Mechanics of Composite Materials

CHAPTER 2.
Micromechanics of Composites

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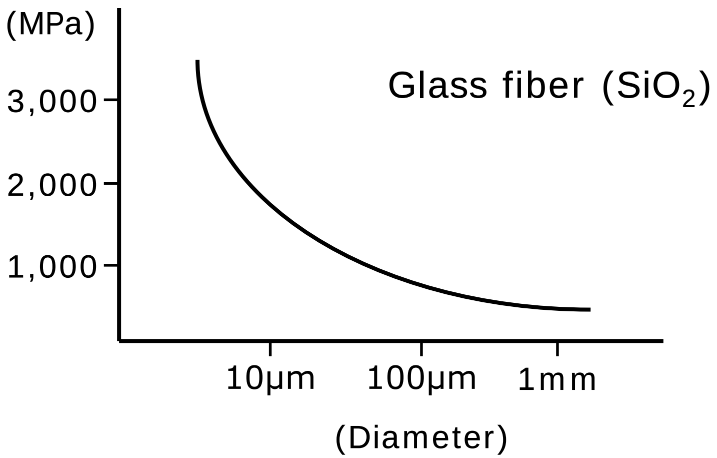
Chap 2. Micromechanics of Composites

2. Micromechanics of Composites

- Lock at fibers, matrix and interactions in a polymer matrix composite
- **Fibers**: very small diameter fibers of glass are much stronger than
  bulk properties of glass
- Griffith Experiment, 1921



- For brittle materials, strength $\propto \frac{1}{\sqrt{a}}$ (flaw size)

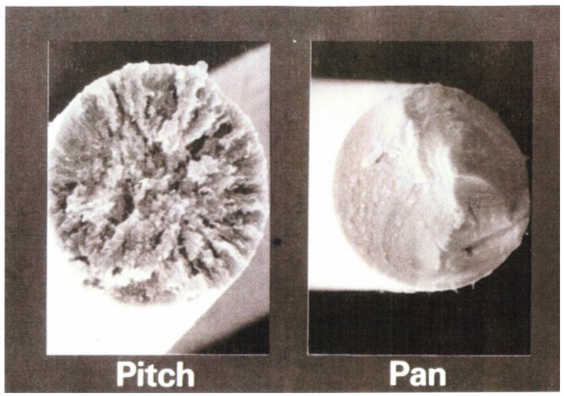
2. Micromechanics of Composites

- Small fibers → smaller flaws, fewer flaws

  → much higher strengths than large fibers, bulk properties

- Similarly for graphite fivers, etc.

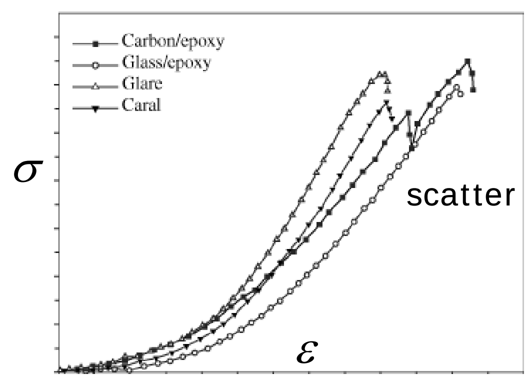
- Fibers for composite – graphite



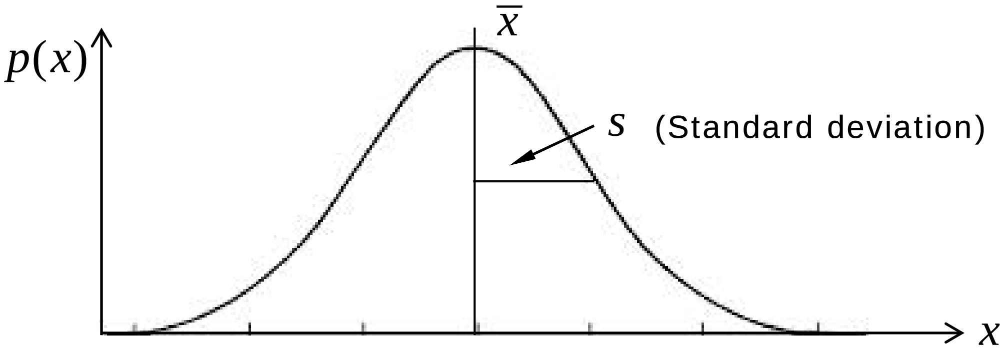
- Strong along fiber direction, weak bond perpendicular to fiber direction

  fiber test

  1. Brittle
  2. Much scatter on σ_{ult}/ρ
     (less scatter on E)



❖ Statistics of Failure

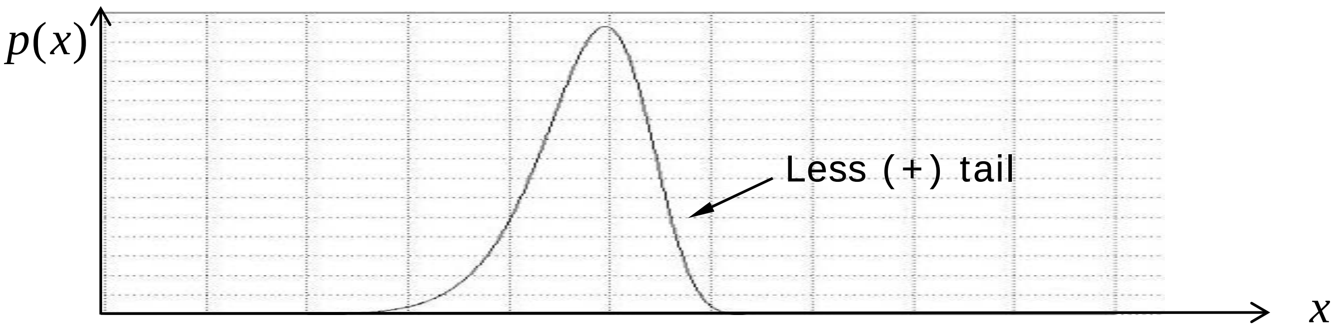


Normal distribution: Convenient for statistics but physical problem

i) Negative tail

ii) Goes to infinite in both direction

- Weibull Distribution



2. Micromechanics of Composites

- Weibull $p(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$ Better for fits here for $x \geq 0$

  β : scale factor (analogous to mean)

  mean, $\bar{x} = \beta \underbrace{\Gamma\left(\frac{1}{\alpha}+1\right)}_{*}$

  $\bar{x} \cong \beta$

α	$*$
5	0.92
25	0.98

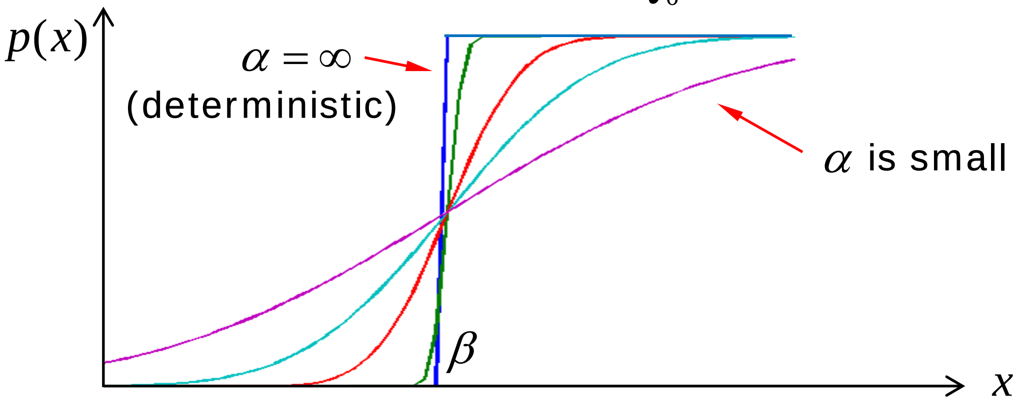
  α : shape factor

S (standard dev.) $= \beta \underbrace{\left[\Gamma\left(\frac{2}{\alpha}+1\right) - \Gamma^3\left(\frac{1}{\alpha}+1\right)\right]^{1/2}}_{**} \cong \beta/\alpha$

α	$**$
5	1.05
25	1.23

coefficient of variation $= \frac{S}{\bar{x}} \cong \frac{1}{\alpha}$

- Cumulative Distribution $P(x) = \int_0^x p(x)dx = 1 - e^{-(x/\beta)^\alpha}$



$P(x)$ is probability that failure will occur before load x is reached.

where, Mean $\cong \beta$, S.D. $\cong \beta(**) \cong \beta/\alpha$, C.O.V. $\cong 1/\alpha$

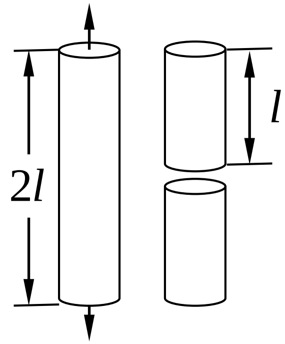
- Typical Values (100 Ksi = 690Mpa)

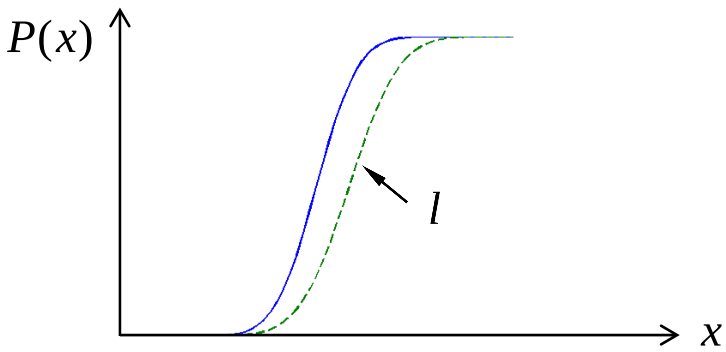
Fiber	β	α	C.O.V.	
Kevlar	~600 Ksi	6	17%	a lot of scatter
Graphite	~450 Ksi	4	25%	
Steel	~200 Ksi	25~50	2~4%	

(100Ksi = 690 MPa)

2. Micromechanics of Composites

- Consider longer fiber, 2l





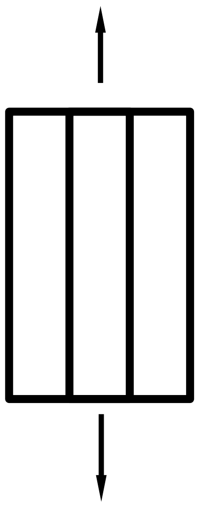
2l weaker, more scatter

- Consider a bundle of fibers,

When one fiber breaks, others carry load.

Stress goes up since net area is down.

Generally, generate less scatter, not more strength



2. Micromechanics of Composites

- Fiber Bundles called tows

  12K tow → 12,000 fibers

  20K tow → 20,000 fibers

  G_r/E_p → fiber 7μm, tow 700μm

- Use of fiber bundles good: high strength, less scatter, but need greater rigidity

  → compression as well as tension

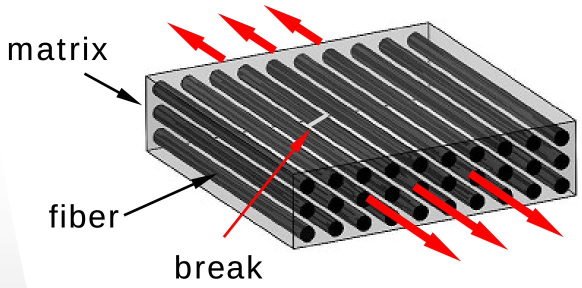
- Use matrix to enclose fiber

2. Micromechanics of Composites

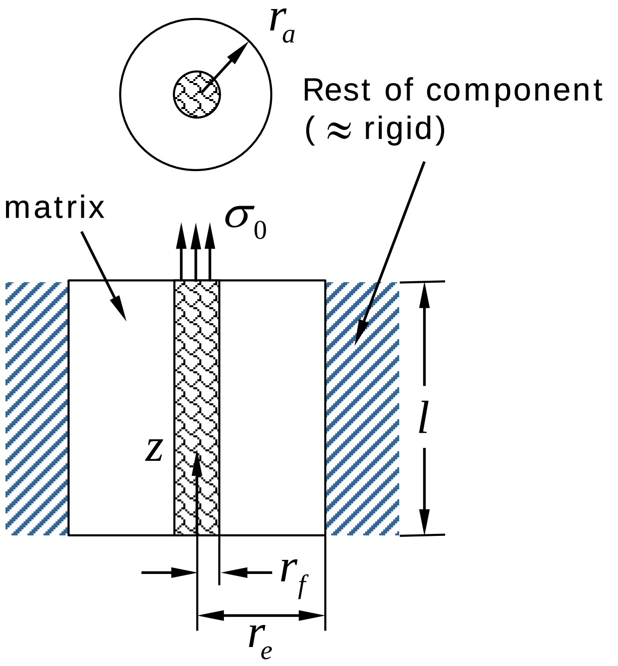
❖ Key role of matrix

1. Protect fibers
2. Provide rigidity for fibers
3. Stress transfer about fiber fracture
4. Reduce stress concentration at break

❖ Role of matrix in stress transfer



If matrix and fiber are well bonded, what happens?
First consider simple pull-out problem



not to scale
Typically, $V_F \approx 70\%$
(matrix really more of a thin sheath around fiber)

2. Micromechanics of Composites

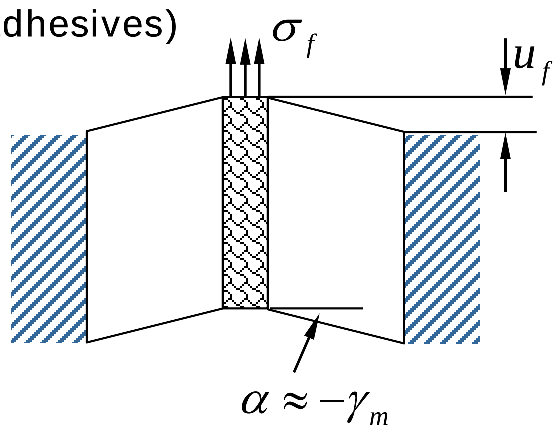
- B.C.`s

$$\begin{cases} @\ z=0,\ \sigma_F=0 \\ @\ z=L,\ \sigma_f=\sigma_0 \\ @\ r=r_a,\ \text{Displacement}=0\ \text{(rigid)} \end{cases}$$

- Assume uniform σ_f

  zero σ_m ($E_m \approx 3.5\text{GPa}$, $E_f \approx 210\text{GPa}$)

  matrix acts only in shear (adhesives)



- Unknowns u_f -displacement

  ε_f, γ_m - strain

  σ_f, τ_m - stress

- Strain-Displacement Equation

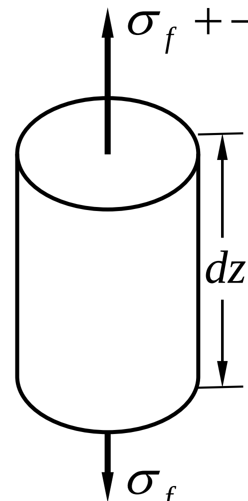
$$\varepsilon_f = \frac{\partial u_f}{\partial \varepsilon} = u_f' \quad \cdots ① \qquad \gamma_m = \frac{u_f}{r_a - r_f} \quad \cdots ②$$

2. Micromechanics of Composites

- Equilibrium Equation

$$\left(\sigma_f + \frac{\partial \sigma_f}{\partial z} dz\right)\pi r_f^2 - \sigma_f \pi r_f^2 + \tau_m 2\pi r_f dz = 0$$

$$\frac{\partial \sigma_f}{\partial z} + \frac{2\tau_m}{r_f} = 0\ , \quad \sigma_f' + \frac{2\tau_m}{r_f} = 0 \qquad \cdots ③$$



- Stress-strain Equation

$$\sigma_f = E_f \varepsilon_f \quad \cdots ④ \qquad \tau_m = G_m \gamma_m \quad \cdots ⑤$$

- 5 Equations → 5 unknowns

⑤ → ③ $$\sigma_f' + \frac{2G_m \gamma_m}{r_f} = 0 \quad \cdots ⑥$$

② → ⑥ $$\sigma_f' + \frac{2G_m}{r_f}\left(-\frac{u_f}{(r_a - r_f)}\right) = 0 \quad \cdots ⑦$$

taking derivative $$\sigma_f'' + \frac{2G_m}{r_f(r_a - r_f)} u_f' = 0 \quad \cdots ⑧$$

2. Micromechanics of Composites

$$① \rightarrow ⑧ \quad \sigma_f'' - \frac{2G_m}{r_f\left(r_a - r_f\right)}\varepsilon_f = 0 \quad \cdots ⑨$$

$$④ \rightarrow ⑨ \quad \sigma_f'' - \frac{2G_m}{r_f\left(r_a - r_f\right)E_t}\sigma_f = 0$$

$$\boxed{\sigma_f'' - \lambda^2\sigma_f = 0} \cdots ⑩ \qquad \text{where} \quad \lambda^2 = \underbrace{\frac{2}{r_f\left(r_a - r_f\right)}}_{\text{Geom.}}\underbrace{\frac{G_m}{E_f}}_{\text{Mat'l}}$$

Solving, $\sigma_f = A\sinh\lambda z + B\cosh\lambda z$

B.C. @ $z = 0,\ \sigma_f = B = 0$

@ $z = L,\ \sigma_f = A\sinh\lambda L = \sigma_0$

Final solution, $\sigma_f = \sigma_0\dfrac{\sinh\lambda z}{\sinh\lambda L}$

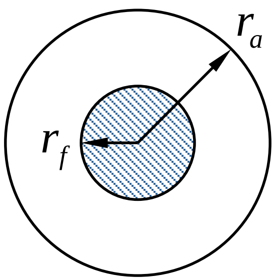
- Useful to non-dimensionalize the problem,

Define, $\begin{cases} \eta = \dfrac{z}{r_f}, \ \eta_{\max} = \dfrac{L}{r_f} \\ \lambda_z = \lambda r_f \eta = \zeta \eta \end{cases}$

then, $\zeta^2 = \lambda^2 r_f^2 = \dfrac{2r_f^2}{r_f\left(r_a - r_f\right)} \dfrac{G_m}{E_f}$ or $\zeta^2 = \dfrac{2\left(r_f / r_a\right)}{1-\left(r_f / r_a\right)} \dfrac{G_m}{E_f}$

- Define fiber volume fraction

$$V_f = \frac{\text{Volume of fibers}}{\text{Total volume}} \qquad V_f = \frac{\pi r_f^2 L}{\pi r_a^2 L} = \frac{r_f^2}{r_a^2}, \quad \zeta^2 = \frac{2\sqrt{V_f}}{1-\sqrt{V_f}} \frac{G_m}{E_f}$$



so, $\boxed{\zeta = \sqrt{\dfrac{2\sqrt{V_f}}{1-\sqrt{V_f}} \dfrac{G_m}{E_f}}}$ Non-dim. parameter in terms of measurable composite properties

$$\sigma_f = \sigma_0 \frac{\sinh \zeta \eta}{\sinh \zeta \eta_{\max}}$$

2. Micromechanics of Composites

- For shear stress in matrix, recall

$$\sigma_f' + \frac{2\tau_m}{r_f} = 0 \ \rightarrow \ \tau_m = -\frac{r_f}{2}\sigma_f' = \sigma_0 \frac{\zeta}{2}\frac{\cosh \zeta\eta}{\sinh \zeta L}$$

Also, from $u_f = -(r_a - r_f)\, r_m$, can show $\frac{u_f}{r_f} = -\left(1 - \frac{r_f}{r_a}\right)\frac{\tau_m}{G_m}$

consider magnitude of ζ (will scale problem)

$$\zeta = \sqrt{\frac{2\sqrt{V_f}}{1-\sqrt{V_f}}}\sqrt{\frac{G_m}{E_f}}$$

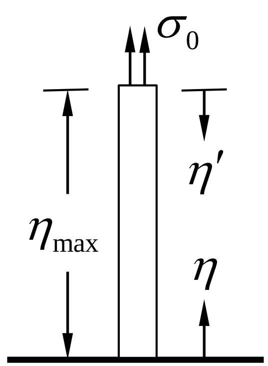
Typical G_r / E_p $\cdots$ $G_m = 133 Gpa,\ E_f = 193 Gpa$ $\sqrt{G_m / E_f} = 0.83$

r_a / r_f	V_f	$\sqrt{2\sqrt{V_f} / 1 - \sqrt{V_f}}$	ζ (typically $\zeta < 1$)
0.16	0.4	1.86	0.154
0.25	0.5	2.20	0.182
0.36	$\boxed{0.6}$	2.26	0.218
0.49	$\boxed{0.7}$	3.20	0.226

practical value

2. Micromechanics of Composites

- Look at stress distribution in fiber



Transform coordination to η'

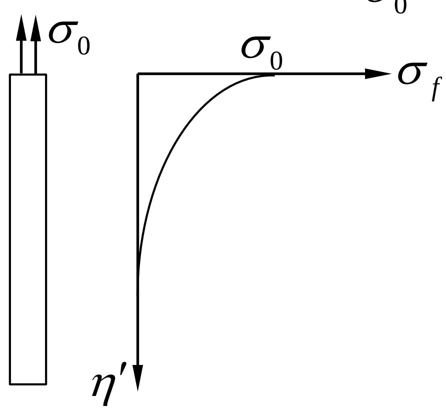
$$\eta = \eta_{\max} - \eta'$$

$$\frac{\sigma_f}{\sigma_0} = \frac{\sinh \zeta\eta}{\sinh \zeta\eta_{\max}} = \frac{1/2\left(e^{\zeta\eta} - e^{-\zeta\eta}\right)}{1/2\left(e^{\zeta\eta_{\max}} - e^{-\zeta\eta_{\max}}\right)}$$

$$= \frac{e^{\zeta\eta_{\max}} e^{-\zeta\eta'} - \cancel{e^{-\zeta\eta_{\max}}} e^{\zeta\eta'}}{e^{\zeta\eta_{\max}} - \cancel{e^{-\zeta\eta_{\max}}}}$$

$$L >> r_f \rightarrow \eta_{\max} >> 1$$

$e^{\zeta\eta_{\max}} >> e^{-\zeta\eta_{\max}}$, $\dfrac{\sigma_f}{\sigma_0} \approx e^{-\zeta\eta'}$ (also similarly, $\dfrac{\tau_m}{\sigma_0} \approx -\dfrac{\zeta}{2} e^{-\zeta\eta'}$)



Decays exponentially,

$\zeta\eta' = 3 \rightarrow$ 5% of σ_0

$\zeta\eta' = 5 \rightarrow <$ 1% of σ_0

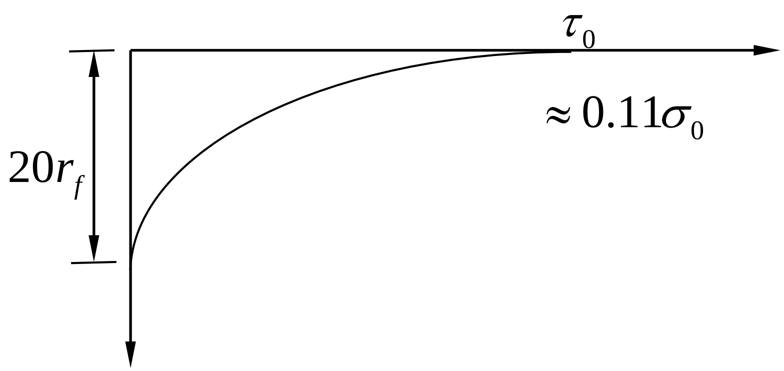
$$\eta' = \frac{5}{\zeta} \rightarrow \frac{z'}{r_f} = \frac{5}{0.218} = 23$$

$V_f = 0.6$

By ~ 20 r_f (10 diam.), σ_f all gone

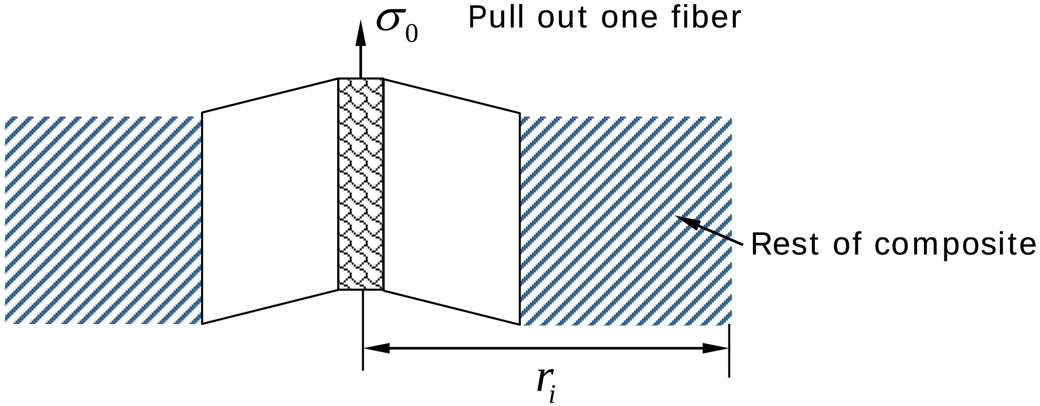
2. Micromechanics of Composites

- Similarly for shear stress



stress concentration in matrix (like adhesives)

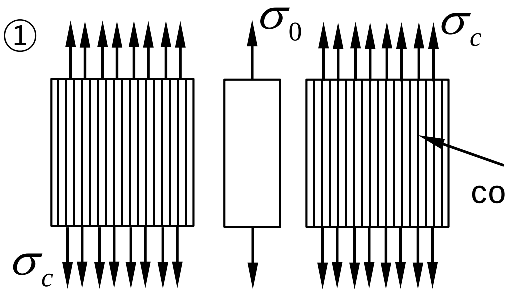
- Have solved this problem



Formed load transfer quickly to matrix

- To examine fiber-break problem, superimpose 2 solutions

①

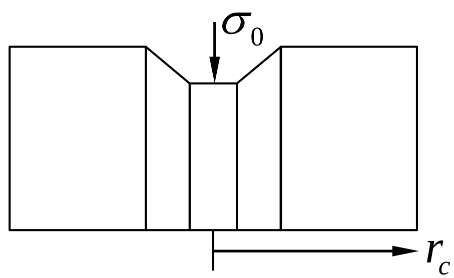


Assume all strain together

$$\varepsilon_f \approx \varepsilon_c$$

$$\frac{\sigma_f}{E_f} \approx \frac{\sigma_c}{E_c}, \ \sigma_c = \frac{E_c}{E_f}\sigma_0$$

②

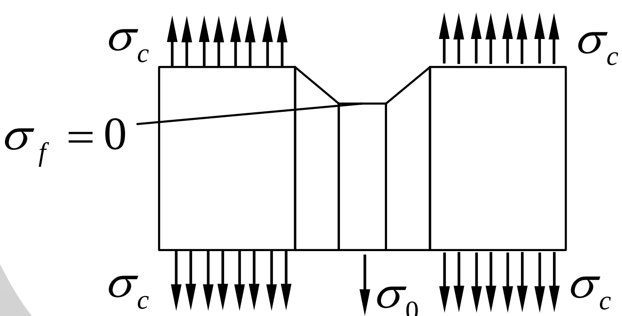


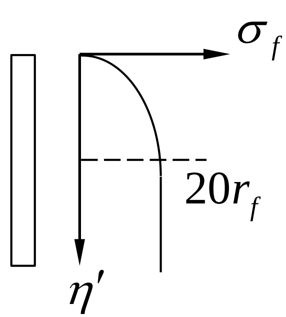
Our problem with $-\sigma_0$

had said $E_c = \infty$

but if $r_c >> r_f$, still rigid

- Adding ① & ② gives





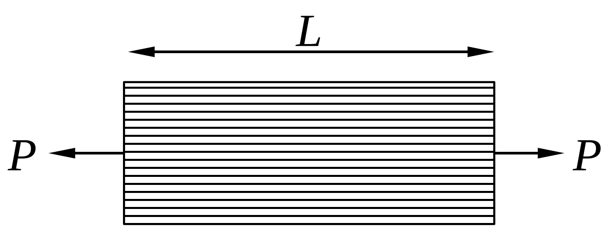
$$\sigma_f = \sigma_0 - \sigma_0 e^{-\zeta\eta'} = \sigma_0\left(1 - e^{-\zeta\eta'}\right)$$

$$\tau_m = \sigma_0 \frac{\zeta}{2} e^{-\zeta\eta'}$$

So fiber picks up load again after break (also τ_m stress concentration)

2. Micromechanics of Composites

- Less than 10 fiber diameters from break,
  stress in fiber reaches $\sim \sigma_0$
- This region called "ineffective zone"
  total ineffective length for one break $\approx 20d_f$ (one zone each side)
- In real lives, a little worse $\begin{cases} \text{- matrix deform plastically} \\ \text{- debonding, sliding} \end{cases}$
- How this affects a composite

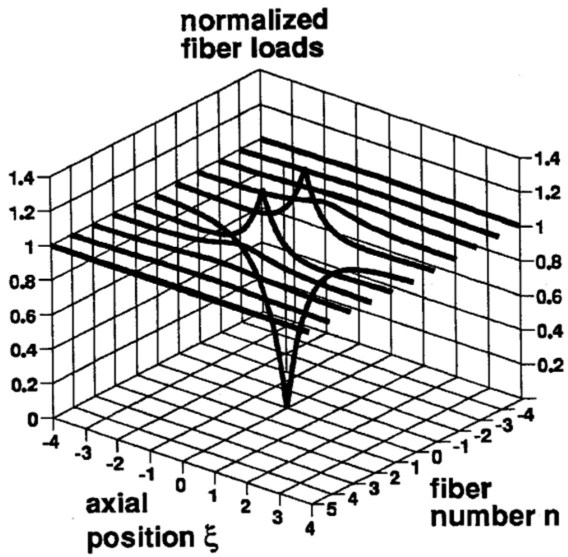


5 fiber, one breaks
Ineffective length $\delta = 40r_f$
$L >> \delta$

No. of breaks	**No Matrix**		**With Matrix**		
	# fiber	Ave. load	Ave. # fiber	Ave. load	
0	5	P/5	5	P/5	
1	4	P/4	5- δ / L	$\frac{P}{5-\delta/L} \approx \boxed{\frac{P}{5}}$	still good

2. Micromechanics of Composites

- Locally, neighboring fibers pick up load,



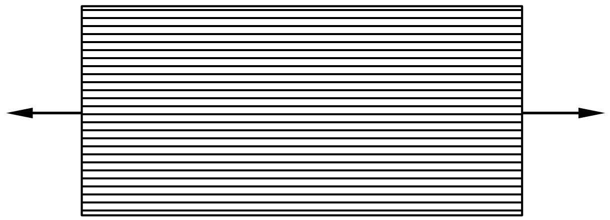
See Sastry and Phoenix

“Shielding and Magnification of Loads in composites”

SAMPE Journal

Vol.30, No.4, July-Aug 1994 p.61

- Locally have load $> P/5$, but it is over small length less chance of break
- Chance of break goes up for larger specimens (more flaws)
  but damage is localized



more length,

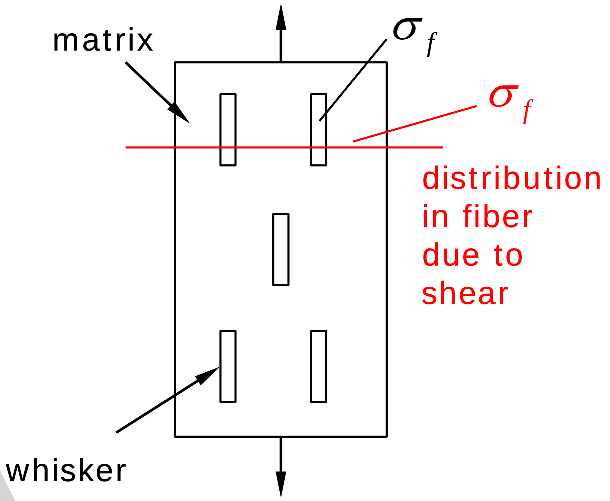
more breaks,

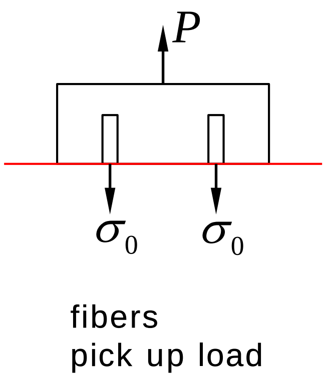
but localized

- So, Matrix transfers load,
  - only local effect when fiber breaks
  - Distribution shift and tightens
  - Length scaling goes down (fewer flaws)

for Kevlar	without matrix	with matrix
Ave. bundle strength	350 Ksi	550 Ksi
C.V.	20~25%	4~5%
α	4~5	20~25

- Also have “Whisker Problem”





2. Micromechanics of Composites

- Effective Properties of a Composite (see Jones, Chap.3)

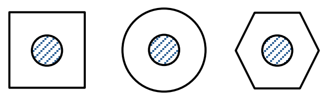
  would like to predict effects of composite constituents and fiber volume fraction

  on macro-properties of a laminate

  (modulus, Poisson`s ratio, strength, thermal expansion, conductivity, etc.)

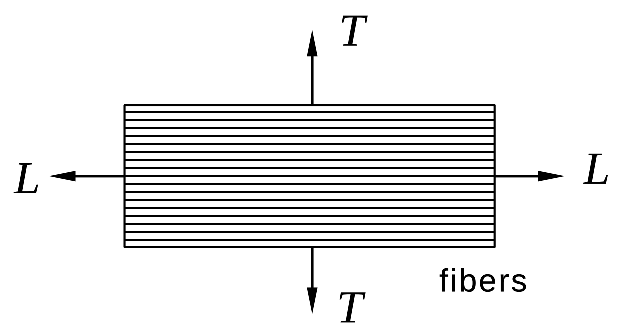
- Use Mechanics of Materials approach (simpler than Theory of Elasticity)

  Basic idea – Choose representative volume element and repeat to form composite
  Analyze element importance of fiber volume fraction



- Define L-T coordinate system

  Length Transverse



- Assumption

  ① Composite (Lamina) is - macroscopically homogeneous
  - linear elastic (but orthotropic)
  - initially stress free

② Fibers are - homogeneous
- linear elastic
- isotropic
- regularly spaced, aligned

③ Matrix is - homogeneous
- linear elastic ?
- isotropic ?
- void free ?

- Matrix and fibers assumed perfectly bonded.

- Measuring volume fraction

1) Cross-section, polish and count fibers in microscope

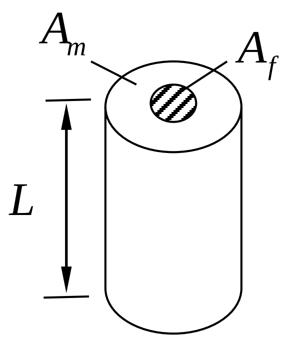
(area fraction → volume fraction)

2) Dissolve matrix, weigh fibers → get mass fraction

From densities → volume fraction

2. Micromechanics of Composites

First property to model, $\rho_c \rightarrow$ density



$$M_f = \rho_f A_f L,\ \ M_m = \rho_m A_m L$$

$$\rho_c = \frac{M_f + M_m}{vol} = \frac{\rho_f A_f L + \rho_m A_m L}{\left(A_f + A_m\right) L}$$

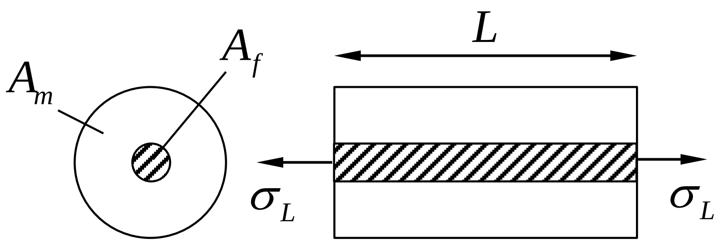
$$\boxed{\rho_c = V_f \rho_f + V_m \rho_m}$$ Rule of Mixtures

where $V_f = \dfrac{A_f}{A_f + A_m} = \dfrac{r_f^2}{r_a^2},\ V_m = 1 - V_f$

Fiber vol. fraction

If no voids

- Look @ E_L - Longitudinal modulus



$$E_L = \frac{\sigma_L}{\varepsilon_L} = ?$$

2. Micromechanics of Composites

Assume Perfect bond $\rightarrow \varepsilon_L = \varepsilon_{Lf} = \varepsilon_{Lm}$

No lateral constraint, $\sigma_T = 0$

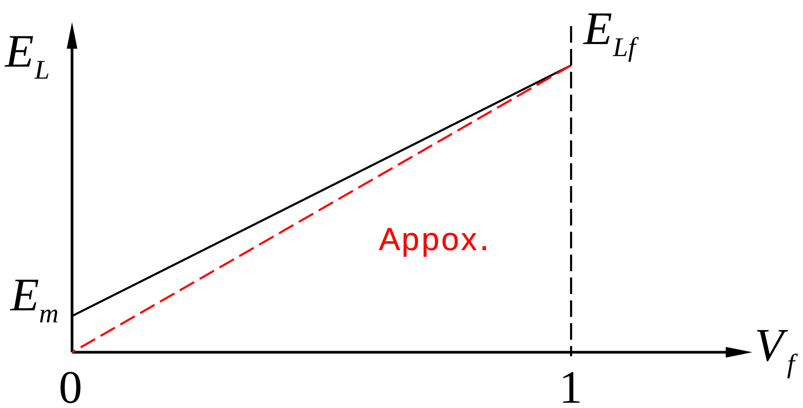
$$\sigma_L = \frac{A_f\sigma_f + A_m\sigma_m}{A_f\sigma_f} = V_f\sigma_f + V_m\sigma_m$$

$$\sigma_f = E_{Lf}\underbrace{\varepsilon_{Lf}}_{\varepsilon_L}, \quad \sigma_m = E_{Lm}\underbrace{\varepsilon_{Lm}}_{\varepsilon_L}$$

$$\frac{\sigma_L}{\varepsilon_L} = \boxed{E_L = V_f E_{Lf} + V_m E_m}$$ R.O.M. again

Note, if $\underbrace{E_{Lf}}_{\sim 230\text{ GPa}} >> \underbrace{E_m}_{\sim 3\text{ GPa}} \rightarrow E_L \approx V_f E_{Lf}$

for reasonable V_f

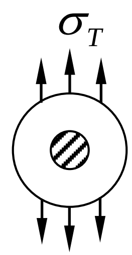


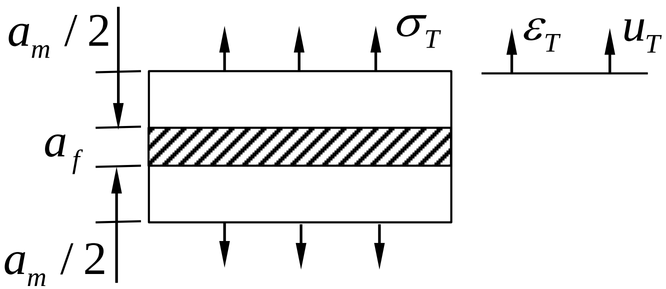
← Real composite not far off

- Look @ E_T - Transverse Modulus

  This looks messy.

  Simplify as lumped series model.





Assume: $\sigma_T = \sigma_f = \sigma_m$

Note $\varepsilon_f = \dfrac{\sigma_f}{E_{Tf}}, \ \varepsilon_m = \dfrac{\sigma_m}{E_m}$

$u_f = \varepsilon_f a_f, \ u_m = \varepsilon_m a_m$

consider displacement, $u_T = u_f + u_m = \varepsilon_f a_f + \varepsilon_m a_m$

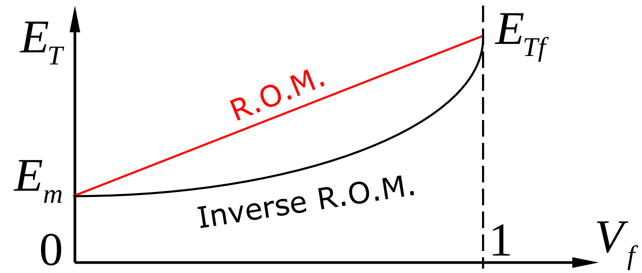
$$\varepsilon_T = \frac{u_T}{a_m + a_f} = \frac{\varepsilon_f a_f + \varepsilon_m a_m}{a_f + a_m}$$

For the same width and depth, $\dfrac{a_f}{a_m + a_f} = V_f, \ \dfrac{a_m}{a_m + a_f} = V_m \qquad \varepsilon_T = \varepsilon_f V_f + \varepsilon_m V_m$

Divide by stress $\sigma_T \left(= \sigma_f = \sigma_m \right)$

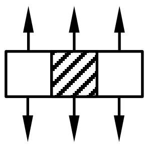
$$\frac{\varepsilon_T}{\sigma_T} = V_f \frac{\varepsilon_f}{\sigma_f} + V_m \frac{\varepsilon_m}{\sigma_m} \rightarrow \boxed{\frac{1}{E_T} = \frac{V_f}{E_{Tf}} + \frac{V_m}{E_m}}$$

Inverse R.O.M.



2. Micromechanics of Composites

- But if we picked parallel model

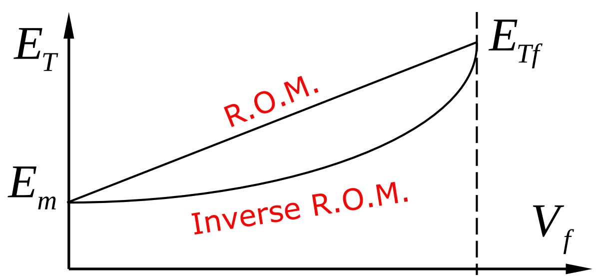


$$\varepsilon_T = \varepsilon_f = \varepsilon_m$$

Get R.O.M. $E_T' = E_{Tf} V_f + E_m V_m$

These 2 cases represent bounds on E_T

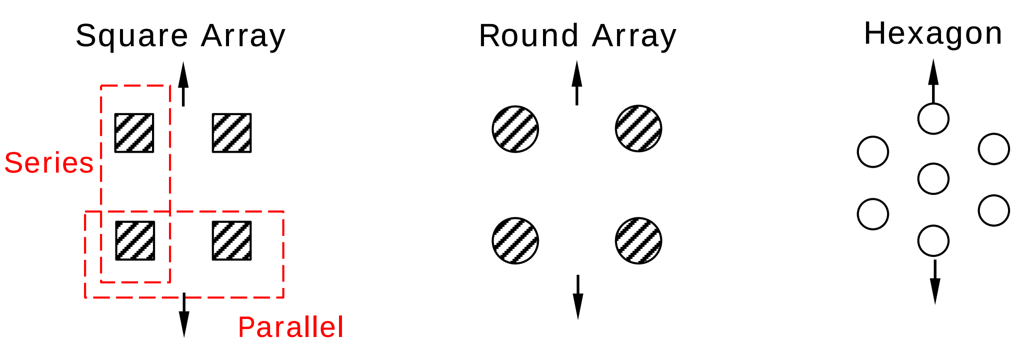
- For transverse properties, wide bounds from R.O.M.



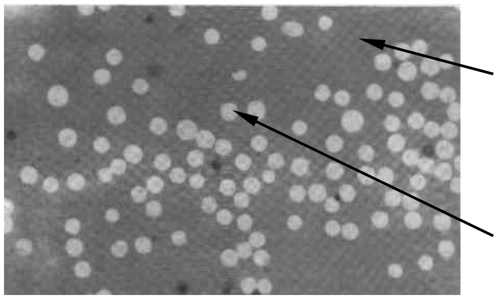
$$\frac{1}{E_T} = \frac{V_f}{E_{Tf}} + \frac{V_m}{E_m}$$: Inverse R.O.M.

$$E_T = V_f E_{Tf} + V_m E_m$$

Many possible theory – depends on model used



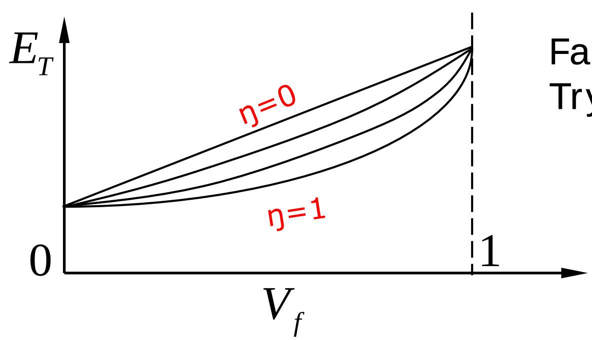
- More complex to analyze / elasticity theorem / F.E.M / Rayleigh-Ritz
  But still might be approx.
- Real composite – statistical distribution of fibers



- Mixed models (empirical)
  simplest idea

$$E_T = E_T(\text{Inverse R.O.M.}) \times \eta + E_T(\text{R.O.M.}) \times (1-\eta)$$

Fit η to data



Family of curves,
Try fit

Much work along these lines

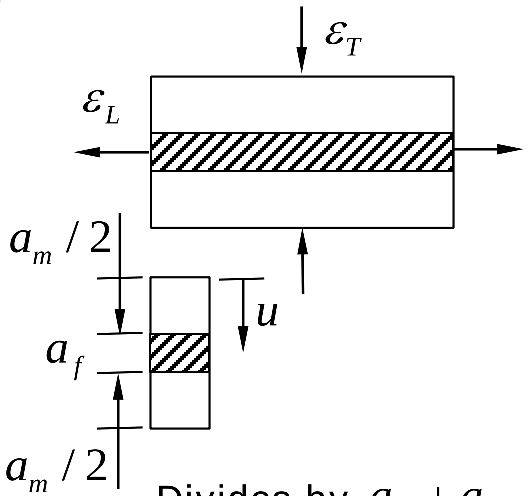
Hahn $E_T = \dfrac{1+V^*}{\dfrac{1}{E_{Tf}} + \dfrac{V^*}{E_m}}, \quad V^* = \eta' \dfrac{V_m}{V_f}$ Chanus $E_T = \dfrac{1}{\dfrac{1-\sqrt{V_f}}{E_{Tf}} + \dfrac{\sqrt{V_f}}{E_m}}, \quad etc$

Another problem, E_{Tf} hard to determine

Practically

1. Find an η that works for $V_f \approx 0.60$
2. Get E_{Tf} as best problem
3. Find E_T for V_f not too far from $0.60 \rightarrow (0.55 \sim 0.70)$

- Look @ ν_{LT} Poisson's Ratio



Assume $\varepsilon_L = \varepsilon_{Lf} = \varepsilon_{Lm}$

$$\varepsilon_{Tf} = -\nu_{LTf}\varepsilon_{Lf}$$

$$\varepsilon_{Tm} = -\nu_m\varepsilon_{Lm}$$

$$u = u_m + u_f = \varepsilon_{Tm}a_m + \varepsilon_{Tf}a_f$$

$$-u = a_m\nu_m\varepsilon_{Lm} + a_f\nu_{LTf}\varepsilon_{Lf}$$

Divides by $a_m + a_f$, $\quad -\dfrac{u}{a_m + a_f} = \left(\dfrac{a_m}{a_m + a_f}\nu_m + \dfrac{a_f}{a_m + a_f}\nu_{LTf}\right)\varepsilon_L$

$$\varepsilon_T = \left(V_m\nu_m + V_f\nu_{LTf}\right)\varepsilon_L$$

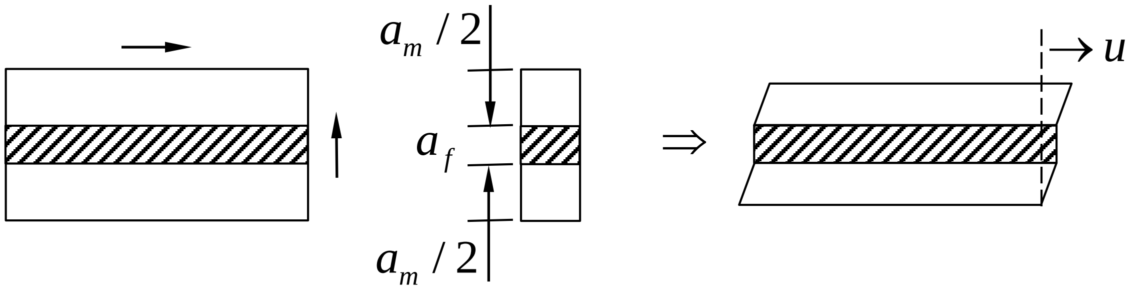
$$-\frac{\varepsilon_T}{\varepsilon_L} = \nu_{LT} = V_m\nu_m + V_f\nu_{LTf}$$: R.O.M

Model seems to work

Anyway $\nu_m \,\&\, \nu_{LTf}$ both ~.3, so anything works

2. Micromechanics of Composites

- Look @ shear modulus,



$$\tau_m = \tau_f = \tau$$

$$\gamma_m = \tau_m / G_m, \;\; \gamma_f = \tau_f / G_{LTf}$$

$$u = \gamma_m a_m + \gamma_f a_f$$

$$\frac{u}{a_m + a_f} = \gamma_m \frac{a_m}{a_m + a_f} + \gamma_m \frac{a_m}{a_m + a_f}$$

$$\bar{\gamma} = \gamma_m V_m + \gamma_f V_f$$

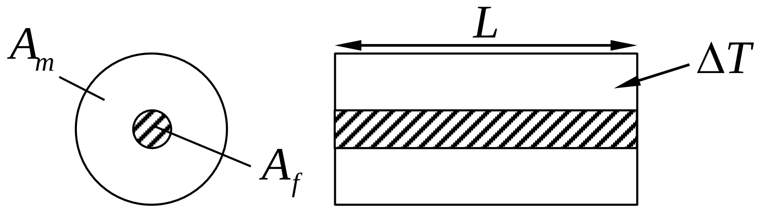
$$\frac{\bar{\gamma}}{\tau} = \underbrace{\frac{\gamma_m}{\tau}}_{=\tau_m} V_m + \underbrace{\frac{\gamma_f}{\tau}}_{=\tau_f} V_f \qquad \frac{1}{G} = \frac{V_m}{G_m} + \frac{V_f}{G_{LTf}}$$

: Inverse R.O.M.

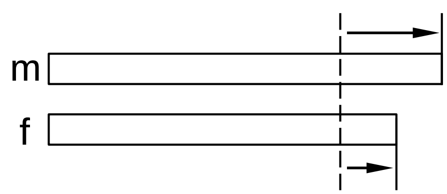
- Could also do a parallel model, get R.O.M. rule
- Use some Mixed model or some experimental fit of η
- Hard to measure G_{LTf}

Another property, Look @ Thermal Expansion, CTE α_L

(Coeff. Thermal Expansion)

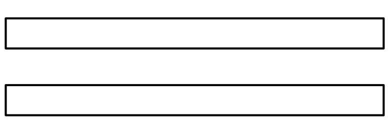


If matrix and fiber were independent,



$$\left.\begin{aligned}\varepsilon_m &= \alpha_m \Delta T \\ \varepsilon_f &= \alpha_f \Delta T\end{aligned}\right\} \text{ Not same}$$

Assume bonded,



$$\rightarrow \sigma_m = E_m(\varepsilon_m - \alpha_m \Delta T) \quad \cdots ①$$

$$\rightarrow \sigma_f = E_{Lf}(\varepsilon_f - \alpha_f \Delta T) \quad \cdots ②$$

No total load over end $\sigma_m A_m + \sigma_f A_f = 0 \quad \cdots ③$

Note $\varepsilon_m = \varepsilon_f = \varepsilon_c = \alpha_c \Delta T \quad \cdots ④$

2. Micromechanics of Composites

- Placing ①, ②, ④ into ③ gives

$$E_m(\varepsilon_c - \alpha_m \Delta T)A_m + E_{Lf}(\varepsilon_c - \alpha_{Lf}\Delta T)A_f = 0$$

Multiple by L and divided by volume and recall $V_m = \dfrac{A_m L}{vol.}, etc, \quad V_f = etc$

$$E_m(\varepsilon_c - \alpha_m \Delta T)v_m + E_{Lf}(\varepsilon_c - \alpha_{Lf}\Delta T)v_f = 0$$

This yield

$$\varepsilon_c = \underbrace{\frac{\alpha_m E_m V_m + \alpha_{Lf} E_{Lf} V_f}{E_m V_m + E_{Lf} V_f}}_{\alpha_{Lc}} \Delta T$$

Modulus weighted R.O.M.

Deal with $\dfrac{E_{Lf}V_f}{E_m V_m + E_{Lf} V_f}$ instead of $\dfrac{V_f}{V_m + V_f}$

- Note: composite stress-free at cure temperature when cools down $(\Delta T < 0)$

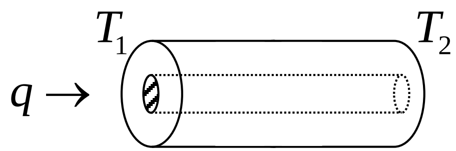
  residual stresses σ_m, σ_{Lf} will be created.

- Transverse C.T.E, α_{Tc}, harder to obtain

  Moisture cause a similar problem

  Matrix swells, fiber doesn`t.

- Look @ Thermal Conductivity, K_L



$$q_f = -K_{Lf}\frac{\partial T}{\partial x}, \quad q_m = -K_m\frac{\partial T}{\partial x}$$

Assume $\left(\frac{\partial T}{\partial x}\right)_f = \left(\frac{\partial T}{\partial x}\right)_m = \frac{\partial T}{\partial x}$ (long geometry)

$$q_c A_T = q_f A_f + q_m A_m$$

$$q_c = -\underbrace{(K_{Lf}V_f + K_m V_m)}_{K_{Lc}}\frac{\partial T}{\partial x}$$

Like stiffness

Note $K_{Lf} >> K_m$ so $K_{Lc} \approx K_{Lf}V_f$

Also K_{Lf} very high for fibers

specific conductivity K/ρ can be greater than Al or Cu

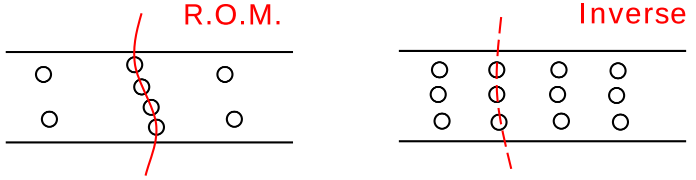
2. Micromechanics of Composites

- Transverse thermal conductivity – K_T

  $K_{Tf} >>> K_m, \quad K_{Tf} > K_{Lf}$

  good conductor

  consider



  V_f`s same, but $K_{Tc}$`s much different

  K_{Tc} very dependent on micro structure. can`t make good simplified model.

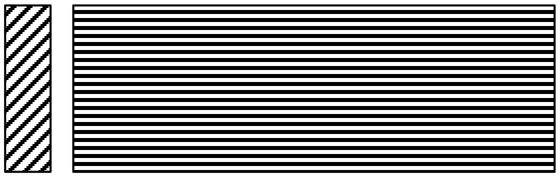
- Electrical Conductivity - C_c somewhat similar

  $C_f >>> C_m$

  so, $C_{Lc} \approx R.O.M. \approx C_{Lf} V_f$

  $C_{Tc} \rightarrow$ difficult to establish (paths @ microstructure)

  Also, electrical behavior is dominated by <u>contact</u>

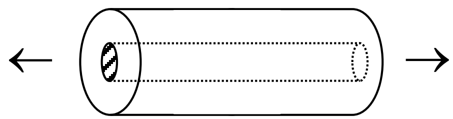


a little glue on space
dominates electrical behavior

2. Micromechanics of Composites

- Strength – difficult to predict

  look @ Tension



Tempting to use R.O.M. $X_t = X_{ft}V_f + \cancel{X_{mt}V_m}$ small

Let`s try this

AS1 / 3501-6 Composite

$$X_{ft} = 1{,}990 MPa \text{ (length)}$$

$$X_{ct} = 1{,}660 MPa \text{ (typical Gr/Ep) } V_f = 0.60$$

$$X_{mt} = 70 MPa$$

$$X_{cT} = 1{,}990(0.6) + 70(0.4) = \cancel{1{,}270 MPa}$$ No!

R.O.M. should have been Upper Bound

Effective fiber strength is increased by load sharing through matrix

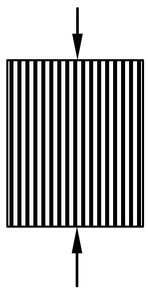
$$X_{fT}^{eff} \approx \frac{X_{cT}}{V_f} = \frac{1{,}660}{0.6} = 2{,}770 MPa$$

For very low fiber volume fractions

actually $V_f < 1.0$ R.O.M. reasonable in practice

2. Micromechanics of Composites

- Compressive strength



Dominated by fiber bucking

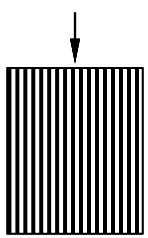
Controlled by fiber & matrix stiffness, fiber geometry

For most composites, material behavior gives

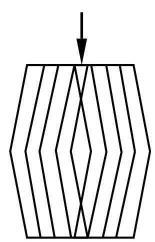
$X_c \approx X_T$

But this is not true for Kevlar, Pitch fiber Gr/Ep,

and in structures, careful of delamination, buckling

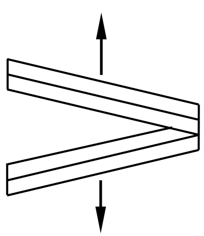


Layers (laminate)



sublaminate

- Transverse Tension Strength



Matrix dominated

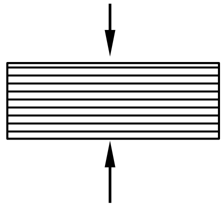
Y_t : very low

crack runs along fibers

impede formation of a plastic zone

2. Micromechanics of Composites

- Transverse Compression



Matrix dominated

but less of problem

$Y_c \approx (4 \sim 7) \times Y_T$

- Shear $\rightarrow$ matrix dominated

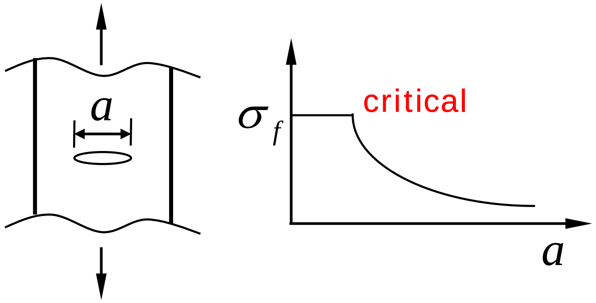
$Y_c > S > Y_T$ typical

- Other strength related properties
  - For fracture

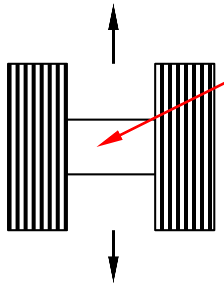
    for metals $\sigma_f = \sigma_y$ or $\dfrac{K_{1c}}{\sqrt{\pi a}}$

    fracture toughness

    ½ crack size



  - For composite



splits in matrix
as well as in fiber

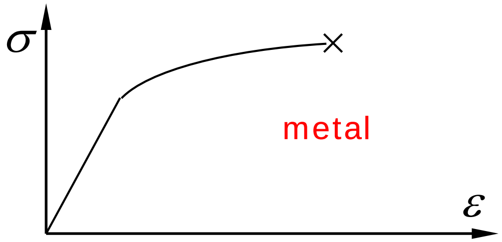
splits can cause delamination of layer

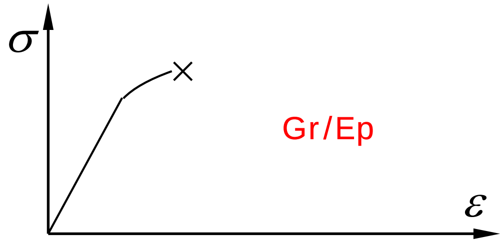
2. Micromechanics of Composites

- Fatigue
  - For metals → crack growth under cyclic loads. a major problem.
  - Carbon fibers are very good in fatigue
    0º dominated structures fatigue resistant (in tension)
    Careful of delaminations in compression and off axis plies

- Impact

  For Gr/Ep → generally low strain to failure, impact a major concern





- Environmental Resistance

  Moisture intake → changes matrix properties

  Temperature → can change matrix properties

People can concern with Hot, Wet, Post Impact, Compression test.

2. Micromechanics of Composites

- Talked about Micromechanics

  (How composites work, trends, and usefulness...)

  Actually will use Experimental Data in design of structures from composites

  will now talk about Macromechanics using composites to design structure