Today's theory

- 2D imaging

Figure 2.1 Makovski

Input: $g_1(x_1, y_1) = \iint g_1(\alpha, \beta) \cdot \delta(x_1 - \alpha, y_1 - \beta) d\alpha d\beta$

Output: $g_2(x_2, y_2) = S\big(g_1(\alpha, \beta)\big)$

$$= \iint g_1(\alpha, \beta) \cdot S[\delta(x_1 - \alpha, y_1 - \beta)] d\alpha d\beta \quad \text{if} \quad \text{linear}$$

Impulse response function or point spread function of the system

$$h(x_2, y_2; \alpha, \beta) = S[\delta(x_1 - \alpha, y_1 - \beta)]$$

$$g_2(x_2, y_2) = \iint g_1(\alpha, \beta) \cdot h(x_2, y_2; \alpha, \beta) d\alpha d\beta$$

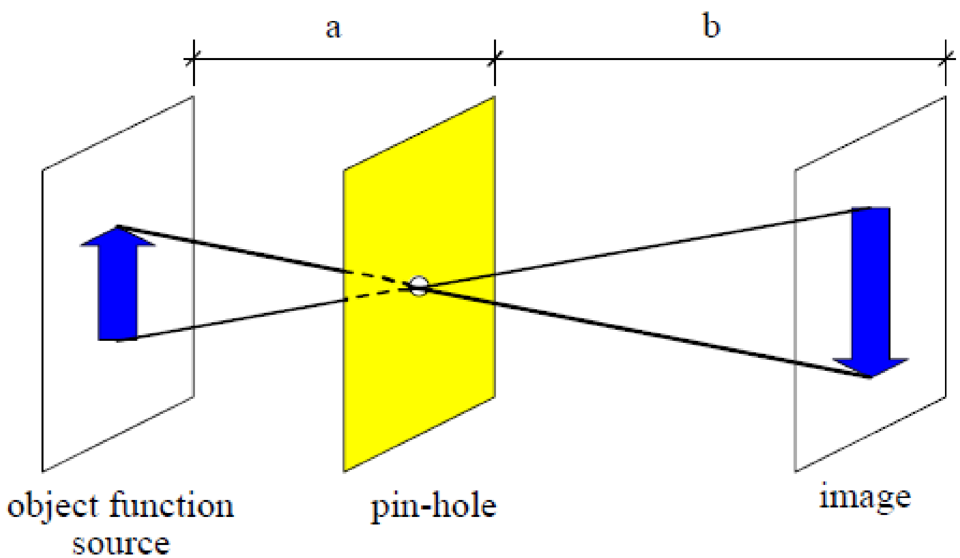
If the system is shift invariant or space invariant

$$h(x_2, y_2; \alpha, \beta) = h(x_2 - \alpha, y_2 - \beta)$$

$$g_2(x_2, y_2) = \iint g_1(\alpha, \beta) \cdot h(x_2 - \alpha, y_2 - \beta) d\alpha d\beta$$

$$= g_1 * h$$

- <u>Pinhole camera</u>



Question 1: Is this an LSI system?

$$h(x_2, y_2; \alpha, \beta) = h(x_2 - M\alpha, y_2 - M\beta)$$

where M = -b/a

$$g_2(x_2, y_2) = \iint g_1(\alpha, \beta) \cdot h(x_2 - M\alpha, y_2 - M\beta) d\alpha d\beta$$

$$\alpha' = M\alpha \text{ and } \beta' = M\beta$$

$$g_2(x_2, y_2) = \frac{1}{M^2} \iint g_1(\frac{\alpha'}{M}, \frac{\beta'}{M}) \cdot h(x_2 - \alpha', y_2 - \beta') d\alpha' d\beta'$$

$$= \frac{1}{M^2} g_1\left(\frac{x_2}{M}, \frac{y_2}{M}\right) * h(x_2, y_2)$$

4.4 Imaging principles

- Parallel geometry

Figure 3.1 Makovski

Assumptions: Parallel x-ray (e.g. x-ray from point source at infinity)

No scattered radiation (detectors are far from the object)

Linear attenuation coefficient (μ)

$$I_d = I_s \cdot \exp[-\mu L]$$

$$I_d(x, y) = \int I_s(E) \cdot \exp\left[-\int \mu(x, y, z, E)dz\right] dE$$

- <u>Geometric effects (point source)</u>

Figure 5.13 Prince

- Inverse square law

$$I_d(0,0) = \frac{I_s}{4\pi d^2}$$

$$I_d(x, y) = I_d(r) = \frac{I_s}{4\pi r^2} \quad where\, r = r(x, y)$$

$since\ cos\theta = d/r$

$$I_d(r) = I_d(0) \cdot \left(\frac{d}{r}\right)^2 = I_d(0) \cdot cos^2\theta$$

- Obliquity

Figure 4.3 Makovski

$$I_d(r) = I_d(0) \cdot cos\theta$$

Hence, due to inverse these effects, the signal in location r is reduced by

$$I_d(r) = I_d(0) \cdot cos^3\theta$$

Question) When does this term becomes problematic?

- Path length

Figure 5.15 Prince

$$I_d(r) = I_d(0) \cdot cos^3\theta \cdot e^{-\mu L/cos\theta}$$

- Depth dependent magnification

Figure 5.16 Prince

$$Magnification: M(z) = d/z$$

Example) Demonstrate signal for a rectangular prism

- Imaging equation with geometric effects

Let's assume the object is infinitely thin and define transmittivity ($t_d(x,y)$) instead of attenuation (i.e $e^{-\mu L(x,y)/cos\theta} = t_d(x,y)$)

If the object is at the detector plan,

$$I_d(x,y) = I_d(0) \cdot cos^3\theta \cdot t_d(x,y)$$

where $cos\theta = d/\sqrt{d^2 + x^2 + y^2}$

In general

$$I_d(x,y) = I_d(0) \cdot cos^3\theta \cdot t_d(x/M(z), y/M(z))$$

where $M(z) = d/z$

- Blurring effects
  - Extended sources

Figure 4.11 Makovski

A pinhole system can be modeled with impulse response function:

$$h_d(x,y) = k \cdot s\left(-x\frac{z}{d-z}, -y\frac{z}{d-z}\right)$$

$$h_d(x,y) = k \cdot s(x/m(z), y/m(z)) \text{ where } m(z) = -z/(d-z)$$

Question) When does this term becomes problematic?

The scaling factor k should contain $cos^3\theta/4\pi d^2$ term to consider geometric effects.

- Film-screen blurring

$$f(x, y)$$

- Overall response of a projection imaging system

$$h_d(x, y) = k \cdot s(x/m, y/m) * t_d(x/M, y/M) * f(x, y)$$