CHAPTER 6. Time-Varying Fields and Maxwell's Equations

Reading assignments: Cheng Ch.6-1~6-4, Ulaby Ch.5, Halliday Ch.32

1. Faraday's Law of Electromagnetic Induction

A. Faraday's Law in Time-Varying Magnetic Fields

1) Fundamental Postulates for Magnetic Induction

Differential form:

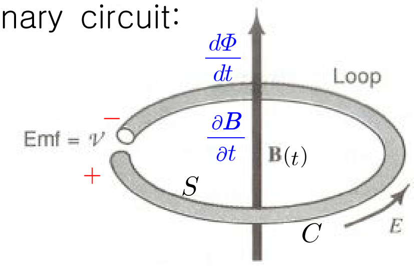
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \text{nonconservative } \boldsymbol{E} \text{ field } (\boldsymbol{E} \neq -\nabla V) \qquad (6\text{-}7)$$

Integral Form:

$$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{s} \qquad (6\text{-}8)$$

2) Faraday's Law in a closed circuit

Electromotive force (emf) induced in a stationary circuit:



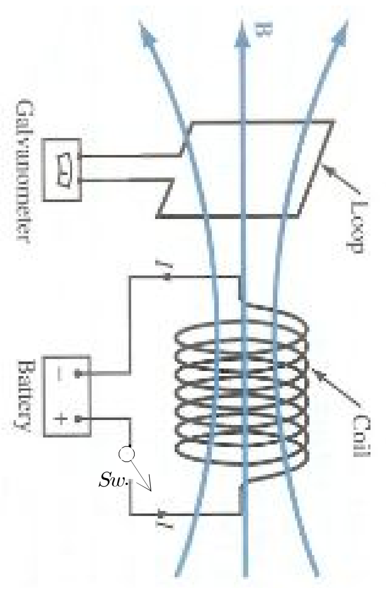
$$\mathscr{V} = \oint_C \boldsymbol{E} \cdot d\boldsymbol{l} \quad \text{(V)} \qquad (6\text{-}10)$$

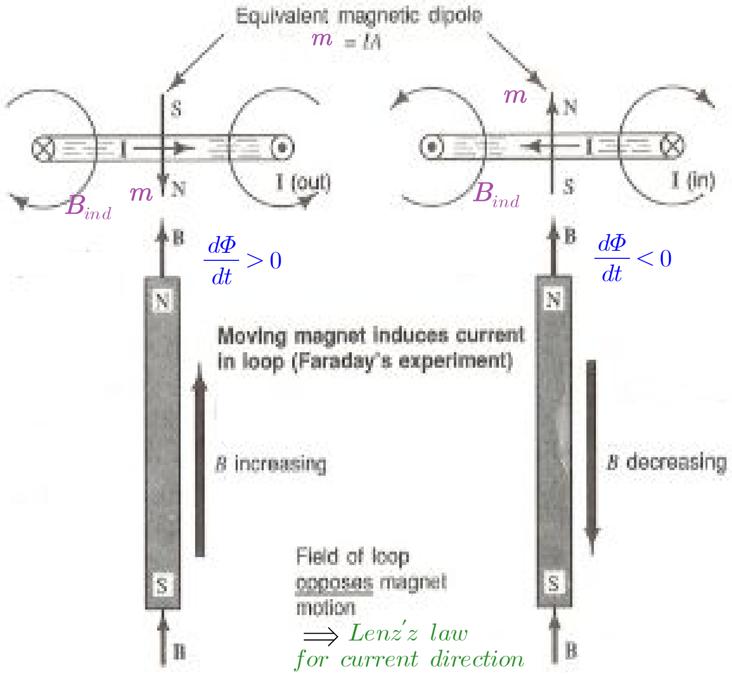
(6-10) in (6-8): $\mathscr{V} = -\frac{d}{dt}\int_S \boldsymbol{B} \cdot d\boldsymbol{s}$

$$\Rightarrow \quad \mathscr{V} = -\frac{d\Phi}{dt}$$: Faraday's law of electromagnetic induction (6-12)

(- sign by Lenz'z law: Induced current flows to oppose the flux change)

(cf) Experimental evidences (1831 M. Faraday, 1834 H.F. Lenz)





(Notes) Realization of Faraday's law

i) Time-varying $\boldsymbol{B}$ in a stationary circuit *(Transformer induction)*

ii) Moving circuit in a static $\boldsymbol{B}$ *(Motional or generator in어ction)*

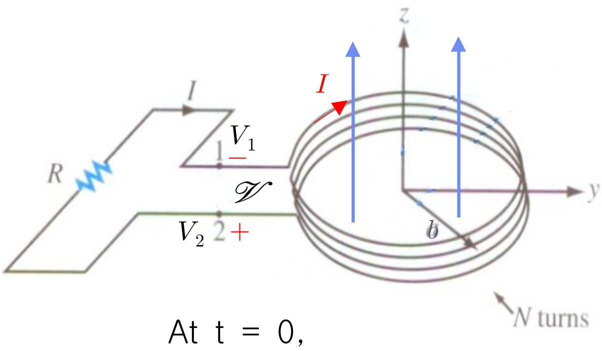
iii) Moving circuit in a time-varying $\boldsymbol{B}$ *(General case)*

B. Stationary Circuit in a Time-Varying Magnetic Field

$\Rightarrow$ Transformer induction ($\boldsymbol{B}$ change only)

$$\mathscr{V} = -\frac{\partial}{\partial t}\int_S \boldsymbol{B}\cdot d\boldsymbol{s} = -\frac{\partial \Phi}{\partial t} \qquad (1),(6\text{-}12)$$

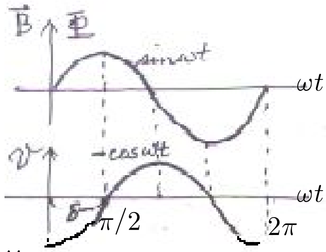
(e.g. 6-1) $B = \hat{z}B_o\cos(\pi r/2b)\sin\omega t$



$$\mathscr{V} = -\frac{\partial}{\partial t}(\sin\omega t)\int_0^b\left[\hat{z}B_o\cos(\pi r/2b)\right]\cdot(\hat{z}2\pi r\,dr)$$

$$= -\frac{8N}{\pi}b^2\left(\frac{\pi}{2}-1\right)B_o\omega\cos\omega t \qquad (\mathrm{V})$$

: $\mathscr{V}$ lags Φ and B by $\pi/2$



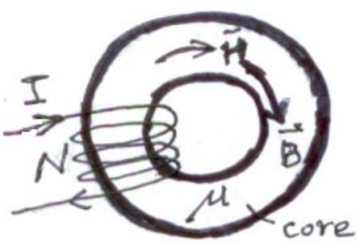
At t = 0,

$\frac{\partial B}{\partial t} > 0$, $\frac{\partial \Phi}{\partial t} > 0$, $\mathscr{V} < 0$, then, $V_2 > V_1$

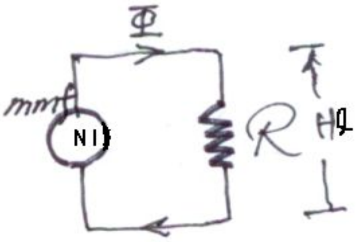
$\Rightarrow$ Current I flows clock-wise

to oppose $\frac{\partial B}{\partial t} > 0$, $\frac{\partial \Phi}{\partial t} > 0$ by Lenz's law.

1) Magnetic circuit



$\left(\begin{array}{c} cross-section\ area = S \\ length = l \end{array}\right)$



Equivalent Circuit

Magnetomotive force (mmf):

$$mmf = \oint_C \boldsymbol{H}\cdot d\boldsymbol{l} = NI \quad (\text{ampere-turn}) \quad \leftrightarrow \mathscr{V} \qquad (2)$$

(*Ampere's law*)

$$\text{Reluctance: } \mathscr{R} = \frac{mmf}{\Phi} = \frac{\oint \boldsymbol{H}\cdot d\boldsymbol{l}}{\int_S \boldsymbol{B}\cdot d\boldsymbol{s}} = \frac{Hl}{BS} = \frac{l}{\mu S} \quad (\mathrm{H}^{-1}) \qquad (3)$$

(cf) In electric circuit, Resistance = $R = \frac{V}{I} = \frac{l}{\sigma S}$ (Ω) (4-16)

2) Transformer

= An a-c device transforming voltages, currents, and impedances consisting of magnetically coupled coils thru a common core

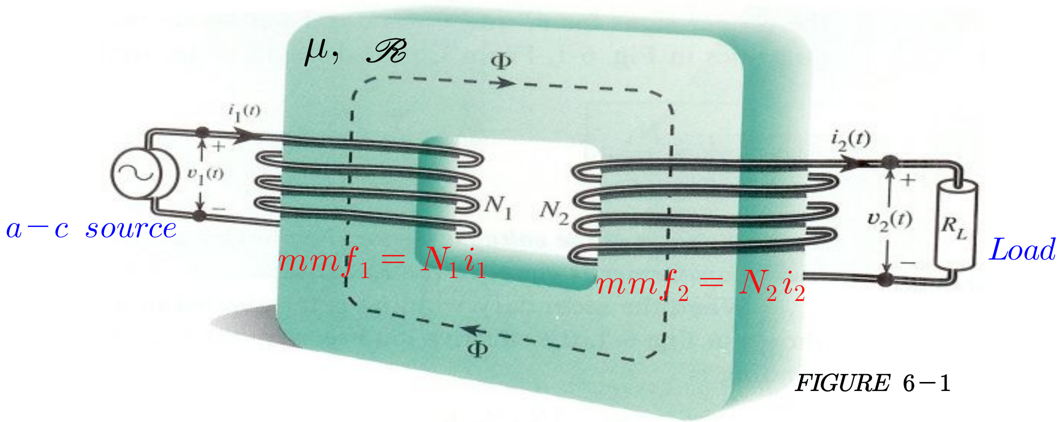


FIGURE 6−1

For the closed flux path in the magnetic circuit,

(3) $\Rightarrow$ $N_1 i_1 - N_2 i_2 = \mathscr{R}\Phi$ (6-13)↔(3-7)* Kirchhoff's voltage law

For ideal transformers ($\mathscr{R} = 0$, $\mu \to \infty$),

(6-13) $\Rightarrow$ $$\frac{i_1}{i_2} = \frac{N_2}{N_1}$$ (6-14)

From Faraday's law, $v_1 = N_1 \dfrac{d\Phi}{dt}$ and $v_2 = N_2 \dfrac{d\Phi}{dt}$ (6-15, 16)

$$\Rightarrow \frac{v_1}{v_2} = \frac{N_1}{N_2}$$ (6-17)

Effective load seen by the source :

$$R_{1,eff} = \frac{v_1}{i_1} = \frac{(N_1/N_2)v_2}{(N_2/N_1)i_2} = \left(\frac{N_1}{N_2}\right)^2 R_L$$ (6-19)

Effective impedance seen by the sinusoidal source :

$$Z_{1,eff} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$ (6-20)

However, in real transformers, Closed-path current induced by mag. induction

$\exists$ $\mathscr{R} \neq 0$, $\mu < \infty$,, $\Phi_{leakage}$, $R_{windings}$, hysteresis, eddy-current losses

Methods for reducing eddy-current power losses:

i) Use high-μ, low-σ core materials $\Rightarrow$ Ferrites

ii) Use laminated cores for low-frequency high-power applications.

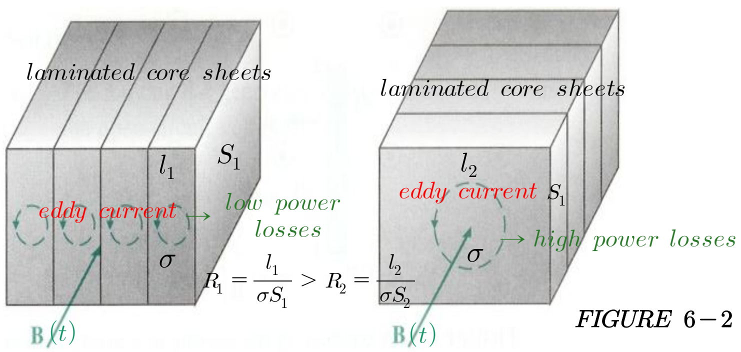
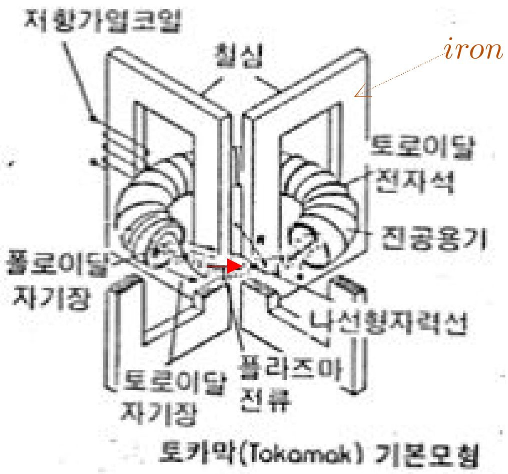
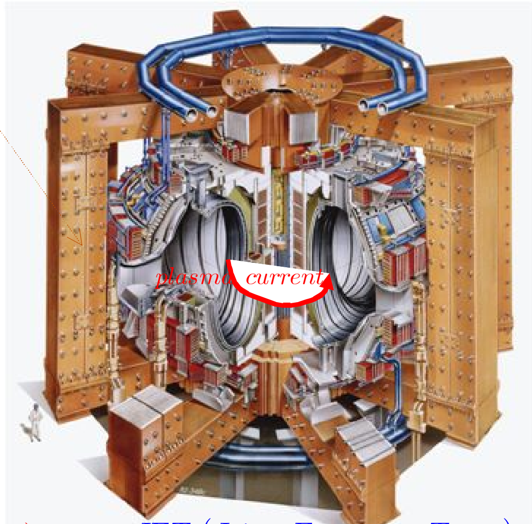


FIGURE 6−2

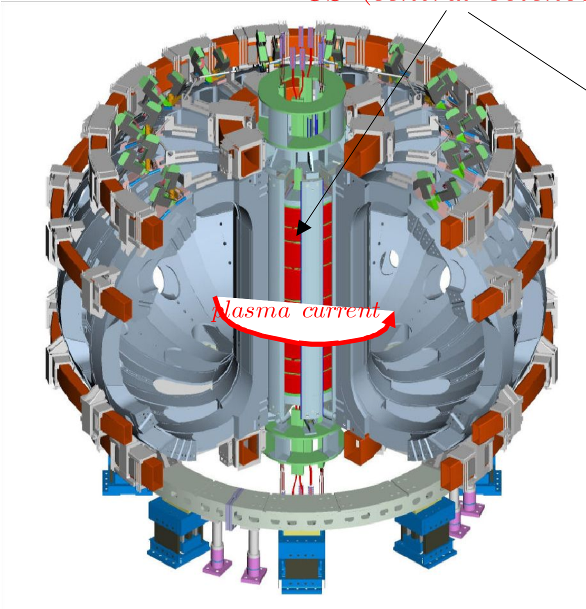
(cf) Plasma generation & current drive by transformer induction in tokamaks



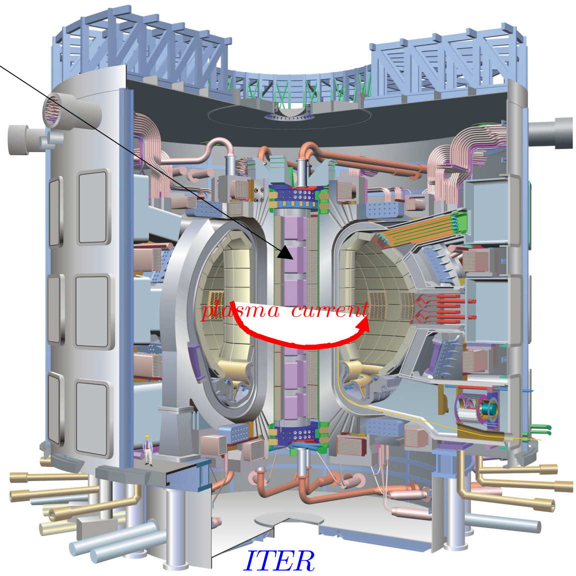


JET (Joint European Torus)

CS (central solenoid) coil



Magnet Systems of KSTAR Tokamak



ITER
(Int'l Thermonuclear Experimental Reactor)

C. Moving Conductor in a Static Magnetic Field

$\Longrightarrow$ Motional induction (motion only)

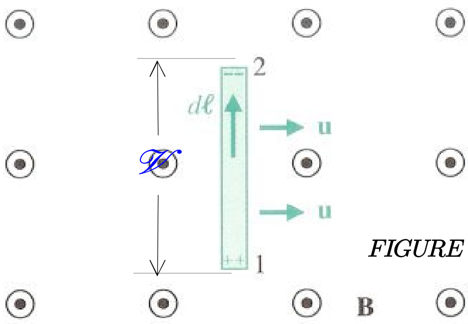


FIGURE 6−3

Force on a charge carrier in a conductor :

$$\boldsymbol{F}_m = q\boldsymbol{u} \times \boldsymbol{B}$$

Induced electric field along the conductor :

$$\boldsymbol{E} = \boldsymbol{F}_m / q = \boldsymbol{u} \times \boldsymbol{B}$$

Induced voltage across the conductor :

$$V_{21} = \int_1^2 \boldsymbol{E} \cdot d\boldsymbol{l} = \int_1^2 (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{l} \quad (6\text{-}21)$$

For a closed circuit conductor,

$$\mathscr{V}' = \oint_C (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{l}$$ (V) : motional (flux-cutting) emf (6−22)

(e.g. 6-2) A metal bar sliding over conducting rails

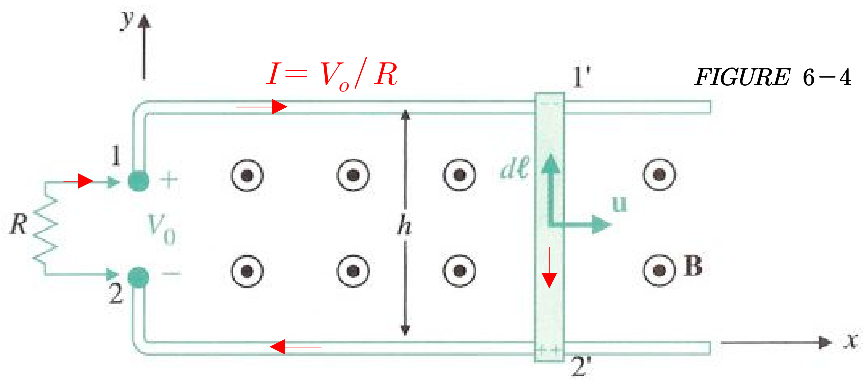


FIGURE 6−4

a) Open circuit voltage

$$V_o = V_1 - V_2 = \oint_C (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{l} = \int_{2'}^{1'} (\hat{\boldsymbol{x}} u \times \hat{\boldsymbol{z}} B_o) \cdot (\hat{\boldsymbol{y}}\, dl) = -\, u B_o h \quad \text{(6-23)}$$

b) Electric power dissipated in R

$$P_e = I^2 R = V_o^2 / R = (u B_o h)^2 / R \quad \text{(6-24)}$$

c) Mechanical power required to move the bar $\quad I = \dfrac{V_o}{R} = \dfrac{u B_o h}{R}$

$$P_m = \boldsymbol{F}_m \cdot \boldsymbol{u} = \left(I \int_{2'}^{1'} d\boldsymbol{l} \times \boldsymbol{B} \right) \cdot \boldsymbol{u} = (-\hat{\boldsymbol{x}} I B_o h) \cdot \hat{\boldsymbol{x}} u = (u B_o h)^2 / R = P_e$$

(e.g. 6-3) Faraday disk generator (Homopolar generator)

Open-circuit voltage:

$$V_o = \oint_C (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{l}$$
$$= \int_3^4 [(\hat{\boldsymbol{\phi}} r \omega) \times \hat{\boldsymbol{z}} B_o] \cdot (\hat{\boldsymbol{r}}\, dr)$$
$$= \omega B_o \int_b^0 r\, dr$$
$$= -\frac{\omega B_o b^2}{2} \quad \text{(V)} \quad \text{(6-28)}$$

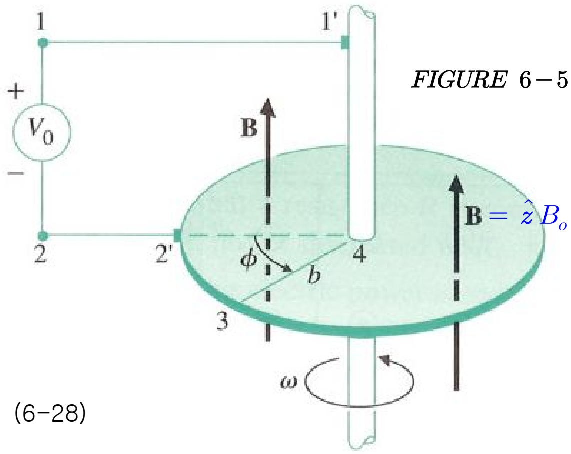


FIGURE 6−5

D. Moving Circuit in a Time-Varying Magnetic Field

$\Longrightarrow$ Total emf in moving frame = Transformer emf + Motional emf

(1),(6-22) $\Longrightarrow$ $$\mathscr{V}' \equiv \oint_C \boldsymbol{E}' \cdot d\boldsymbol{l} = -\frac{\partial}{\partial t} \int_S \boldsymbol{B} \cdot d\boldsymbol{s} + \oint_C (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{l} \quad \text{(6-32)}$$

: General form of Faraday's law

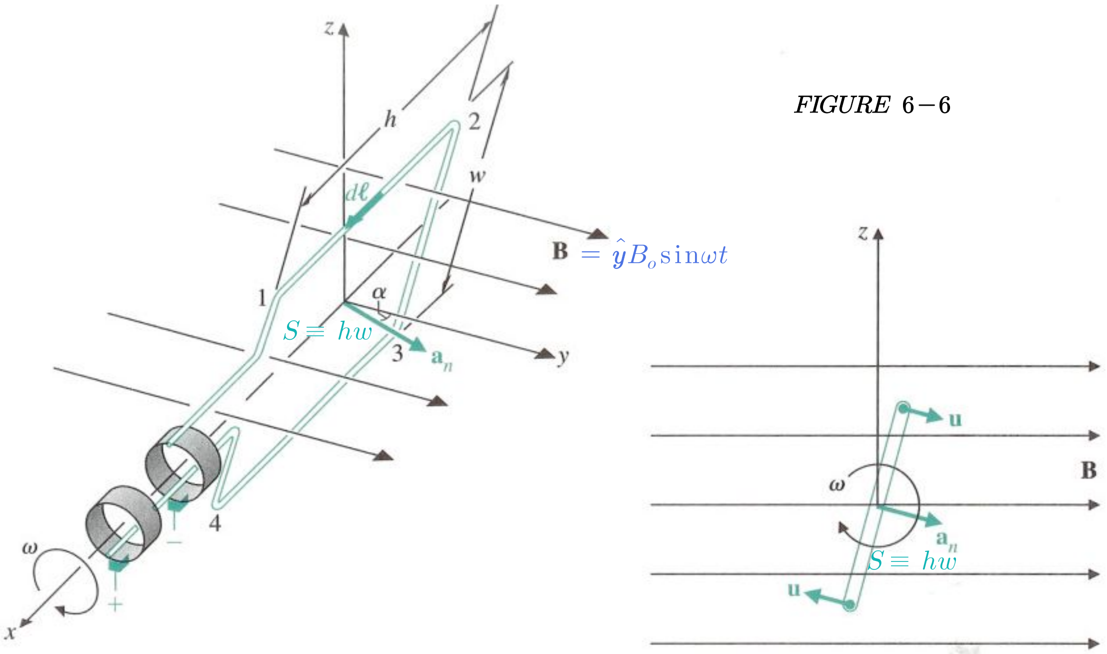
$$\Longrightarrow \oint_C \boldsymbol{E}' \cdot d\boldsymbol{l} = \oint_C [\boldsymbol{E} + (\boldsymbol{u} \times \boldsymbol{B})] \cdot d\boldsymbol{l} \Longrightarrow \boldsymbol{E}' = \boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} \quad \text{(6-31)}$$

(6-32) $\Longrightarrow$ $$\mathscr{V}' = -\frac{d}{dt} \int_S \boldsymbol{B} \cdot d\boldsymbol{s} = -\frac{d\Phi}{dt} \quad \text{(v)} \quad \text{(6-34)}$$

(ex. 6-3) *(e.g. 6-2)* using (6-34)

$$V_o = -\frac{d}{dt}\int_S \boldsymbol{B}\cdot d\boldsymbol{s} = -\frac{d\Phi}{dt} = -B_o\left(\frac{dx}{dt}h\right) = -B_o u h \quad \equiv \text{(6-23)}$$

(e.g. 6-5) *2nd harmonic* generator



(a) Perspective view. (b) View from $+x$ direction.

a) For the loop at rest ($\boldsymbol{u}=0,\ \partial B/\partial t \neq 0$),

(1),(6-12) $\Rightarrow$ $\mathscr{V}_a = -\frac{\partial}{\partial t}\int_S \boldsymbol{B}\cdot d\boldsymbol{s} = -\frac{\partial}{\partial t}[(\hat{\boldsymbol{y}}B_o \sin\omega t)\cdot(\hat{\boldsymbol{n}}hw)]$

$\hat{y}\cdot\hat{n}hw = S\cos\alpha$

$= -B_o S\omega\cos\omega t\cos\alpha$: transformer emf (6-37)

b) For the rotating loop ($\boldsymbol{u}\neq 0,\ \partial B/\partial t \neq 0$),

Motional emf in (6-32):

$$\mathscr{V}_a^* = \oint_C (\boldsymbol{u}\times\boldsymbol{B})\cdot d\boldsymbol{l} = \int_2^1 \left(\hat{\boldsymbol{n}}\frac{w}{2}\omega\right)\times(\hat{\boldsymbol{y}}B_o\sin\omega t)\cdot(\hat{\boldsymbol{x}}dx)$$

$$+\int_4^3\left(-\hat{\boldsymbol{n}}\frac{w}{2}\omega\right)\times(\hat{\boldsymbol{y}}B_o\sin\omega t)\cdot(\hat{\boldsymbol{x}}dx)$$

$$= B_o S\omega\sin\omega t\sin\alpha \qquad \text{(6-38)}$$

Total emf for $\alpha = 0$ at $t=0$:

$$\mathscr{V}_t = \mathscr{V}_a + \mathscr{V}_a^* = -B_o S\omega(\cos\omega t\cos\omega t + \sin\omega t\sin\omega t)$$

$$= -B_o S\omega\, \underline{\cos 2\omega t} \qquad \text{(6-39)}$$

(cf) Other way using (6-34) directly:

$$\Phi(t) = \boldsymbol{B}(t)\cdot[\hat{\boldsymbol{n}}(t)S] = B_o S\sin\omega t\cos\alpha = B_o S\sin\omega t\cos\omega t$$
$$= (1/2)B_o S\sin 2\omega t$$

$$\mathscr{V}_t' = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\frac{1}{2}B_o S\sin 2\omega t\right) = -B_o S\omega\,\underline{\cos 2\omega t} \quad \text{(6-39)}$$

2. Ampere's Law in Electromagnetic Fields

A. Generalization of Ampere's Law in Time-Varying Electric Field

Ampere's law in magnetostatics,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \qquad (5\text{-}61)(6\text{-}5)(6\text{-}40b)$$

free currents: coduction($\sigma \boldsymbol{E}$) and convection($\rho_v \boldsymbol{u}$) currents

$$\Rightarrow \quad \nabla \cdot \boldsymbol{J} = \nabla \cdot (\nabla \times \boldsymbol{H}) = 0 \qquad (5\text{-}8)(6\text{-}42)$$

which disagree with current continuity equation (charge conservation):

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho_v}{\partial t} \qquad (4\text{-}20)(6\text{-}41)$$

Suppose an unknown term $\mathscr{J}$ in the time-varying case,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \mathscr{J}$$

$$\Rightarrow \quad \nabla \cdot (\nabla \times \boldsymbol{H}) = 0 = \nabla \cdot \boldsymbol{J} + \nabla \cdot \mathscr{J}$$

$$\Rightarrow \quad \nabla \cdot \mathscr{J} = -\nabla \cdot \boldsymbol{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \boldsymbol{D}) = \nabla \cdot \left(\frac{\partial \boldsymbol{D}}{\partial t}\right)$$

(6-41) $\quad \nabla \cdot \boldsymbol{D} = \rho_v$

$$\Rightarrow \quad \mathscr{J} = \frac{\partial \boldsymbol{D}}{\partial t} \equiv \boldsymbol{J}_D \text{ : Displacement current density} \qquad (4)$$

$$\therefore \quad \nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad (6\text{-}44)$$

: Generalized Ampere's law which is consistent with the charge conservation

Integral form : $\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = \int_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot d\boldsymbol{s} = I_C + I_D$ (6-46b)

B. Concept of Displacement current

Displacement current: $\boldsymbol{J}_D \equiv \frac{\partial \boldsymbol{D}}{\partial t}$ (A/m^2) (4)

- does not carry real charge, but behaves like real current.
- is extension of current concept to include the charge-free space.
- was first introduced by J.C. Maxwell in 1873 to unified connection between electric and magnetic fields under time-varying conditions.

(e.g. 6-6)

a)
$$\begin{cases} i_C = C_1 \dfrac{dv_c}{dt} = \left(\dfrac{\epsilon A}{d}\right)\dfrac{dv_c}{dt} \\ i_D = \displaystyle\int_A \left(\frac{\partial D}{\partial t}\right) \cdot ds = \int_A \epsilon \left(\frac{\partial E}{\partial t}\right) \cdot ds \end{cases}$$

$E = v_c / d$

$$= \left(\frac{\epsilon A}{d}\right)\frac{dv_c}{dt} \quad \Rightarrow \quad i_C = i_D$$

b) $\oint_C H \cdot dl = \int_{S_1} J_C \cdot ds$ or $\int_{S_2} J_D \cdot ds$

$$\Rightarrow \quad 2\pi r H_\phi = i_C \text{ or } i_D \quad \Rightarrow \quad H_\phi = i_C / 2\pi r = i_D / 2\pi r$$

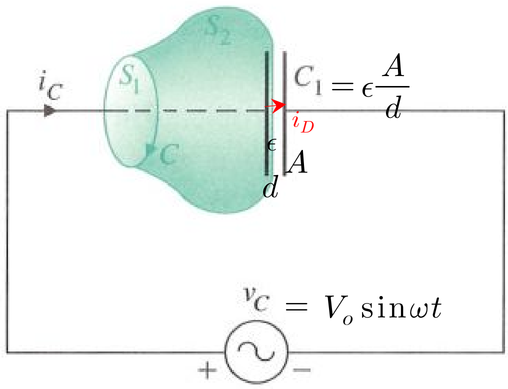


FIGURE 6−7

3. Summary of Maxwell's Equations for Electromagnetics

TABLE 6-1

Differential Form	Integral Form	Significance	
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E}\cdot d\boldsymbol{\ell} = -\dfrac{d\Phi}{dt} = -\int_S \dfrac{\partial B}{\partial t}\cdot ds$	Faraday's law	$(6-45,46a)$
$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_C \mathbf{H}\cdot d\boldsymbol{\ell} = I + \int_S \dfrac{\partial \mathbf{D}}{\partial t}\cdot ds$ (with $I = \int_S J\cdot ds$)	Ampère's circuital law	$(6-45,46b)$
$\nabla\cdot\mathbf{D} = \rho_v$	$\oint_S \mathbf{D}\cdot ds = Q = \int_V \rho_v dv$	Gauss's law	$(6-45,46c)$
$\nabla\cdot\mathbf{B} = 0$	$\oint_S \mathbf{B}\cdot ds = 0$	No isolated magnet	$(6-45,46d)$

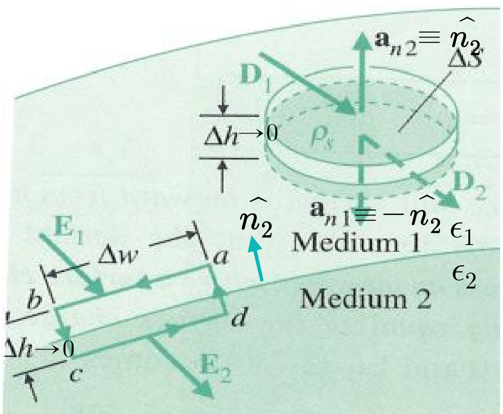
Notes) i) J & ρ_v are free sources.

ii) $\nabla\cdot(6-45b)$ in $\dfrac{\partial}{\partial t}(6-45c)$ yields (6-41) $\nabla\cdot J = -\dfrac{\partial\rho_v}{\partial t}$

iii) (6-45a) and (6-45d) are not independent.

4. Summary of Boundary Conditions for Electromagnetics

Field Components	General Form	TABLE 6-2: Medium 1 Dielectric	TABLE 6-2: Medium 2 Dielectric	TABLE 6-3: Medium 1 Dielectric	TABLE 6-3: Medium 2 Conductor
Tangential E	$\hat{n}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$	$E_{1t} = E_{2t}$		$E_{1t} = E_{2t} = 0$	
Normal D	$\hat{n}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$	$D_{1n} - D_{2n} = \rho_s$		$D_{1n} = \rho_s$	$D_{2n} = 0$
Tangential H	$\hat{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$	$H_{1t} = H_{2t}$		$H_{1t} = J_s$	$H_{2t} = 0$
Normal B	$\hat{n}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$	$B_{1n} = B_{2n}$		$B_{1n} = B_{2n} = 0$	



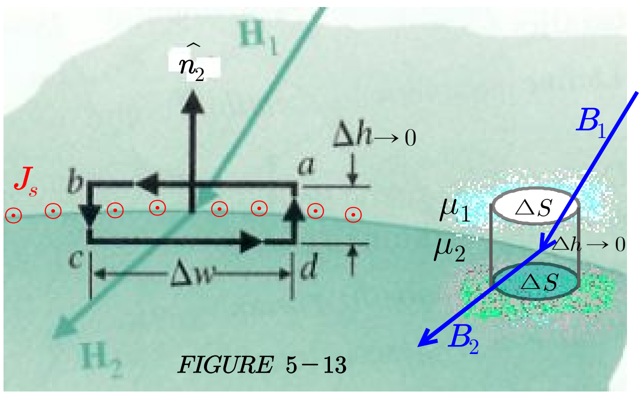


FIGURE 5−13

5. Electromagnetic Potentials

Vector magnetic potential (5-14):

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \qquad (6\text{-}50)$$

(6-50) in (6-45a) $\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$:

$$\nabla \times \left(\boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t}\right) = 0 \qquad (6\text{-}52)$$

$$\Rightarrow \quad \boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t} = -\nabla V$$

$$\Rightarrow \quad \boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t} \equiv \boldsymbol{E}_V + \boldsymbol{E}_A \quad \text{(V/m)} \qquad (6\text{-}53)$$

(6-53) in (6-45c):

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} - \frac{\partial}{\partial t}(\nabla \cdot \boldsymbol{A}) \qquad (6\text{-}45c)*$$

(6-50), (6-53) in (6-45b) using $\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A}$:

$$\nabla^2 \boldsymbol{A} - \mu\varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu \boldsymbol{J} + \nabla\left(\nabla \cdot \boldsymbol{A} + \mu\varepsilon \frac{\partial V}{\partial t}\right) \qquad (6\text{-}55)$$

Lorentz gauge transformations:

$$\boldsymbol{A} \rightarrow \boldsymbol{A}' = \boldsymbol{A} + \nabla R \quad (R = \textit{gauge function})$$

$$V \rightarrow V' = V - \frac{\partial R}{\partial t}$$

do not affect *B* and *E* in (6-50) and (6-53) : Gauge invariance

(Proof) $\boldsymbol{B} = \nabla \times \boldsymbol{A}' = \nabla \times \boldsymbol{A} + \nabla \times \nabla R = \nabla \times \boldsymbol{A}$

$$\boldsymbol{E} = -\nabla V' - \frac{\partial \boldsymbol{A}'}{\partial t} = \left(-\nabla V + \cancel{\frac{\partial \nabla R}{\partial t}}\right) - \left(\frac{\partial \boldsymbol{A}}{\partial t} + \cancel{\frac{\partial \nabla R}{\partial t}}\right)$$

$$= -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

Choose *R* so that

$$\nabla \cdot \boldsymbol{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0 \quad : \text{ Lorentz condition} \qquad (6\text{-}56)$$

$$\Rightarrow \quad \nabla^2 R - \mu\varepsilon \frac{\partial^2 R}{\partial t^2} = 0 \qquad (6\text{-}56)*$$

Then (6-45c)*, (6-55):

$$\left(\nabla^2 - \mu\varepsilon \frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix} V \\ \boldsymbol{A} \end{Bmatrix} = \begin{Bmatrix} -\rho_v/\varepsilon \\ -\mu \boldsymbol{J} \end{Bmatrix} \quad : \text{ Wave equation} \qquad (6\text{-}57)(6\text{-}58)$$

(cf) i) Time-independent solutions :

$$V(\boldsymbol{R}) = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho_v(\boldsymbol{R}')}{|\boldsymbol{R} - \boldsymbol{R}'|} dv' \quad \text{(V)} \qquad (3\text{-}38)$$

$$\boldsymbol{A}(\boldsymbol{R}) = \frac{\mu}{4\pi}\int_{V'} \frac{\boldsymbol{J}(\boldsymbol{R}')}{|\boldsymbol{R} - \boldsymbol{R}'|} dv' \quad \text{(Wb/m)} \qquad (5\text{-}22)$$

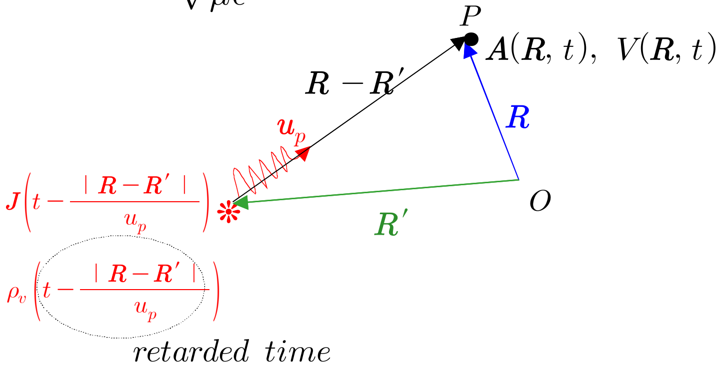
ii) Time-dependent solutions $\Rightarrow$ Retarded potentials

$$V(\boldsymbol{R},\ t) = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho_v(t - |\boldsymbol{R} - \boldsymbol{R}'|/u_p)}{|\boldsymbol{R} - \boldsymbol{R}'|}\,dv' \quad \text{(V)} \qquad \text{(6-67)}$$

$$\boldsymbol{A}(\boldsymbol{R},\ t) = \frac{\mu}{4\pi}\int_{V'} \frac{\boldsymbol{J}(t - |\boldsymbol{R} - \boldsymbol{R}'|/u_p)}{|\boldsymbol{R} - \boldsymbol{R}'|}\,dv' \quad \text{(Wb/m)} \qquad \text{(6-68)}$$

where

$$u_p = \frac{1}{\sqrt{\mu\epsilon}}$$: wave phase velocity in the medium (6-63)



Homework Set 8

1) P.6-4
2) P.6-6
3) P.6-7
4) P.6-9