Today's focus

- 2D delta function

$$\delta(x-1, y-1)$$

$$\delta(x-y)$$

$$\delta(x \cdot cos\theta + y \cdot cos\theta - l)$$

- 2D Fourier transform pair

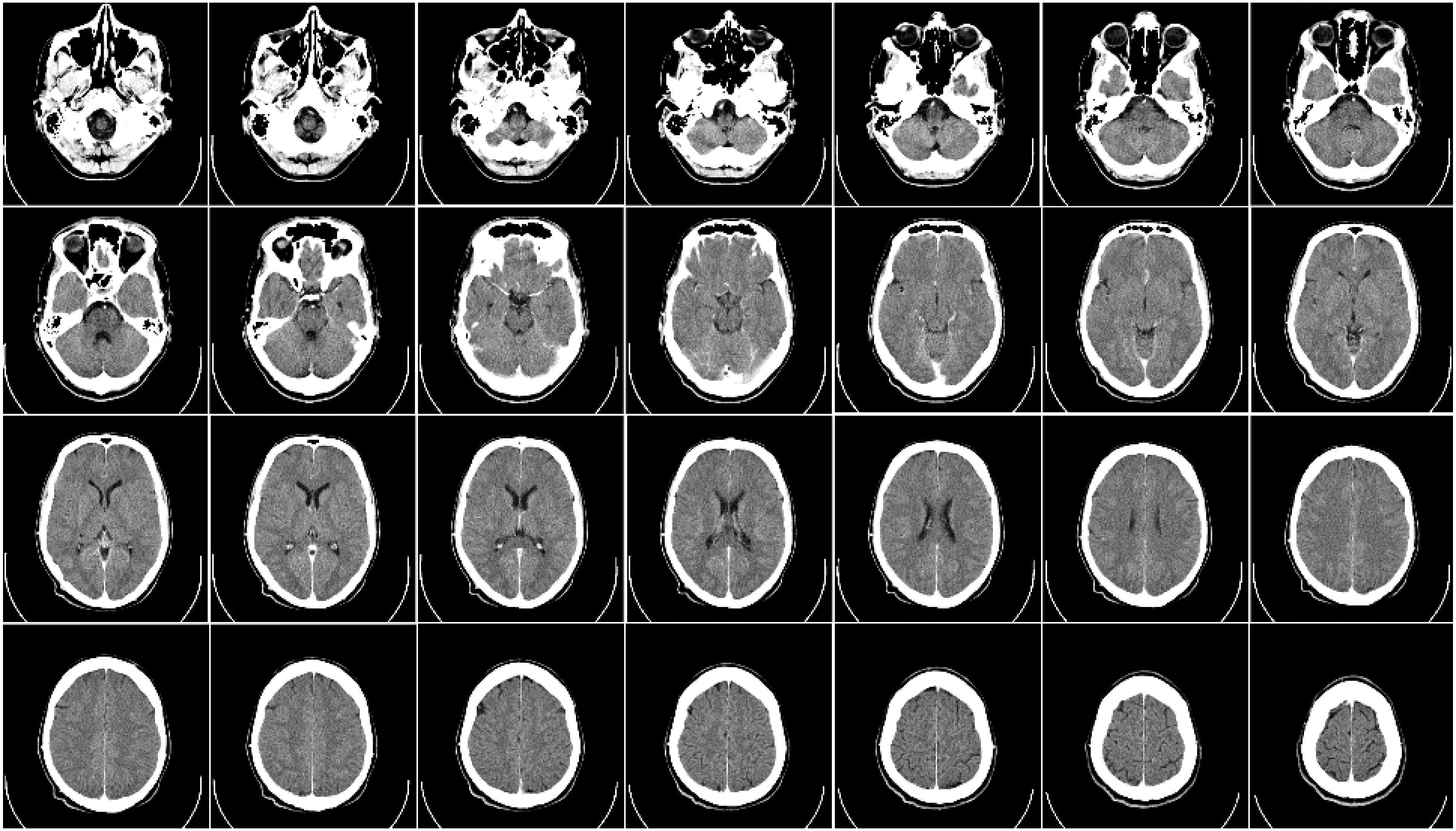
$$f_\theta(x, y) = f(x \cdot cos\theta - y \cdot sin\theta, x \cdot sin\theta + y \cdot cos\theta)$$

$$F_\theta\left(k_x, k_y\right) = F(k_x \cdot cos\theta - k_y \cdot sin\theta, k_x \cdot sin\theta + k_y \cdot cos\theta)$$

Prove this! (More importantly memorize this property)

Chapter 5 – Computed Tomography (CT or CAT)

5.1. Overview



Idea

- Use multi-anlge x-ray images to reconstruct tomogram (image of a "cut" or "section")

Advantages

- Cross-sectional images (2D or 3D)
- High resolution (sub-mm)
- Short scan time (~ 30 sec)

Disadvantage

- High radiation exposure (2-10 mSv; a few times of annual background radiation)

Applications

- Cancer, infraction, haemorrhage …

Contrast agent

- Various contrast agents (e.g. iodine) with various side effects

Source

- Single X-ray source

Collimation

- Use lead to form a slit to shape X-ray

Filtering

- Copper + Aluminum to narrow the energy spectrum ("harden beam")

Detector

- Scintillation crystal (X-ray to photoelectron) + solid-state photodiode

CT number

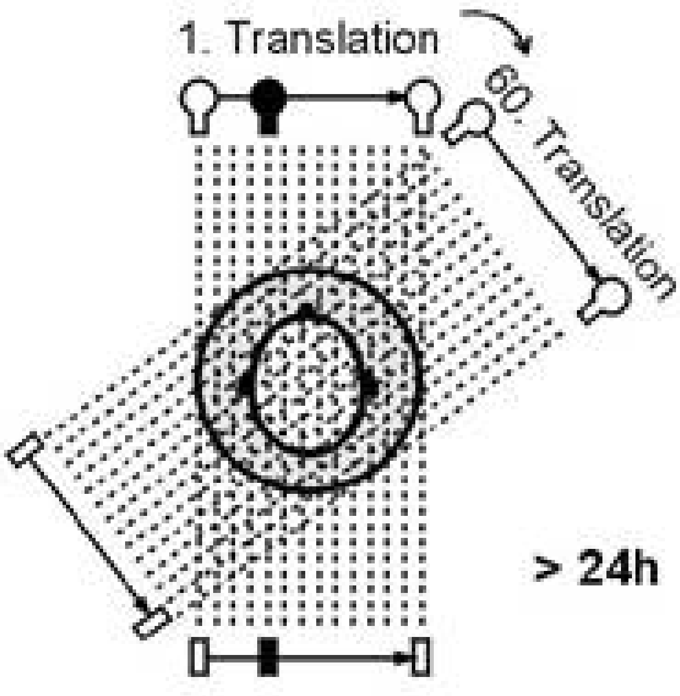
- CT generate μ value after calibration

$$h\ (Hounsfield\ unit; HU) = 1000\ \times \frac{\mu - \mu_{water}}{\mu_{water}}$$

air = -1000 HU, water = 0 HU,
bone ~ 1000 HU, metal/contrast agent ~ 3000 HU

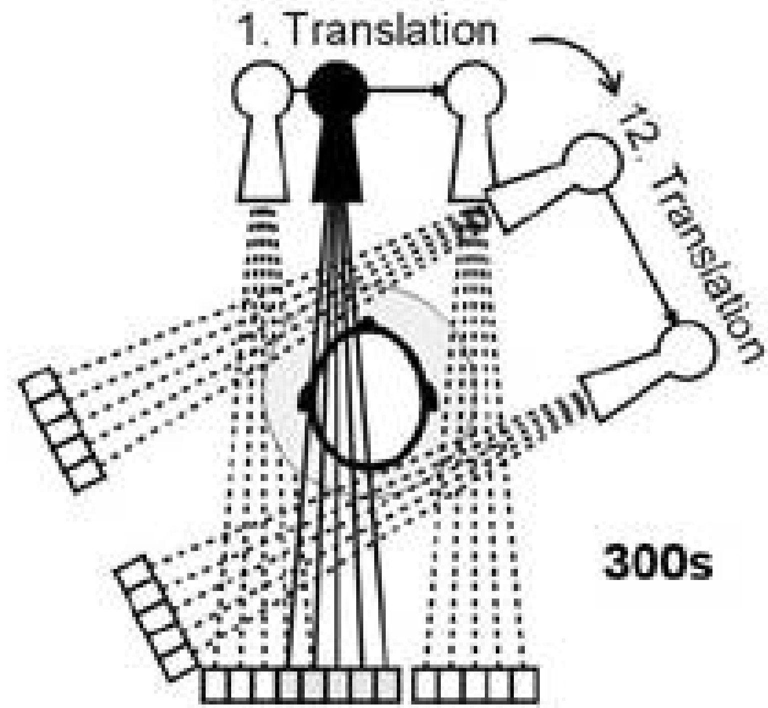
5.3 Generation

Pencil Beam (1970)

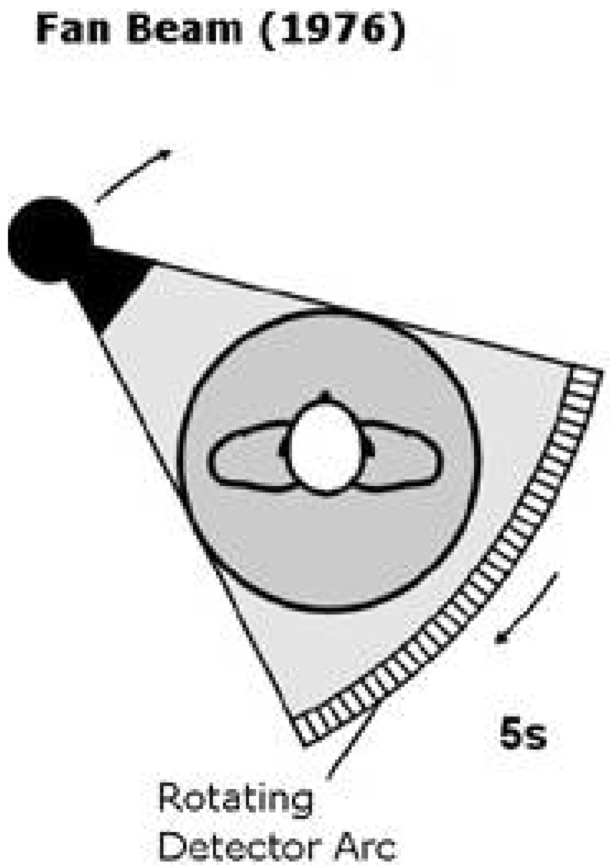


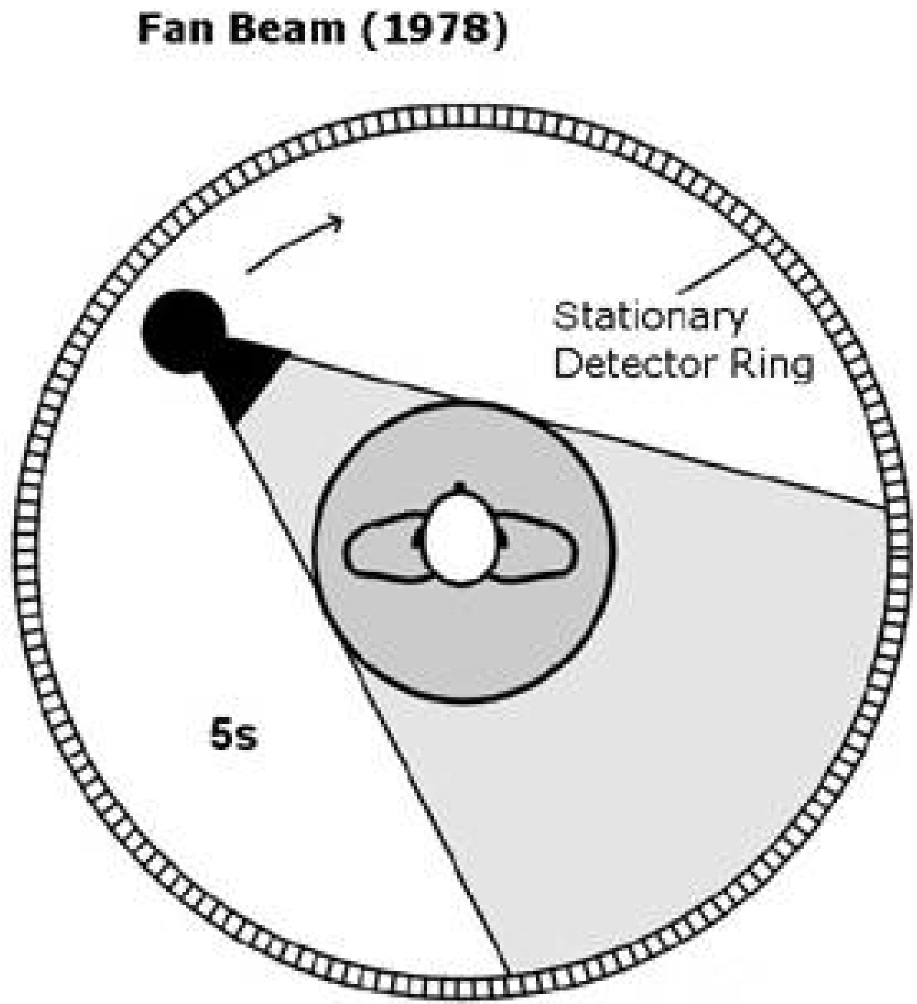
Generation 1: Translation / Rotation

Partial Fan Beam (1972)



Generation 2: Translation / Rotation





5G, 6G (spiral CT), 7G (multislice CT using cone beam)…

5.2 An example of image reconstruction

Iterative algorithm

$$f_{ij}^{q+1} = f_{ij}^{q} + \frac{g_j - \sum_{i=1}^{N} f_{ij}^{q}}{N}$$

	$f_1 =$ 5	$f_2 =$ 7	
	$f_3 =$ 6	$f_4 =$ 2	

7	11	9	13
	$f_1 =$ 5	$f_2 =$ 7	12
	$f_3 =$ 6	$f_4 =$ 2	8

	0	0	
	$f_1 =$ 0	$f_2 =$ 0	
	$f_3 =$ 0	$f_4 =$ 0	

$$f_1^1 = f_3^1 = 0 + \frac{11 - 0}{2} = 5.5; \qquad f_2^1 = f_4^1 = 0 + \frac{9 - 0}{2} = 4.5;$$

	$f_1 =$ 5.5	$f_2 =$ 4.5	10
	$f_3 =$ 5.5	$f_4 =$ 4.5	10

$$f_1^2 = 5.5 + \frac{12 - 10}{2} = 6.5; \qquad f_2^2 = 4.5 + \frac{12 - 10}{2} = 5.5;$$

$$f_3^2 = 5.5 + \frac{8 - 10}{2} = 4.5; \qquad f_4^2 = 4.5 + \frac{8 - 10}{2} = 3.5;$$

10			10
	$f_1 =$ 6.5	$f_2 =$ 5.5	
	$f_3 =$ 4.5	$f_4 =$ 3.5	

$$f_1^3 = 6.5 + \frac{7 - 10}{2} = 5; \qquad f_2^3 = 5.5 + \frac{13 - 10}{2} = 7;$$

$$f_3^3 = 4.5 + \frac{13 - 10}{2} = 6; \qquad f_4^3 = 3.5 + \frac{7 - 10}{2} = 2;$$

	$f_1 =$ 5	$f_2 =$ 7	
	$f_3 =$ 6	$f_4 =$ 2	

5.4 Reconstruction algorithms

- <u>Central Section Theorem</u>

Figure 7.6 Makovski

$$g(y) = \int f(x, y) dx$$

$$F(k_x, k_y) = \iint f(x, y) \cdot \exp[-i2\pi(k_x x + k_y y)] dx\, dy$$

$$F(0, k_y) = \iint f(x, y) \cdot \exp[-i2\pi k_y y] dx\, dy$$

$$F(0, k_y) = \int \left[\int f(x, y) dx \right] \cdot \exp[-i2\pi k_y y] dy$$

$$= \mathrm{FT}_{1\mathrm{D}}[g(y)]$$

Figure 7.7 Makovski

This means 1D FT of 1D projected object is one line in 2D FT domain

Let's generalize this for an arbitrary angle

Figure 7.8 Makovski

$$g_\theta(R) = \iint f(x, y) \cdot \delta(x \cdot cos\theta + y \cdot cos\theta - R) dxdy$$

$$= \iint f(r, \varphi) \cdot \delta(r \cdot \cos(\theta - \varphi) - R) r dr d\varphi$$

1) 1D FT of 1D projected image

$$\int g_\theta(R) \cdot \exp(-i2\pi\rho R)\, dR$$

$$\int \iint f(x,y) \cdot \delta(x \cdot cos\theta + y \cdot cos\theta - R) \cdot \exp(-i2\pi\rho R)\, dxdydR$$

We need to show this is identical to one line (of the same angle) in 2D FT domain

First, 2D Fourier transform can be written as

$$F(\rho,\theta) = \iint f(x,y) \cdot \exp[-i2\pi\rho \cdot (x \cdot cos\theta + y \cdot sin\theta)]dx\, dy$$

One line (of the same angle) in 2D FT domain is

$$\int F(\rho,\theta)\,\delta(x\cdot cos\theta + y\cdot cos\theta - R)dR$$

$$= \int \iint f(x,y)\cdot \exp[-i2\pi\rho\cdot(x\cdot cos\theta + y\cdot sin\theta)]\cdot$$

$$\delta(x\cdot cos\theta + y\cdot cos\theta - R)dxdydR$$

$$= \int \iint f(x,y)\cdot\delta(x\cdot cos\theta + y\cdot cos\theta - R)\cdot \exp(-i2\pi\rho R)\,dxdydR$$

This means in an arbitrary angle, 1D FT of a projected object is the same angle in 2D FT domain

If we acquire enough angles (and do 1D FT), we can fill out 2D FT domain. Then the object can be "reconstructed" by using 2D FT!!!

More approaches for CT reconstruction will be discussed by an expert, Professor Jong-Duk Baek, Yonsei University on next Wednesday (March 26th)