Mechanics of Composite Materials

CHAPTER 5.
Failure of Composite

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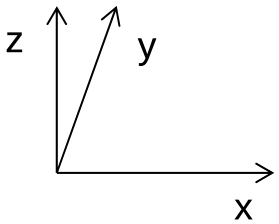
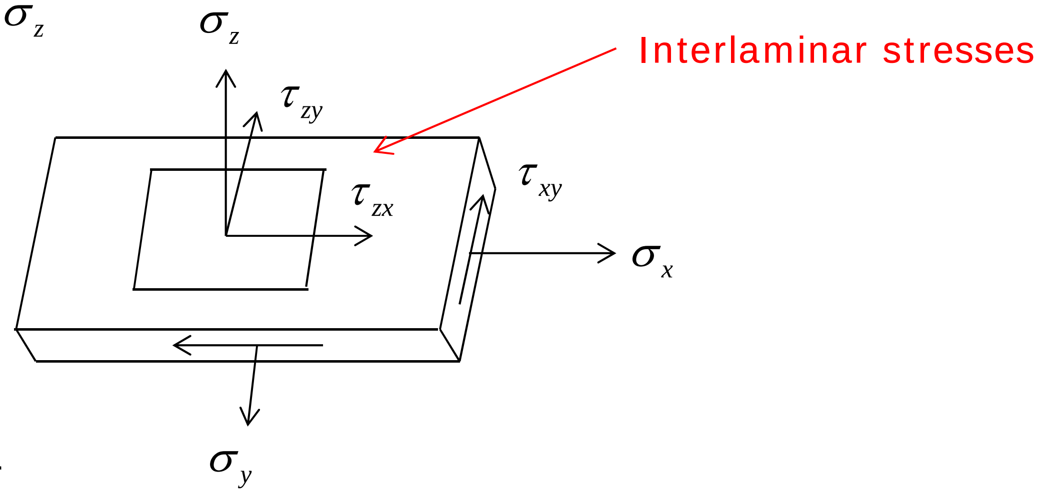
5. Failure of Composites

Have calculated ply stresses σ_x σ_y τ_{xy}

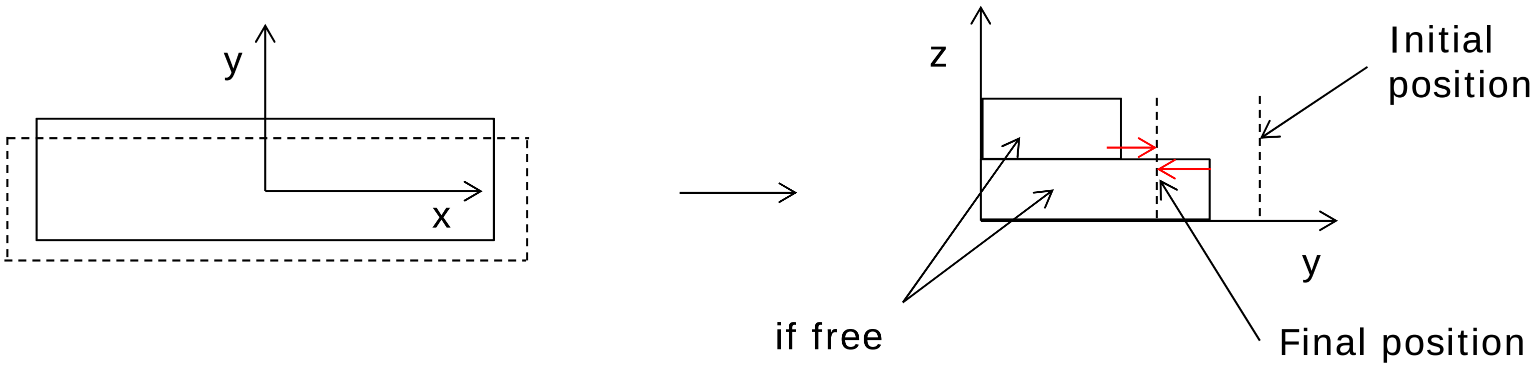
Also have inter-laminar stress τ_{zx} τ_{zy} σ_z



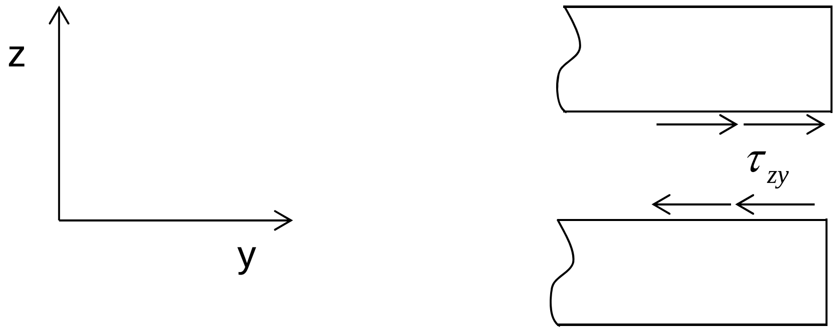


Interlaminar stresses hold plies together

Consider $[0/90]_s$ lay up



5. Failure of Composites

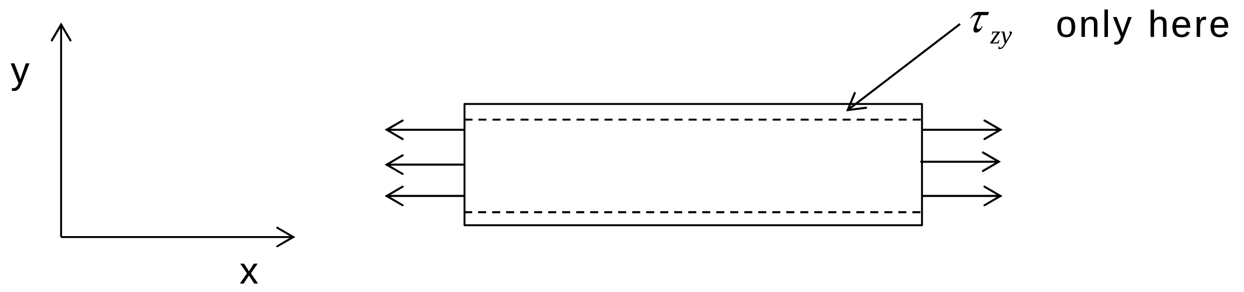


"glue" stress keep plate together

Actually τ_{zy} only develops at edge (~ 1 ply thickness)

- Edge zone

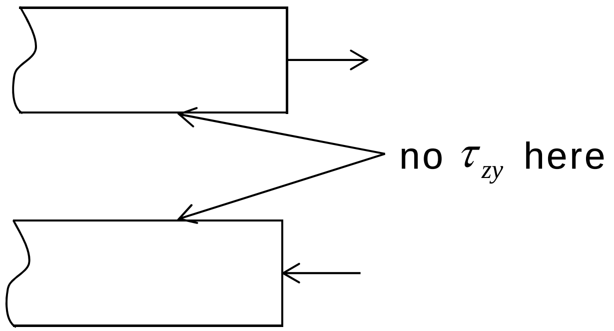
After ~1 ply thickness τ_{zy} disappears



See Pipes and Pagano, J. Composite Material Oct. 1970;
Jones book, Chap. 4. p. 210

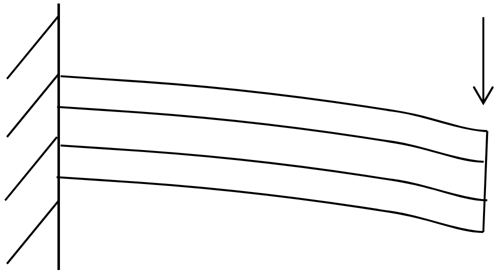
5. Failure of Composites

So cutting of edges plate behaves as if



Need elaborate 3-D analysis

τ_{zy}, Important for edge effects and also plate bending



Layers slide over each other unless restrained by τ_{zx}

will discuss Interlaminar Stress later.

5. Failure of Composites

Now return to ply stresses from CLPT, and how they can cause failure

For laminate, have found ply stresses and strains on ply-by-ply basis

Try to use these to assess failure.

For a single ply: At least 5 failure mechanisms → 5 quantities need to be measured from unidirectional tests.

X_t - longitudinal (L-direction) ultimate tensile stress

X_c - longitudinal (L-direction) ultimate compressive stress

Y_t - transverse (T-direction) ultimate tensile stress

Y_c - transverse (T-direction) ultimate compressive stress

S - Ultimate shear stress

: Generally all different for a composite

- Basic Building Blocks

  Try to extend these to combined stress states.

  Several theories proposed.

5. Failure of Composites

(a) Maximum Stress Theory

Failure occurs if any one of following occurs

$$\sigma_1 \geq X_t$$
$$\sigma_1 \leq X_c$$
$$\sigma_2 \geq Y_t$$
$$\sigma_2 \leq Y_c$$
$$|\sigma_6| > S$$

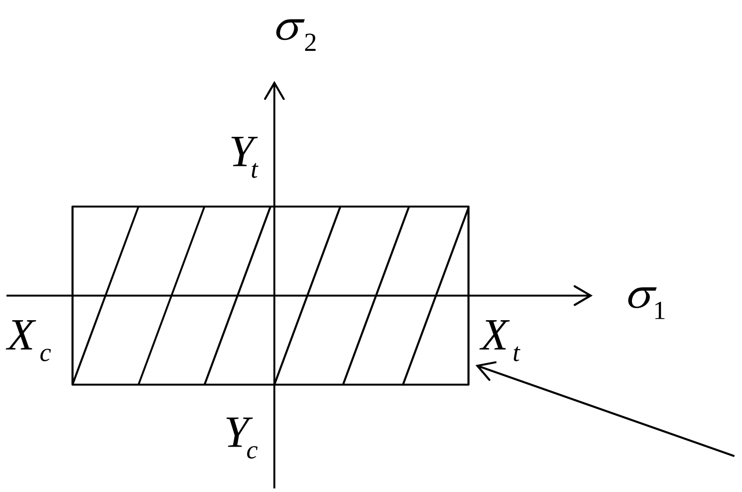
where,

$\sigma_1,\ \sigma_2,\ \sigma_6$	→	Stresses in ply coordinates
$X_c,\ Y_c$	→	Negative
σ_6	→	Sign unimportant

Then, allowable stress values are

$$X_c \leq \sigma_1 \leq X_t$$
$$Y_c \leq \sigma_2 \leq Y_t$$
$$-S \leq \sigma_6 \leq S$$

Can draw failure envelope in σ_1, σ_2 space



Also check $-S \le \sigma_6 \le S$ (3-Dimensional)

Previous Example with $[0/\pm45/90]_s$ laminate

ply, θ	σ_1	σ_2	σ_6
0°	-206	0.3	0
$+45^\circ$	-73	-6.5	9.5
-45°	-73	-6.5	-9.5
$+90^\circ$	60	-13	-9.5

Ksi

5. Failure of Composites

Compare with strengths

X_t	190 Ksi
X_c	-160 Ksi
Y_t	6 Ksi
Y_c	-25 Ksi
S	10 Ksi

Note

$X_t > X_c$

$Y_t < Y_c$

and $Y_t << X_t$

S small

Crushed the tube.

if pulled tube, all stresses reversed

Failure of 90° plies (cracking)

May not be failed, but fiber failure in 0° much more fatal

5. Failure of Composites

(b) Maximum Strain Theory

Look at strains rather than stresses

Define,

ε_{xt} - longitudinal (L-direction) ultimate tensile strain

ε_{xc} - longitudinal (L-direction) ultimate compressive strain

ε_{yt} - transverse (T-direction) ultimate tensile strain

ε_{yc} - transverse (T-direction) ultimate compressive strain

ε_{s} - Ultimate shear strain

Then allowable strain values are

$$\varepsilon_{xc} \le \varepsilon_1 \le \varepsilon_{xt}$$

$$\varepsilon_{xc} \le \varepsilon_2 \le \varepsilon_{yt}$$

$$-\varepsilon_s \le \varepsilon_6 \le \varepsilon_s$$

Since stress-strain curves tend to be linear to failure, one can say roughly,

$$\varepsilon_{xt} = \frac{X_t}{E_t}, \quad \varepsilon_{xc} = \frac{X_c}{E_L}, \quad \varepsilon_{yt} = \frac{Y_t}{E_T}, \; etc$$

For T300/934 this would give

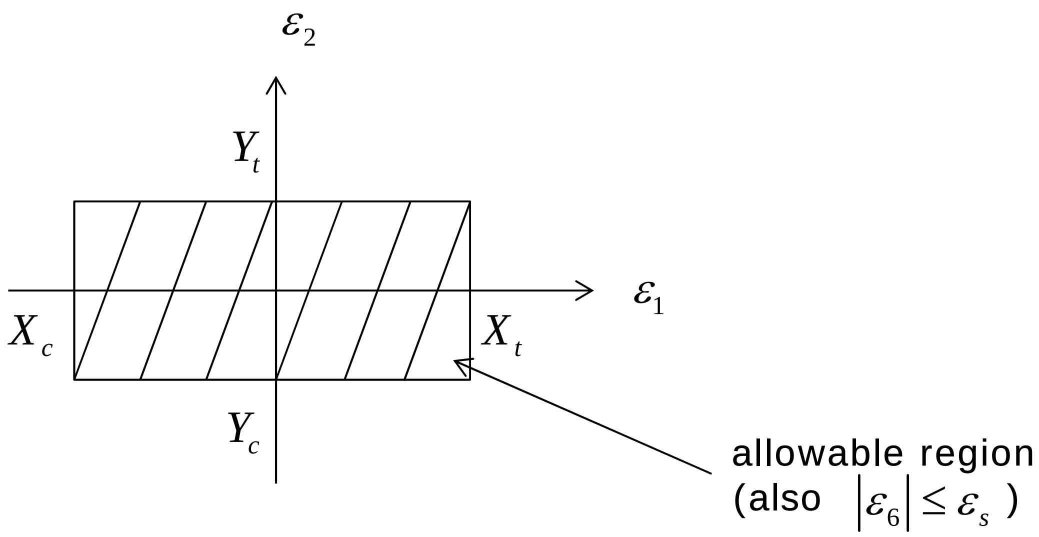
$$\varepsilon_{xt} = \frac{X_t}{E_t} = \frac{190,000}{20,000,000} = 0.0095 \ (0.95\ \%\ strain)$$

$$\varepsilon_{xc} = \frac{-160,000}{20,000,000} = -0.0080$$

$$\varepsilon_{yt} = \frac{6,000}{1,400,000} = 0.0043$$

$$\varepsilon_{yc} = \frac{-25,000}{1,400,000} = -0.0179$$

$$\varepsilon_{yc} = \frac{10,000}{700,000} = 0.0143$$



Or, can use direct test values of strain

Max. Strain criteria will be similar to Max. Stress

5. Failure of Composites

For the previous example

ply, θ	ε_1	ε_2	ε_6
0°	-0.0103	0.0032	0
$+45^\circ$	-0.0036	-0.0036	0.0136
-45°	-0.0036	-0.0036	-0.0136
$+90^\circ$	-0.0032	-0.0103	0
	↑ 0.0095 / -0.008	↑ 0.0043 / -0.0179	↑ $\pm$ 0.0143

close (ε_6 = 0.0136 and -0.0136)

Similar results as Max. Stress

5. Failure of Composites

(c) Tsai-Wu Interaction Theory

Want to account for potential interactions between failure mechanisms in $\sigma_1,\ \sigma_2,\ \sigma_6$

Think back to von Mises criterion for isotropic materials

von Mises (isotropic)

$$(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+3\tau_{xy}^{\ 2}+3\tau_{yz}^{\ 2}+3\tau_{zx}^{\ 2}=2\sigma^2_{\ yield}$$

For 2-D Plane Stress

$$\sigma_z=\tau_{zx}=\tau_{yz}=0$$

$$\sigma_x^{\ 2}-2\sigma_x\sigma_y+\sigma_y^{\ 2}+\sigma_y^{\ 2}+\sigma_x^{\ 2}+3\tau_{xy}^{\ 2}=2\sigma^2_{\ yield}$$

or

$$\sigma_x^{\ 2}-\sigma_x\sigma_y+\sigma_y^{\ 2}+\frac{3}{2}\tau_{xy}^{\ 2}=\sigma^2_{\ yield}=X^2$$

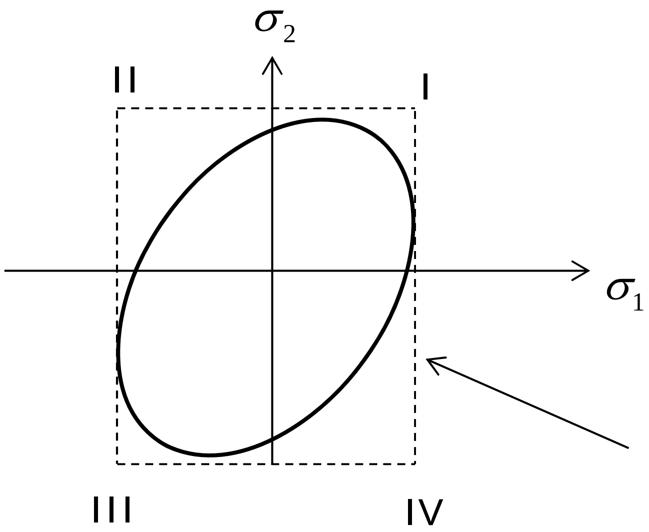
$$\left(\frac{\sigma_x}{X}\right)^2-\left(\frac{\sigma_x\sigma_y}{X}\right)^2+\left(\frac{\sigma_y}{X}\right)^2+\left(\frac{\tau_{xy}}{\sqrt{\frac{3}{2}}X}\right)^2=1$$

one failure value *X*

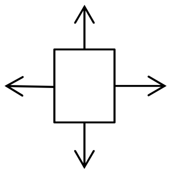
5. Failure of Composites

For Principal Stresses,

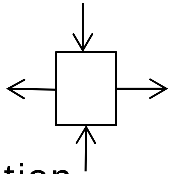
$\sigma_x = \sigma_1, \ \sigma_y = \sigma_2, \ \tau_{xy} = 0,$ this gives



In I, Max. Stress and Interaction similar



In II, much different



wish to include interaction

Hill generalized von Mises for Orthotropic material.

$$(\frac{\sigma_1}{X})^2 - (\frac{\sigma_1 \sigma_2}{X})^2 + (\frac{\sigma_2}{Y})^2 + (\frac{\tau_{12}}{S})^2 = 1$$

↑ Interaction term

X: strength in longitudinal

Y: strength in Transverse

Tsai-Wu proposed more general criteria

$$\sigma_1 F_1 + \sigma_2 F_2 + \sigma_1{}^2 F_{11} + \sigma_2{}^2 F_{22} + \sigma_6{}^2 F_{66} + 2F_{12}\sigma_1\sigma_2 = 1$$

where F_i, F_{ij} are coefficients to be found from tests

To find coefficients

a) consider σ_1 only

$$\sigma_1 F_1 + \sigma_1{}^2 F_{11} = 1$$

5. Failure of Composites

for tension $X_t F_1 + X_t^2 F_{11} = 1$

for compression $X_c F_1 + X_c^2 F_{11} = 1$

Solving gives

$$F_1 = \frac{1}{X_t} + \frac{1}{X_c}$$

$$F_{11} = -\frac{1}{X_t X_c}$$

Similarly for σ_2 only

$$F_1 = \frac{1}{Y_t} + \frac{1}{Y_c}$$

$$F_{11} = -\frac{1}{Y_t Y_c}$$

for σ_6 only $F_6 = 0$

$$F_{66} = \frac{1}{S_2}$$

5. Failure of Composites

Correction on von Mises criterion

1) $(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6\tau_{xy}{}^2+6\tau_{yz}{}^2+6\tau_{zx}{}^2=2\sigma^2{}_{yield}$

2) $\sigma_x{}^2-2\sigma_x\sigma_y+\sigma_y{}^2+\sigma_y{}^2+\sigma_x{}^2+6\tau_{xy}{}^2=2\sigma^2{}_{yield}$

3) $\left(\dfrac{\sigma_x}{X}\right)^2-\left(\dfrac{\sigma_x\sigma_y}{X}\right)^2+\left(\dfrac{\sigma_y}{X}\right)^2+\left(\dfrac{\tau_{xy}}{\frac{1}{\sqrt{3}}X}\right)^2=1$

$$F_1\sigma_1+F_2\sigma_2+F_6\sigma_6+F_{11}\sigma_2{}^2+F_{66}\sigma_6{}^2+2F_{12}\sigma_1\sigma_2=1$$

$$\begin{cases}\sigma_1=X_t\\ \sigma_1=X_c\end{cases}\qquad\begin{cases}\sigma_2=Y_t\\ \sigma_2=Y_c\end{cases}\qquad\begin{cases}\sigma_6=+S\\ \sigma_6=-S\end{cases}$$

For obtaining F_{12}, should use a biaxial $\sigma_1=\sigma_2=X_B$

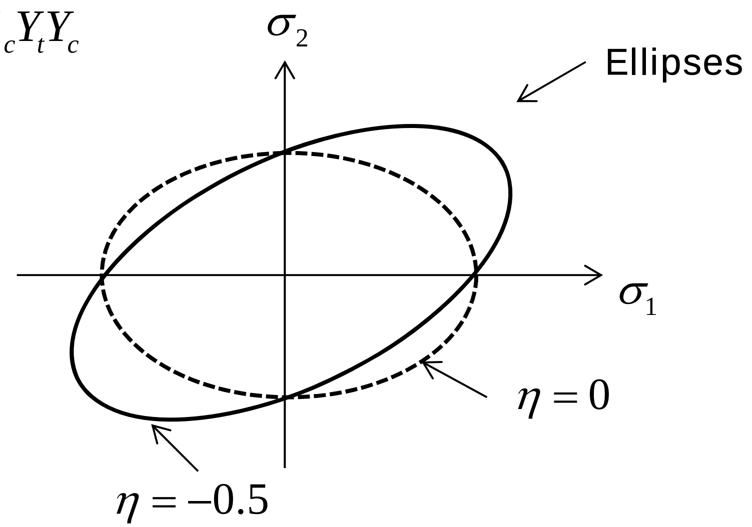
$$(F_1+F_2)X_B+(F_{11}+F_{22})X_B{}^2+2F_{12}X_B{}^2=1$$

$$F_{12}=\frac{1}{2X_B}[1-(F_1+F_2)X_B-(F_{11}+F_{22})X_B{}^2]$$

5. Failure of Composites

convenient to express F_{12} as $F_{12} = \eta / \sqrt{X_t X_c Y_t Y_c}$

where η is between 0 and -1



Biaxial Tests for F_{12} hard to do.

For simplicity, sometimes assume

$$F_{12} \cong -0.5 / \sqrt{X_t X_c Y_t Y_c}$$

comes from analogy with von Mises criteria for isotropic materials.

Summarizing, Tsai-Wu Criteria becomes

$$F_1\sigma_1 + F_2\sigma_2 + F_6\sigma_6 + F_{11}\sigma_2{}^2 + F_{66}\sigma_6{}^2 + 2F_{12}\sigma_1\sigma_2 = 1$$

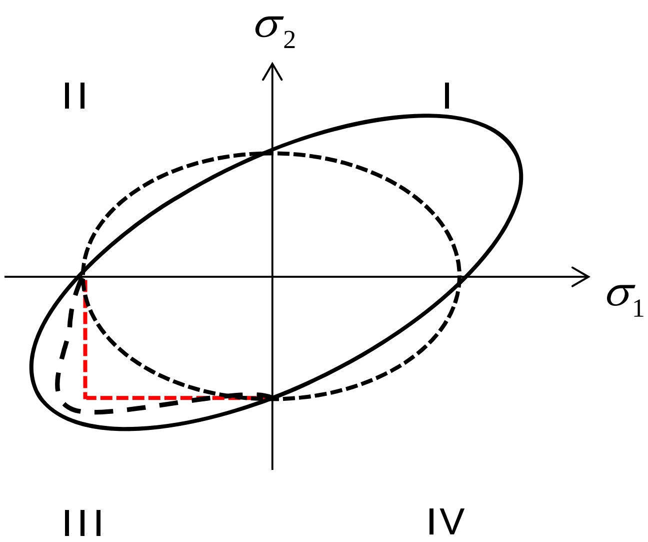
Based on tests for $X_t,\ X_c,\ Y_t,\ Y_c,\ S$

Gives reasonable empirical fit to data

See Jones p. 80, Tsai and Hahn

- Word of Caution

For two materials, different Y_t



- Quads I II IV reasonable
- Quads III – increased strength in biaxial compression?

Careful, if designing submarine hulls

- Max. Stress Theory for Quad III

Discussion of Failure Criteria given by

Hart – Smith, Composite (24), 1, 1993, p. 53-

5. Failure of Composites

- Remarks on Failure Criteria

1. Failure criteria useful to interpolate experimental data.
2. Don't use to extrapolate – particularly into quadrants without test data
3. Useful for preliminary laminate design
4. Failure is complicated.
   - Ply behavior in a laminate may differ from simple lamina (delamination, edge effects, ···)
   - Size effects, holes
   - Fatigue
   - Environment

5. But Failure criteria have their uses, easily employed, simple equations. Don't want to do full set of strength tests for each layup.

- Calculating Laminate Failure

Up to here, looked at failure of a ply in isolation.

Combine with laminate analysis (CLPT)

Procedure

1. Analyze structure → get $\underset{\sim}{N},\ \underset{\sim}{A},\ \underset{\sim}{\varepsilon}^{o}\ (also\ \underset{\sim}{B},\ \underset{\sim}{D},\ \underset{\sim}{M},\ \underset{\sim}{\kappa})$

   → later on

2. Determine ply stresses (strains) in ply axes.
3. Apply appropriate criteria to each ply
4. Determine first ply to fail. (First Ply Failure)
5. To work out, ply stress

$$\underset{\sim}{\sigma} = \underset{\sim}{T}_{\sigma}\underset{\sim}{\bar{\sigma}} = \underset{\sim}{T}_{\sigma}\underset{\sim}{\bar{Q}}\underset{\sim}{\varepsilon}^{o} = \underset{\sim}{T}_{\sigma}\underset{\sim}{\bar{Q}}\underset{\sim}{a}\underset{\sim}{N}$$

Ply coords. (σ)   Laminate coords. ($\bar{\sigma}$)   Linear eqn. (aN)

5. Failure of Composites

Using Max. Stress Theory

one part of failure criterion

$$\sigma_1 \le X_t$$

$$\frac{\sigma_1(\underset{\sim}{N})}{X_t} = 1$$

1 equation, 3 unknowns, N_x, N_y, N_{xy}

To solve for failure loads

Say

$$\underset{\sim}{N} = \lambda \begin{Bmatrix} 1000 \\ 2000 \\ 0 \end{Bmatrix}$$

← define proportional loads

scalar factor

$\underset{\sim}{N}^o$ Load case (dummy)

then

$$\underset{\sim}{\sigma} = \lambda \underset{\sim}{T_\sigma}^k \bar{Q}^k \underset{\sim}{a} \underset{\sim}{N}^o$$

$$\frac{\lambda \sigma_1^o}{X_t} = 1 \longrightarrow \lambda = \frac{X_t}{\sigma_1^o}$$

5. Failure of Composites

Failure occurs at $\underset{\sim}{N} = \lambda \underset{\sim}{N}^o$

Repeat for others

Using Tsai-Wu

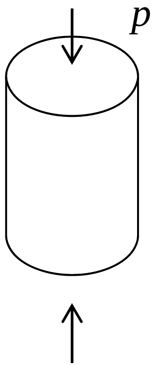
$$\lambda F_1 \sigma_1^o + \lambda F_2 \sigma_2^o + \lambda^2 \sigma_1^{o^2} F_{11} + \lambda^2 \sigma_2^{o2} F_{22} + \lambda^2 \sigma_6^{o^2} F_{66} + \lambda^2 \sigma_1^o \sigma_2^o F_{12} = 1$$

Quadratic for λ , solve

Failure at $N = \lambda N^o$

Alternatively, do incremental computation

- First-ply Failure for the previous example



Applied loading was (for *P = 20,000 lb*)

$$\underset{\sim}{N}^o = \begin{Bmatrix} -3183 \\ 0 \\ 0 \end{Bmatrix}$$

5. Failure of Composites

Using Max. Stress → $\lambda = \frac{X_c}{\sigma_1} = \frac{-160}{-206} = 0.78$

So permissible loading is

$$\underset{\sim}{N} = 0.78\begin{Bmatrix} -3183 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -2483 \\ 0 \\ 0 \end{Bmatrix}$$

$$N_x = \frac{P}{2\pi r}, P = 2\pi(1)(-2483) = 15,600\ compression$$

Note if applied Tsai-Wu

$$\lambda = 0.76$$

$$N_x = 0.76(-3183) = -2419$$

$$P = 15,190\ lbs$$

← small effect of transverse tension on *0°* ply

Generally repeat to check for Y_c, S, etc.

(above for σ is max. stress)

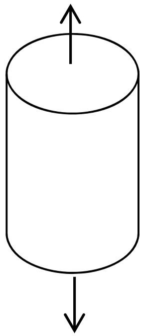
5. Failure of Composites

- Progressive Failure

Does failure of a ply = failure of the laminate?

Not necessarily.

If tube loaded in tension



$P = 20,000\ lb$

same stress state with all signs changed

ply, θ	σ_1	σ_2	σ_6
0°	206	-0.3	0
$+45^\circ$	+73	+6.5	-9.5
-45°	+73	+6.5	+9.5
$+90^\circ$	+60	+13	0

Allowables	+190	+6	$\pm$ 10
	-160	-25	

90°s would crack in tension at low load.

$$\sigma_2 = +13\ ksi,\ Y_t = +6,$$

$$\lambda = Y_t / \sigma_2 = 6/13 = 0.46$$

$$N_x = 0.46(+3183) = 1464\ lb/in$$

$$or\ \ P = 2\pi r N_x = 9{,}213\ lb$$

But does this kill the laminate?

Assume 90° ply cracks and looses transverse stiffness $E_t' = kE_T$ ← knock down factor

More generally assume all properties at cracked ply are knocked down.

$$E_t' = k_T E_T, E_L' = k_L E_L, \nu_{LT}' = K_\nu \nu_{LT}, G_{LT}' = K_G G_{LT}$$

Select *K*'s somewhat arbitrary

Type	K_L	K_T	K_ν	K_G
Drastic crack	1	0.001	0.001	0.001
Typical crack	1	0.5	1	0.5
Some Research	1	0.5	1	1
Fiber	10^{-6}	10^{-6}	10^{-6}	10^{-6}

5. Failure of Composites

- Do all calculation once again
- Recompute stresses, get new λ

  new failure loads

- Automate on computer

Typical Progression for $[0/\pm 45/90]_s$

λ	N_x $(=3183\lambda)$	P $(=628N_x)$	Failure		$\overline{E_x}$ $(=7.7\ Msi,\ orig)$
0.42	1340	8400	cracking	90° plies	7.5
0.65	2070	13,000	cracking	±45° plies	7.2
0.86	2740	17,200	cracking	0° plies	0

Last ply to fail (0.86)

5. Failure of Composites

Jones Sec 4

- Other Remarks on Failure

Failure of laminates is complicated phenomenon.

Many modes of failure

1. Fiber failure
2. Matrix cracking
3. Delamination
4. Interlaminar stress effects near free edge
5. Effects of holes and notches
6. Residual initial stresses due to thermal contraction

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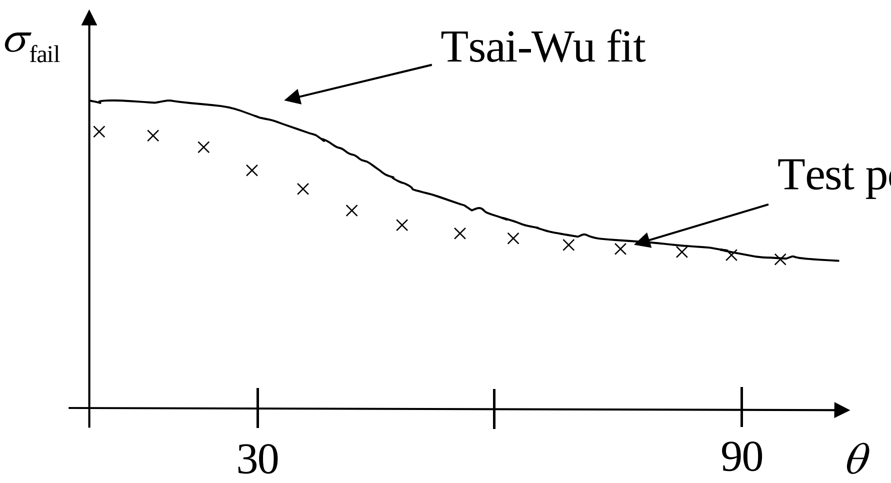
5. Failure of Composites

From cure temperature,

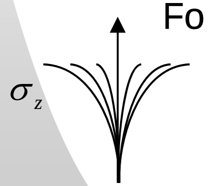
some observations

- Interlaminar stresses at free edge can cause delamination.

Tests on $[\pm\theta]_s$ laminate show



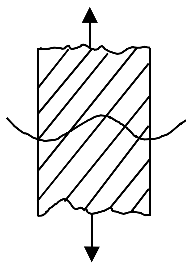
For $\theta > 30°$, Delamination failure



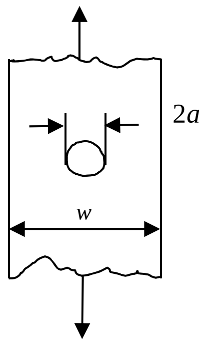
"Brooming" strong σ_z develops

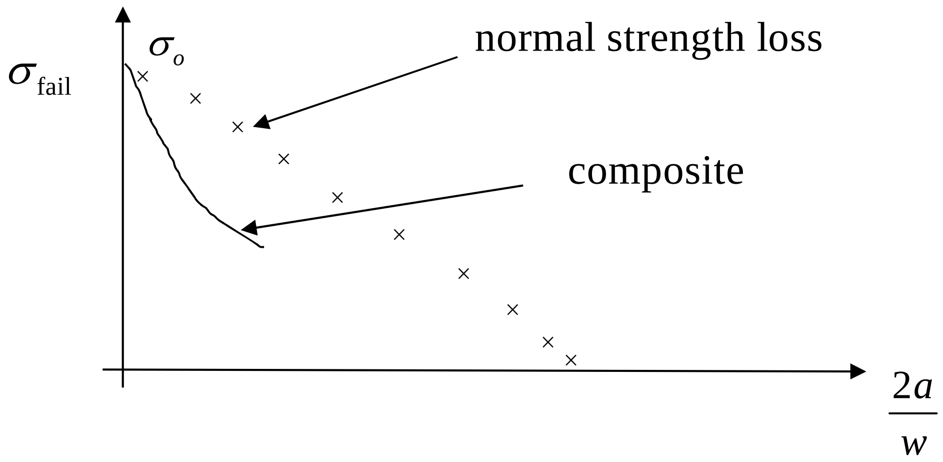
5. Failure of Composites

For $\theta > 30°$, Regular fracture



- Holes in composites lower strength





Holes in composites behave like cracks in metals.

5. Failure of Composites

Much work on failure still going on.

Research on Impact, Fatigue, Environment (temperature, moisture)

For example with conservative safety factor, can design reasonable composite structure.

Better understanding → lower safety factors

more efficient design

See Jones Sections 6.1 ~ 6.4