy$$

$$= \int_0^b \frac{\omega\varepsilon\beta}{2h^2} A_n^2 \cos^2(\frac{n\pi y}{b}) dy = \frac{\omega\varepsilon\beta b}{4h^2} A_n^2$$

Energy-transport Velocity (4)

$$cf)\ \bar{P}_{av} = \frac{1}{2}\Re e(\bar{E} \times \bar{H}^*)$$

$$= \frac{1}{2}\Re e(-\hat{z}E_y^0 H_x^{0*} + \hat{y}E_z^0 H_x^{0*})$$

$$(\bar{P}_{av})\bullet\hat{z} = -\frac{1}{2}\Re e(E_y^0 H_x^{0*})$$

$$= \frac{\omega\varepsilon\beta}{2h^2} A_n^2 \cos^2(\frac{n\pi y}{b})$$

$$u_{en} = \frac{\omega\beta}{k^2} = \frac{\omega}{k}(\frac{\beta}{k}) = u\sqrt{1-(\frac{f_c}{f})^2}$$

Attenuation in Parallel-plate Waveguides (1)

* Attenuation in parallel-plate waveguide

- Losses are very small
- $\alpha = \alpha_d + \alpha_c$ → Ohmic losses (α_c); ↓ Dielectric losses (α_d)

For TEM mode

cf) For a lossy transmission line the time-average power loss per unit length

$$P_L(z) = \frac{1}{2}[|I(z)|^2 R + |V(z)|^2 G] = \frac{V_0^2}{2|Z_0|^2}(R + G|Z_0|^2)e^{-2\alpha z}$$

$$P(z) = \frac{1}{2}\Re e[V(z)I^*(z)] = \frac{V_0^2}{2|Z_0|^2}R_0 e^{-2\alpha z}$$

Attenuation in Parallel-plate Waveguides (2)

$$-\frac{\partial P(z)}{\partial z} = P_L(z) = 2\alpha P(z)$$

$$\therefore \alpha = \frac{P_L(z)}{2P(z)} = \frac{1}{2R_0}(R + G|Z_0|^2)$$

$$\therefore \alpha_d = \frac{G}{2}R_0 = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma}{2}\eta \quad (\because Z_0 \simeq R_0 \text{ for low loss conductor})$$

$\frac{\sigma}{2}\eta$ → independent of frequency

$$\text{where} \begin{cases} G = \sigma\dfrac{\omega}{b} \\ R_0 = \dfrac{b}{\omega}\eta \end{cases}$$

Attenuation in Parallel-plate Waveguides (3)

$$\therefore \alpha_c = \frac{R}{2R_0} = \frac{1}{b}\sqrt{\frac{\pi f \varepsilon}{\sigma_c}} \propto \sqrt{f}$$

cf) $$R_0 = \frac{b}{\omega}\eta = \frac{b}{\omega}\sqrt{\frac{\mu}{\varepsilon}}$$

$$R = \frac{2}{\omega}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$$

For TM mode

to find dielectric losses, α_d at $f > f_c$

- $\varepsilon_d = \varepsilon + (\frac{\sigma}{j\omega})$

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Attenuation in Parallel-plate Waveguides (4)

$$\gamma = j[\omega^2\mu\varepsilon(1 - \frac{j\sigma}{\omega\varepsilon}) - (\frac{n\pi}{b})^2]^{1/2}$$

$$= j\sqrt{\omega^2\mu\varepsilon - (\frac{n\pi}{b})^2}\{1 - j\omega\mu\sigma[\omega^2\mu\varepsilon - (\frac{n\pi}{b})^2]^{-1}\}^{1/2}$$

$$\cong j\sqrt{\omega^2\mu\varepsilon - (\frac{n\pi}{b})^2}\{1 - \frac{j\omega\mu\sigma}{2}[\omega^2\mu\varepsilon - (\frac{n\pi}{b})^2]^{-1}\}$$

Assumption that $\omega\mu\sigma \ll \omega^2\mu\varepsilon - (\frac{n\pi}{b})^2$

Attenuation in Parallel-plate Waveguides (5)

For cutoff frequency $\frac{n\pi}{b} = 2\pi f_c \sqrt{\mu\varepsilon}$

$$\Rightarrow \sqrt{\omega^2 \mu\varepsilon - (\frac{n\pi}{b})^2} = \omega\sqrt{\mu\varepsilon}\sqrt{1-(\frac{\omega_c}{\omega})^2}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-(\frac{f_c}{f})^2}$$

$$\therefore \gamma = \alpha_d + j\beta = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1-(\frac{f_c}{f})^2}} + j\omega\sqrt{\mu\varepsilon}\sqrt{1-(\frac{f_c}{f})^2}$$

Attenuation in Parallel-plate Waveguides (6)

We obtain $\alpha_d = \frac{\sigma\eta}{2\sqrt{1-(\frac{f_c}{f})^2}}$ $\Rightarrow$ decreases when frequency increases

and $\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-(\frac{f_c}{f})^2}$

To find α_c

$$\alpha_c = \frac{P_L(z)}{2P(z)}$$

$$P(z) = w\int_0^b -\frac{1}{2}(E_y^0)(H_x^0)^* dy$$

$$= \frac{w\omega\varepsilon\beta}{2}(\frac{bA_n}{n\pi})^2 \int_0^b \cos^2(\frac{n\pi y}{b})dy = w\omega\varepsilon\beta b(\frac{bA_n}{2n\pi})^2$$

Attenuation in Parallel-plate Waveguides (7)

$$P_L(z) = 2w(\frac{1}{2}\left|J_{sz}^0\right|^2 R_s)$$

$$= w(\frac{\omega\varepsilon b A_n}{n\pi})^2 R_s \quad \text{where } \left|J_{sz}^0\right| = \left|H_x^0(y=0)\right| = \frac{\omega\varepsilon b A_n}{n\pi}$$

$$\therefore \alpha_c = \frac{P_L(z)}{2P(z)} = \frac{2\omega\varepsilon}{\beta b} R_s = \frac{2R_s}{\eta b\sqrt{1-(\frac{f_c}{f})^2}}$$

$$R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$$

$$\therefore \alpha_c = \frac{2}{\eta b}\sqrt{\frac{\pi\mu_c f_c}{\sigma_c}} \frac{1}{\sqrt{(\frac{f_c}{f})[1-(\frac{f_c}{f})^2]}}$$

Attenuation in Parallel-plate Waveguides (8)

TE modes

α_d : the same as TM

$$\alpha_c: \quad P(z) = w\int_0^b \frac{1}{2}(E_x^0)(H_y^0)^* dy$$

$$= \frac{w\omega\mu\beta}{2}(\frac{bB_n}{n\pi})^2 \int_0^b \sin^2(\frac{n\pi y}{b})dy = w\omega\mu\beta b(\frac{bB_n}{2n\pi})^2$$

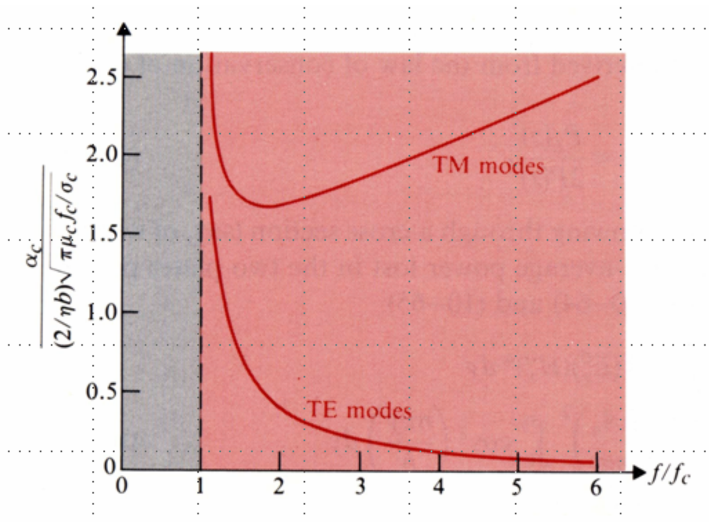
$$P_L(z) = 2w(\frac{1}{2}\left|J_{sx}^0\right|^2 R_s)$$

$$= w\left|H_z^0(y=0)\right|^2 R_z = wB_n^2 R_s$$

$$\therefore \alpha_c = \frac{P_L(z)}{2P(z)} = \frac{2R_s}{\omega\mu\beta b}(\frac{n\pi}{b})^2 = \frac{2R_s f_c^2}{\eta b f^2\sqrt{1-(\frac{f_c}{f})^2}}$$

$\Rightarrow$ decreases monotonically as frequency increases

Attenuation in Parallel-plate Waveguides (9)



Homework

H.W

10-2, 10-4, 10-5, 10-8, 10-9, 10-11, 10-14