CHAPTER 1. INTRODUCTION

Reading assignments: Cheng Ch.1, Ulaby Ch.1,

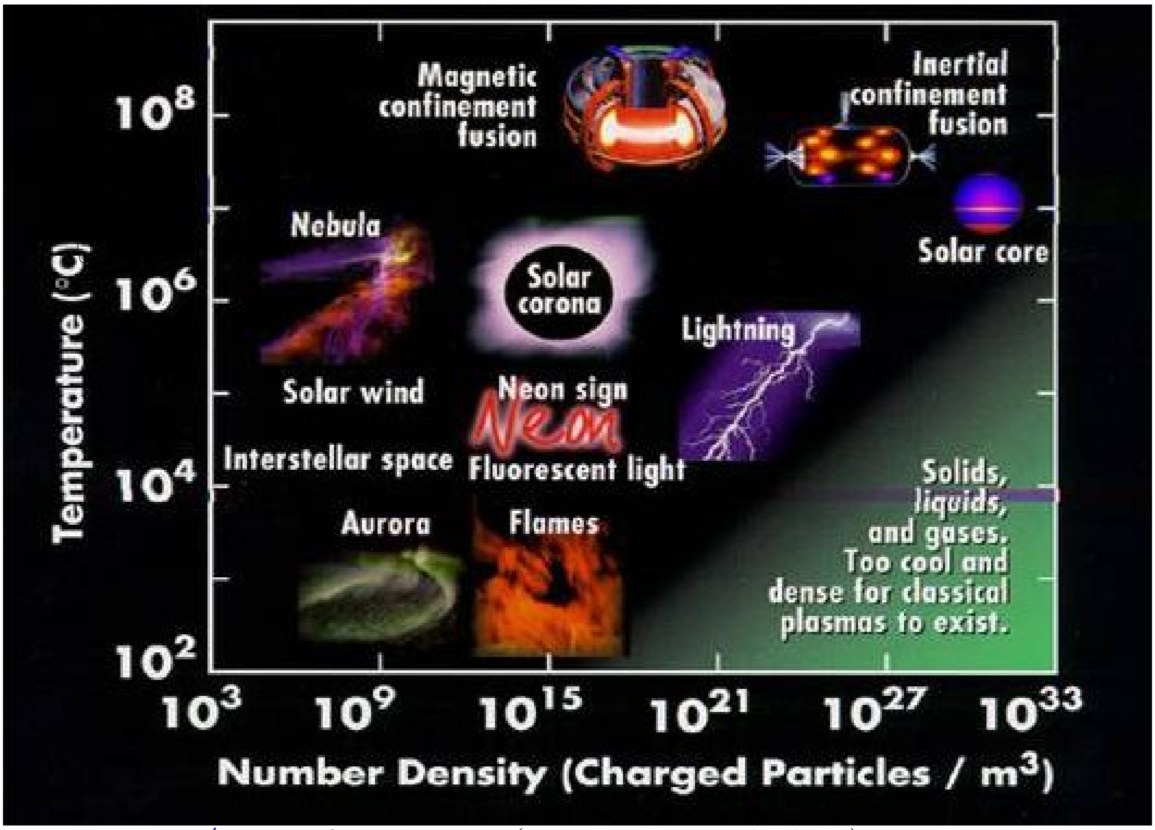
1. Electromagnetics in Plasmas

A. Plasma Electrodynamics

Plasma state: The 4th state of matter

	W ↘ fusion →		W ↘ vaporization →		W ↘ ionization →	
(s)olid		(l)iquid		(g)as		(p)lasma
	← solidification W ↵		← liquefaction W ↵		← recombination W ↵	

Plasma = Quasi-neutrally ionized gas (electrons + ions + atoms & molecules)
exhibiting collective behavior by long-range Coulomb forces



Laboratory/Processing plasmas (low T, weakly-ionized)

- Thermal (Arc): $T_i \approx T_e \approx T_a$ (LTE), high n, high heat (hot)
- Non-equilibrium (Glow, Corona): $T_e > T_i, T_a$; low n, low heat (cold)

Fusion plasmas: High T, fully-ionized, high power ($\gtrsim$ 100 MW)

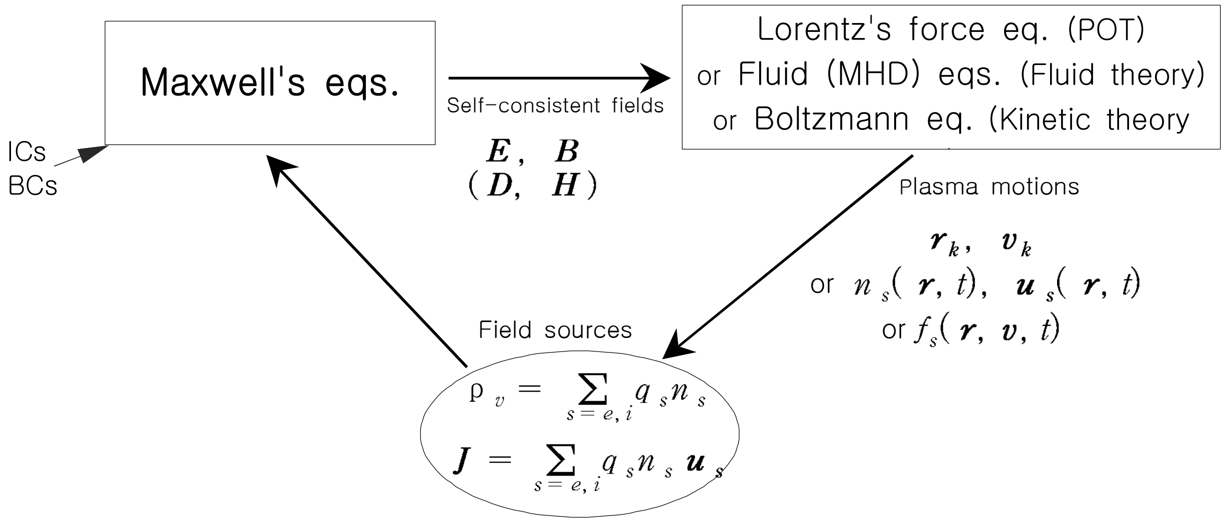
Space & astrophysical plasmas: Low T, low n ; Atm. & astro spaces
High T, high n ; Stars

Plasma dynamics

⇒ Electromagnetics + Fluid or Statistical mechanics (Thermodynamics)

↓ ↓ ↓ ↓

Maxwell's eqs. + Fluid eqs. Kinetic eq. (w/ therm. laws & coeffs.)



B. Role of Electromagnetics in Plasmas and Nuclear Engineering

- Fundamental plasma properties (Debye shielding, plasma oscillation, quasi-neutrality, ...)
- Basic plasma behavior in e.m. fields (lab, industry, fusion, space)
- Plasma generation by discharges (electrode, electrodeless, arc, glow, corona ...)
- Control of fusion and processing plasmas (circuits, devices, sensors)
- Magnetic confinement of plasmas (J x B forces, orbit motions, field coils, magnets, magnetic flux surfaces, divertors, ...)
- Plasma waves (O, X, R, L, Afven, magnetosonic, ...)
- Plasma instabilities (current driven, resistive, disruption, ...)
- Plasma heating and current drive by RF (LH, ICRF, ECRF, ..., transmission lines, wave guides, antennas)
- Radiation losses (bremsstrahlung, cyclotron, impurity, ...)
- Plasma diagnostics by charged particles and waves (beams, MW, lasers, spectroscopy, Langmuir & magnetic probes, ...)
- Plasma processing (radical generation, surface modification, physical and chemical reactions, decomposition, ...)
- Radiation detection and shielding in nuclear engineering
- Nuclear reactor control and instrumentation
- Generation, transmission, and supply of electric power

2. Electromagnetics and Electromagnetic Model

A. *Electromagnetics* *(Electromagnetism)*

Electromagnetics:

Study of (static or time-varying) electric and magnetic phenomena by the effects (fields) of electric charges at rest (charge densities) or in motion (currents).

▶ Maxwell's equations:

Integral form (global expression)	Vector form (point expression)		
$\oint_S \boldsymbol{D}\cdot d\boldsymbol{s} = \int_V \rho_v dv$	$\nabla\cdot\boldsymbol{D} = \rho_v$	Gauss's law	Ch.3
$\oint_S \boldsymbol{B}\cdot d\boldsymbol{s} = 0$	$\nabla\cdot\boldsymbol{B} = 0$	No isolated mag. pole	Ch.5
$\oint_C \boldsymbol{E}\cdot d\boldsymbol{l} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t}\cdot d\boldsymbol{s}$	$\nabla\times\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	Faraday's law	Chs.3,4,6
$\oint_C \boldsymbol{H}\cdot d\boldsymbol{l} = \int_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right)\cdot d\boldsymbol{s}$	$\nabla\times\boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$	Ampere's law	Chs.5,6

where constitutive relations are

$\boldsymbol{D} = \varepsilon\boldsymbol{E},\ \ \boldsymbol{B} = \mu\boldsymbol{H},$ ($\boldsymbol{J} = \sigma\boldsymbol{E}$: Ohm's law) for linear isotropic medium

$\boldsymbol{D} = \overleftrightarrow{\varepsilon}\cdot\boldsymbol{E},\ \ \boldsymbol{B} = \overleftrightarrow{\mu}\cdot\boldsymbol{H},$ ($\boldsymbol{J} = \overleftrightarrow{\sigma}\cdot\boldsymbol{E}$) in general

Notes)

i) $\epsilon = \epsilon_o$ in free space; scalar in dielectric medium; $\overleftrightarrow{\epsilon}$ in plasma, some crystal, ...
$\mu = \mu_o$ in free space and plasmas; scalar in magnetic medium
$\sigma = 0$ in free space; scalar in conductor; $\overleftrightarrow{\sigma}$ in plasma

ii) In general, all fields and sources are functions of position $\boldsymbol{r}$ and/or time t.
Static: $\partial/\partial t = 0$ (Stationary charges → Electrostatics – Ch.3;
Steady currents → Magnetostatics – Ch.5)

▶ Vector magnetic potential $\boldsymbol{A}$ and scalar electric potential V defined by

$$\boldsymbol{B} = \nabla\times\boldsymbol{A}\ ,\qquad \boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

in Maxwell's equations yield the wave equations for potentials $\boldsymbol{A}$ and V:

$$\left(\nabla^2 - \frac{1}{\mu\varepsilon}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix} V \\ \boldsymbol{A} \end{Bmatrix} = \begin{Bmatrix} -\rho_v/\varepsilon \\ -\mu\boldsymbol{J} \end{Bmatrix}$$

Notes) In static cases, $\nabla^2\begin{Bmatrix} V \\ \boldsymbol{A} \end{Bmatrix} = \begin{Bmatrix} -\rho_v/\varepsilon \\ -\mu\boldsymbol{J} \end{Bmatrix}$: Poisson's equations

▶ Maxwell's equations or wave equations are subject to the appropriate initial and boundary conditions in the e.m. system
⇒ (Initial) Boundary Value Problems for $\boldsymbol{E}$, $\boldsymbol{B}$ or $\boldsymbol{A}$, V

▶ Lorentz's force on an individual charge: $\boldsymbol{F} = \boldsymbol{F}_e + \boldsymbol{F}_m = q(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B})$
Electromagnetic body force in plasmas: $\boldsymbol{f} = \boldsymbol{f}_e + \boldsymbol{f}_m = \rho_v \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}$

▶ Electromagnetic energy:

$$W = W_e + W_m = \frac{1}{2}\int_{V'} (\boldsymbol{D}\cdot\boldsymbol{E} + \boldsymbol{H}\cdot\boldsymbol{B})\, dv = \frac{1}{2}\int_{V'} \left(\epsilon E^2 + \frac{B^2}{\mu}\right) dv$$

Electromagnetic energy density:

$$w = w_e + w_m = \frac{1}{2}(\boldsymbol{D}\cdot\boldsymbol{E} + \boldsymbol{H}\cdot\boldsymbol{B}) = \frac{\epsilon E^2}{2} + \frac{B^2}{2\mu}$$

▶ Conservation of electric charge (Equation of current continuity):

Gauss's law in $\nabla\cdot$(Ampere's law) $\Rightarrow$ $\nabla\cdot\boldsymbol{J} = -\frac{\partial \rho_v}{\partial t}$

For steady currents ($\partial\rho_v/\partial t = 0$),

$$\nabla\cdot\boldsymbol{J} = 0 \ \Rightarrow\ \oint_S \boldsymbol{J}\cdot d\boldsymbol{s} = 0 \ \Rightarrow\ \sum_j I_j = 0$$: Kirchhoff's current law

▶ Conservation of electromagnetic energy = Poynting's theorem:
H · (Faraday's) − E · (Ampere's) $\Rightarrow$ Energy conservation

$$\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H}) + \frac{\partial}{\partial t}\left(\frac{\boldsymbol{E}\cdot\boldsymbol{D}}{2} + \frac{\boldsymbol{H}\cdot\boldsymbol{B}}{2}\right) + \boldsymbol{E}\cdot\boldsymbol{J} = 0$$

Notes)

i) $\boldsymbol{E}\times\boldsymbol{H} \equiv \boldsymbol{S}$: *Poynting vector = e.m. power flux*

ii) energy inflow rate + e.m. energy accumulation rate + Joule dissipation = 0

B. Electromagnetic Model

1) Theoretical approaches to electromagnetics

Inductive approach

- ▹ Obtains general principles from particular facts or phenomena.
- ▹ Experimental observations → inferring laws & theorems from them
  (Experimental laws → generalizing them in steps
  → synthesizing in the form of Maxwell's eqs.)

Deductive (axiomatic) approach

- ▹ Conclusion reached from general laws to a particular case
- ▹ Postulates (axioms) of fundamental relations for an idealized model
  → derive particular laws & theorems
  → verified with experimental observations
  (Maxwell's eqs. → identifying each with the appropriate experimental law
  → specializing general eqs. to static & time-varying analysis)

cf.) Circuit model:
Lumped elements (DC/RF sources, resistor, capacitor, inductor, ...)
Basic circuit quantities (*V*, *I*, *R*, *L*, *C*, ...)

Field (Electromagnetic) model:
Distributed space sources (ρ_v rest charges, $\boldsymbol{J}$ moving charges)
Basic field quantities ($\boldsymbol{E}$, $\boldsymbol{D}$, $\boldsymbol{H}$, $\boldsymbol{B}$, $\boldsymbol{J}$, $\eta(1/\sigma)$, μ, ϵ, ...)

2) Axiomatic approach in three steps for developing a field theory

Step 1) Define the basic quantities of electromagnetics

Step 2) Specify the rules of mathematical operation

(Vector algebra & calculus, PDEs, Integrals, …)

Step 3) Present fundamental postulates (axioms)

in electrostatics, magnetostatics, and electromagnetics.

3) Source quantities

▹ Electric charge q (or Q) exists only in positive of negative integral multiples of an electron charge

$$e = 1.60 \times 10^{-19} \quad \text{(C)}, \tag{1-1}$$

▹ Charge sources at rest:

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v} \quad (\text{C/m}^3), \quad \text{volume charge density} \tag{1-2}$$

$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta q}{\Delta s} \quad (\text{C/m}^2), \quad \text{surface charge density} \tag{1-3}$$

$$\rho_\ell = \lim_{\Delta \ell \to 0} \frac{\Delta q}{\Delta \ell} \quad (\text{C/m}). \quad \text{line charge density} \tag{1-4}$$

▹ Current I is defined by

$$I = \frac{dq}{dt} \quad (\text{C/s or A}), \tag{1-5}$$

▹ Charge sources in motion (moving charges):

$$J = \lim_{\Delta s \to 0} \frac{\Delta I}{\Delta s} = nqu = \rho_v u \qquad (\text{A}/m^2), \quad \text{current density}$$

4) Field quantities

A field is a spatial distribution of a quantity generated by a charge source, which may of may not be a function of time.

TABLE 1-1 FUNDAMENTAL ELECTROMAGNETIC FIELD QUANTITIES

Symbols and Units for Field Quantities	Field Quantity	Symbol	Unit
Electric	Electric field intensity	**E**	V/m
	Electric flux density (Electric displacement)	**D**	C/m^2
Magnetic	Magnetic flux density	**B**	T
	Magnetic field intensity	**H**	A/m

3. Units and Universal Constants

A. SI (International System of Units) or MKSA Units

1) Dimension: defines physical characteristics

▹ Fundamental dimensions:
length(L), mass(M), time(T), electric current(I),
temperature($\mathfrak{I}$), luminous intensity($\mathcal{J}$)

▹ Other dimensions can be defined in terms of these fundamental dims.
(e.g.) volume – L^3, velocity – L/T, force – ML/T^2

2) Unit: Reference for numerical expression of dimension

TABLE **1-2** FUNDAMENTAL SI UNITS

Quantity	Unit	Abbreviation	
Length	meter	m	MKSA
Mass	kilogram	kg	MKSA
Time	second	s	MKSA
Current	ampere	A	MKSA
Temperature	kelvin	K	
Luminous intensity	candela	cd	

▹ Other units used in electromagnetics are derived units expressed in terms of m, kg, s, and A.
(e.g.) i) Units in Table 1–1:
C = A · s, V/m = kg · m/A · s^3 , T = kg/A · s^2
ii) Units of derived quantities: Appendix A–2

▹ Other unit systems: Gaussian, Electrostatic(esu), Electromagnetic(emu)

B. Universal Constants

The velocity of light in free space:

$$c \cong 3 \times 10^8 \quad \text{(m/s)}. \quad \text{(in free space)} \tag{1-6}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad \text{(m/s)}, \quad \text{(in free space)} \tag{1-10}$$

TABLE 1-3 UNIVERSAL CONSTANTS IN SI UNITS

Universal Constants	Symbol	Value	Unit
Velocity of light in free space	c	$\cong 3 \times 10^8$	m/s
Permeability of free space	μ_0	$4\pi \times 10^{-7}$	H/m
Permittivity of free space	ϵ_0	$\cong \frac{1}{36\pi} \times 10^{-9}$ $\cong 8.854 \times 10^{-12}$	F/m

C. Symbols and Notation

Physical quantities: scalar, *italic*; q, ρ_v, n, c, ϵ_o, R, L, ...

Dimensions: scalar, *italic*; L/T , ML/T^2 ...

Units: scalar, roman; C, V/m = kg · m/A · s^3, K...

Metric prefixes: scalar, roman; Appendix A-3: G(10^9), M(10^6), k(10^3), m(10^{-3}), μ(10^{-6}), n(10^{-9})

Fields & Positions: vector, boldface; ***E, D, B, H, v***, ..., ***r***

Unit vectors: boldface with a hat on it; $\hat{r}$, $\hat{x}$, $\hat{\theta}$, $\hat{\boldsymbol{B}}$ (or $\boldsymbol{a}_r$, $\boldsymbol{a}_x$, $\boldsymbol{a}_\theta$, $\boldsymbol{a}_B$)

Coordinate systems: Cartesian (Rectangular) coordinates (x, y, z)
Cylindrical coordinates (r, ϕ, z)
Spherical coordinates (R, θ, ϕ)

cf.) In toroidal plasmas,
Toroidal coordinates (r, θ, ϕ)
Flux coordinates (ψ, χ, ϕ)