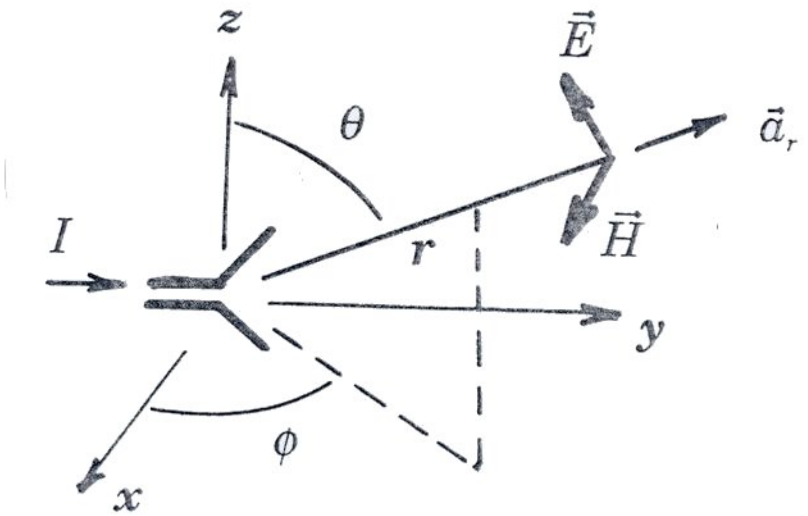
Spherical Waves and Radiation

- For large r,

  Localized current sources radiate fields in the form of <u>Spherical Waves</u>

$$\vec{E} = \vec{a}_E ZI \frac{e^{-jkr}}{r} f(\theta,\phi)$$

$$\vec{H} = \frac{1}{\eta}\vec{a}_r \times \vec{E} \qquad \left|\vec{a}_E\right| = \left|\vec{a}_r\right| = 1$$



- Radiated power flux (watts/m^2)

$$\vec{P} = \frac{1}{2}\mathrm{Re}\left\{\vec{E}\times\vec{H}^*\right\} = \vec{a}_r \frac{1}{2\eta}\frac{|ZI|^2}{r^2}\left|f(\theta,\phi)\right|^2$$

I = terminal current

Z = constant with units of ohms

η = 120π

- Antenna pattern = $\left|f(\theta,\phi)\right|^2$

Total Radiated Power

$$P = \int \int_{sphere} \vec{P} \cdot \vec{a}_r dA$$

$$dA = r^2 \sin\theta d\theta d\phi$$

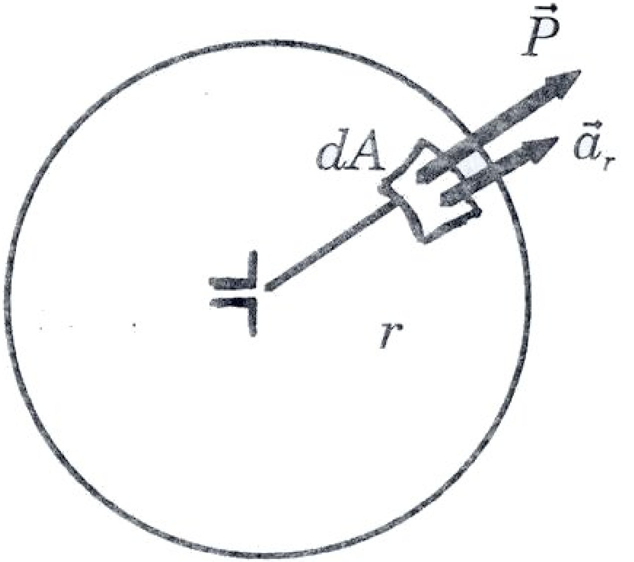
$$P_T = \frac{1}{2\eta} |ZI|^2 \int_0^{2\pi} \int_0^{\pi} |f(\theta,\phi)|^2 \sin\theta d\theta d\phi$$

P_T is independent of r

- Normalization for f(θ,ϕ) is:

$$\int_0^{2\pi} \int_0^{\pi} |f(\theta,\phi)|^2 \sin\theta d\theta d\phi = 4\pi$$

- Then: $\vec{P} = \vec{a}_r P_T \dfrac{|f(\theta,\phi)|^2}{4\pi r^2}$



Antenna Gain and Radiation Resistance

- Directive gain = $g(\theta,\phi) = |f(\theta,\phi)|^2$
- Antenna Gain = G = Max. value of $g(\theta,\phi)$
- If isotropic antennas could exist, then $|f(\theta,\phi)|^2 = 1, G = 1$
- Radiation Resistance R_r: effective resistance seen at antenna terminals

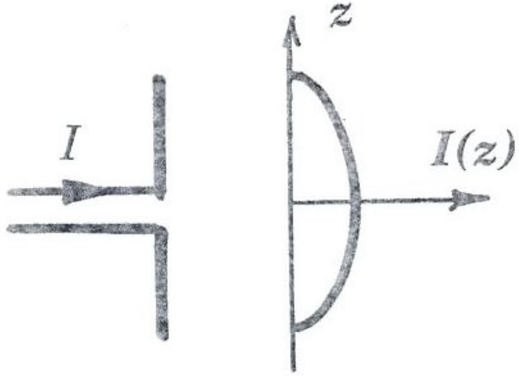
$$\frac{1}{2}|I|^2 R_r = P_T = \frac{4\pi}{2\eta}|ZI|^2$$

$$R_r = \frac{4\pi}{\eta}|Z|^2$$

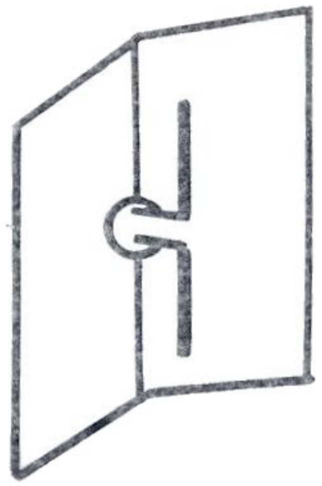
$$|Z| = \sqrt{\frac{R_r\eta}{4\pi}}$$

Examples of Antennas

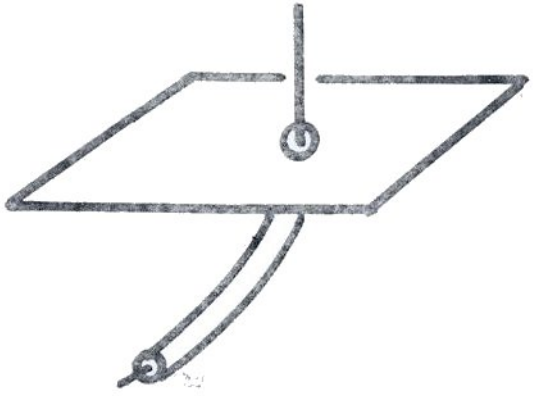
Half wave dipole



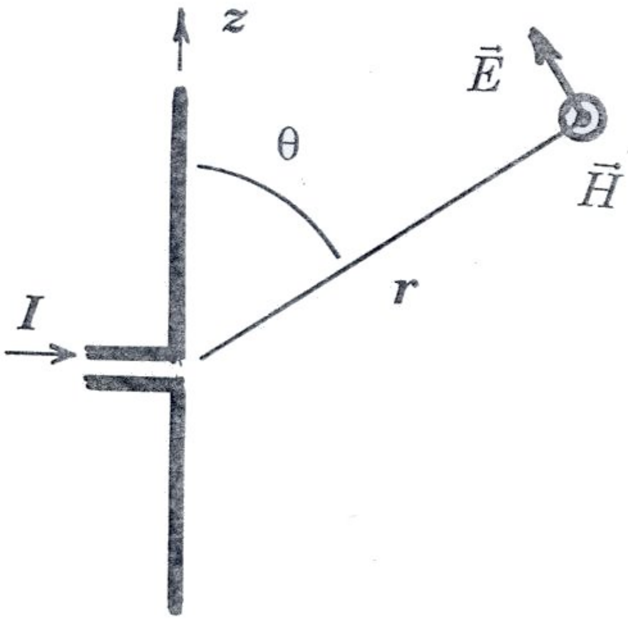
Dipole in corner reflector

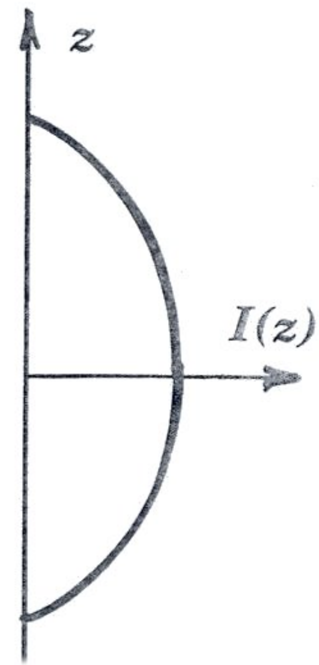


Quarter wave
dipole above
ground plane



Half Wave Dipole Antenna





$$\vec{E} = \vec{a}_\theta Z I \frac{e^{-jkr}}{r} f(\theta)$$

$$f(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{0.781\sin\theta}$$

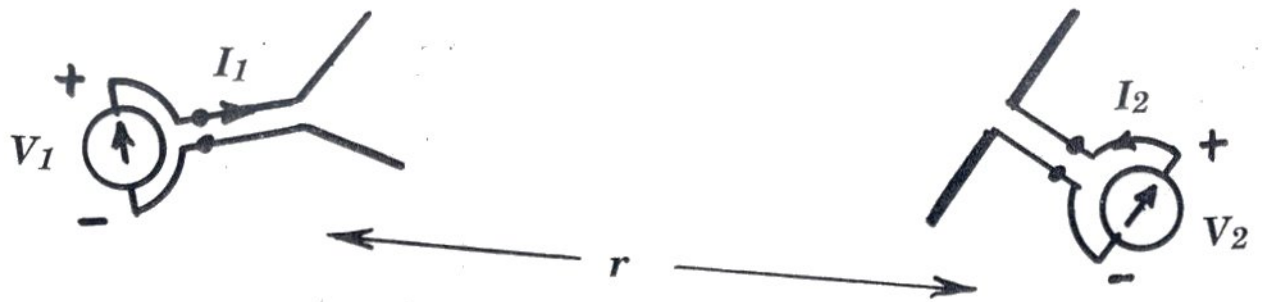
$$Z = j\frac{0.781}{2\pi}\eta$$

$$G = \left| f(90°)\right|^2 = 1.64$$

$$10 Log G = 2.2\ dB$$

$$R = \frac{4\pi}{\eta}\left| j\frac{0.781}{2\pi}\eta\right|^2 = 73\,\Omega$$

Receiving Antennas and Reciprocity



For a linear two-port

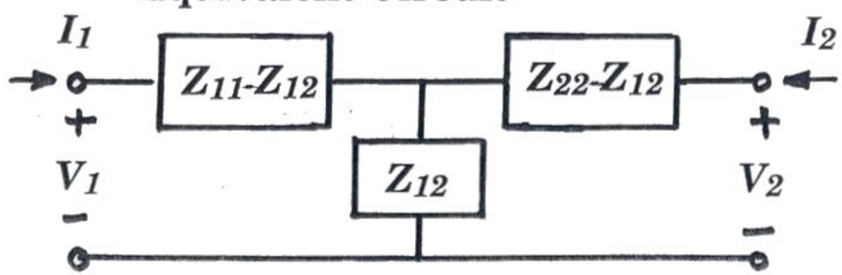
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Reciprocity

$$Z_{12} = Z_{21}$$

Equivalent Circuit

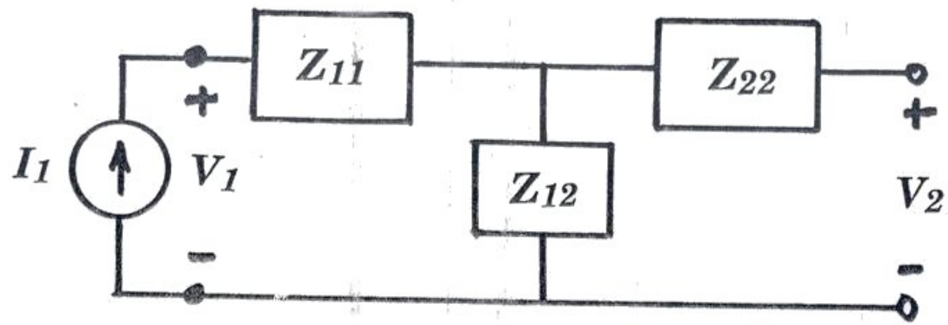


If $I_2 = 0,\ V_2 = Z_{12}I_1 \sim 1/r$

For r large,

$$|Z_{12}| << |Z_{11}|,\ |Z_{22}|$$

Circuit relation for Radiation into Free Space



Open Circuit

$V_1 = Z_{11}I_1$ and $V_2 = V_{oc} = Z_{12}I_1$

$$P_T = (1/2)Re(V_1 I_1^*) = (1/2)Re(Z_{11}|I_1|^2) = (1/2)R_{r1}|I_1|^2$$

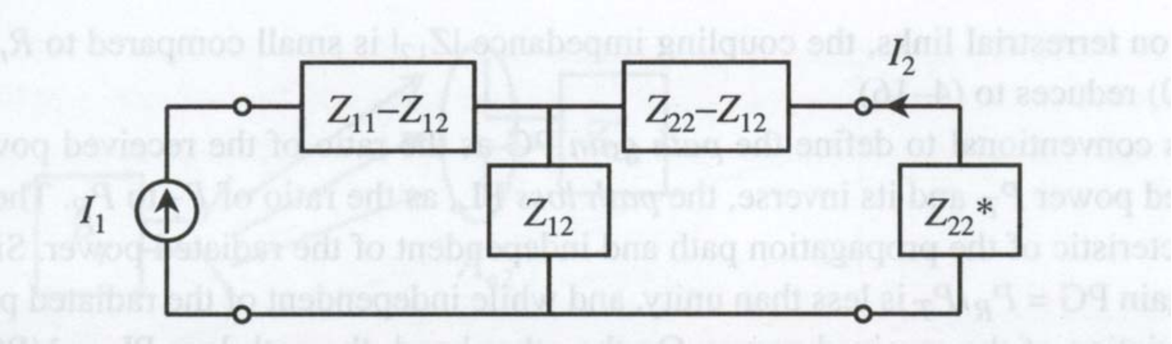
where R_{r1} = radiation resistance of antenna 1

Therefore: $Z_{11} = R_{r1} + jX_1$

Similarly: $Z_{22} = R_{r2} + jX_2$

where R_{r2} = radiation resistance of antenna 2

Received Power and Path Loss Ratio



$$I_2 = -I_1 \frac{Z_{12}}{\left(Z_{22} - Z_{12} + Z_{22}^*\right) + Z_{12}} = -I_1 \frac{Z_{12}}{2R_{r2}}$$

$$P_R = \frac{1}{2} R_{r2} \left|I_2\right|^2 = \frac{1}{8} \frac{\left|Z_{12}\right|^2}{R_{r2}} \left|I_1\right|^2 = \frac{1}{8} \frac{\left|V_{oc}\right|^2}{R_{r2}}$$

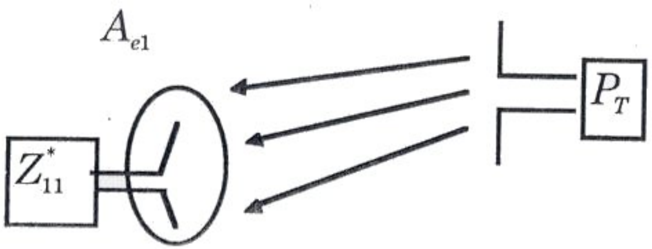
$$P_T = \frac{1}{2}\left|I_1\right|^2 \operatorname{Re}\left\{\left(Z_{11} - Z_{12}\right) + \frac{\left(2R_{r2} - Z_{12}\right) Z_{12}}{\left(2R_{r2} - Z_{12}\right) + Z_{12}}\right\} = \frac{1}{2}\left|I_1\right|^2 \frac{2R_{r1}R_{r2} - \operatorname{Re}\left\{Z_{12}^2\right\}}{2R_{r2}}$$

$$PG = \frac{P_R}{P_T} = \frac{\left|Z_{12}\right|^2}{4R_{r2}R_{r1} - 2\operatorname{Re}\left\{Z_{12}^2\right\}}$$

Effective Area of Receiving Antenna

Effective Area $= A_e$

$$P_R = \vec{P} \bullet \vec{a}_r A_e = P_T \frac{g(\theta,\phi)}{4\pi r^2} A_e$$



$$PL = \frac{P_R}{P_T} = \frac{g_2 A_{e1}}{4\pi r^2}$$

and by reciprocity

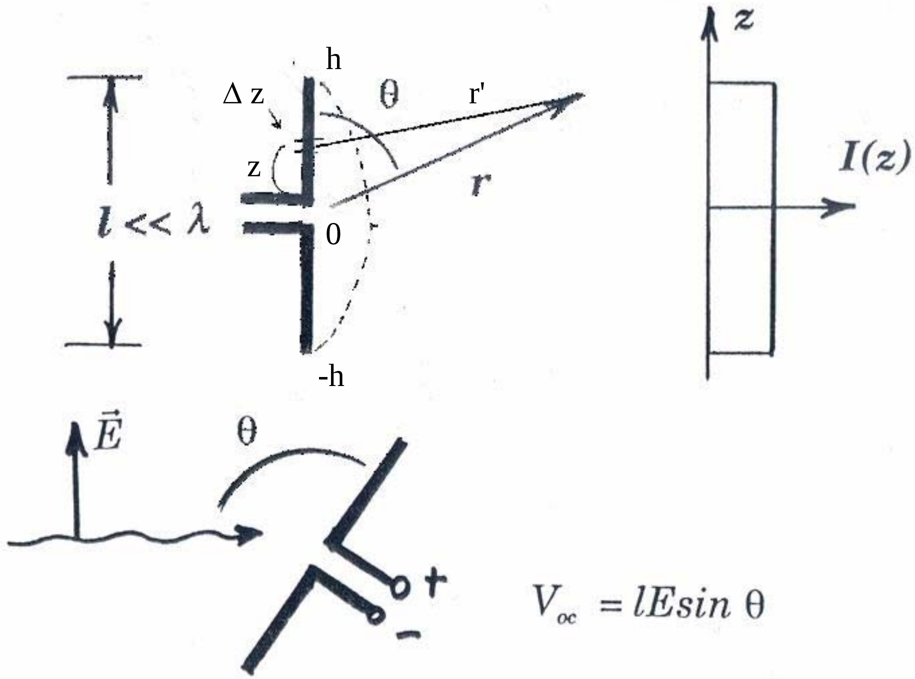
$$PL = \frac{P_R}{P_T} = \frac{g_1 A_{e2}}{4\pi r^2}$$

$$g_2 A_{e1} = g_1 A_{e2} \quad \textbf{or} \quad \frac{A_{e1}}{g_1} = \frac{A_{e2}}{g_2}$$

For an elemental dipole $\dfrac{A_e}{g} = \dfrac{\lambda^2}{4\pi}$

Therefore for any antennas $PL = g_1 g_2 \left(\dfrac{\lambda}{4\pi r}\right)^2$

Effective Area for Hertzian Dipole



$$\vec{E} = \vec{a}_\theta \, ZI \frac{e^{-jkr}}{r} f(\theta)$$

$$f(\theta) = \sqrt{3/2}\,\sin\theta$$

$$Z = j\frac{l}{2\lambda}\eta\sqrt{2/3}$$

$$g(\theta) = (3/2)\,\sin^2\theta$$

$$R_r = \eta\frac{4\pi}{6}\left(\frac{l}{\lambda}\right)^2$$

For matched termination:

$$P_R = \vec{P}\cdot\vec{a}_r\, A_e = A_e\left(\frac{1}{2\eta}|E|^2\right)$$

$$P_R = \frac{|V_{OC}|^2}{8R_r} = \frac{(l\,E\sin\theta)^2}{8\eta(l/\lambda)^2(4\pi/6)}$$

$$A_e = \frac{3}{8\pi}(\lambda\sin\theta)^2 = g(\theta)\frac{\lambda^2}{4\pi}$$