3. Reflection and Transmission of Plane EM Waves

A. *Normal Incidence of Plane Waves at Plane Boundaries*

1) Reflection and transmission of TEM waves

In two unbounded lossless (or lossy) media contacting at a plane boundary,

Note) In lossless media,
$\hat{\gamma} = \hat{k}$ and $\beta = k$

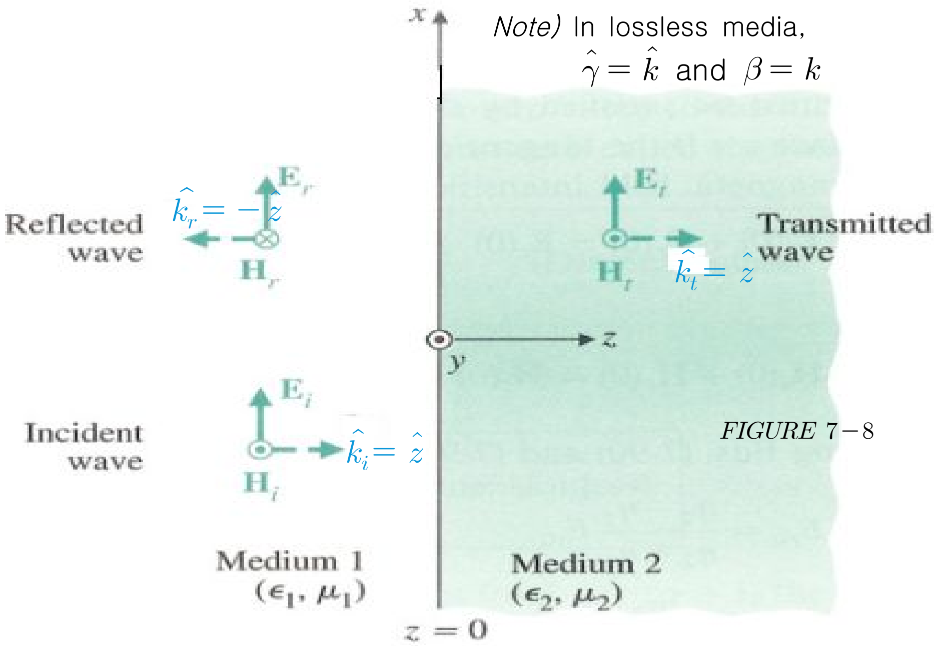


FIGURE 7-8

Incident wave $(\boldsymbol{E}_i, \boldsymbol{H}_i)$: $\hat{k}_i = \hat{z}$

$$\boldsymbol{E}_i(z) = \hat{x}\, E_{io} e^{-j\beta_1 z}, \quad \boldsymbol{H}_i(z) = \frac{\hat{k}_i \times \boldsymbol{E}_i(z)}{\eta_1} = \hat{y}\,\frac{E_{io}}{\eta_1} e^{-j\beta_1 z} \qquad (7\text{-}84,\ 85)$$

Reflected wave $(\boldsymbol{E}_r, \boldsymbol{H}_r)$: $\hat{k}_r = -\hat{z}$

$$\boldsymbol{E}_r(z) = \hat{x}\, E_{ro} e^{+j\beta_1 z}, \quad \boldsymbol{H}_r(z) = \frac{\hat{k}_r \times \boldsymbol{E}_r(z)}{\eta_1} = -\hat{y}\,\frac{E_{ro}}{\eta_1} e^{+j\beta_1 z} \qquad (7\text{-}86,\ 87)$$

Transmitted wave $(\boldsymbol{E}_t, \boldsymbol{H}_t)$: $\hat{k}_t = \hat{z}$

$$\boldsymbol{E}_t(z) = \hat{x}\, E_{to} e^{-j\beta_2 z}, \quad \boldsymbol{H}_t(z) = \frac{\hat{k}_t \times \boldsymbol{E}_t(z)}{\eta_2} = \hat{y}\,\frac{E_{to}}{\eta_2} e^{-j\beta_2 z} \qquad (7\text{-}88,\ 89)$$

Total fields in medium 1 : incident + reflected waves

$$\boldsymbol{E}_1(z) = \hat{x}\,(E_{io} e^{-j\beta_1 z} + E_{ro}\, e^{+j\beta_1 z}), \quad \boldsymbol{H}_1(z) = \hat{y}\,\frac{1}{\eta_1}(E_{io} e^{-j\beta_1 z} - E_{ro}\, e^{+j\beta_1 z}) \qquad (11)$$

Total fields in medium 2 : transmitted waves

$$\boldsymbol{E}_2(z) = \hat{x}\, E_{to} e^{-j\beta_2 z}, \qquad \boldsymbol{H}_2(z) = \hat{y}\,\frac{E_{to}}{\eta_2} e^{-j\beta_2 z} \qquad (12)$$

Boundary conditions at $z = 0$:

$$\hat{n} \times (\boldsymbol{E}_1 - \boldsymbol{E}_2) = 0, \quad \hat{n} \times (\boldsymbol{H}_1 - \boldsymbol{H}_2) = 0 \quad \Rightarrow \quad \text{Continuity of } E_{\parallel}\ \&\ H_{\parallel}$$

$$\Rightarrow \quad E_{io} + E_{ro} = E_{to}, \qquad (E_{io} - E_{ro})/\eta_1 = E_{to}/\eta_2 \qquad (7\text{-}90,\ 91)$$

(11),(12)

Solving (7-90, 91),

$$E_{ro} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_{io} \equiv \Gamma E_{io} \ , \quad E_{to} = \left(\frac{2\eta_2}{\eta_2 + \eta_1}\right) E_{io} \equiv \tau E_{io} \qquad (7\text{-}92,\ 93)$$

where Reflection coefficient :

$$\Gamma \triangleq \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad (7\text{-}94)$$

Transmission coefficient :

$$\tau \triangleq \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1} \qquad (7\text{-}95)$$

$$\Rightarrow \quad 1 + \Gamma = \tau \quad \text{for normal incidence} \qquad (7\text{-}96)$$

Note) In lossy media, η (real) is replaced by η_c (complex).
Then, Γ and τ become complex (phase shift at the interface).

Rewriting (11) for total fields in medium 1,

$$\boldsymbol{E}_1(z) = \hat{x}\, E_{io} e^{-j\beta_1 z}\,(1 + \Gamma e^{+j2\beta_1 z}) \qquad (7\text{-}97)$$

Standing (envelope) wave

$$\boldsymbol{H}_1(z) = \hat{y}\, \frac{E_{io}}{\eta_1}\, e^{-j\beta_1 z}\,(1 - \Gamma e^{+j2\beta_1 z}) \qquad (7\text{-}100)$$

Rewriting (12) for total fields in medium 2,

$$\boldsymbol{E}_2(z) = \boldsymbol{E}_t(z) = \hat{x}\,\tau E_{io} e^{-j\beta_2 z} \qquad (7\text{-}101)$$

$$\boldsymbol{H}_2(z) = \boldsymbol{H}_t(z) = \hat{y}\,\frac{\tau}{\eta_2} E_{io} e^{-j\beta_2 z} \qquad (7\text{-}102)$$

If the standing-wave ratio (SWR) is defined as

$$S \triangleq \frac{|E|_{max}}{|E|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad (1 \le S < \infty) \qquad (7\text{-}98)$$

then

$$|\Gamma| = \frac{1 - S}{1 + S} \qquad (-1 \le \Gamma \le 1) \qquad (7\text{-}99)$$

Time-ave. power density = time-ave. Poynting vector (7-79)

$$\mathscr{P}_{av}(z) = \frac{1}{2} Re[\boldsymbol{E} \times \boldsymbol{H}^*] \quad (\mathrm{W/m^2}) \qquad (7\text{-}79,\ 103)$$

(7-97, 100) in (7-103) :

$$(\mathscr{P}_{av})_1 = \frac{1}{2} Re[\boldsymbol{E}_1 \times \boldsymbol{H}_1{}^*] = \hat{z}\,\frac{E_{io}^2}{2\eta_1}(1 - \Gamma^2) \qquad (7\text{-}104)$$

(7-101, 102) in (7-103) :

$$(\mathscr{P}_{av})_2 = \frac{1}{2} Re[\boldsymbol{E}_2 \times \boldsymbol{H}_2{}^*] = \hat{z}\,\frac{E_{io}^2}{2\eta_2}\tau^2 \qquad (7\text{-}105)$$

Power (or Energy) conservation in lossless media : $(\mathscr{P}_{av})_1 = (\mathscr{P}_{av})_2$

$$\Rightarrow \quad 1 - \Gamma^2 = (\eta_1/\eta_2)\,\tau^2 \qquad (7\text{-}107)$$

(cf) Reflectivity (or reflectance in optics):

$$R \triangleq \frac{(P_{av})_{ro}}{(P_{av})_{io}} = \frac{S_{ro}(\mathscr{P}_{av})_{ro}}{S_{io}(\mathscr{P}_{av})_{io}} = \Gamma^2 \qquad (7\text{-}94)^*$$

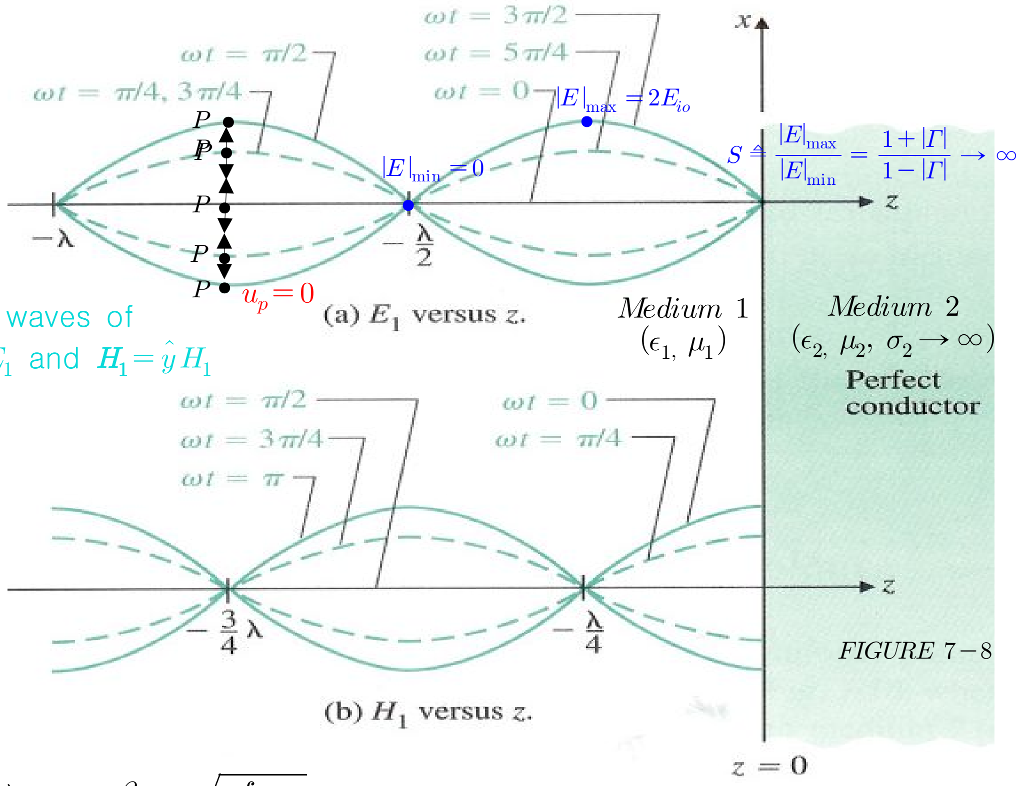
Transmissivity (or transmittance in optics):

$$T \triangleq \frac{(P_{av})_{to}}{(P_{av})_{io}} = \frac{S_{to}(\mathscr{P}_{av})_{to}}{S_{io}(\mathscr{P}_{av})_{io}} = \frac{\eta_1}{\eta_2}\tau^2 \qquad (7\text{-}95)^*$$

(7-107) $\Rightarrow$ $R + T = 1$ (7-107)*

2) Normal incidence on a good conductor and standing waves

For medium 2 of perfect conductor ($\sigma_2 \to \infty$, $i.e.$, $\sigma_2/\omega\epsilon \gg 1$),



Standing waves of $E_1 = \hat{x} E_1$ and $H_1 = \hat{y} H_1$

(7-52): $\alpha = \beta \cong \sqrt{\pi f \mu \sigma_2} \to \infty$

(7-53): $\eta_{2c} = (1+j)\sqrt{\pi f \mu / \sigma_2} = 0$ (short-circuit boundary)

(7-94): $\Gamma = \dfrac{E_{ro}}{E_{io}} = \dfrac{\eta_{2c} - \eta_1}{\eta_{2c} + \eta_1} = -1$ [total reflection and out-of-phase between E_r & E_i $(-1 = e^{j\pi})$: phase shift = π]

(7-95): $\tau = \dfrac{E_{to}}{E_{io}} = \dfrac{2\eta_2}{\eta_2 + \eta_1} = 0$ (no transmission across perfect conductor surface)

Incident wave: $\mathbf{E}_i(z) = \hat{x} E_{io} e^{-j\beta_1 z}$, $\mathbf{H}_i(z) = \hat{y} \dfrac{E_{io}}{\eta_1} e^{-j\beta_1 z}$ (7-84, 85)(7-108, 109)

Reflected wave: $\mathbf{E}_r(z) = -\hat{x} E_{io} e^{+j\beta_1 z}$, $\mathbf{H}_r(z) = \hat{y} \dfrac{E_{io}}{\eta_1} e^{+j\beta_1 z}$ (7-110, 111)

Total wave phasors in medium 1 (7-97, 100) :

$\mathbf{E}_1(z) = \hat{x} E_{io} (e^{-j\beta_1 z} - e^{+j\beta_1 z}) = -j\hat{x} 2 E_{io} \sin\beta_1 z$ ($\mathbf{E}_1$ lags behind $\mathbf{H}_1$ by $\pi/2$)(7-112)

$\mathbf{H}_1(z) = \hat{y} \dfrac{E_{io}}{\eta_1} (e^{-j\beta_1 z} + e^{+j\beta_1 z}) = \hat{y} 2 \dfrac{E_{io}}{\eta_1} \cos\beta_1 z$ (7-113)

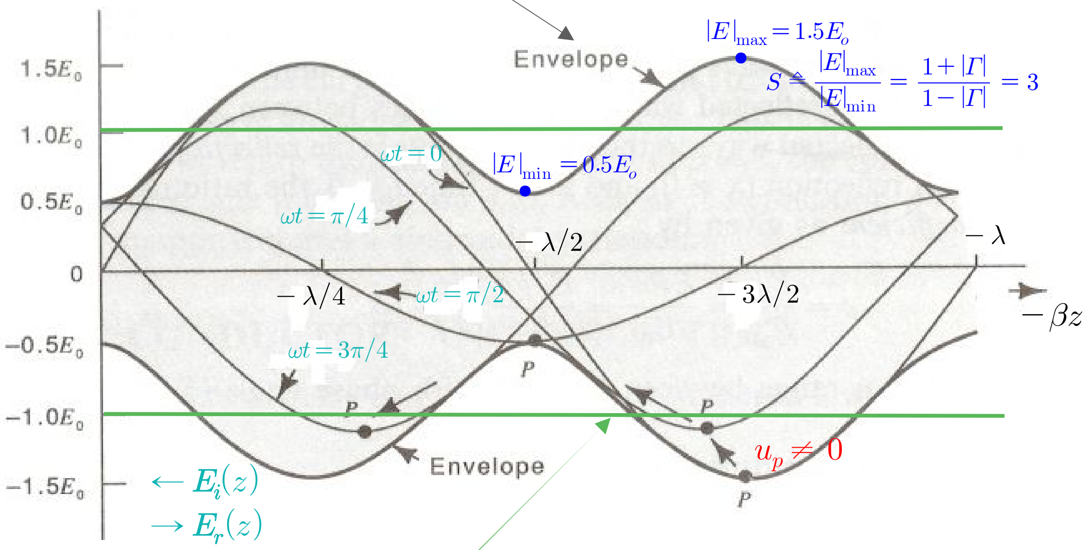
Total instantaneous waves in medium 1 ⇒ Standing wave resulted from the interference of two (incid. and reflec.) waves

$\mathbf{E}_1(z,t) = Re[\mathbf{E}_1(z) e^{j\omega t}] = \hat{x}\, 2 E_{io} \sin\beta_1 z \sin\omega t = \hat{x}\, E_1 \sin\omega t$ (7-114)

$\mathbf{H}_1(z,t) = Re[\mathbf{H}_1(z) e^{j\omega t}] = \hat{y}\, 2 \dfrac{E_{io}}{\eta_1} \cos\beta_1 z \cos\omega t = \hat{y}\, H_1 \cos\omega t$ (7-115)

Standing wave curve = Envelope of instantaneous curve

Note) (7-104) ⇒ $(\mathscr{P}_{av})_1 = 0$: No EM power flow in medium 1 (cf) Standing wave envelope for $E_{ro}=\frac{1}{2}E_{io}$ $\quad (\Gamma=1/2)$



Envelope for (medium 1) = (medium 2) : no boundary

$E_{ro}=0,\quad E_{to}=E_{io},\quad \Gamma=0,\quad \tau=1,\quad S=1$

B. Oblique Incidence of Plane Waves at Plane Boundaries

1) Snell's laws (ind. of wave polarization)

Traveling with same u_{p1} in medium 1

$\Rightarrow \quad \overline{OA'} = \overline{AO'}$

$\Rightarrow \quad \overline{OO'}\,sin\theta_r = \overline{OO'}\,sin\theta_i$

$\Rightarrow \quad \theta_r = \theta_i$: Snell's law of reflection (7-116)

Traveling with different u_p's in med. 1 & 2,

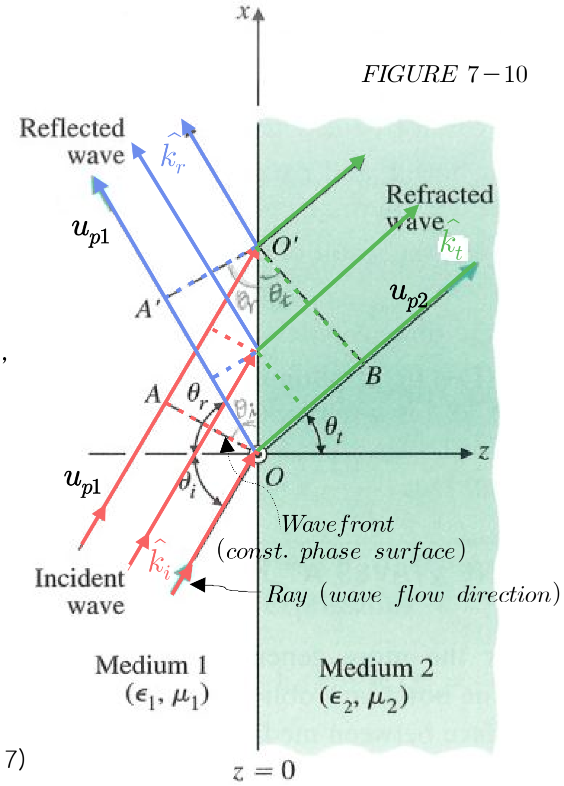
$$\Rightarrow \quad \frac{\overline{OB}}{u_{p2}} = \frac{\overline{AO'}}{u_{p1}}$$

$$\Rightarrow \quad \frac{\overline{OO'}\,sin\theta_t}{\overline{OO'}\,sin\theta_i} = \frac{u_{p2}}{u_{p1}}$$

$u_p=\frac{\omega}{\beta}=\frac{c}{n_r}$

$$\Rightarrow \quad \frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}} = \frac{\beta_1}{\beta_2} = \frac{n_{r1}}{n_{r2}}$$

: Snell's law of refraction (7-117)



For $\mu_1 = \mu_2$,

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}} = \frac{n_{r1}}{n_{r2}} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{\eta_2}{\eta_1} \qquad (7\text{-}118)$$

$u_p=\frac{1}{\sqrt{\mu\epsilon}}$ $\qquad \eta=\sqrt{\frac{\mu}{\epsilon}}$

2) Total (internal) reflection and surface wave

: wave behavior of internal reflection and surface wave if the wave is incident with $\theta_i > \theta_c$ on a less dense medium $(\epsilon_1 > \epsilon_2)$

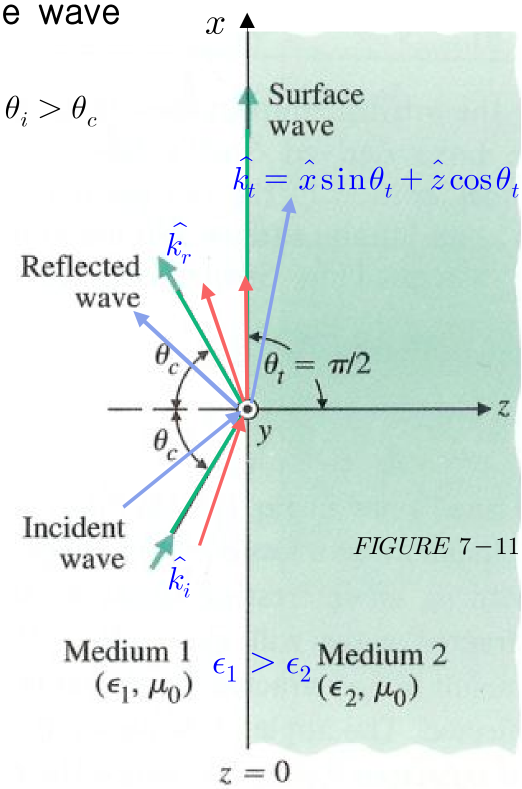


FIGURE 7−11

For $\epsilon_1 > \epsilon_2$, from (7−118) $\theta_t > \theta_i$.

Then, critical angle of incidence for total reflection $(\theta_t = \pi/2)$ from (7−118):

$$\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sin^{-1}\left(\frac{n_{r2}}{n_{r1}}\right) \qquad (7\text{−}120)$$

If $\theta_i > \theta_c$, from (7−118)

$$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\, sin\theta_i > 1 \qquad (7\text{−}121)$$

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t}$$

$$= \pm j\sqrt{\frac{\epsilon_1}{\epsilon_2} sin^2\theta_i - 1} \qquad (7\text{−}122)$$

Transmitted waves $\boldsymbol{E}_t(x,z)$, $\boldsymbol{H}_t(x,z)$ from (7−20, 23) :

$$\boldsymbol{E}_t(x,z),\ \boldsymbol{H}_t(x,z) \propto e^{-j\boldsymbol{k}_t \cdot \boldsymbol{R}}$$

$k = \omega\sqrt{\mu\epsilon} = \beta$

$$\Rightarrow e^{-j\boldsymbol{k}_t \cdot \boldsymbol{R}} = e^{-jk_2\hat{k}_t \cdot \boldsymbol{R}} = e^{-j\beta_2(\hat{x}\sin\theta_t + \hat{z}\cos\theta_t)\cdot(\hat{x}x + \hat{z}z)}$$

$$= e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

(7−121), (7−122)

$$= e^{-\alpha_2 z} e^{-j\beta_{2x} x} \qquad (7\text{−}125)$$

$e^{-\alpha_2 z}$: *exponentially attenuating amplitude in the normal direction (z)*

$e^{-j\beta_{2x} x}$: *traveling wave along the interface (x)*

where $\alpha_2 = \beta_2\sqrt{\frac{\epsilon_1}{\epsilon_2} sin^2\theta_i - 1}$, $\beta_{2x} = \beta_2\sqrt{\frac{\epsilon_1}{\epsilon_2}}\, sin\theta_i$ (7−125a, b)

⇒ Surface wave = Evanescent wave along the interface for $\theta_i > \theta_c$

(e.g. 7−9) Underwater light source

$$\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$= \sin^{-1}\sqrt{\frac{1}{1.75}} = 49.2^o$$

$$d = \frac{\overline{O'P}}{\tan\theta_c} = \frac{5}{\tan 49.2^o}$$

$= 4.32$ (m)

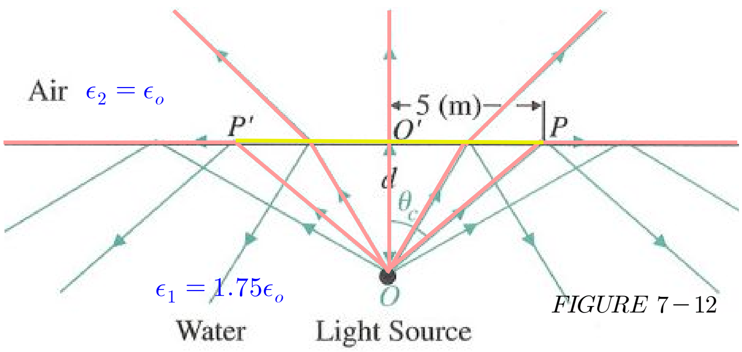


FIGURE 7−12

(e.g. 7-10) Optical fiber

= Dielectric rod or fiber guiding EM wave by total internal reflection

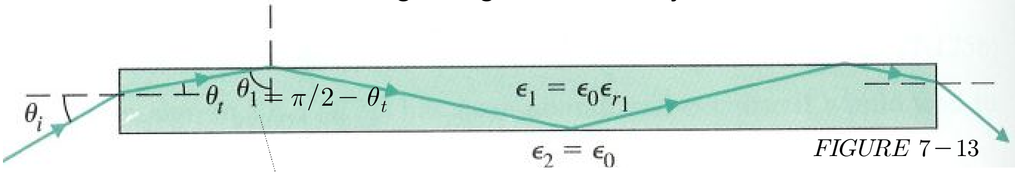


FIGURE 7−13

For total internal reflection in the fiber,

$$\sin\theta_1 \geq \sin\theta_c \quad \Rightarrow \quad \cos\theta_t \geq \sin\theta_c = \epsilon_{r1}^{-1/2} \qquad (7\text{-}127)$$

Snell's law of refraction : $\sin\theta_t \geq \epsilon_{r1}^{-1/2}\sin\theta_i$ (7-128)

(7-127) in (7-128) : $1 - \epsilon_{r1}^{-1}\sin^2\theta_i \geq \epsilon_{r1}^{-1}$

$$\Rightarrow \quad \epsilon_{r1} \geq 1 + \sin^2\theta_i \qquad (7\text{-}129)$$

For the total internal reflection at any incident angle,

$$\theta_i \leq \pi/2 \quad \Rightarrow \quad (\epsilon_{r1})_{\min} = 1 + \sin^2(\pi/2) = 2$$

or $(n_{r1})_{\min} = \sqrt{(\epsilon_{r1})_{\min}} = \sqrt{2}$

Note) ϵ_r = 4~10 (glass), 3.4 (flexiglass)

3) Perpendicular(⊥) polarization

$\boldsymbol{E}_i \perp (incidence\ plane)$

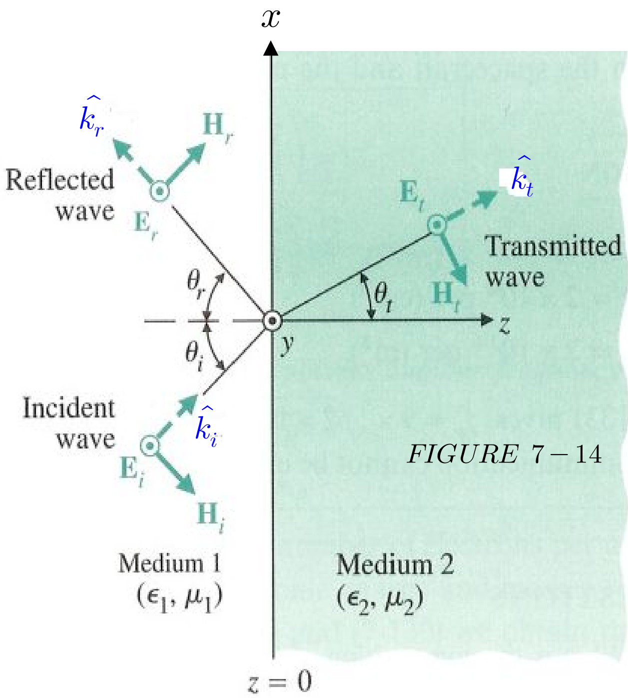


FIGURE 7−14

Incident wave $(\boldsymbol{E}_i, \boldsymbol{H}_i)$:

$$\hat{k}_i = \hat{x}\sin\theta_i + \hat{z}\cos\theta_i$$

$$\boldsymbol{E}_i(x,z) = \hat{y}\,E_{io}\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\boldsymbol{H}_i(x,z) = \frac{E_{io}}{\eta_1}(-\hat{x}cos\theta_i + \hat{z}sin\theta_i) \times e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

Reflected wave $(\boldsymbol{E}_r, \boldsymbol{H}_r)$:

$$\hat{k}_r = \hat{x}\sin\theta_r - \hat{z}\cos\theta_r$$

$$\boldsymbol{E}_r(x,z) = \hat{y}\,E_{ro}\,e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\boldsymbol{H}_r(x,z) = \frac{E_{ro}}{\eta_1}(\hat{x}cos\theta_r + \hat{z}sin\theta_r) \times e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

Transmitted wave $(\boldsymbol{E}_t, \boldsymbol{H}_t)$: $\hat{k}_t = \hat{x}\sin\theta_t + \hat{z}\cos\theta_t$

$$\boldsymbol{E}_t(x,z) = \hat{y}\,E_{to}\,e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\boldsymbol{H}_t(x,z) = \frac{E_{to}}{\eta_2}(-\hat{x}cos\theta_t + \hat{z}sin\theta_t)\,e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \qquad (7\text{-}134)\sim(7\text{-}142)$$

Boundary conditions at $z=0$ $\Rightarrow$ Continuity of $E_{\parallel}(x,0)$ & $H_{\parallel}(x,0)$

$$\begin{cases} E_{iyo}+E_{ryo}=E_{tyo}: \quad E_{io}e^{-j\beta_1 x\sin\theta_i}+E_{ro}e^{-j\beta_1 x\sin\theta_r}=E_{to}e^{-j\beta_2 x\sin\theta_t} & (7\text{-}143) \\ H_{ixo}+H_{rxo}=H_{txo}: \quad \eta_1^{-1}(-E_{io}\cos\theta_i e^{-j\beta_1 x\sin\theta_i}+E_{ro}\cos\theta_r e^{-j\beta_1 x\sin\theta_r}) & \\ \qquad\qquad =-\eta_2^{-1}E_{to}\cos\theta_t e^{-j\beta_2 x\sin\theta_t} & (7\text{-}144) \end{cases}$$

$\Rightarrow$ Phase-matching condition for all x :

$$\beta_1 x\sin\theta_i=\beta_1 x\sin\theta_r=\beta_2 x\sin\theta_t$$

$\Rightarrow$ $\theta_r=\theta_i$ and $\sin\theta_t=(\beta_1/\beta_2)\sin\theta_i$: Snell's laws (7-116, 117)

Then, rewriting of (7-143) and (7-144) yields

$$\begin{cases} E_{io}+E_{ro}=E_{to} & (7\text{-}145) \\ \eta_1^{-1}(E_{io}-E_{ro})\cos\theta_i=\eta_2^{-1}E_{to}\cos\theta_t & (7\text{-}146) \end{cases}$$

Solving (7-145, 145),

$$E_{ro}=\Gamma_{\perp}E_{io}\,, \qquad E_{to}=\tau_{\perp}E_{io}$$

where $\Gamma_{\perp}\triangleq\dfrac{E_{ro}}{E_{io}}=\dfrac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}$

: Fresnel's refl. coeff. for perpendicular polarization (7-147)

$$\tau_{\perp}\triangleq\frac{E_{to}}{E_{io}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}$$

: Fresnel's trans. coeff. for perpendicular polarization (7-148)

$\Rightarrow$ $1+\Gamma_{\perp}=\tau_{\perp}$ for perpendicular polarization (7-149)

Notes)

i) For $\theta_i=0$ (normal incidence), $\theta_t=0$, $\Gamma_{\perp}=\Gamma$ and $\tau_{\perp}=\tau$

ii) If med. 2 is perfect conductor, $\eta_2=0$, $\Gamma_{\perp}=-1$ and $\tau_{\perp}=0$

iii) For lossless $(\sigma=0)$, nonmagnetic $(\mu=\mu_o)$ dielectrics,

using $\eta_1/\eta_2=\sqrt{\epsilon_2/\epsilon_1}$ and $\cos\theta_t=\sqrt{1-\sin^2\theta_t}=\sqrt{1-(\epsilon_1/\epsilon_2)\sin^2\theta_i}$

(7-147) $\Rightarrow$ $\Gamma_{\perp}=\dfrac{\cos\theta_i-\sqrt{(\epsilon_2/\epsilon_1)-\sin^2\theta_i}}{\cos\theta_i+\sqrt{(\epsilon_2/\epsilon_1)-\sin^2\theta_i}}$ (7-147)*

For $\epsilon_1<\epsilon_2$ (Med. 2 is optically denser), $\Gamma_{\perp}=real$

For $\epsilon_1>\epsilon_2$ (Med. 1 is optically denser) and $\theta_i>\theta_c=\sin^{-1}\sqrt{\epsilon_2/\epsilon_1}$,

$\Gamma_{\perp}=complex$, $|\Gamma_{\perp}|=1$: total internal reflection

(7-141) $\Rightarrow$ $\mathbf{E}_t(x,z)=\hat{y}\,\tau_{\perp}E_{io}e^{-j\beta_2(x\sin\theta_t+z\cos\theta_t)}\propto e^{-\alpha_2 z}e^{-j\beta_{2x}x}$ (7-125)

: accompanied by a surface wave

4) Parallel(∥) polarization

$\boldsymbol{E}_i \parallel (incidence\ plane)$

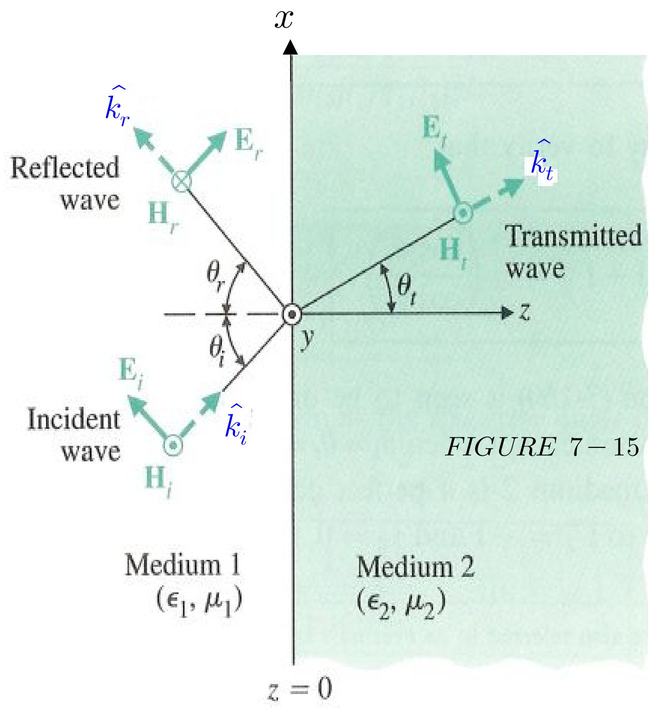


FIGURE 7-15

Incident wave $(\boldsymbol{E}_i, \boldsymbol{H}_i)$:

$$\hat{k}_i = \hat{x}\sin\theta_i + \hat{z}\cos\theta_i$$

$$\boldsymbol{E}_i(x,z) = E_{io}(\hat{x}cos\theta_i - \hat{z}sin\theta_i) \times e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\boldsymbol{H}_i(x,z) = \hat{y}\frac{E_{io}}{\eta_1}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

Reflected wave $(\boldsymbol{E}_r, \boldsymbol{H}_r)$:

$$\hat{k}_r = \hat{x}\sin\theta_r - \hat{z}\cos\theta_r$$

$$\boldsymbol{E}_r(x,z) = E_{ro}(\hat{x}cos\theta_r + \hat{z}sin\theta_r) \times e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\boldsymbol{H}_r(x,z) = -\hat{y}\frac{E_{ro}}{\eta_1}e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

Transmitted wave $(\boldsymbol{E}_t, \boldsymbol{H}_t)$: $\hat{k}_t = \hat{x}\sin\theta_t + \hat{z}\cos\theta_t$

$$\boldsymbol{E}_t(x,z) = E_{to}(\hat{x}cos\theta_t - \hat{z}sin\theta_t)\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\boldsymbol{H}_t(x,z) = \hat{y}\frac{E_{to}}{\eta_2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$ (7-150)~(7-155)

Boundary conditions at $z=0$ ⇒ Continuity of $E_{\parallel}(x,0)$ & $H_{\parallel}(x,0)$

⇒ Phase-matching condition for all x :

⇒ $\theta_r = \theta_i$ and $\sin\theta_t = (\beta_1/\beta_2)\sin\theta_i$: Snell's laws (7-116, 117)

⇒
$$(E_{io} - E_{ro})\cos\theta_i = E_{to}\cos\theta_t \quad (7\text{-}156)$$
$$\eta_1^{-1}(E_{io} - E_{ro}) = \eta_2^{-1}E_{to} \quad (7\text{-}157)$$

Solving (7-156, 157),

$$E_{ro} = \Gamma_{\perp} E_{io}, \qquad E_{to} = \tau_{\perp} E_{io}$$

where $\Gamma_{\parallel} \triangleq \dfrac{E_{ro}}{E_{io}} = \dfrac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$

: Fresnel's refl. coeff. for parallel polarization (7-158)

$$\tau_{\perp} \triangleq \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_t}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

: Fresnel's trans. coeff. for parallel polarization (7-159)

⇒ $1 + \Gamma_{\parallel} = \tau_{\parallel}\left(\dfrac{\cos\theta_t}{\cos\theta_i}\right)$ for parallel polarization (7-160)

Notes)

i) For $\theta_i = 0$ (normal incidence), $\theta_t = 0$, $\Gamma_\parallel = \Gamma$ and $\tau_\parallel = \tau$

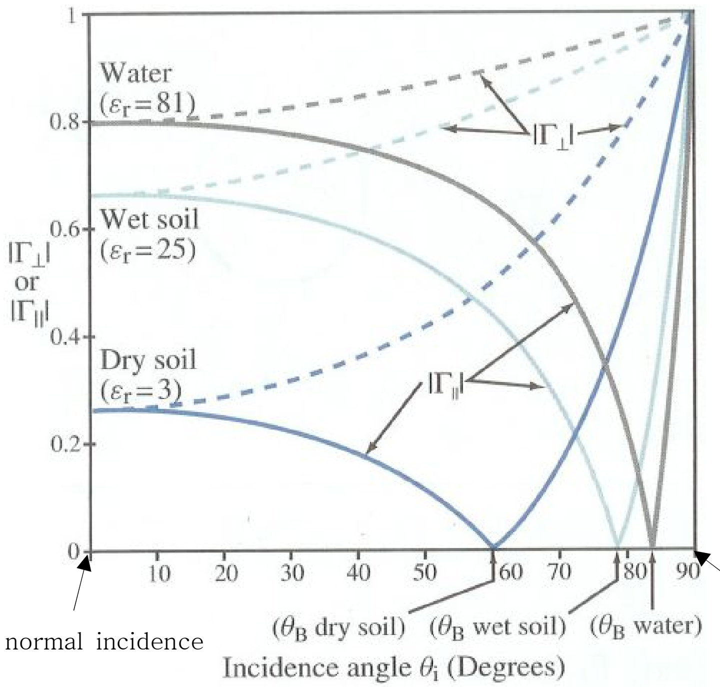
ii) If med. 2 is perfect conductor, $\eta_2 = 0$, $\Gamma_\parallel = -1$ and $\tau_\parallel = 0$

iii) For lossless $(\sigma = 0)$, nonmagnetic $(\mu = \mu_o)$ dielectrics,

using $\eta_1 / \eta_2 = \sqrt{\epsilon_2 / \epsilon_1}$ and $\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - (\epsilon_1/\epsilon_2)\sin^2\theta_i}$

(7-158) $\Rightarrow$ $$\Gamma_\parallel = \frac{-(\epsilon_2/\epsilon_1)\cos\theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}{+(\epsilon_2/\epsilon_1)\cos\theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}$$ (7-158)*

iv) Comparison between $|\Gamma_\perp|$ and $|\Gamma_\parallel|$



normal incidence

grazing incidence

$|\Gamma_\perp| \geq |\Gamma_\parallel|$

As $\epsilon_r^\uparrow$,

$|\Gamma|^\uparrow$ and $\theta_B^\uparrow$

Applications :

Polaroid sunglass design

Polarization control

v) Brewster angle θ_B (or polarizing angle) of no reflection

= Threshold incidence angle at which $\Gamma = 0$ (no reflection).

Parallel polarization :

(7-158) for $\Gamma_\parallel = 0$ $\Rightarrow$ $\eta_1 \cos\theta_{B\parallel} = \eta_2 \cos\theta_t$ (7-161)

$$\Rightarrow \sin^2\theta_{B\parallel} = \frac{1 - \mu_2\epsilon_1/\mu_1\epsilon_2}{1 - (\epsilon_1/\epsilon_2)^2}$$ (7-162)

For $\mu_1 = \mu_2$, $\theta_{B\parallel} = \sin^{-1}\frac{1}{\sqrt{1 + (\epsilon_1/\epsilon_2)}} = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = \tan^{-1}\left(\frac{n_{r2}}{n_{r1}}\right)$ (7-163)

Perpendicular polarization :

(7-147) for $\Gamma_\perp = 0$ $\Rightarrow$ $\eta_2 \cos\theta_{B\perp} = \eta_1 \cos\theta_t$ (7-165)

$$\Rightarrow \sin^2\theta_{B\perp} = \frac{1 - \mu_1\epsilon_2/\mu_2\epsilon_1}{1 - (\mu_1/\mu_2)^2}$$ (7-166)

For $\mu_1 = \mu_2$, $\theta_{B\perp}$ does not exist for nonmagnetic medium.