2007 Fall: Electronic Circuits 2

CHAPTER 13

Signal Generators and Waveform-Shaping Circuits

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Introduction

- In this chapter, we will be covering…
  - Basic Principles of Sinusoidal Oscillators
  - Op-Amp RC Oscillator Circuits
  - LC and Crystal Oscillators
  - Bistable Multivibrators
  - Generation of Square and Triangular Waveforms Using Astable Multivibrators

13.1 Basic Principles Of Sinusoidal Oscillators

- In this section, we study the basic principles of the design of linear sine-wave oscillators.
- In spite of the name *linear oscillator*, some form of **nonlinearity has to be employed** to control the **amplitude** of the output sine wave. (S-transform method is not able to apply directly).
- Nevertheless, techniques have been developed by which the design of sinusoidal oscillators can be performed in two steps.
  : Frequency-domain methods of feedback circuit analysis → A **nonlinear** mechanism for **amplitude** control

13.1.1 The Oscillator Feedback Loop

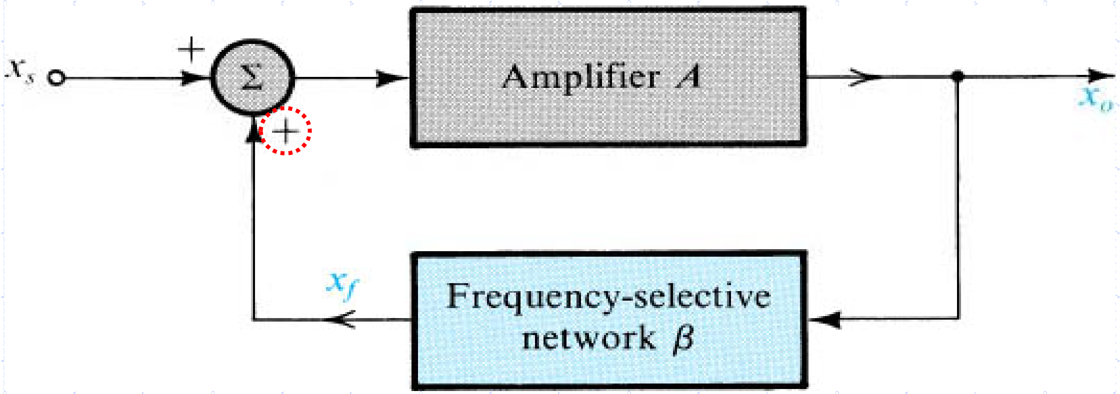


Figure 13.1 The basic structure of a sinusoidal oscillator. A positive-feedback loop is formed by an amplifier and a frequency-selective network . In an actual oscillator circuit, no input signal will be present; here an input signal x_s is employed to help explain the principle of operation.

- Sinusoidal oscillator = Amplifier, ***A*** + Frequency-selective network, ***ß*** connected in a positive-feedback loop.
- In an actual oscillator circuit, no input signal will be present.
- The loop gain(Chapter 8) of the circuit is –A(s)ß(s). However, for our purposes here, it is more convenient to drop the minus sign.

$$A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)}$$

$$L(s) \equiv A(s)\beta(s)$$

13.1.1 The Oscillator Feedback Loop

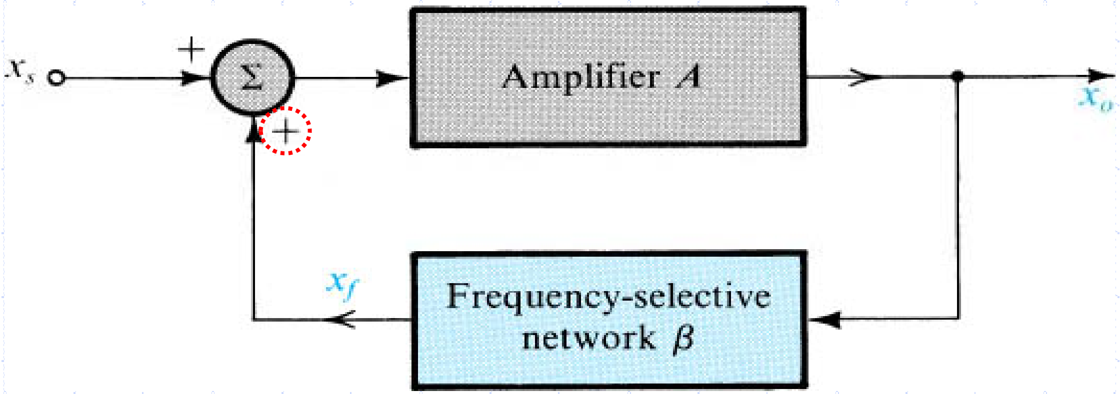


Figure 13.1 The basic structure of a sinusoidal oscillator. A positive-feedback loop is formed by an amplifier and a frequency-selective network . In an actual oscillator circuit, no input signal will be present; here an input signal x_s is employed to help explain the principle of operation.

- The characteristic equation thus becomes

$$\boxed{1 - L(s) = 0}$$

13.1.2 The Oscillation Criterion

- If at a specific frequency f_0 the loop gain Aß is equal to unity, A_f will be infinite.

$$A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)}$$

- That is, at this frequency the circuit will have a finite output for zero input signal. Such a circuit is by definition an oscillator.
- The condition of sinusoidal oscillations of frequency ω_0 for the feedback loop is

$$L(j\omega_0) \equiv A(j\omega_0)\beta(j\omega_0) = 1$$

- **Barkhausen criterion** : at ω_0 *the phase of the loop gain should be zero and the magnitude of the loop gain should be unity* for zero input signal.

13.1.2 The Oscillation Criterion

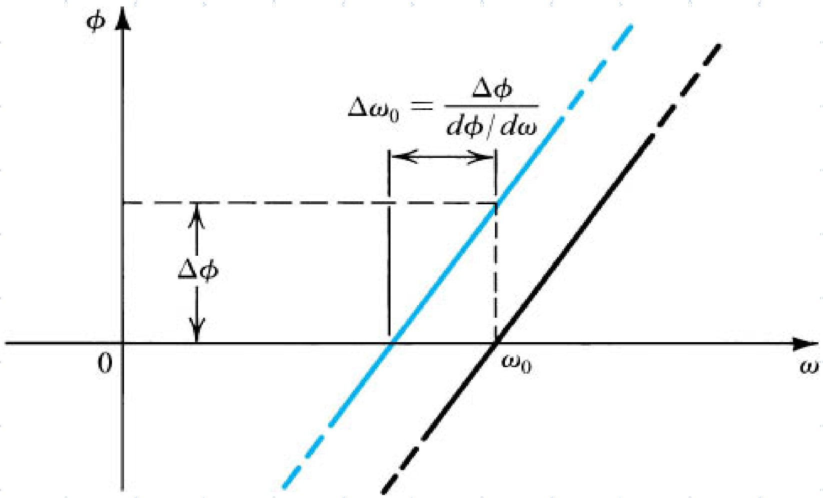


Figure 13.2 Dependence of the oscillator-frequency stability on the slope of the phase response. A steep phase response (i.e., large $d\phi/d\omega$) results in a samll $\Delta\omega_0$ for a given change in phase $\Delta\phi$ (resulting from a change (due, for example, to temperature) in a circuit component).

- It should be noted that the *frequency of oscillation* ω_0 is determined solely by the phase characteristics of the feedback loop
  : **The loop oscillates at the frequency for which the phase is zero.**
- A "steep" function $\Phi(\omega)$ will result in a more stable frequency.
  : If a change in phase $\Delta\Phi$ due to a change in one of the circuit components(due, for example, to temperature), larger $d\Phi/d\omega$ results in a smaller ω_0 change.

13.1.3 Nonlinear Amplitude Control

- **Problem** : the parameters of any physical system cannot be maintained constant for any length of time(due, for example, to temperature).
→ Aß becomes slightly less than unity : oscillation will cease.
→ Aß exceeds unity : oscillation will grow in amplitude.
- **Solution** : a nonlinear circuit for gain control
The function of the gain-control mechanism is
  - 1. First, to ensure that oscillations will start, one designs the circuit such that Aß is slightly greater than unity.(poles are in the right half of the s plane.)
  - 2. Thus as the power supply is turned on, oscillations will grow in amplitude.
  - 3. When the amplitude reaches the desired level, the nonlinear network comes into action and causes the loop gain to be reduced to exactly unity(the poles will be "pulled back" to the $j\omega$ axis.).
  - 4. If, for some reason, the loop gain is reduced below unity, the amplitude of the sine wave will diminish. This will be detected by the nonlinear network, which will cause the loop gain to increase to exactly unity.

13.1.3 Nonlinear Amplitude Control

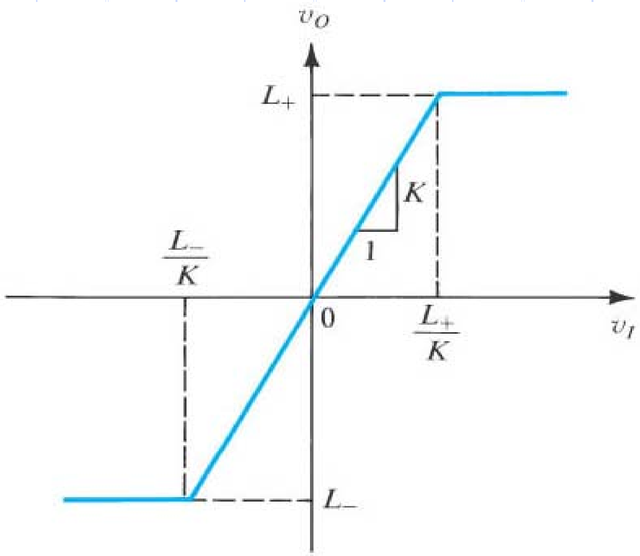


Figure 3.32 General transfer characteristic for a limiter circuit.

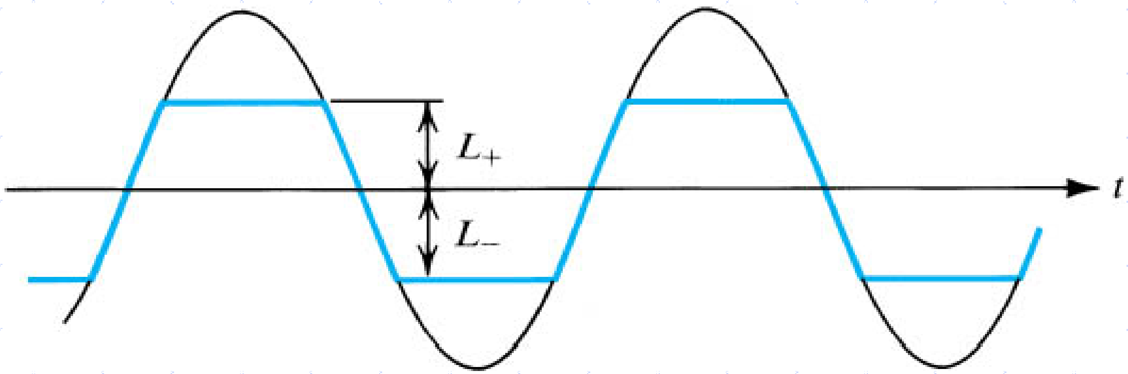
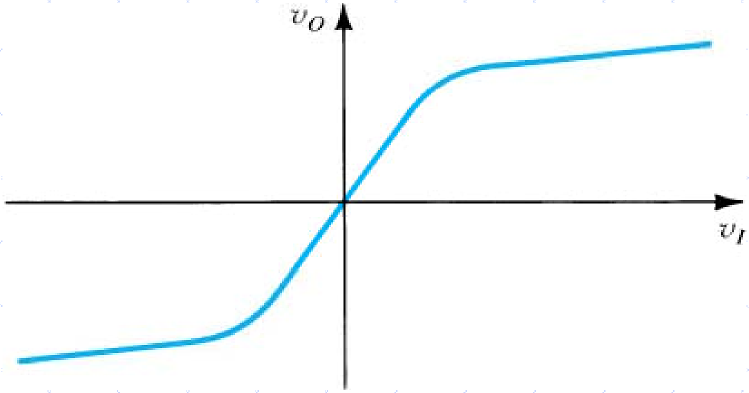
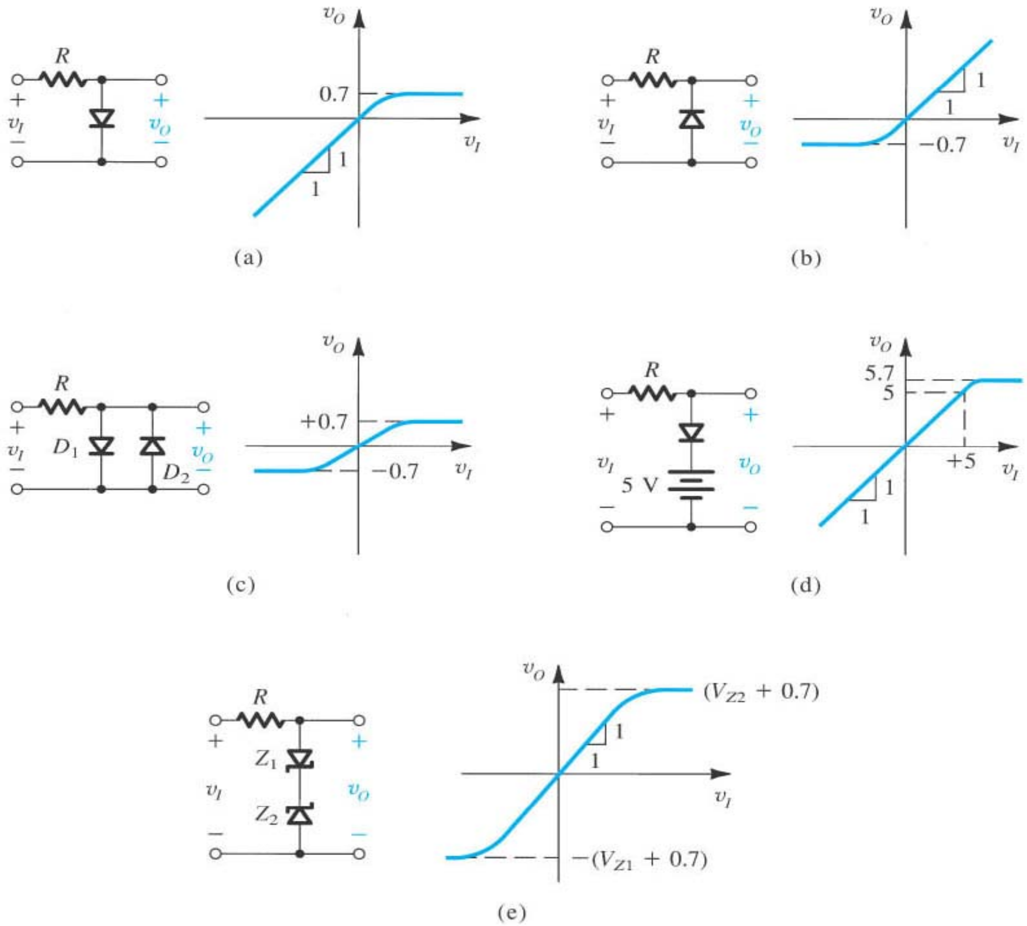


Figure 3.33 Applying a sine wave to a limiter can result in clipping off its two peaks.

- **Implementation** of the nonlinear amplitude-stabilization mechanism
  1. Limiter circuit (Chapter 3, p184~187)
     - Double Limiter & Hard Limiter

13.1.3 Nonlinear Amplitude Control





- Double Limiter & Soft Limiter

13.1.3 Nonlinear Amplitude Control

- **Implementation** of the nonlinear amplitude-stabilization mechanism

  1. Limiter circuit
     - Oscillations are allowed to grow until the amplitude reaches the level to which the limiter is set.
     - When the limiter comes into operation, the amplitude remains constant.
     - To minimize nonlinear distortion, the limiter should be “soft” and such distortion is reduced by the filtering action of the frequency-selective network in the feedback loop.
     - The hard limited sine waves are applied to a bandpass filter present in the feedback loop. The “purity” of the output sine waves will be a function of the selectivity of this filter. That is, the higher the Q of the filter, the less the harmonic content of the sine-wave output(Section 13.2).

  2. Amplitude control utilizing an element whose **resistance** can be **controlled** by the amplitude of the output sinusoidal.
     - By placing this element in the feedback circuit so that its resistance determines the loop gain
     - The circuit can be designed to ensure that the loop gain reaches unity at the desired output amplitude(Diodes or JFET in the triode region).

13.1.4 A Popular Limiter Circuit for Amplitude Control

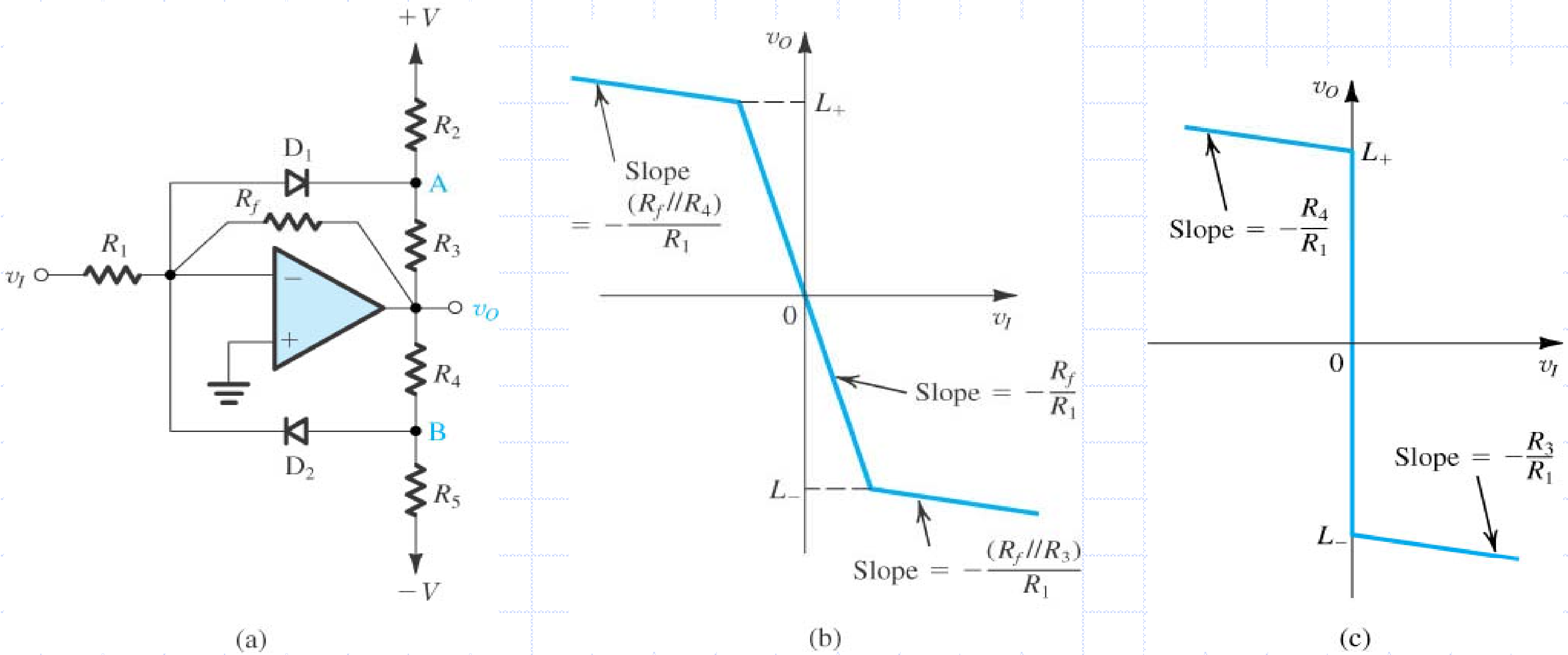
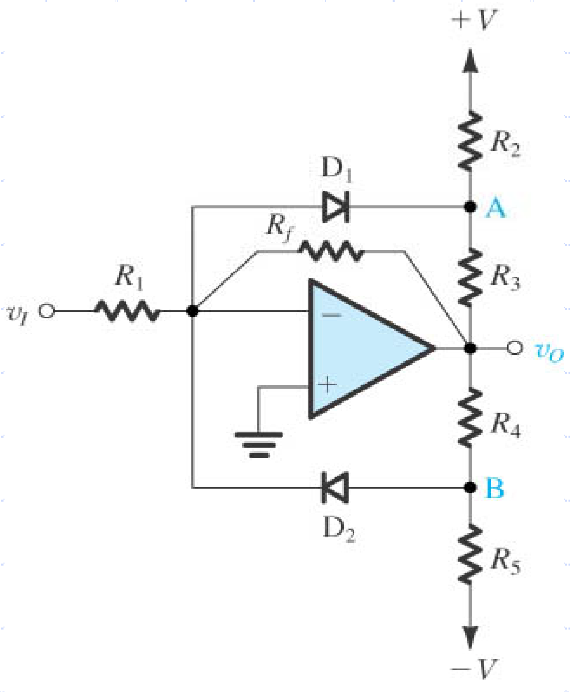


Figure 13.3 **(a)** A popular limiter circuit. **(b)** Transfer characteristic of the limiter circuit; L_- and L_+ are given by Eqs. (13.8) and (13.9), respectively. **(c)** When R_f is removed, the limiter turns into a comparator with the characteristic shown.

- The circuit is more precise and versatile than those presented in Chapter 3.

13.1.4 A Popular Limiter Circuit for Amplitude Control



(a)

Transfer characteristic

- Consider first the case of a small(close to zero) input signal v_I and a small output voltage v_O.
  → v_A is positive & v_B is negative.
  → Diodes D_1 and D_2 is off.
  → Thus all of the input current v_1/R_1 flows through the feedback resistance R_f.

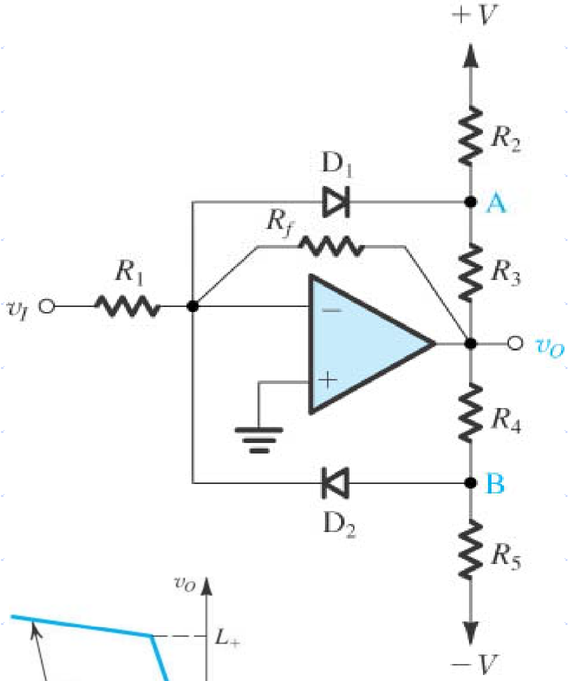
$$v_O = -(R_f / R_1) v_I$$

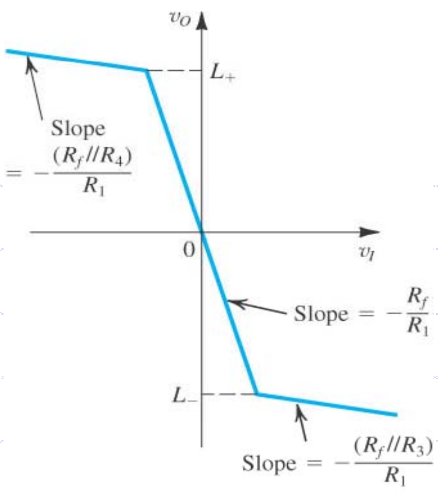
  → To find the voltages at node A and B using superposition.

$$v_A = V \frac{R_3}{R_2 + R_3} + v_O \frac{R_2}{R_2 + R_3}$$

$$v_B = -V \frac{R_4}{R_4 + R_5} + v_O \frac{R_5}{R_4 + R_5}$$

13.1.4 A Popular Limiter Circuit for Amplitude Control





(b)

- **Transfer characteristic**
  - As v_I goes positive, v_O goes negative and v_B will become more negative, thus keeping D_2 off.
  - v_A becomes less positive.

$$v_O = -(R_f / R_1) v_I$$

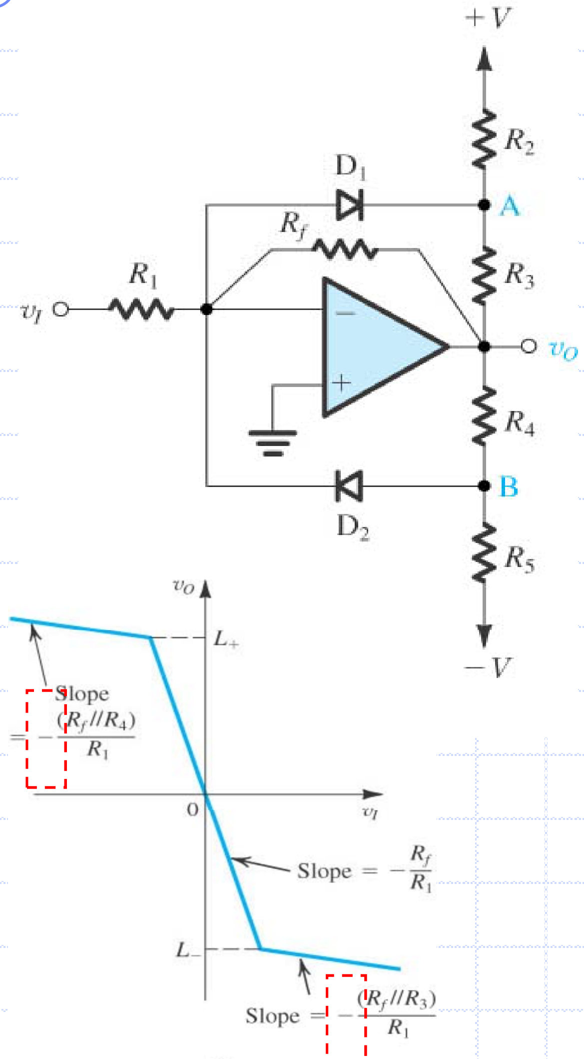
$$v_A = V \frac{R_3}{R_2 + R_3} + v_O \frac{R_2}{R_2 + R_3} \quad ...(Eq.13.6)$$

$$v_B = -V \frac{R_4}{R_4 + R_5} + v_O \frac{R_5}{R_4 + R_5} \quad ...(Eq.13.7)$$

  - If we continue to increase v_I, a negative value of v_O will be reached at which v_A becomes -0.7V or so and diode **D_1 conducts.**
  - The negative limiting level from Eq.(13.6): L_

$$L_- = -V \frac{R_3}{R_2} - V_D \left(1 + \frac{R_3}{R_2}\right)$$

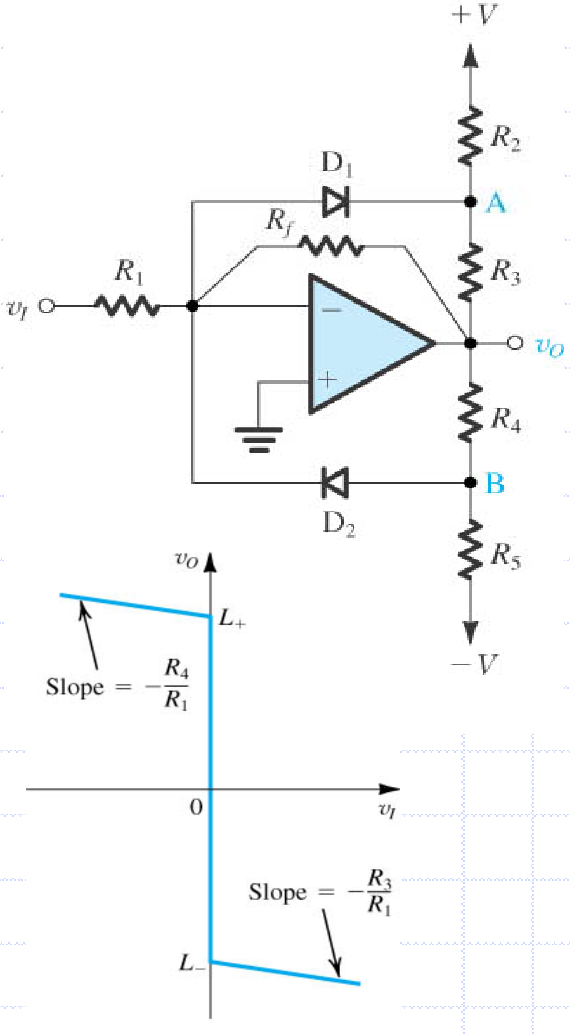
13.1.4 A Popular Limiter Circuit for Amplitude Control



(b)

- **Transfer characteristic**
  - $$v_I = \frac{L_-}{-(R_f/R_1)}$$
  - If v_I is increased beyond this value.
    → More current is injected into D_1 and v_A remains at approximately $-V_D$.
    → The additional diode current flows through R_3 and thus R_3 appears in effect in parallel with R_f.
    $$\frac{v_O}{v_I} = -\frac{R_f // R_3}{R_1}$$
    → To make the slope small in the limiting region, a low value should be selected for R_3.
    $$L_+ = V\frac{R_4}{R_5} + V_D\left(1 + \frac{R_4}{R_5}\right)$$ ...Positive limiting level
    → Increasing R_f results in a higher gain in the linear region

13.1.4 A Popular Limiter Circuit for Amplitude Control



(c)

- **Transfer characteristic**
  - Removing R_f → comparator
    : The circuit compares v_I with the comparator <u>reference value of 0V</u>
    : $v_I > v_O$, $v_O \approx L_-$ and $v_I < v_O$, $v_O \approx L_+$

13.2.1 The Wien-Bridge Oscillator

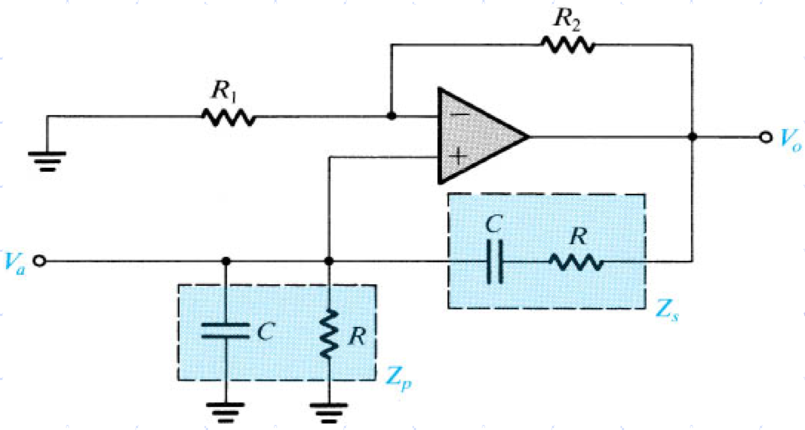


Figure 13.4 A Wien-bridge oscillator without amplitude stabilization.

- The circuit consists of an op amp connected in the non-inverting configuration with a closed-loop gain of $1+R_2/R_1$.
- In the feedback path RC network is connected
- The loop gain

$$L(s) = \left[1+\frac{R_2}{R_1}\right]\frac{Z_P}{Z_P+Z_S} = \frac{1+R_2/R_1}{3+sCR+1/sCR}$$

$$L(j\omega) = \frac{1+R_2/R_1}{3+j(\omega CR-1/\omega CR)}$$

13.2.1 The Wien-Bridge Oscillator

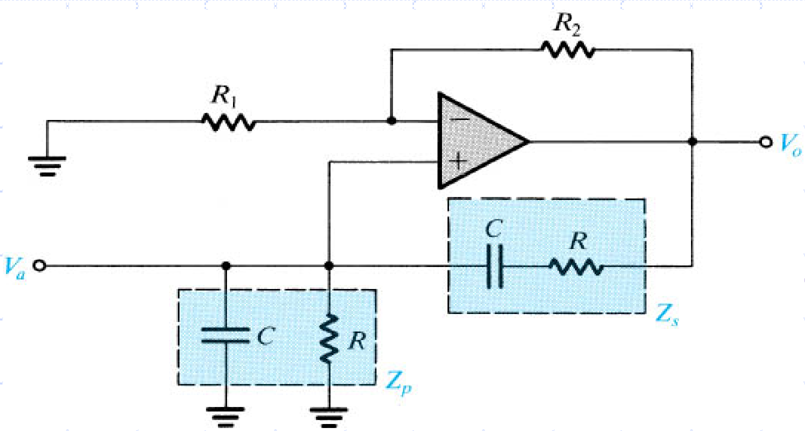


Figure 13.4 A Wien-bridge oscillator without amplitude stabilization.

- The loop gain will be a real number (i.e., the phase will be zero) at

$$\boxed{\omega_0 = 1/CR}$$

- To set the magnitude of the loop gain to unity (to obtain sustained oscillations at this frequency)

$$\boxed{R_2 / R_1 = 2}$$

- If $R_2 / R_1 = 2 + \delta$, (δ is a small number)
  - the roots of the characteristic equation

    $$\boxed{1 - L(s) = 0}$$

    will be in the right half of the s plane.
  - oscillations will start.

13.2.1 The Wien-Bridge Oscillator

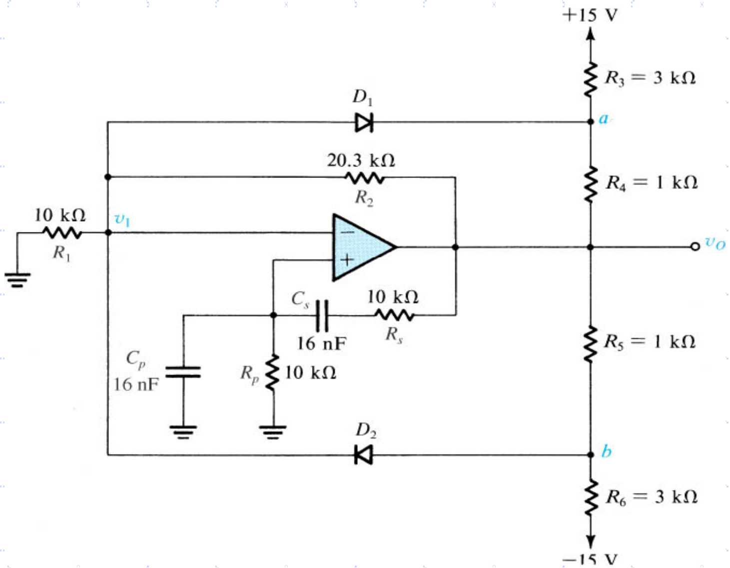


Figure 13.5 A Wien-bridge oscillator with a limiter used for amplitude control.

- Symmetrical feedback limiter formed by diodes D_1 and D_2, resistors R_3, R_4, R_5 and R_6.

- [Operation]
  - ① At the positive peak of the output voltage v_O, the voltage at node b will exceed the voltage v_1 and diode D_2 conducts.
  - ② Clamp the positive peak to a value determined by R_5, R_6, and the negative power supply.

13.2.1 The Wien-Bridge Oscillator

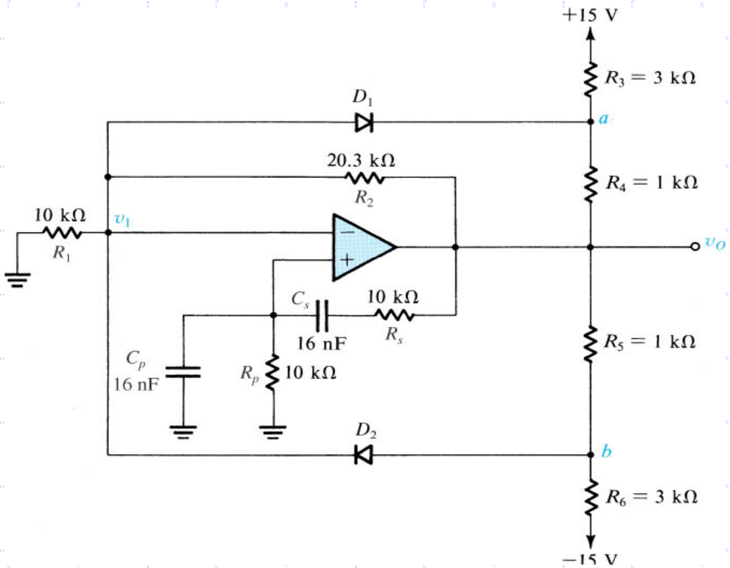


Figure 13.5 A Wien-bridge oscillator with a limiter used for amplitude control.

- Positive peak can be determined by setting $v_b = v_1 + V_{D2}$ and writing a node equation at node b while neglecting the current through D_2.

- Negative peak can be determined by setting $v_a = v_1 - V_{D1}$ and writing a node equation at node a while neglecting the current through D_1.

- To obtain a symmetrical output waveform,
  - R_3 is chosen equal to R_6
  - R_4 is chosen equal to R_5.

13.2.1 The Wien-Bridge Oscillator

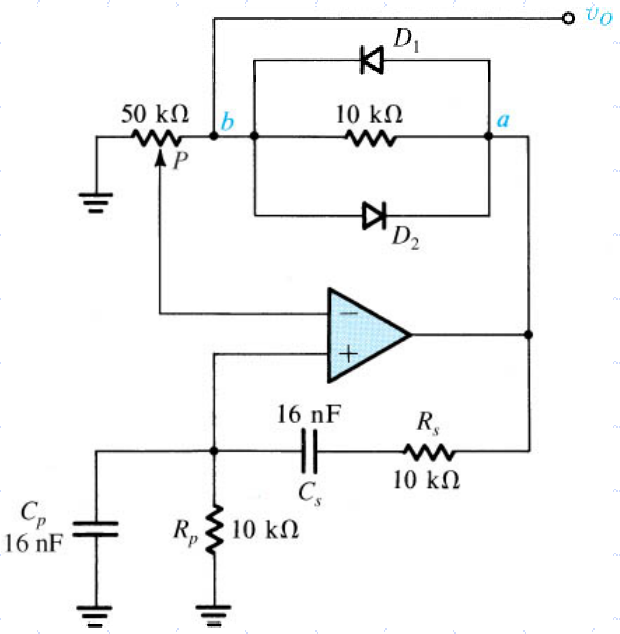


Figure 13.6 A Wien-bridge oscillator with an alternative method for amplitude stabilization.

- Inexpensive implementation of the parameter-variation mechanism of amplitude control.
- [Operation]
  - ① Potentiometer *P* is adjusted until oscillations just start to grow.
  - ② As the oscillations grow, the diodes start to conduct, causing the effective resistance between *a* and *b* to decrease.
- The output amplitude can be varied by adjusting potentiometer *P*.
- The output is taken at point *b* rather than at the op-amp output terminal.

  (∵Signal at *b* has lower distortion than that at *a*.)

13.2.1 The Wien-Bridge Oscillator

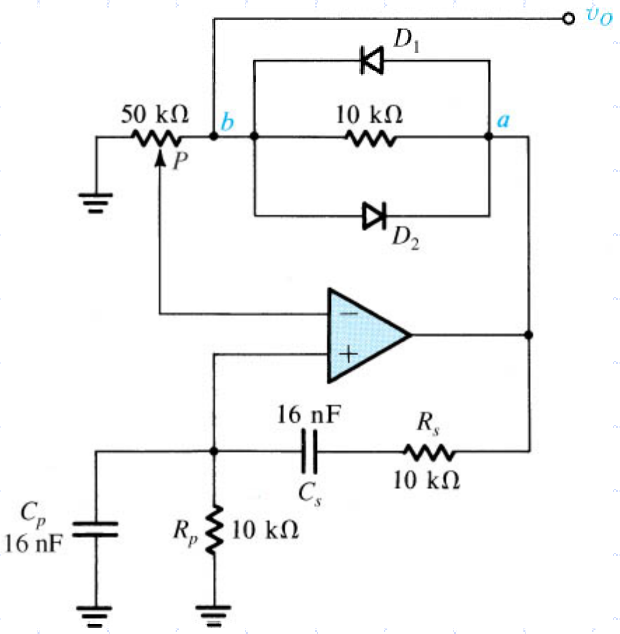


Figure 13.6 A Wien-bridge oscillator with an alternative method for amplitude stabilization.

- The voltage at *b* is proportional to the voltage at the op-amp input terminals.
- The voltage at *b* is a filtered version of the voltage at node *a*.
- Node *b*, is a high-impedance node, and a buffer will be needed if a lead is to be connected.

13.2.1 The Wien-Bridge Oscillator

- Exercise 13.3 For the circuit; Disregarding the limiter circuit, find the location of the closed-loop poles. (b) Find the frequency of oscillation. (c) with the limiter in phase, find the amplitude of the output sine wave

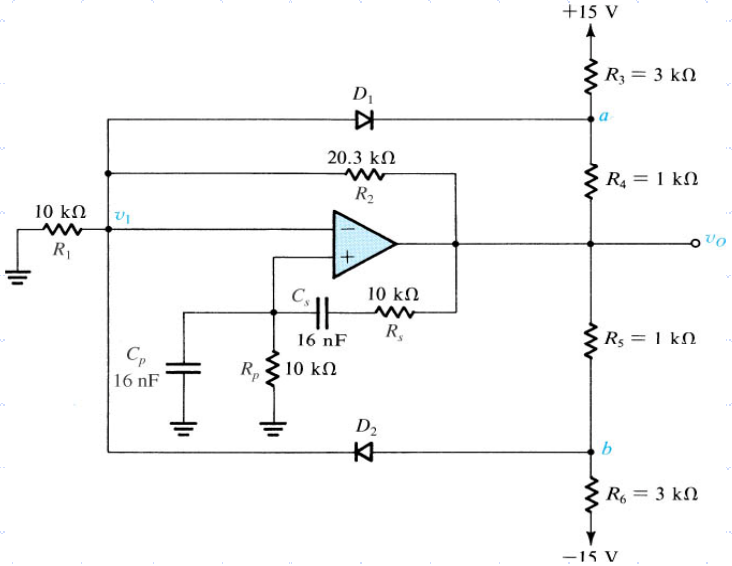


Figure 13.5 A Wien-bridge oscillator with a limiter used for amplitude control.

(a) $$L(s) = \left(1+\frac{R_2}{R_1}\right)\frac{Z_p}{Z_p+Z_s}$$

$$= \left(1+\frac{R_2}{R_1}\right)\frac{1}{1+Z_sY_p}$$

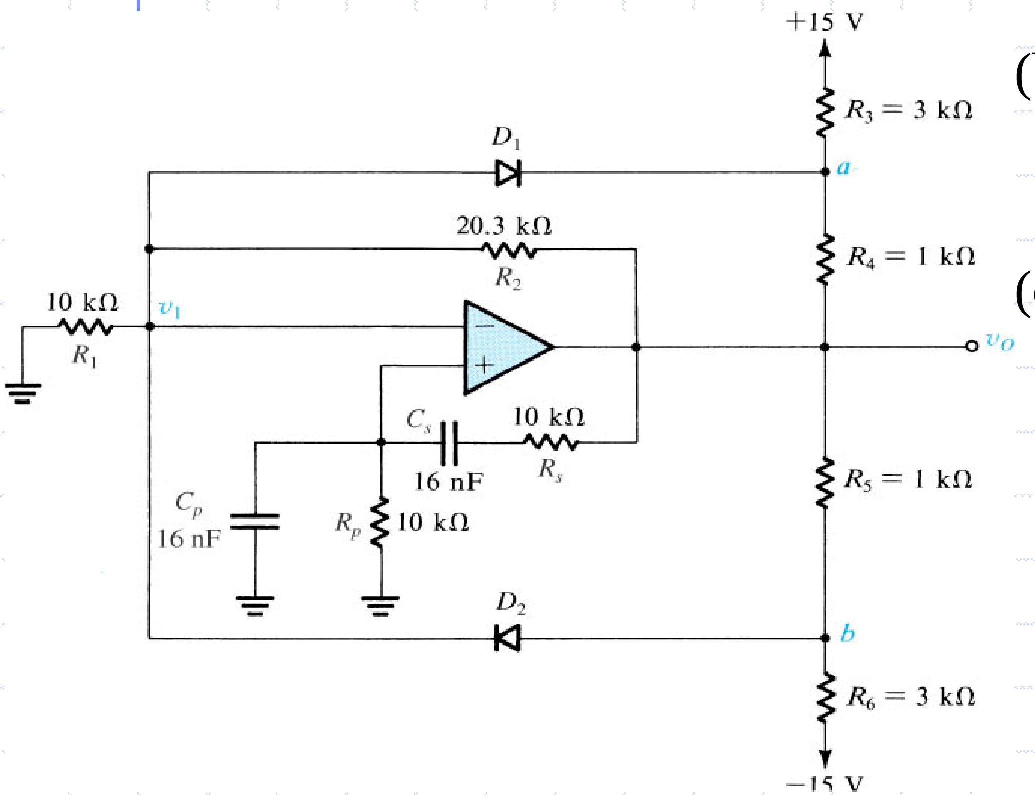
$$= \left(1+\frac{20.3}{10}\right)\frac{1}{1+\left(R+\frac{1}{SC}\right)\left(\frac{1}{R}+SC\right)}$$

$$= \frac{3.03}{1+s16\times10^{-5}+\frac{1}{s16\times10^{-5}}}$$

We can find the closed loop poles by setting L(s)=1 → $s=\frac{10^5}{16}(0.015\pm j)$

13.2.1 The Wien-Bridge Oscillator

- Exercise 13.3 For the below circuit; Disregarding the limiter circuit, find the location of the closed-loop poles. (b) Find the frequency of oscillation. (c) with the limiter in phase, find the amplitude of the output sine wave



(b) The frequency of oscilltion is $10^5/16$ rad/s or 1kHz

(c) voltage at node b $v_b = 0.7 + V_{peak}/3$

Neglecting the current through D_2,

$$\frac{V_{peak} - v_b}{R_3} = \frac{v_b - (-15)}{R_6}$$

combining two equations, we obtain

$V_{peak} = 10.68V$

$\rightarrow V_{pp} = 2V_{peak} = 21.36V$

13.2.2 The Phase-Shift Oscillator

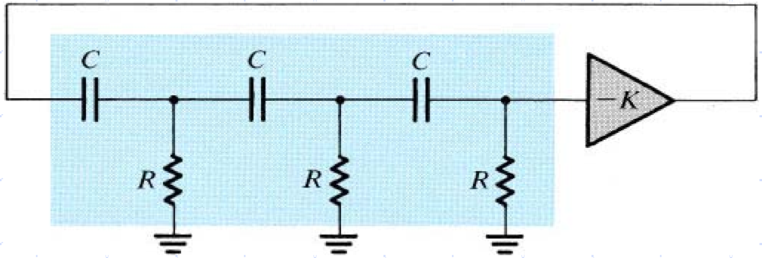


Figure 13.7 A phase-shift oscillator.

- Consists of a negative gain amplifier(-K) with a three-section (three-order) RC ladder network in the feedback.
- Oscillate at the frequency for which the phase shift of the RC network is 180°.
- At this (phase shift of the RC network is 180°) frequency will the total phase shift around the loop be 0° or 360°.
- Three is the minimum number of RC network that is capable of producing a 180° phase shift at a finite frequency.

13.2.2 The Phase-Shift Oscillator

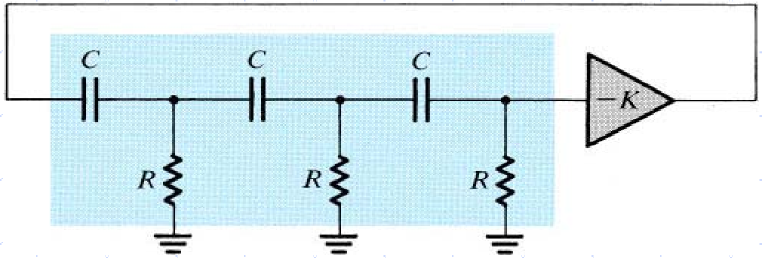


Figure 13.7 A phase-shift oscillator.

- For oscillation to be sustained, the value of K must be greater than the inverse of the magnitude of the RC network transfer function at the frequency of oscillation.

13.2.2 The Phase-Shift Oscillator

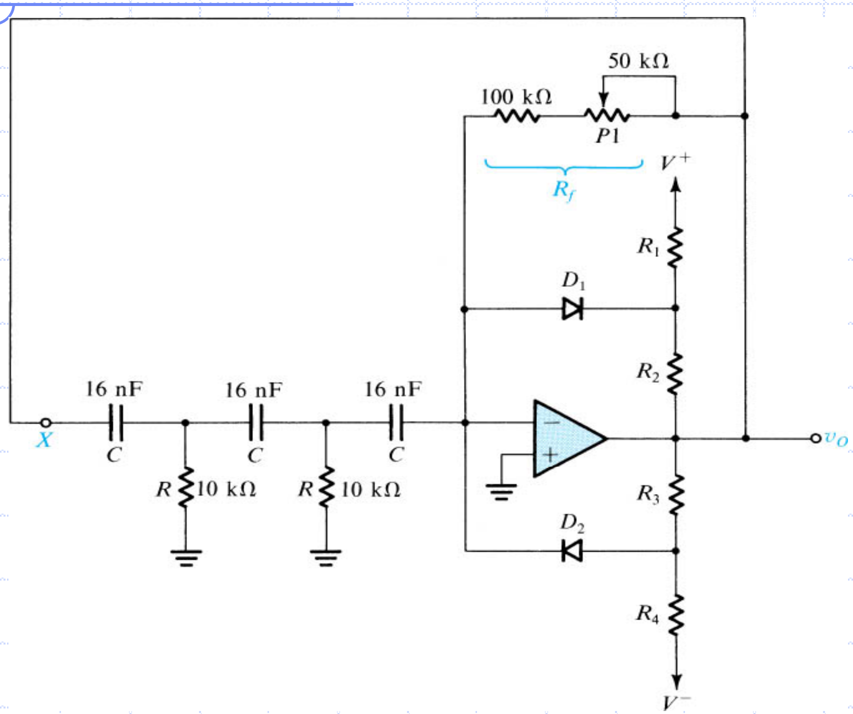


Figure 13.8 A practical phase-shift oscillator with a limiter for amplitude stabilization.

- Diodes D_1 and D_2 and resistors R_1, R_2, R_3, and R_4 for amplitude stabilization.
- To start oscillations, R_f has to be made slightly greater than the minimum required value.

(장점) The circuit stabilizes more rapidly

(장점) Provides sine waves with more stable amplitude

(단점) The price paid is an increased output distortion.

13.2.3 The Quadrature Oscillator

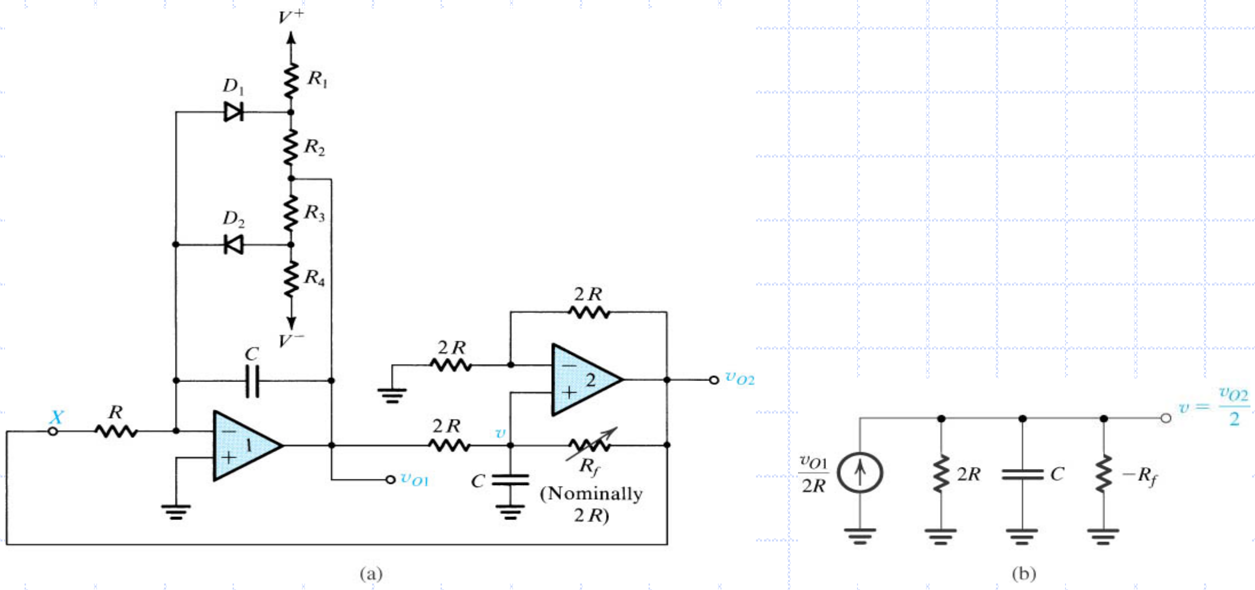


Figure 13.9 **(a)** A quadrature-oscillator circuit. **(b)** Equivalent circuit at the input of op amp 2.

- Based on the two-integrator loop.
- To ensure that oscillations start, the poles are initially located in the right half-plane and then "pulled back" by the nonlinear gain control.

13.2.3 The Quadrature Oscillator

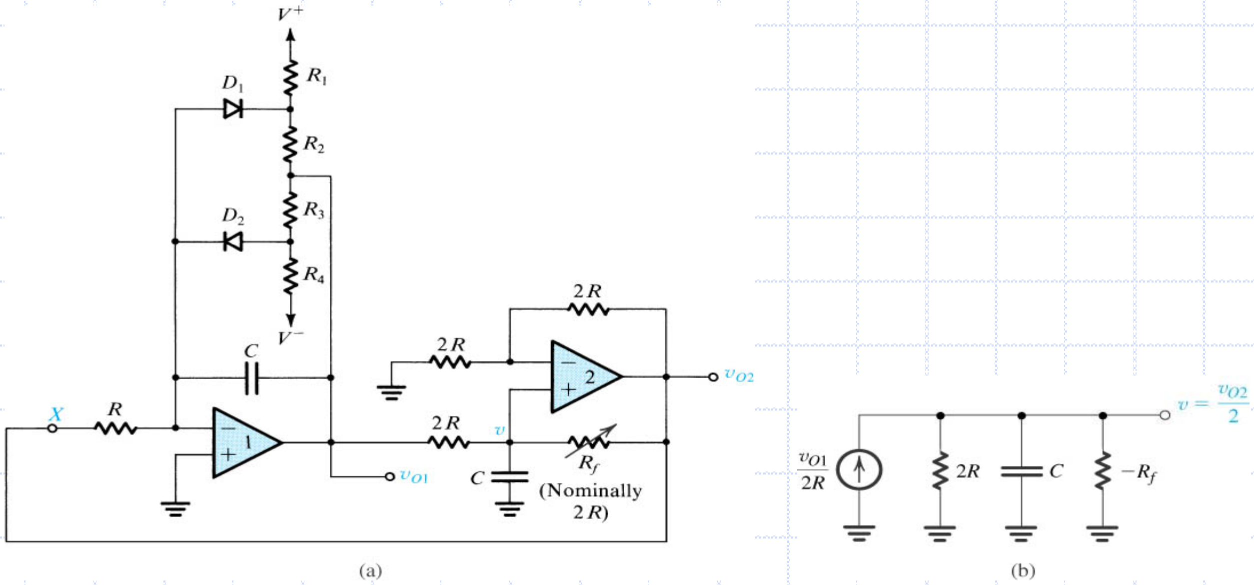


Figure 13.9 **(a)** A quadrature-oscillator circuit. **(b)** Equivalent circuit at the input of op amp 2.

- Amplifier 1 is connected as an inverting Miller integrator with a limiter in the feedback for amplitude control.
- Amplifier 2 is connected as a non-inverting integrator.

13.2.3 The Quadrature Oscillator

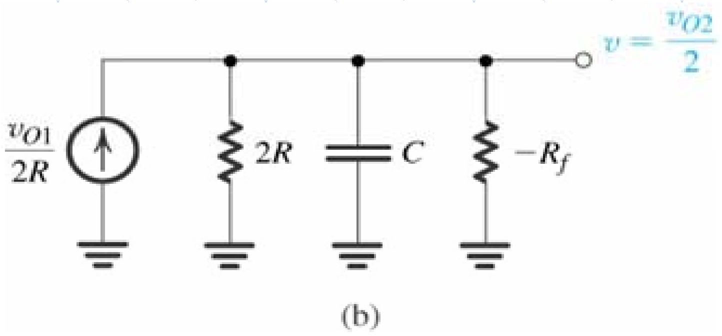


Figure 13.9 (b) Equivalent circuit at the input of op amp 2.

- The integrator input voltage v_{O1} and the series resistance 2R
  → The Norton equivalent composed of a current source v_{O1}/2R and a parallel resistance 2R.
- Since $v_{O2} = 2v$, the current through R_f is
  $$\boxed{(2v - v)/R = v/R}$$
  (the direction from output to input).
- R_f cancels 2R, and $v_{O1}/2R$ feeding a capacitor *C*.

13.2.3 The Quadrature Oscillator

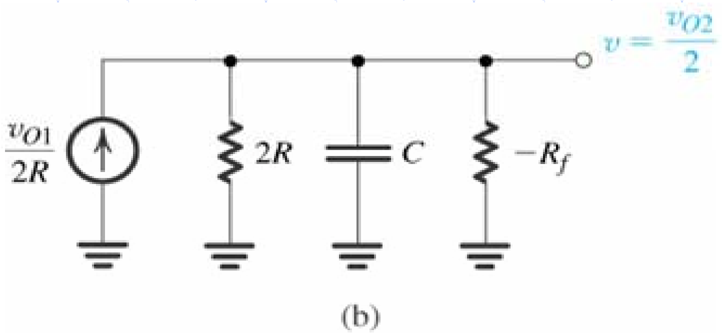


Figure 13.9 (b) Equivalent circuit at the input of op amp 2.

- The result is

$$v = \frac{1}{C}\int_0^t \frac{v_{O1}}{2R}dt \quad \text{and} \quad v_{O2} = 2v = \frac{1}{CR}\int_0^t v_{O1}dt$$

(non-inverting integrator).

13.2.3 The Quadrature Oscillator

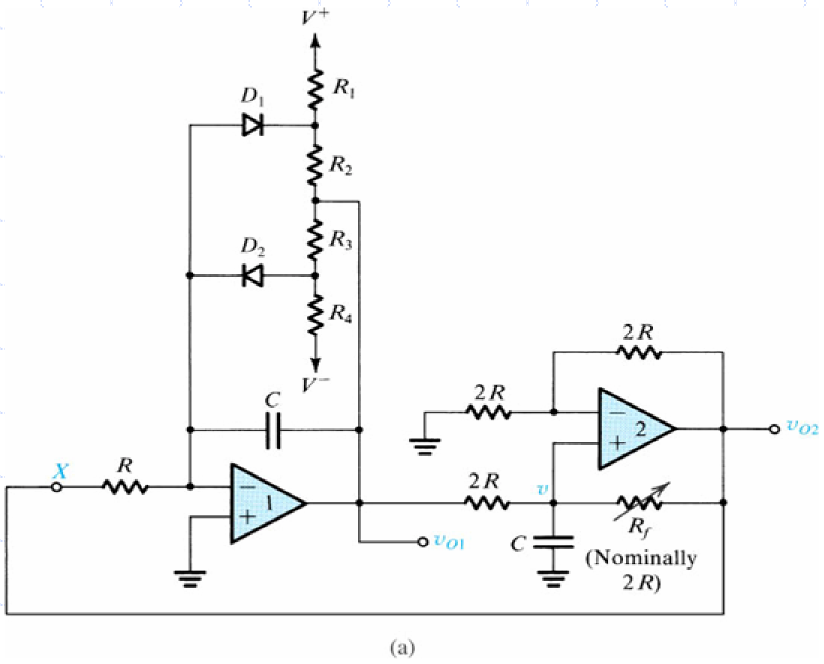


Figure 13.9 **(a)** A quadrature-oscillator circuit.

- The resistance R_f in the positive-feedback path is made variable.
- Decreasing the value of R_f ensures that the oscillations start.
- The loop gain

$$\boxed{L(s) \equiv \frac{V_{O2}}{V_x} = -\frac{1}{s^2C^2R^2}}$$

- The loop will oscillate at frequency

$$\boxed{\omega_0 = \frac{1}{CR}}$$

13.2.4 The Active-Filter-Tuned Oscillator

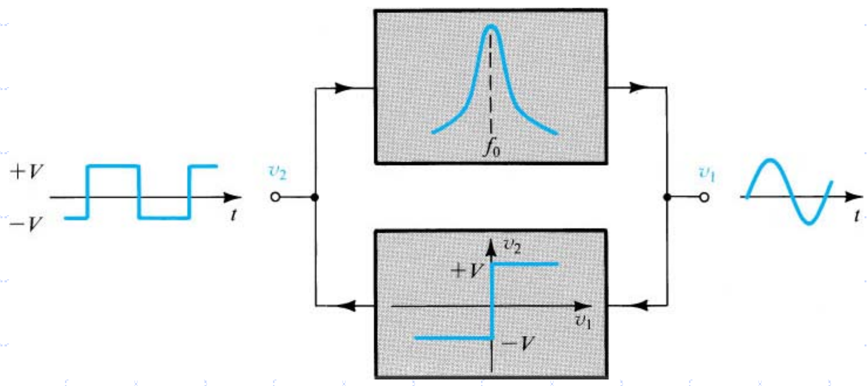


Figure 13.10 Block diagram of the active-filter-tuned oscillator.

- The circuit consists of a high-*Q* bandpass filter connected in a positive-feedback loop with a hard limiter.
- Assume that oscillations have already started.
- The output of the bandpass filter will be a sine wave whose frequency is f_0.
- The sine-wave signal v_1 is fed to the limiter.
- The square wave is fed to the bandpass filter.
- Independent control of frequency and amplitude as well as of distortion of the output sinusoid.

13.2.4 The Active-Filter-Tuned Oscillator

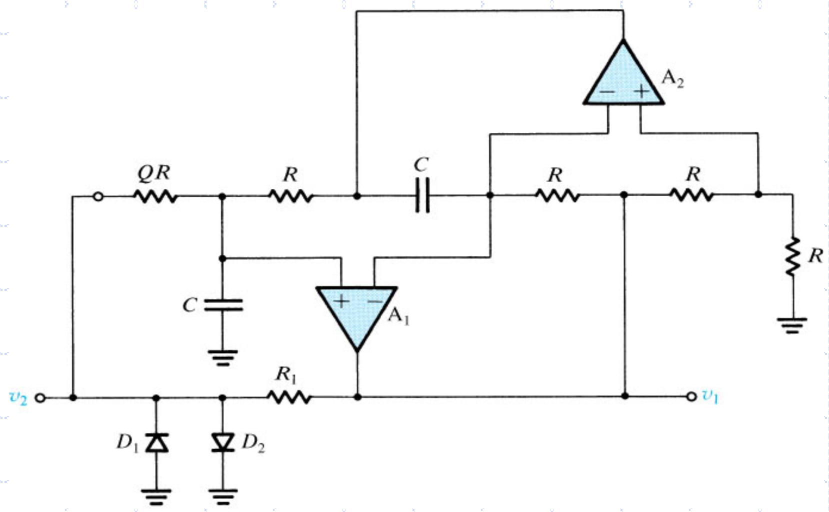


Figure 13.11 A practical implementation of the active-filter-tuned oscillator.

- Resistor R_2 and capacitor C_4 make the output of the lower op amp directly proportional to the voltage across the resonator.
- Limiter : resistance R1 and two diodes.

13.2.5 A Final Remark

- Useful for operation in the range 10Hz to 100kHz (or perhaps 1MHz at most).
- The lower frequency limit is dictated by the size of passive components required
- the upper limit is governed by the frequency-response and slew-rate limitations of op amps.
- For higher frequencies, transistors together with LC tuned circuits or crystals are frequently used.

13.3 LC and Crystal Oscillators

- Oscillators utilizing transistors(FETs or BJTs), with LC-tuned circuits or crystals as feedback elements, are used in the frequency range of **100kHz to hundreds of megahertz**.

- They exhibit **higher Q** than the RC types

- LC oscillators are difficult to tune over wide ranges, and crystal oscillators operate a single frequency.

13.3.1 LC-Tuned Oscillators

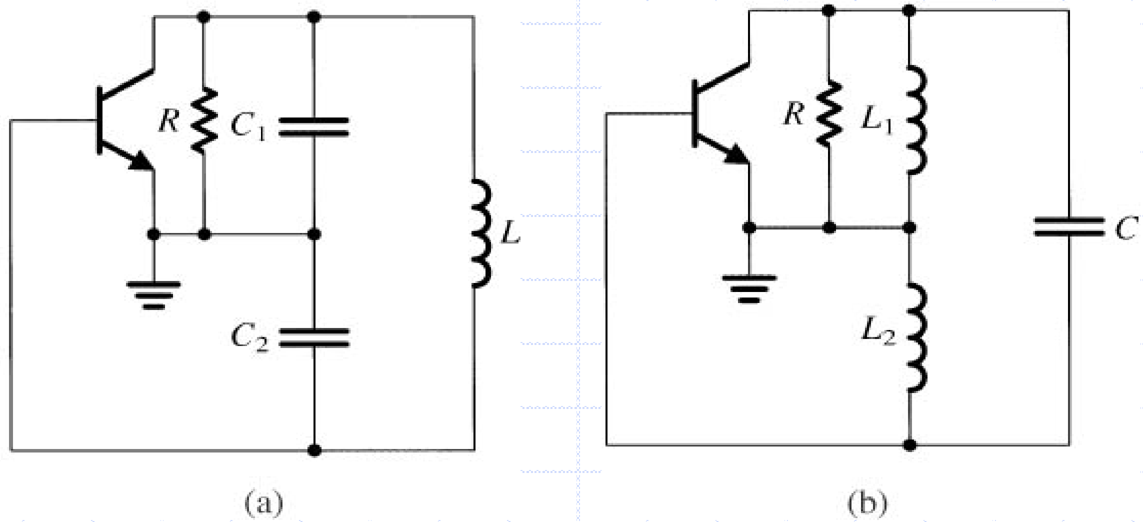


Figure 13.12 Two commonly used configurations of LC-tuned oscillators: **(a)** Colpitts and **(b)** Hartley.

- They are known as the **Colpitts oscillator(a)** and the **Hartley oscillator(b)**.
  - This feedback is achieved by way of a capacitive divider in the Colpitts oscillator and by way of an inductive divider in the Hartley circuit.
  - The resistor R models the combination of the losses of the inductors, the load resistance of the oscillator, and the output resistance of the transistor.

13.3.1 LC-Tuned Oscillators

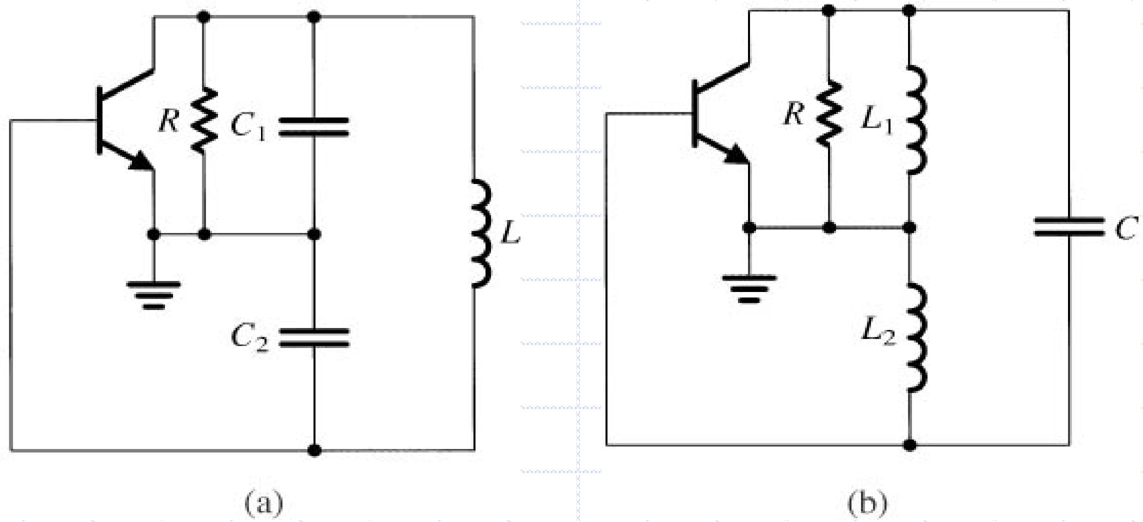


Figure 13.12 Two commonly used configurations of LC-tuned oscillators: **(a)** Colpitts and **(b)** Hartley.

- If the frequency of operation is sufficiently low that we can neglect the transistor capacitances, the frequency of oscillation will be determined by the resonance frequency of the parallel-tuned circuit
- The Colpitts oscillator

$$\omega_0 = \frac{1}{\sqrt{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}}$$

The Hartley oscillator

$$\omega_0 = \frac{1}{\sqrt{(L_1 + L_2)C}}$$

13.3.1 LC-Tuned Oscillators - Colpitts

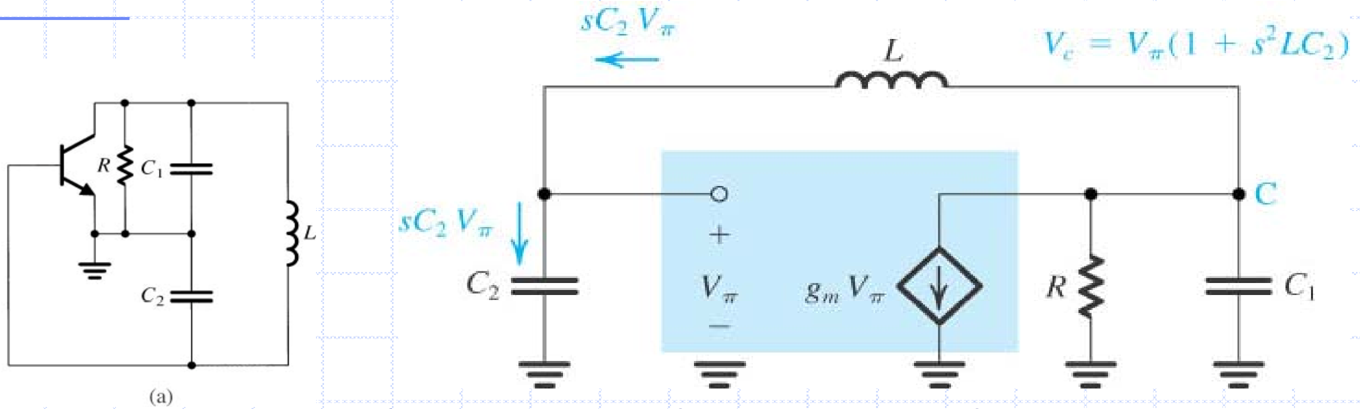


Figure 13.13 Equivalent circuit of the Colpitts oscillator of Fig. 13.12(a). To simplify the analysis, C_μ and r_π are neglected. We can consider C_π to be part of C_2, and we can include r_o in R.

- The ratio L_1/L_2 or C_1/C_2 determines the feedback factors.
- Capacitance C_μ is neglected & capacitance C_π is included in C_2
- Input resistance r_π is neglected assuming that at the frequency of oscillation $r_\pi \gg (1/\omega C_2)$.
- Resistance R includes r_0 of the transistor.
- **To find the loop gain:** break the loop at the transistor base, apply an input voltage Vπ and find the returned voltage that appears across the input terminals of the transistor.
- **To analyze the circuit:** eliminate all current and voltage variables, and thus obtain one equation.
- The resulting equation will give us the conditions for oscillation.

13.3.1 LC-Tuned Oscillators - Colpitts

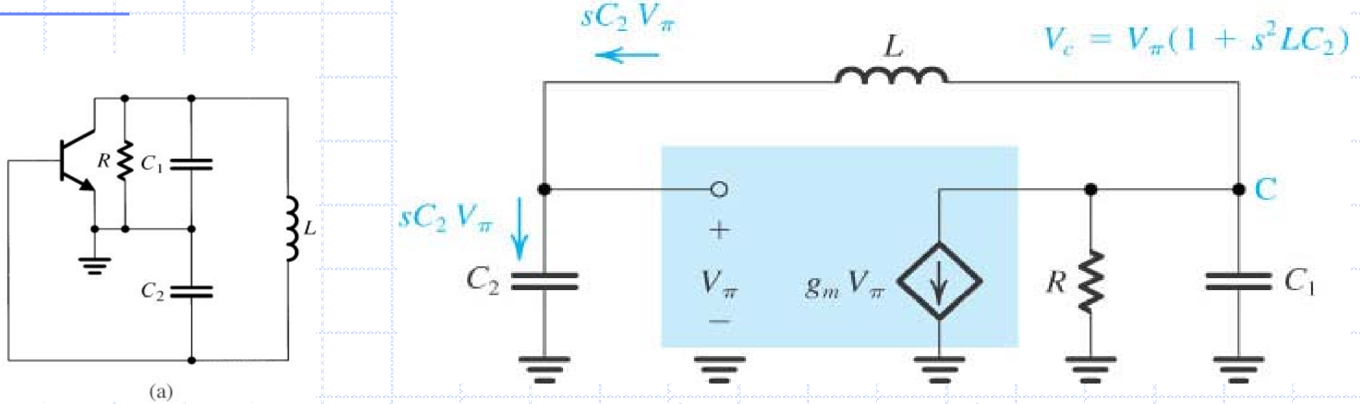


Figure 13.13 Equivalent circuit of the Colpitts oscillator of Fig. 13.12(a). To simplify the analysis, C_μ and r_π are neglected. We can consider C_π to be part of C_2, and we can include r_o in R.

- A node equation at node C is

$$sC_2V_\pi + g_mV_\pi + \left(\frac{1}{R} + sC_1\right)(1 + s^2LC_2)V_\pi = 0$$

- Since $V_\pi \neq 0$(oscillations have started), it can be eliminated,

$$s^3LC_1C_2 + s^2(LC_2/R) + s(C_1 + C_2) + \left(g_m + \frac{1}{R}\right) = 0$$

- Substituting s=jω gives,

$$\left(g_m + \frac{1}{R} - \frac{\omega^2 LC_2}{R}\right) + j[\omega(C_1 + C_2) - \omega^3 LC_1C_2] = 0$$

13.3.1 LC-Tuned Oscillators - Colpitts

- For oscillations to start, both the real and imaginary parts must be zero

$$\omega(C_1+C_2)-\omega^3 LC_1C_2=0$$

$$\omega_0=\frac{1}{\sqrt{L\left(\frac{C_1C_2}{C_1+C_2}\right)}}$$

- Substituting s=jω gives,

$$\left(g_m+\frac{1}{R}-\frac{\omega^2 LC_2}{R}\right)=0$$

$$g_mR+1-\frac{1}{L\left(\frac{C_1C_2}{C_1+C_2}\right)}LC_2=0$$

$$g_mR+1-\frac{C_1+C_2}{C_1}=0$$

$$\therefore C_2/C_1=g_mR$$

• For sustained oscillations, the magnitude of the gain from base to collector (g_mR) must be equal to the inverse of the voltage ratio provided by the capacitive divider($v_{eb}/v_{ce}=C_1/C_2$).

For oscillations to start, the loop gain must be greater than unity.

• As oscillations grow in amplitude, the transistor's **nonlinear characteristic reduces** the effective value of $\mathbf{g_m}$ and reduce the loop gain to unity.

13.3.1 LC-Tuned Oscillators - Hartley

◆ The Hartley circuit analysis(Exercise 13.8)

At high frequencies, more accurate transistor models must be used.

: The y parameters(the short-circuit admittance) of the transistor can be measured at the intended frequency ω_0, and the analysis can then be carried out using the y-parameter model(Appendix B).

: This is usually simpler and more accurate, especially at frequencies above about 30% of the transistor f_T.

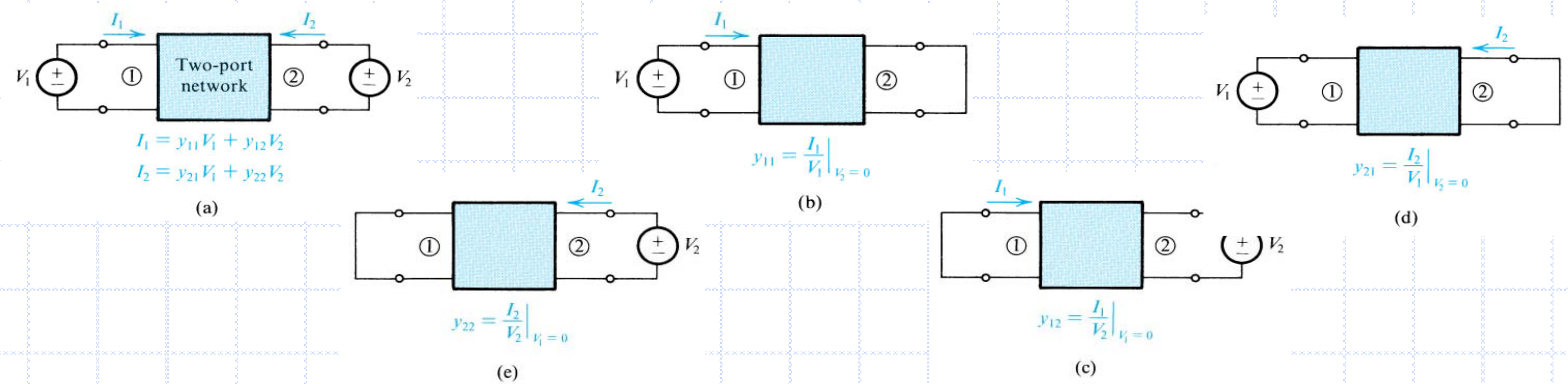


Figure B.2 Definition and conceptual measurement circuits for *y* parameters.

13.3.1 LC-Tuned Oscillators

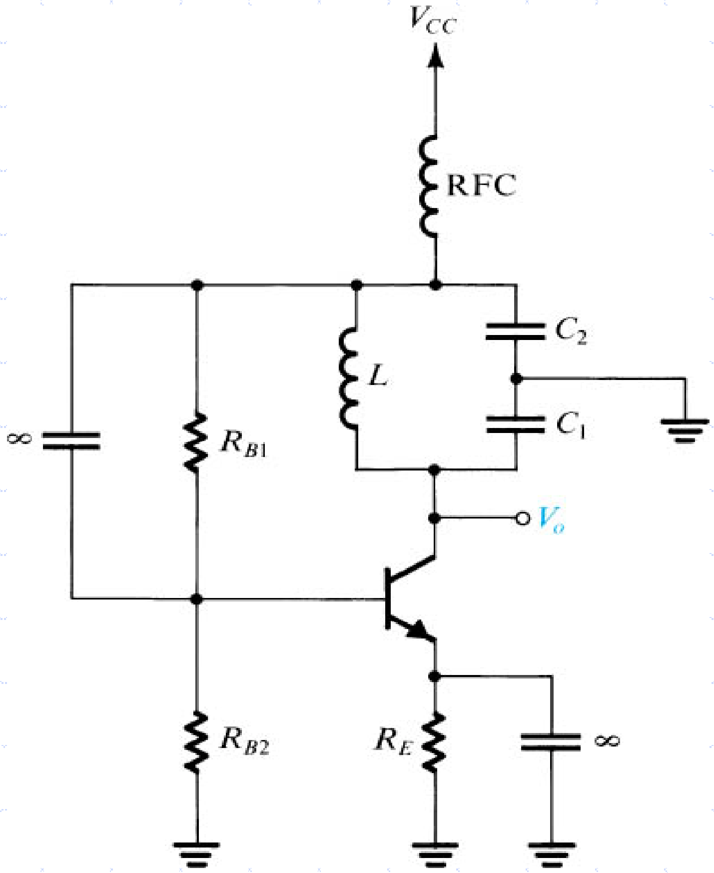


Figure 13.14 Complete circuit for a Colpitts oscillator.

- An example of a practical LC oscillator(Colpitts)
- The radio-frequency choke(RFC) provides a high reactance at ω_0 but a low dc resistance.

13.3.1 LC-Tuned Oscillators

- Determining the amplitude of oscillation

Unlike the op-amp oscillators that incorporate special amplitude-control circuitry, LC-tuned oscillators utilize the nonlinear i_C-v_{BE} characteristics of the BJT(*self-limiting oscillators*).

→ As the oscillations grow in amplitude, the effective gain of the transistor is reduced below its small-signal value.

→ Eventually, an amplitude is reached at which the effective gain is reduced to the point that the Barkhausen criterion is satisfied exactly.

→ The amplitude then remains constant at this value.

13.3.2 Crystal Oscillators

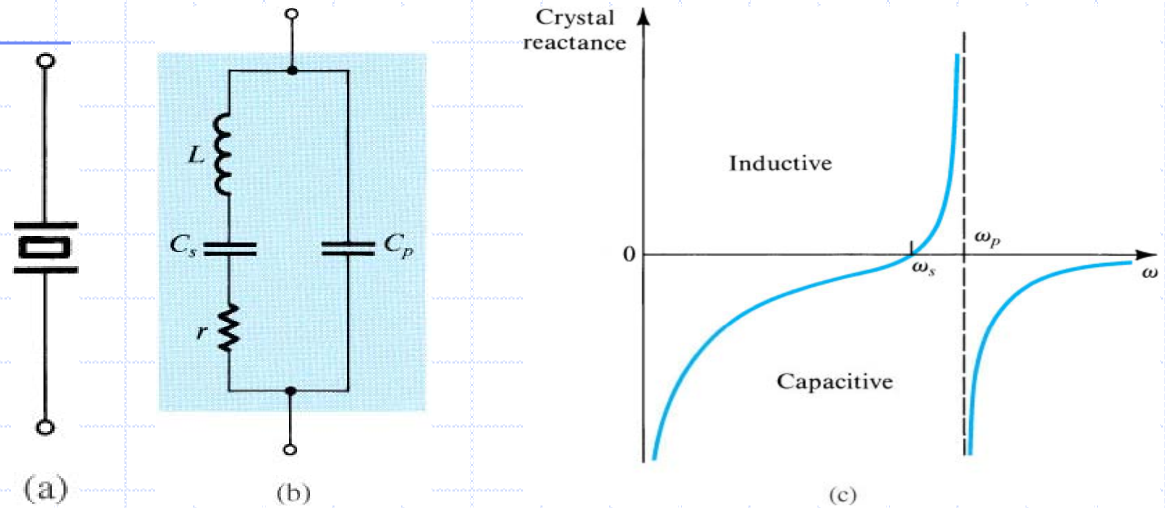


Figure 13.15 A piezoelectric crystal. **(a)** Circuit symbol. **(b)** Equivalent circuit. **(c)** Crystal reactance versus frequency [note that, neglecting the small resistance r, $Z_{crystal} = jX(\omega)$].

- A piezoelectric crystal(quartz) exhibits electromechanical-resonance characteristics that are very stable(with time and temperature) and highly selective(having very high Q factors).
- The resonance properties are characterized by

: large inductance L(as high as hundreds of henrys), very small series capacitance C_S(as small as 0.0005pF), series resistance *r* representing a Q factor $\omega_0 L/r$(can be as high as a few hundred thousand) and parallel capacitance C_P(a few pF, $C_P \gg C_S$)

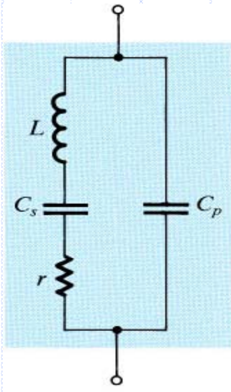
13.3.2 Crystal Oscillators

◆ Since the Q factor is very high, we may neglect the resistance *r*. The crystal impedance is

$$Z(s) = 1/\left[sC_P + \frac{1}{sL + 1/sC_S}\right]$$
$$= \frac{1}{sC_P}\frac{s^2 + (1/LC_S)}{s^2 + [(C_P + C_S)/LC_S C_P]} \quad ...Eq.(13.23)$$

◆ From Eq.(13.23) and from Fig. 13.15(b) we see that the crystal has two resonance frequencies.

- Series resonance at ω_S $\quad \omega_S = 1/\sqrt{LC_S} \quad ...Eq.13.24$
- Parallel resonance at ω_P $\quad \omega_P = 1/\sqrt{L\left(\frac{C_S C_P}{C_S + C_P}\right)} \quad ...Eq.13.25$



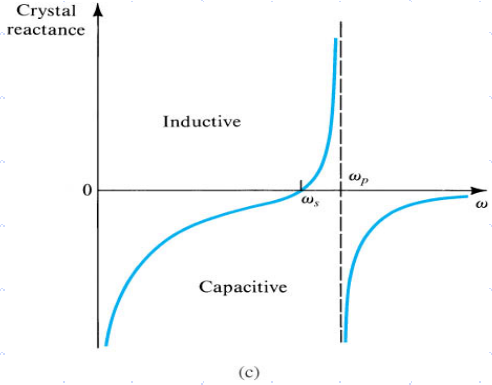
(b)

◆For s=jω

$$Z(j\omega) = -j\frac{1}{\omega C_P}\left(\frac{\omega^2 - \omega_S^2}{\omega^2 - \omega_P^2}\right)$$

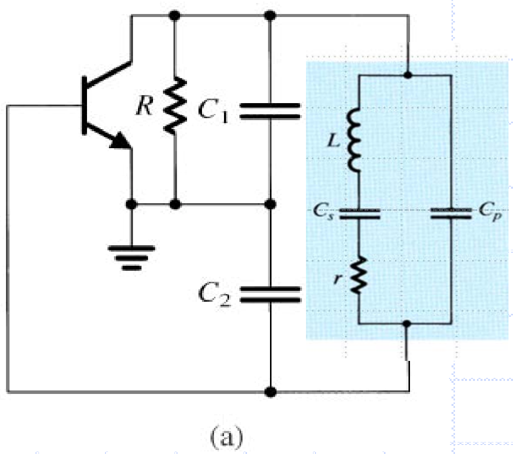
13.3.2 Crystal Oscillators

- Expressing Z(jω)=jX(ω), the crystal reactance X(ω) will have the shape,



$$Z(j\omega)=jX(\omega)=-j\frac{1}{\omega C_P}\left(\frac{\omega^2-\omega_S{}^2}{\omega^2-\omega_P{}^2}\right)$$

- We observe that the crystal reactance is inductive over the very narrow frequency band between ω_S and ω_P.



- For a given crystal, this frequency band is well defined. Thus we may use the crystal to replace the inductor of the Colpitts oscillator.
- The resulting circuit will oscillate at the resonance frequency of the crystal inductance L with the series equivalent of C_S and $(C_P+C_1C_2/(C_1+C_2))$.
- Since C_S is much smaller than the three other capacitances,

$$\omega_0 \approx 1/\sqrt{LC_S}=\omega_S$$

13.3.2 Crystal Oscillators

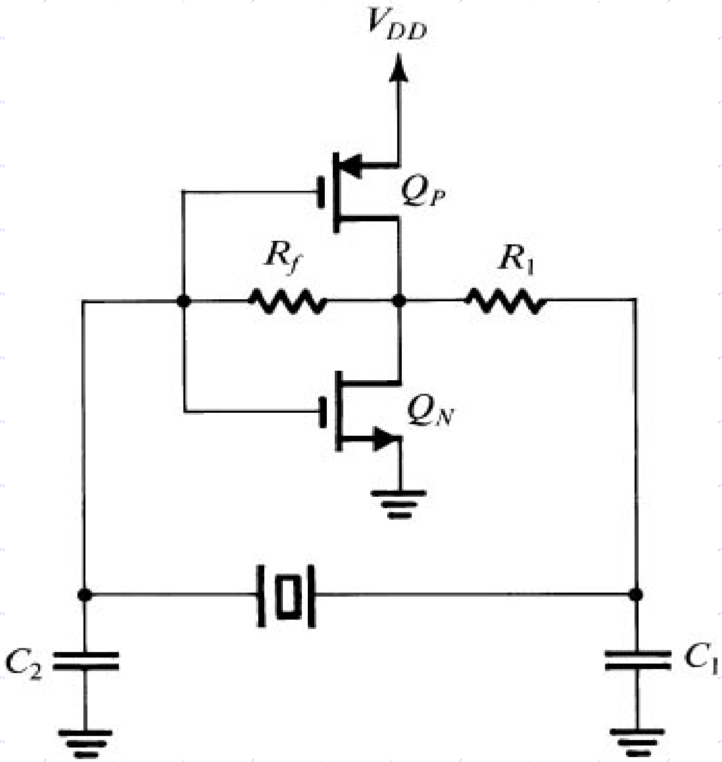


Figure 13.16 A Pierce crystal oscillator utilizing a CMOS i nverter as an amplifier.

- Pierce oscillator
  - Utilizing CMOS inverter(Section 4.10) as amplifier
  - Resistor R_f determines a dc operating point in the high-gain region of the CMOS inverter
  - Resistor R_1 and capacitor C_1 provide a low-pass filter that discourages the circuit from oscillating at a higher harmonic of the crystal frequency

13.4 Bistable Multivibrators

- Multivibrators.
  - Bistable.
  - Monostable.
  - Astable.

- Bistable vibrator has two stable states.
  - ① can remain in stable state indefinitely.
  - ② moves to the other stable state only when appropriately triggered.

13.4 Bistable Multivibrators

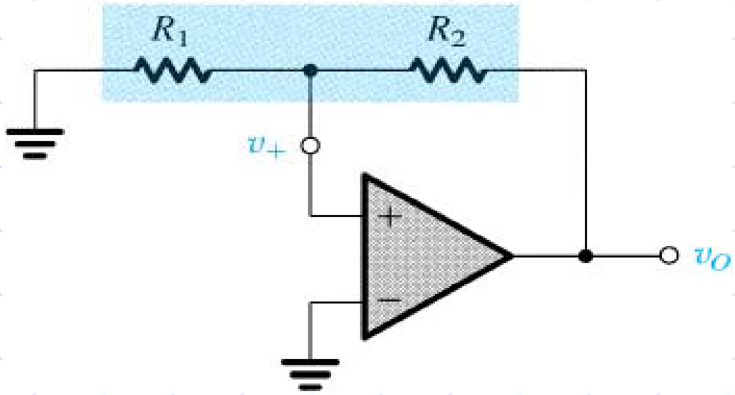


Figure 13.17 A positive-feedback loop capable of bistable operati on.

- Consists of an op amp and a resistive voltage divider in the positive-feedback path.

$$\beta \equiv R_1 / (R_1 + R_2)$$

- Assume that the electrical noise causes a small positive increment in the voltage v_+.

① Positive increment occurred in v+.

$$v_O = L_+ \qquad v_+ = L_+ R_1 / (R_1 + R_2)$$

② Negative increment occurred in v+.

$$v_O = L_- \qquad v_+ = L_- R_1 / (R_1 + R_2)$$

13.4 Bistable Multivibrators

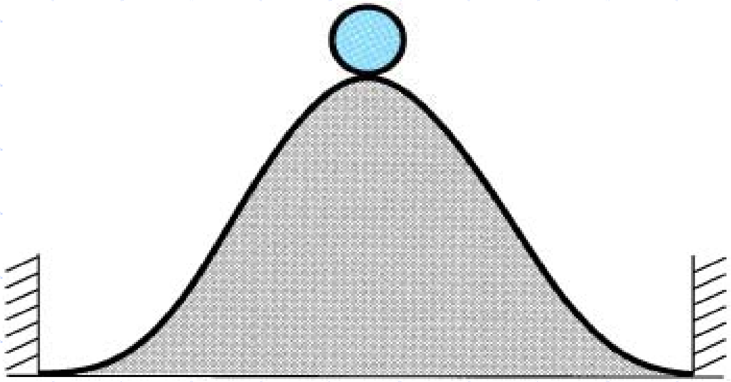


Figure 13.18 A physical analogy for the operation of the bistable circuit. The ball cannot remain at the top of the hill for any length of ti me (a state of unstable equilibrium or metastability); the inevitably present disturbance will cause the ball to fall to one side or the other, where it can remain indefinitely (the two stable states).

- The circuit cannot exist in the state for which $v_+ = 0$ and $v_O = 0$ (state of unstable equilibrium, metastable state) for any length of time.
- Any disturbance (electrical noise) causes the bistable circuit to switch to one of its two stable states (positive saturation or negative saturation).

13.4.2 Transfer Characteristics of the Bistable Circuit

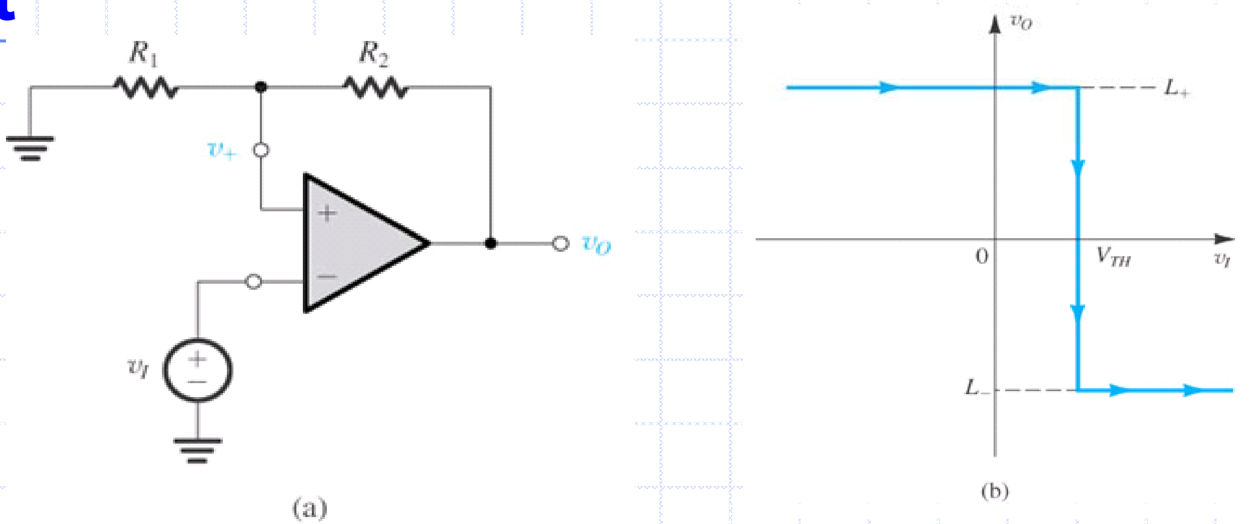


Figure 13.19 **(a)** The bistable circuit of Fig. 13.17 with the negative input terminal of the op amp disconnected from ground and connected to an input signal v_I. **(b)** The transfer characteristic of the circuit in (a) for increasing v_I.

① Assume that v_I is increased from 0V,

$$v_O = L_+ \text{ and } v_+ = \beta L_+$$

- As v_I begins to exceed v_+, a net negative voltage develops between the input terminals of the op amp and thus v_O goes negative.

13.4.2 Transfer Characteristics of the Bistable Circuit

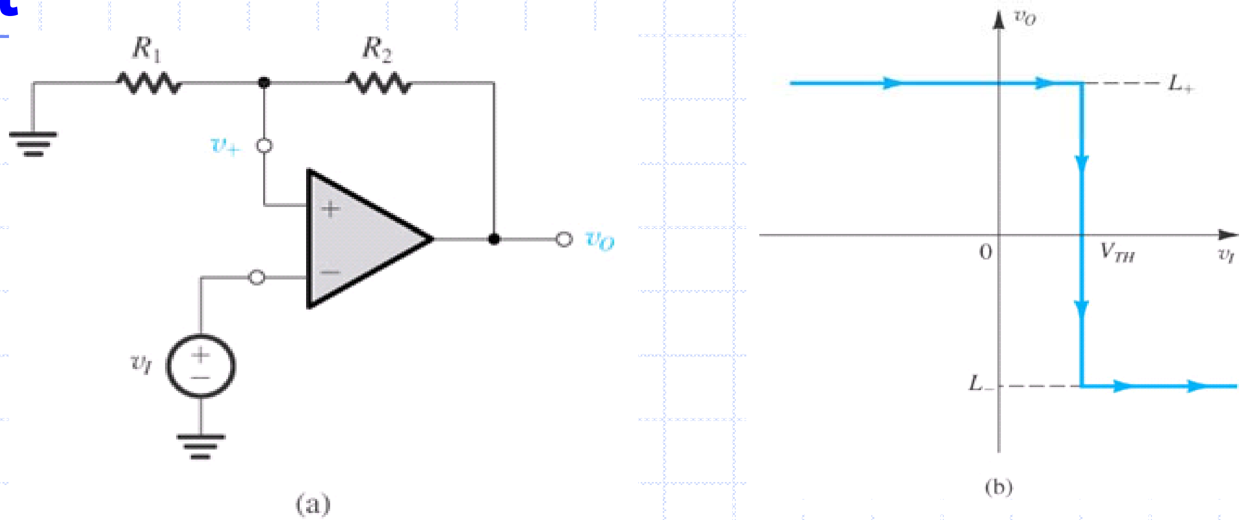


Figure 13.19 **(a)** The bistable circuit of Fig. 13.17 with the negative input terminal of the op amp disconnected from ground and connected to an input signal v_I. **(b)** The transfer characteristic of the circuit in (a) for increasing v_I.

- v_+ goes negative, increasing the net negative input to the op amp.
- The process culminates in the op amp saturating in the negative direction. $\boxed{v_O = L_- \text{ and } v_+ = \beta L_-}$
- Threshold voltage : $\boxed{V_{TH} = \beta L_+}$

13.4.2 Transfer Characteristics of the Bistable Circuit

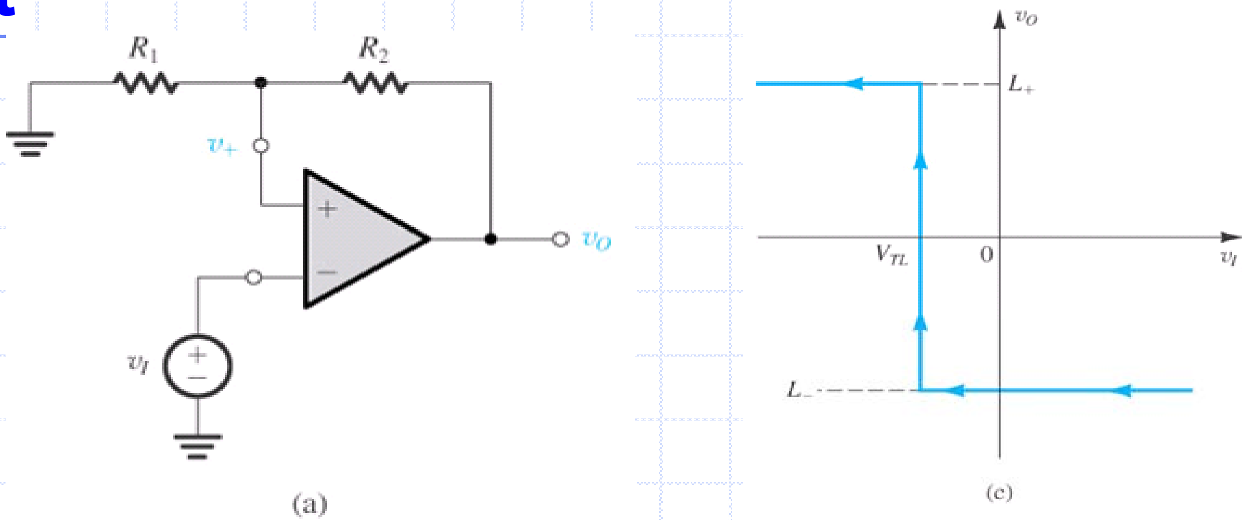


Figure 13.19 **(a)** The bistable circuit of Fig. 13.17 with the negative input terminal of the op amp disconnected from ground and connected to an input signal v_I. **(b)** The transfer characteristic of the circuit in (a) for increasing v_I.

② Consider v_I is decreased.

- Circuit remains in the negative-saturation state until $v_I \geq \beta L_-$
- $v_I < \beta L_-$ ⟶ Net positive voltage appears between the op amp's input terminals ⟶ Positive-saturation state
- Threshold voltage : $\boxed{V_{TL} = \beta L_-}$

13.4.2 Transfer Characteristics of the Bistable Circuit

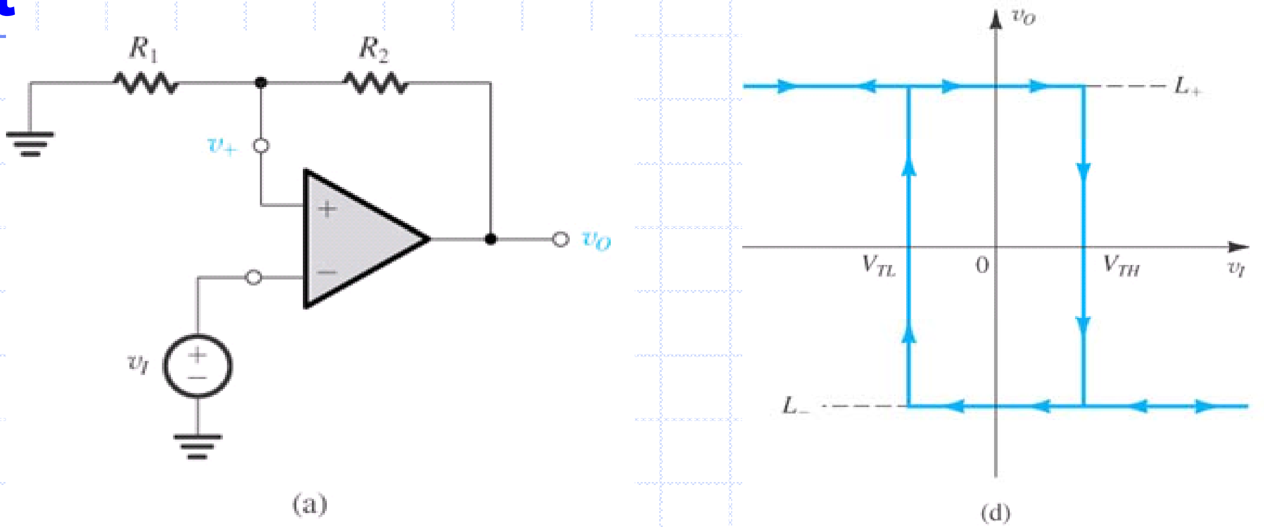


Figure 13.19 **(a)** The bistable circuit of Fig. 13.17 with the negative input terminal of the op amp disconnected from ground and connected to an input signal v_I. **(d)** The complete transfer characteristics.

- The circuit changes state at different values of v_I, depending on whether v_I is increasing or decreasing.
- The width of the *hysteresis* is the difference between the high threshold V_{TH} and the low threshold V_{TL}.
- Inverting circuit.

13.4.3 Triggering the Bistable Circuit

- If the circuit is in the L_+ state.
  - → Applying an input v_I of value greater than $V_{TH} \equiv \beta L_+$
  - → The circuit can be switched to the L_- state.

- If the circuit is in the L_- state.
  - → Applying an input v_I of value smaller than $V_{TL} \equiv \beta L_-$
  - → The circuit can be switched to the L_+ state.

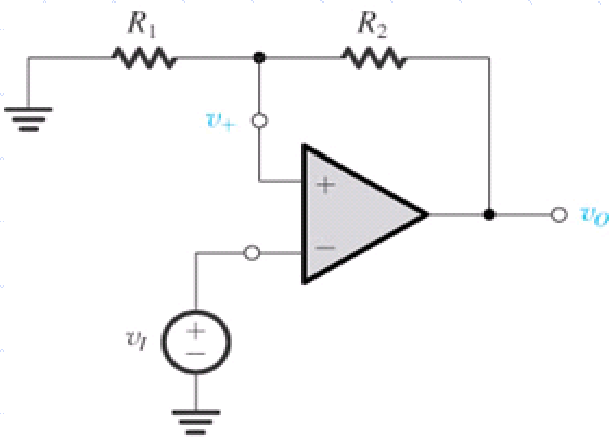
$\therefore$ v_I : trigger signal.

13.4.4 The Bistable Circuit as a Memory Element

- For certain input range, the output is determined by the previous value of the trigger signal.
- The bistable multivibrator is the basic *memory* element of digital systems.

13.4.2 Transfer Characteristics of the Bistable Circuit

- Exercise 13.11
  The op amp in the circuit of Fig.13.19(a) has output saturation voltages of ± 13V, Design the circuit to obtain threshold voltages of ± 5V. For R_1=10kΩ, find the value required for R_2.



(a)

$$\frac{V_{peak} - v_b}{R_3} = \frac{v_b - (-15)}{R_6}$$

$$V_{TH} = V_{TL} = \beta |L|$$

$$5 = \frac{R_1}{R_1 + R_2} \times 13$$

$$\frac{R_2}{R_1} = 1.6 \qquad \therefore R_2 = 16k\Omega$$

13.4.5 A Bistable Circuit with noninverting Transfer Characteristics

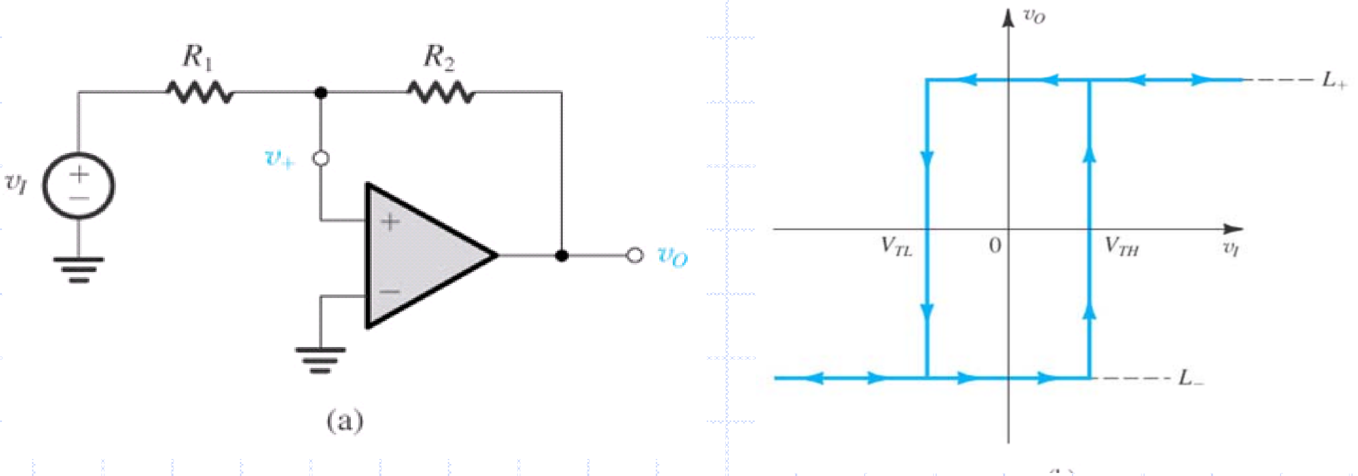


Figure 13.20 **(a)** A bistable circuit derived from the positive-feedback loop of Fig. 13.17 by applying v_I through R_1. **(b)** The transfer characteristic of the circuit in (a) is noninverting. (Compare it to the inverting characteristic in Fig. 13.19d.)

- Transfer characteristics,

$$v_+ = v_I \frac{R_2}{R_1 + R_2} + v_O \frac{R_1}{R_1 + R_2}$$

- If the circuit is in the positive stable state, $v_I = V_{TL} = -L_+(R_1 / R_2)$ will trigger the circuit into the L_- state.

13.4.5 A Bistable Circuit with noninverting Transfer Characteristics

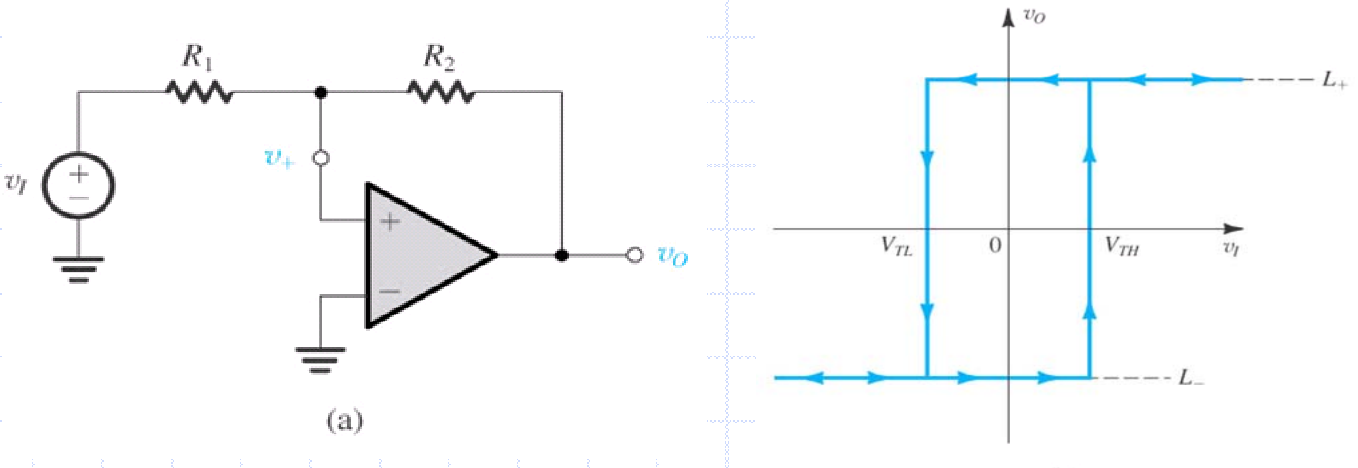


Figure 13.20 **(a)** A bistable circuit derived from the positive-feedback loop of Fig. 13.17 by applying v_I through R_1. **(b)** The transfer characteristic of the circuit in (a) is noninverting. (Compare it to the inverting characteristic in Fig. 13.19d.)

- If the circuit is in the negative stable state, $v_I = V_{TH} = -L_-(R_1 / R_2)$ will trigger the circuit into the L_+ state.
- Negative triggering signal $\longrightarrow$ Negative state.
- Positive triggering signal $\longrightarrow$ Positive state.

∴ The transfer characteristic of this circuit is non-inverting.

13.4.6 Application of the Bistable Circuit as a Comparator

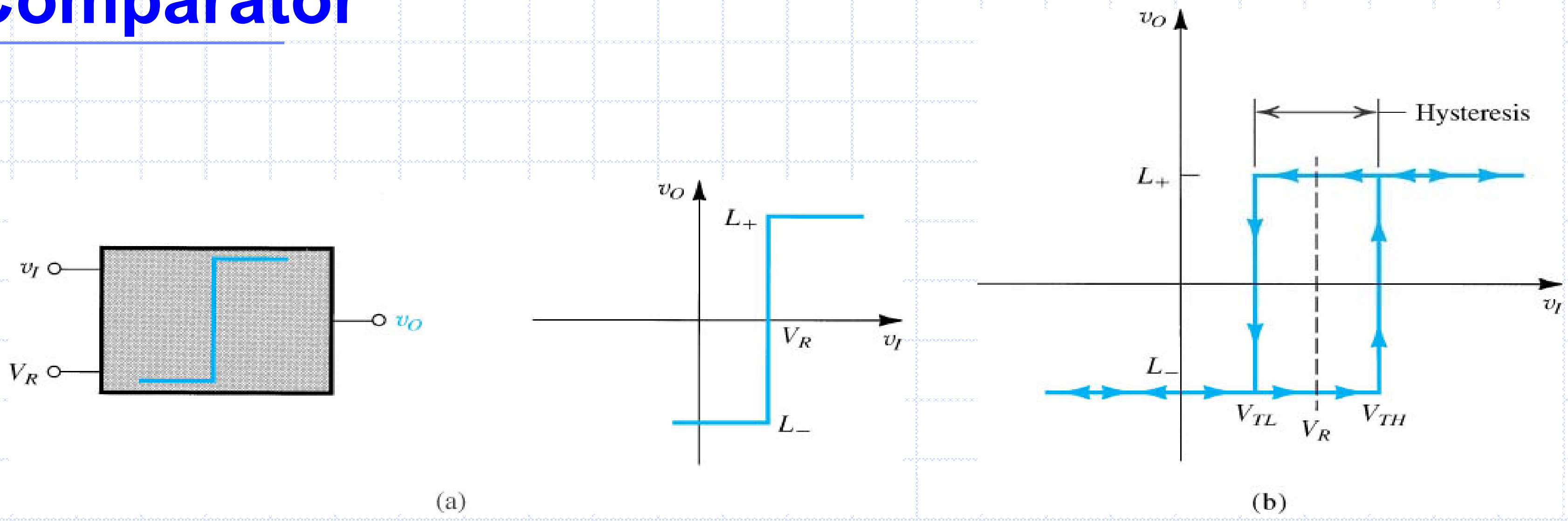


Figure 13.21 **(a)** Block diagram representation and transfer characteristic for a comparator having a reference, or threshold, voltage V_R. **(b)** Comparator characteristic with hysteresis.

- It is useful in many applications to add hysteresis to the comparator characteristics.
- The comparator exhibits two threshold values, V_{TL} and V_{TH}.
- Usually V_{TH} and V_{TL} are separated by a small amount(100mV).

13.4.6 Application of the Bistable Circuit as a Comparator

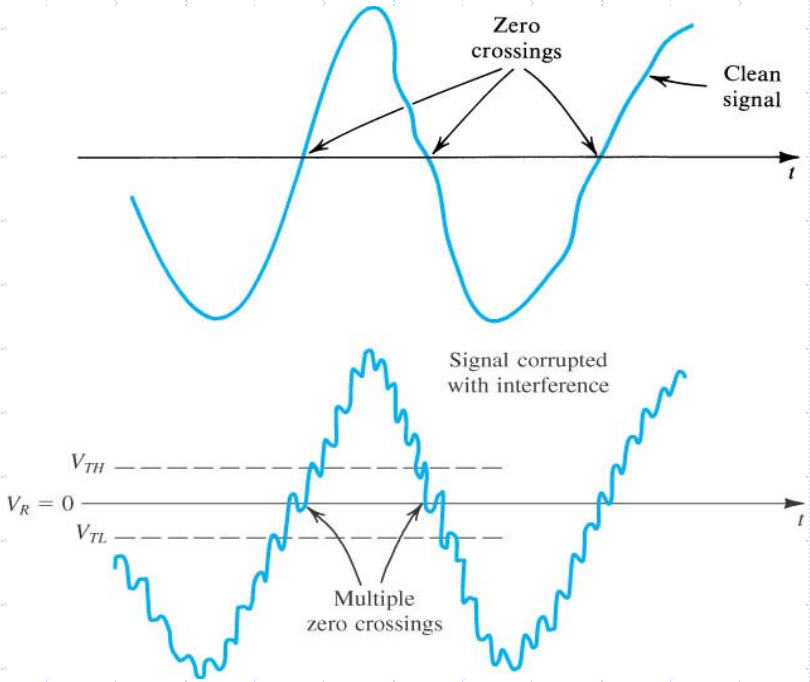


Figure 13.22 Illustrating the use of hysteresis in the comparator characteristics as a means of rejecting interference.

- To design a circuit that detects and counts the zero crossings of an arbitrary waveform.
- The comparator provides a step change at its output every time a zero crossing occurs.
- If the signal being processed has interference superimposed on it.

Solved by introducing hysteresis of appropriate width in the comparator characteristics.

13.4.7 Making the Output Levels more Precise

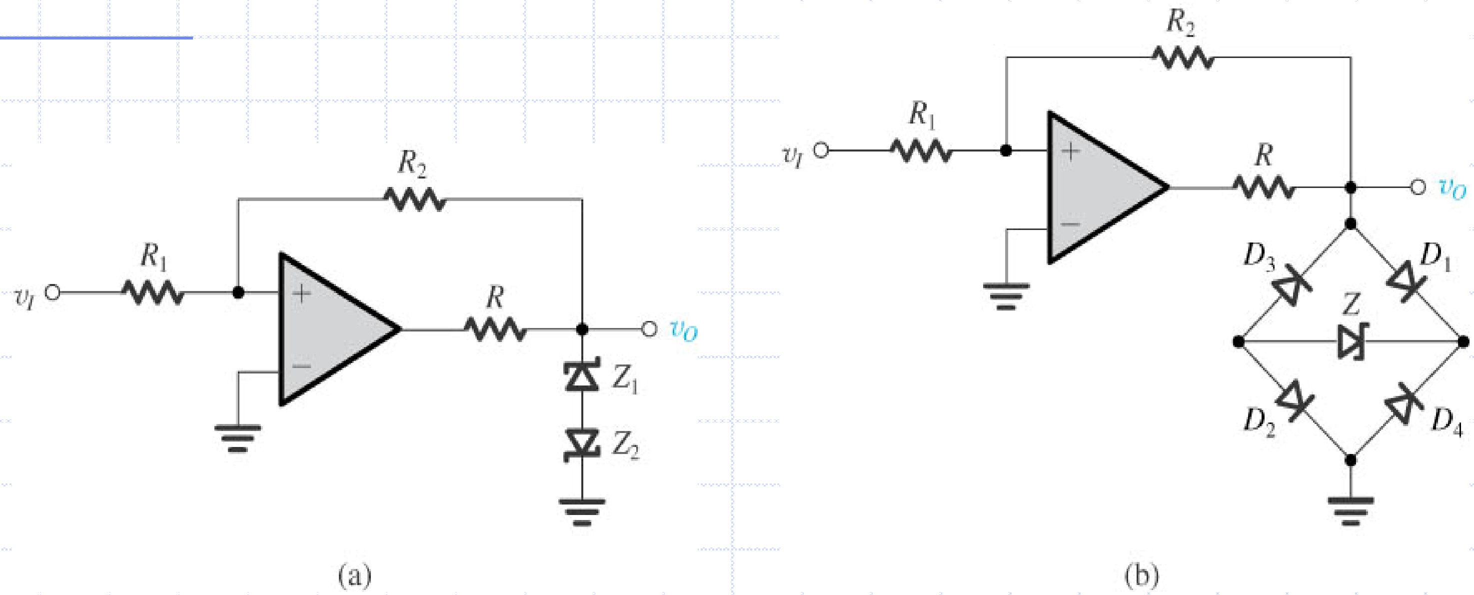
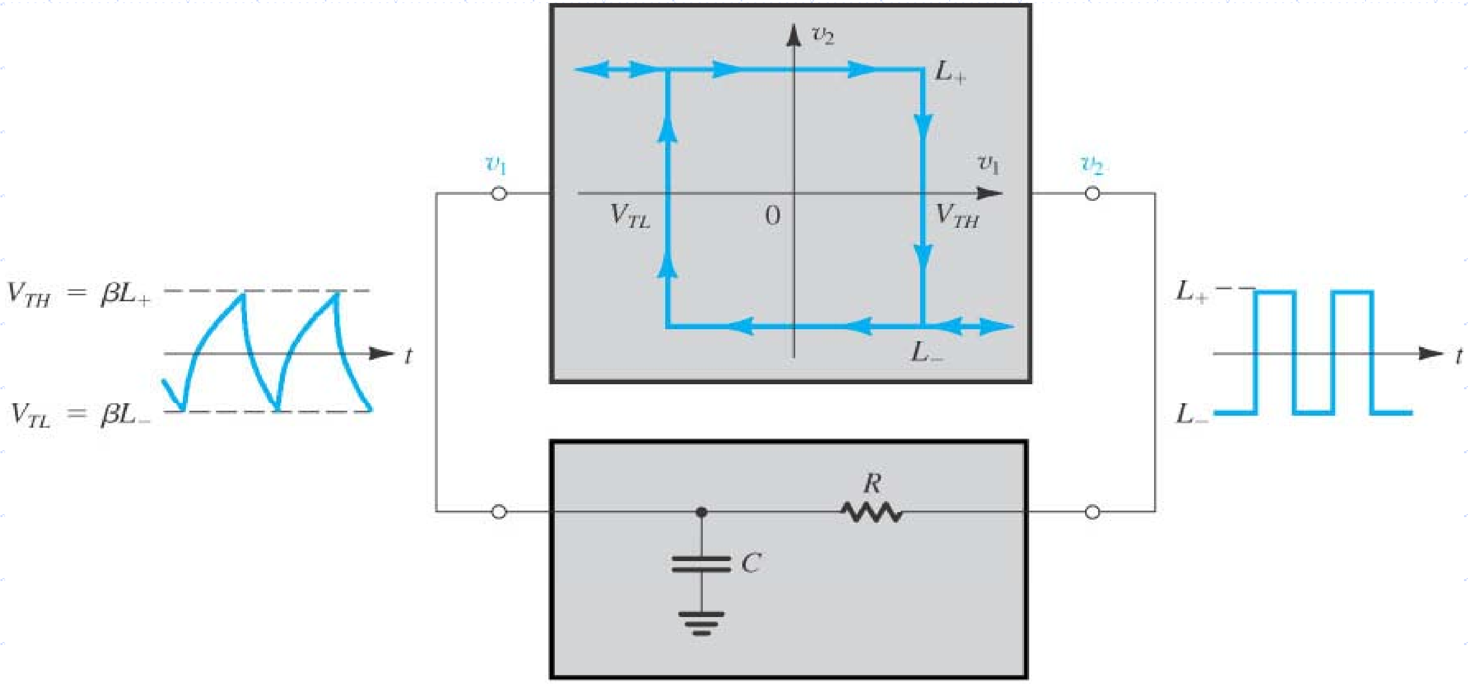


Figure 13.23 Limiter circuits are used to obtain more precise output levels for the bistable circuit. In both circuits the value of R should be chosen to yield the current required for the proper operation of the zener diodes. **(a)** For this circuit $L_+ = V_{Z_1} + V_D$ and $L_- = -(V_{Z_2} + V_D)$, where V_D is the forward diode drop. **(b)** For this circuit $L_+ = V_Z + V_{D_1} + V_{D_2}$ and $L_- = -(V_Z + V_{D_3} + V_{D_4})$.

- By cascading the op amp with a limiter circuit.

  → The output levels of the bistable circuit can be made more precise.

13.5 Generation of Square and Triangular Waveforms Using Astable Multivibrators



- Operation of the Astable Multivibrator
  - The bistable multivibrator with inverting transfer characteristics in a feedback loop with an RC circuit results in a square-wave generator.
  - The circuit has no stable state → Astable multivibrator

13.5.1 Operation of the Astable Multivibrator

- During the charging interval $T_1(\tau = RC)$

$$v_- = L_+ - (L_+ - \beta L_-)e^{-t/\tau}$$

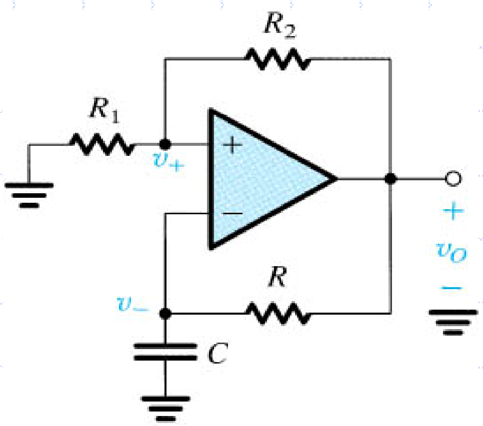
$$T_1 = \tau \ln \frac{1 - \beta(L_-/L_+)}{1 - \beta}$$

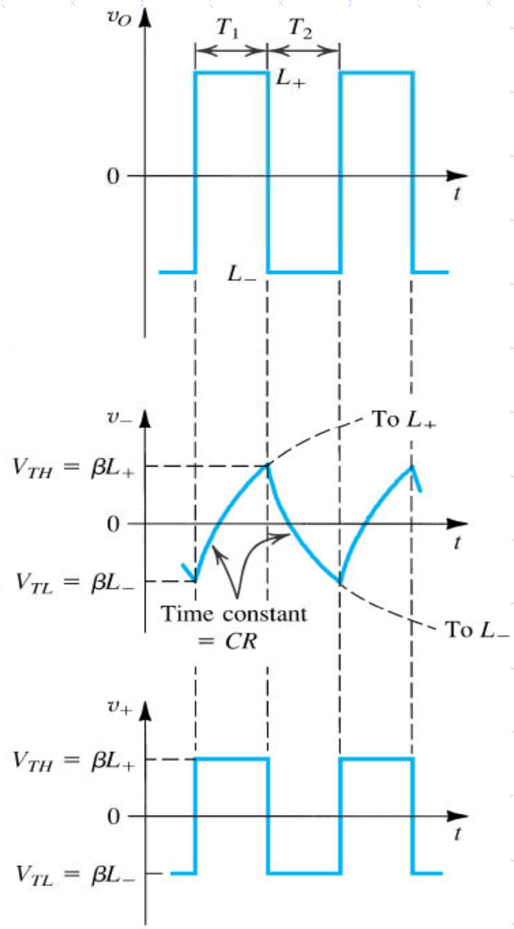
- Similarly T_2

$$v_- = L_- - (L_- - \beta L_+)e^{-t/\tau}$$

$$T_2 = \tau \ln \frac{1 - \beta(L_+/L_-)}{1 - \beta}$$

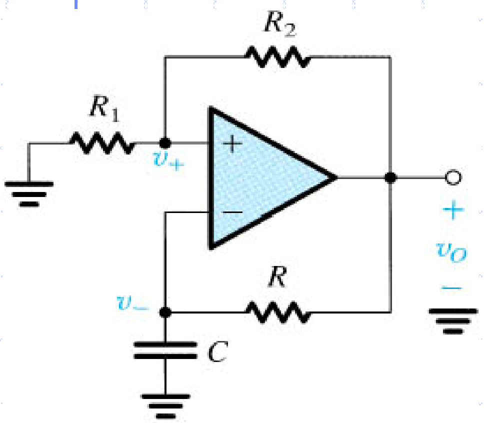
$$\therefore T = T_1 + T_2 = 2\tau \ln \frac{1 + \beta}{1 - \beta}$$

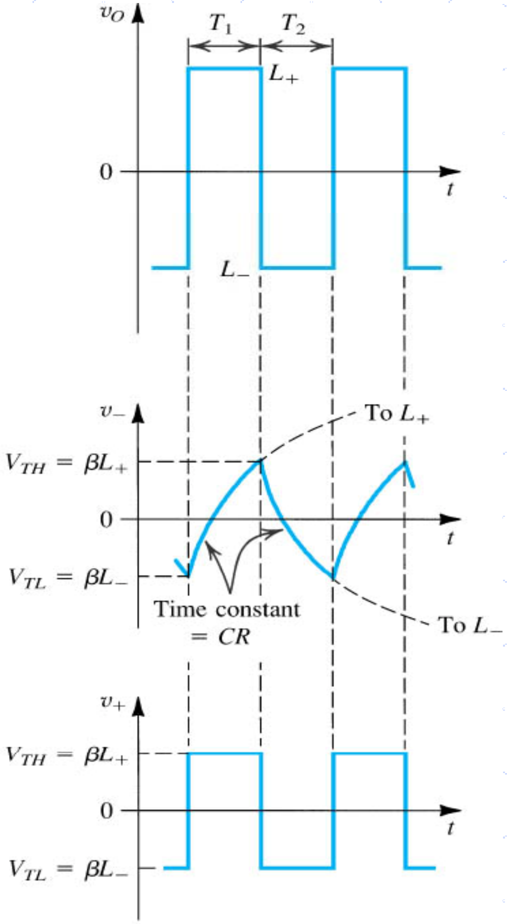




13.5.1 Operation of the Astable Multivibrator

- **Exercise 13.16**
  For the below circuit, let the op-amp saturation voltages be ±10V, R_1=100KΩ, R_2=R=1MΩ, and C=0.01uF. Find the Frequency of oscillation





$$\beta = \frac{R_1}{R_1 + R_2} = 0.091V/V$$

$$T = 2\tau \ln\left(\frac{1+\beta}{1-\beta}\right) = 0.00365\,\mathrm{sec}$$

$$f_o = \frac{1}{T} = 274Hz$$

13.5.2 Generation of Triangular Waveforms

- During the internal T_1

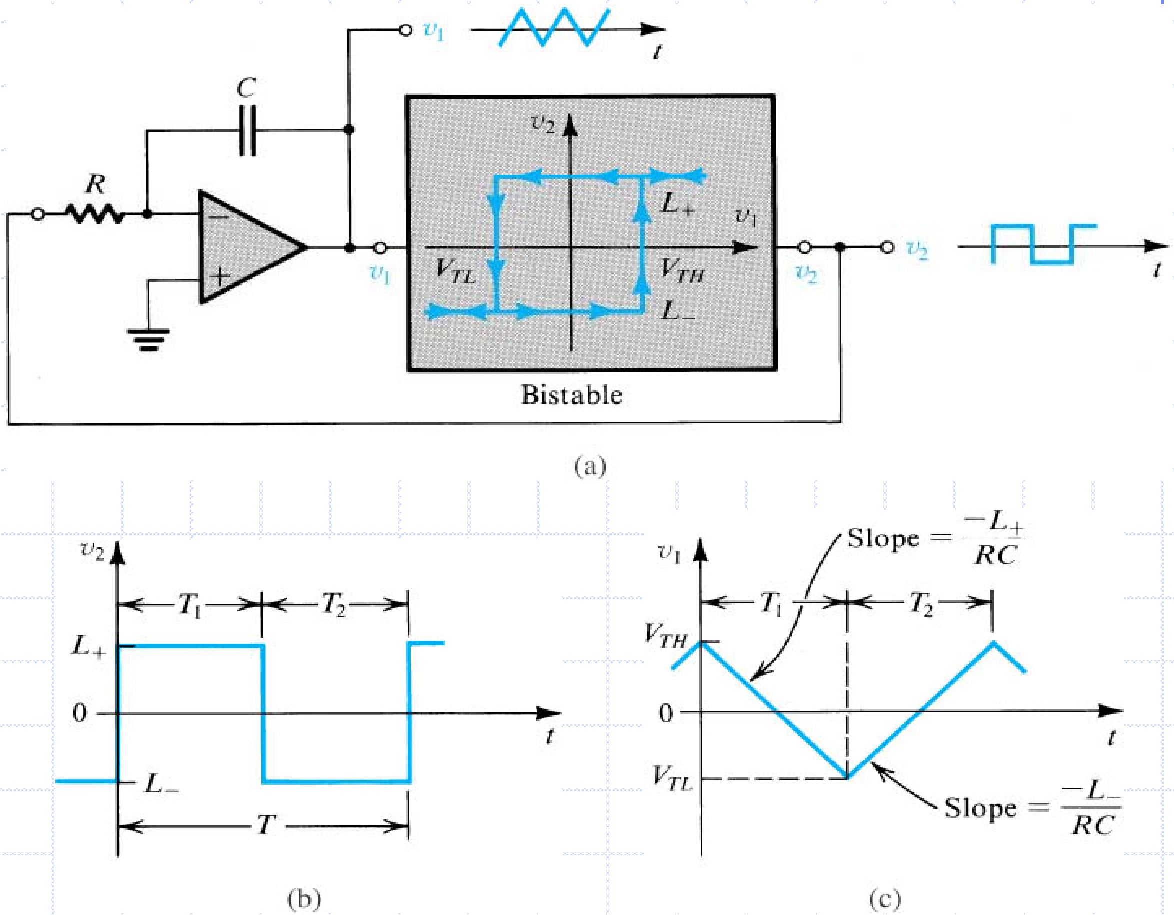
$$\frac{V_{TH} - V_{TL}}{T_1} = \frac{L_+}{CR}$$

$$T_1 = CR\,\frac{V_{TH} - V_{TL}}{L_+}$$

- Similarly

$$\frac{V_{TH} - V_{TL}}{T_2} = \frac{-L_-}{CR}$$

$$T_2 = CR\,\frac{V_{TH} - V_{TL}}{-L_-}$$



13.6 Generation of a Standardized Pulse – The Monostable Multivibrator

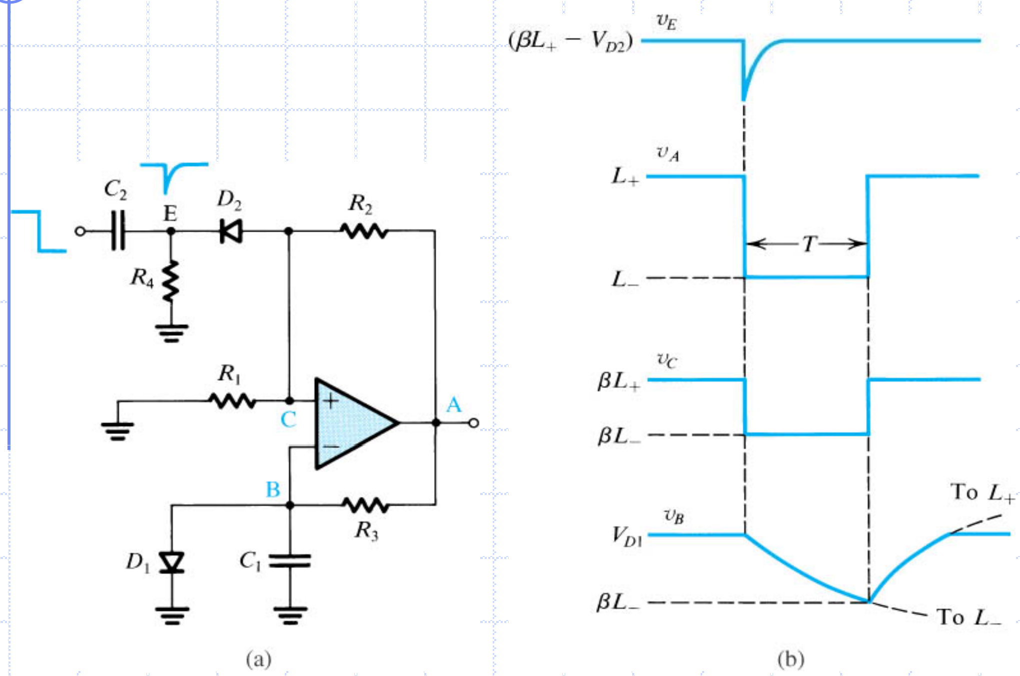


Figure 13.26 **(a)** An op-amp monostable circuit. (**b**) Signal waveforms in the circuit of (a).

- First, the multivibrator is at its stable state.
- Negative triggering edge pushes node E down.
- D_2 will conduct heavily, thus pulls node C down.
- If node C goes below B, the amp switches output to L_-.
- Now, D_1 does not conduct, so C_1 starts to discharge with time constant C_1R_3.
- If node B is discharged below C, the amp will switch output to L_+.
- The multivibrator goes back to its stable state.

13.6 Generation of a Standardized Pulse – The Monostable Multivibrator

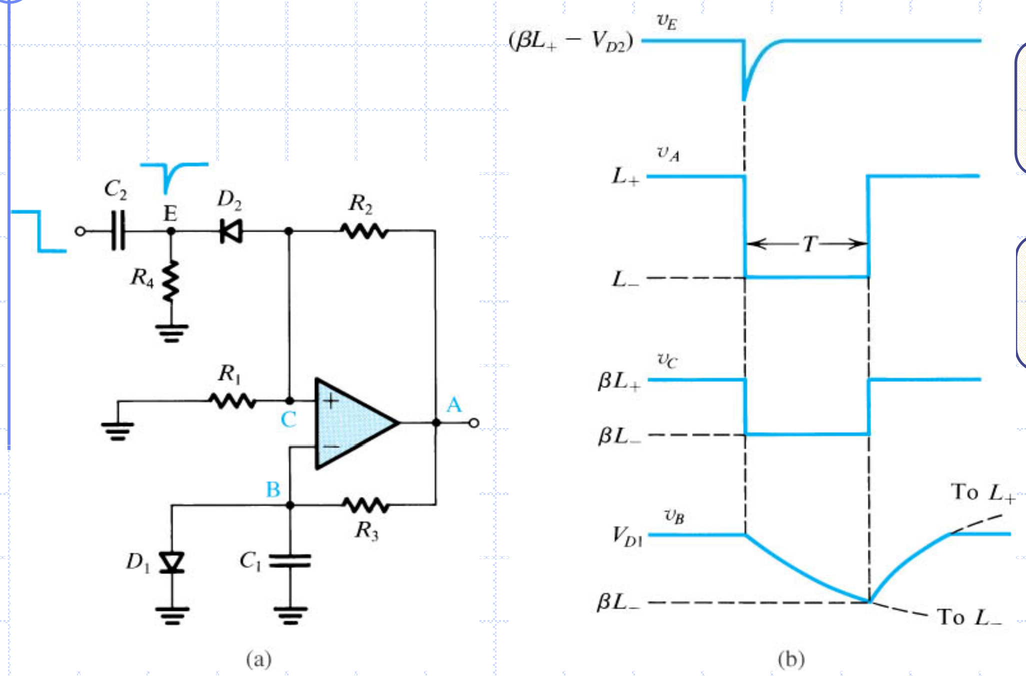


Figure 13.26 **(a)** An op-amp monostable circuit. (**b**) Signal waveforms in the circuit of (a).

$$v_B(t) = L_- - (L_- - V_{D1})e^{-t/C_1R_3}$$

$$\beta L_- = L_- - (L_- - V_{D1})e^{-T/C_1R_3}$$

(T : Recovery Period)

$$T = C_1R_3 \ln\left(\frac{V_{D1} - L_-}{\beta L_- - L_-}\right)$$

If $V_{D1} << |L_-|$

$$T \approx C_1R_3 \ln\left(\frac{1}{1-\beta}\right)$$