Numerical Methods in Rock Engineering - Introduction to numerical methods (Week1, 1 Sept)

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Numerical Approach in Rock Engineering
Methodology in Rock Engineering

- Empirical Method
  - RMR, Q, empirical system
- Analytical Method
  - Mathematical exact solution
- Experimental Method
  - Conduct experiment in the lab and insitu
- Numerical Method
  - Solve equations (often PDE) numerically using computer to obtain solution

Numerical Methods

- Continuum method
  - Finite Element Method
  - Finite Difference Method
  - Boundary Element Method
- Discontinuum Method
  - Discrete Element Method (explicit & Implicit)
- Hybrid Continuum/Discontinuum Method

Numerical Approach in Rock Engineering
Physical variables for THMC problems

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Physical problem	Conservation Principle $\nabla \cdot q = 0$	State Variable u	Flux σ	Material properties k	Source f	Constitutive equation $\sigma = ku'$
Elasticity	Conservation of linear momentum (equilibrium)	Displacement u	Stress σ	Young's modulus & Poisson's ratio	Body forces	Hooke's law
Heat conduction	Conservation of energy	Temperature T	Heat flux Q	Thermal conductivity k	Heat sources	Fourier's law
Porous media flow	Conservation of mass	Hydraulic head h	Flow rate Q	Permeability k	Fluid source	Darcy's law
Mass transport	Conservation of mass	Concentration C	Diffusive flux q	Diffusion coefficient D	Chemical source	Fick's law

Structure of state variables and fluxes are mathematically similar –
a convenient truth!

Advantage/Usefulness – analytical approach

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An example of analytical solution: Kirsch solution (1898)

$$\sigma_r = \frac{S_{H\max} + S_{h\min}}{2}\left(1 - \frac{R^2}{r^2}\right) + \frac{S_{H\max} - S_{h\min}}{2}\left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{S_{H\max} + S_{h\min}}{2}\left(1 + \frac{R^2}{r^2}\right) - \frac{S_{H\max} - S_{h\min}}{2}\left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta$$

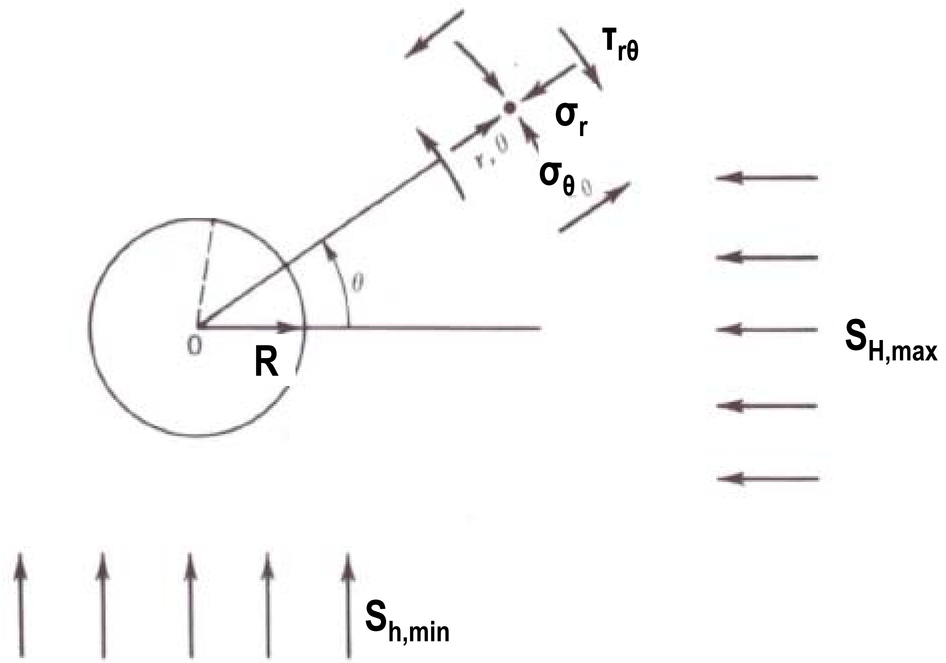
$$\tau_{r\theta} = \frac{S_{H\max} - S_{h\min}}{2}\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\theta$$

R: radius of well

r: radial distance from the center of the well

θ: measured from $S_{H,max}$

$S_{H,max}$ and $S_{H,max}$: maximum and minimum horizontal insitu stress



Numerical Approach in Rock Engineering
Advantage/Usefulness

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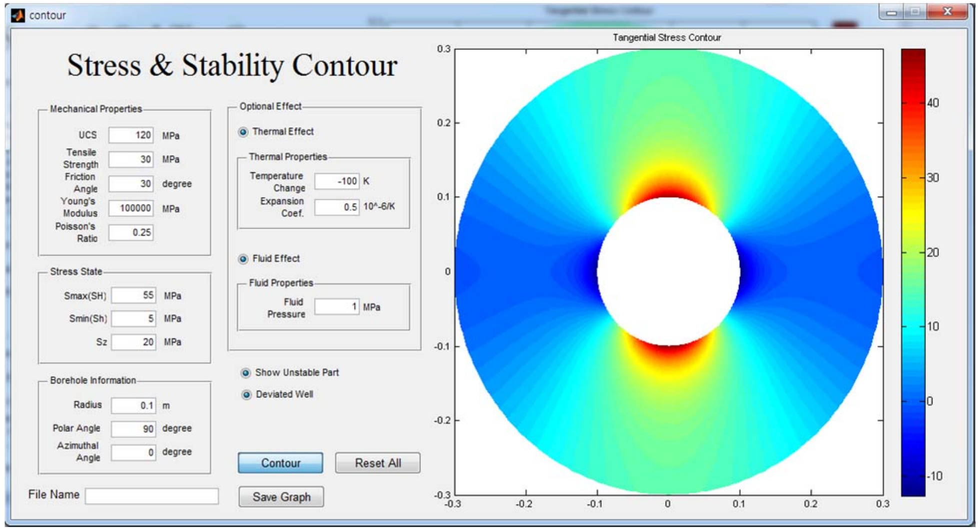
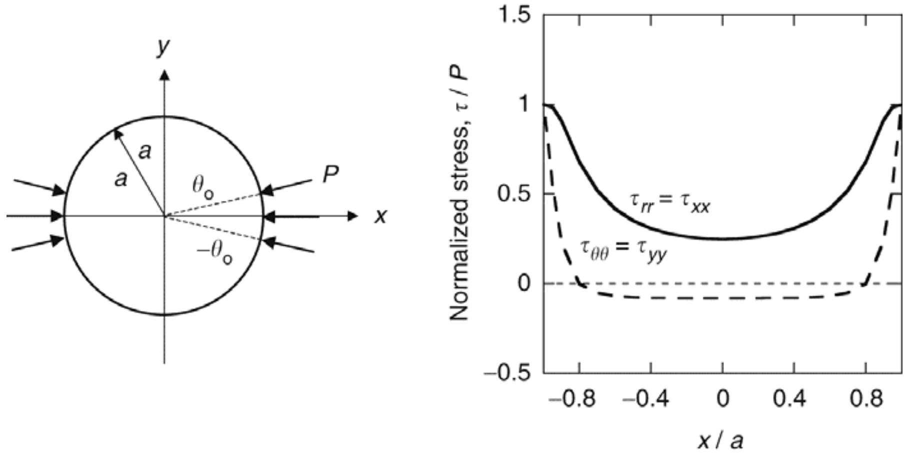
Kirsch solution



Diametral compression

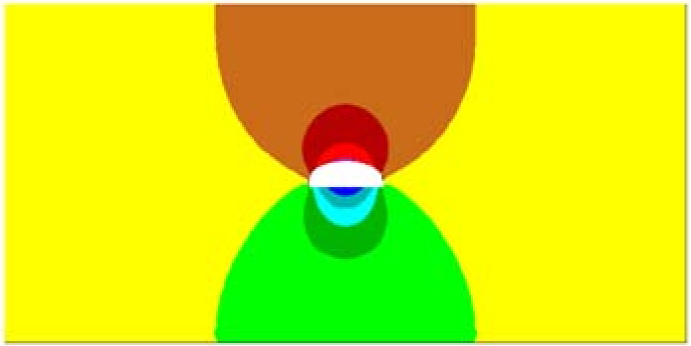


- Numerous analytical solutions exist – fast evaluation & still powerful
- However, 1) complex geometry, 2) multiple formation, 3) complex boundary condition, 4) complex process cannot be handled accurately.

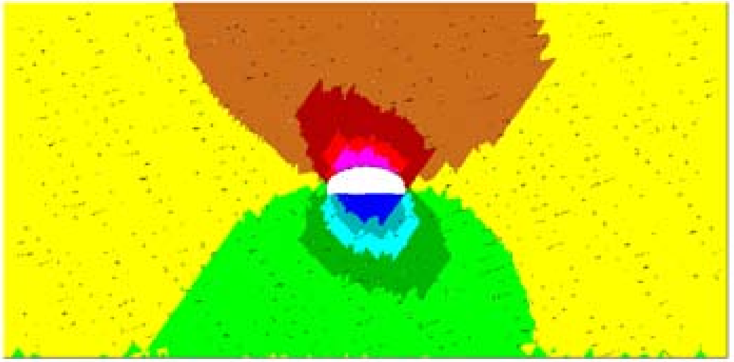
Courtesy of Kwon S

Numerical Approach in Rock Engineering
Advantage/Usefulness

- When geometry is not simply circular,



- When fractures around rock is considered,



Numerical Approach in Rock Engineering
Examples (1) – Underground construction

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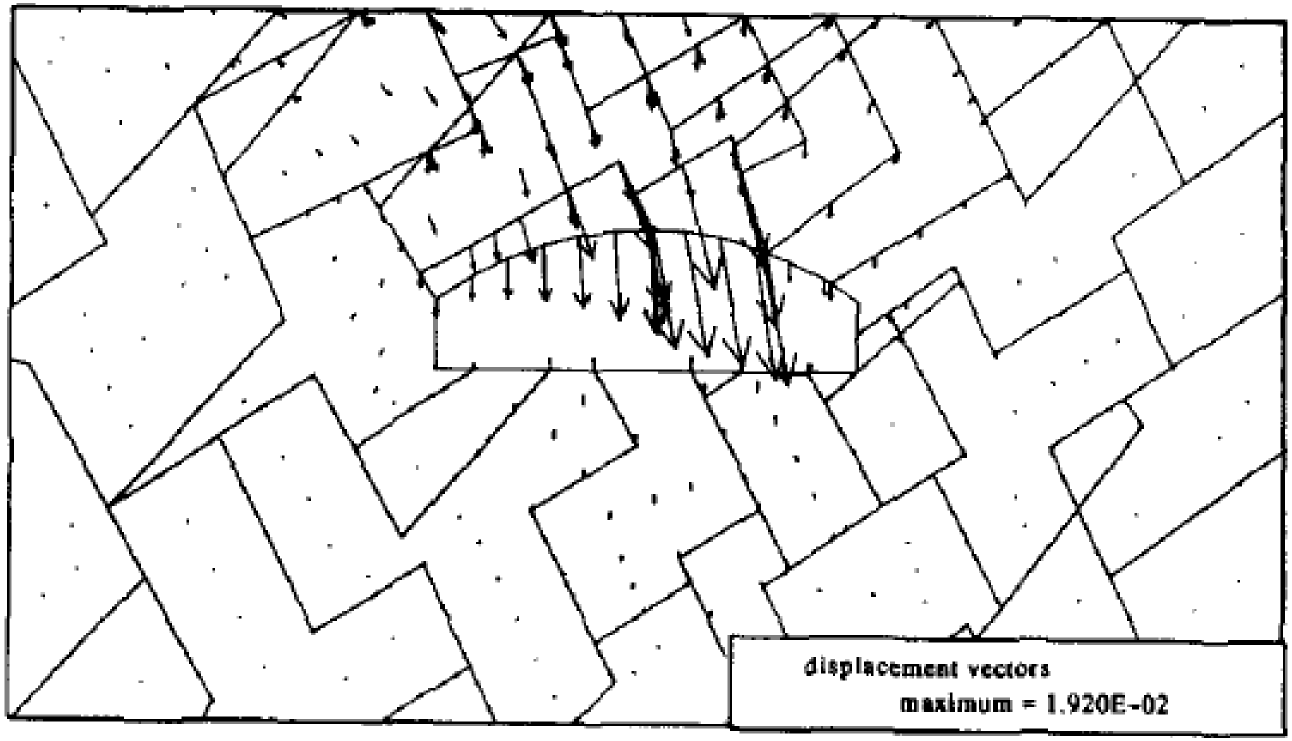
- Underground ice hockey stadium in Norway – discontinuum method was used for design



노르웨이 여빅 지하아이스하키 경기장 **(Barton et al., 1994)**

Numerical Approach in Rock Engineering
Examples (1) – underground construction

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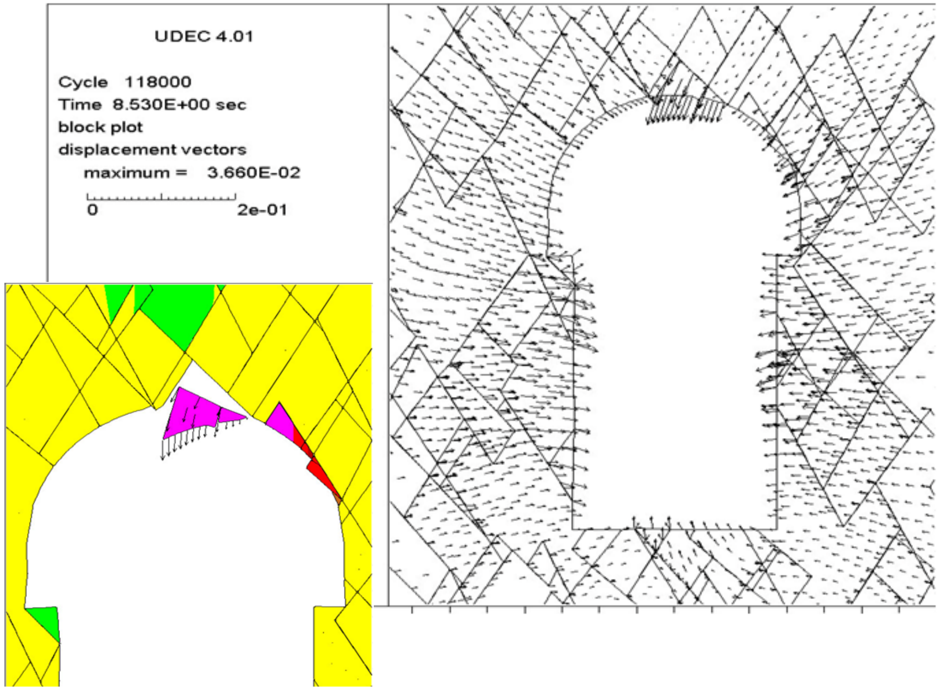
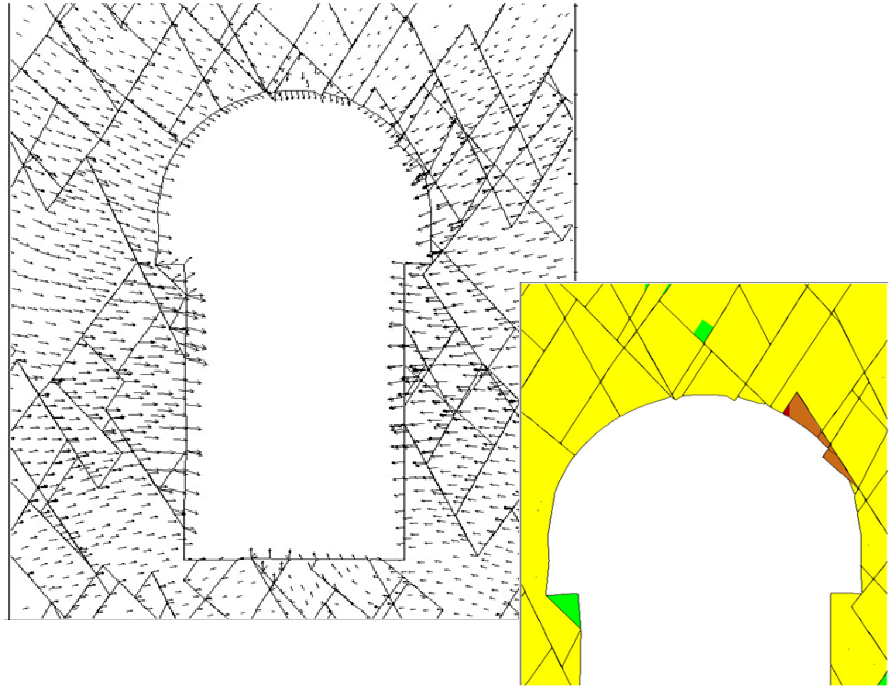


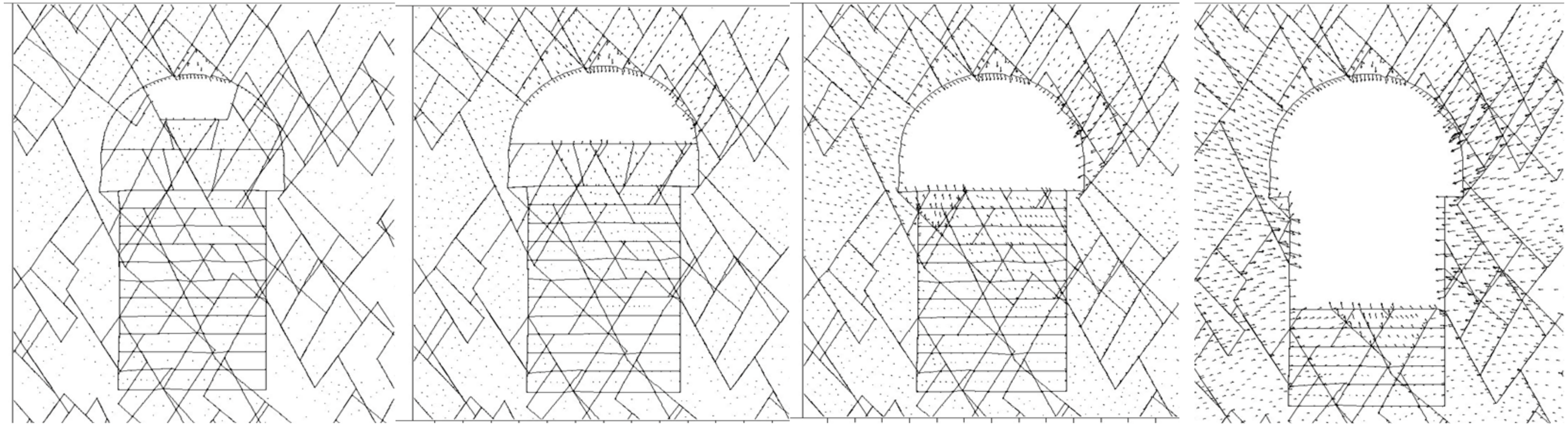
절리면 마찰각 **25**도

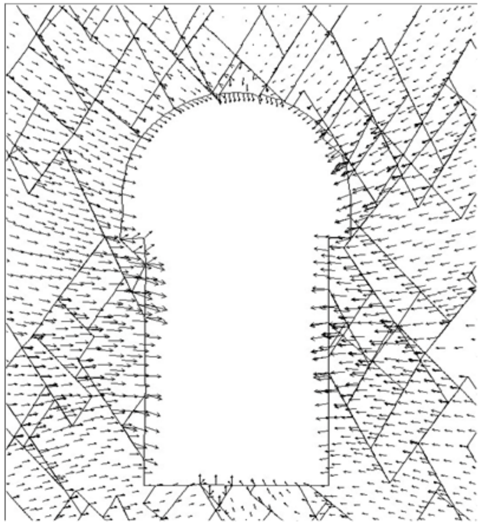


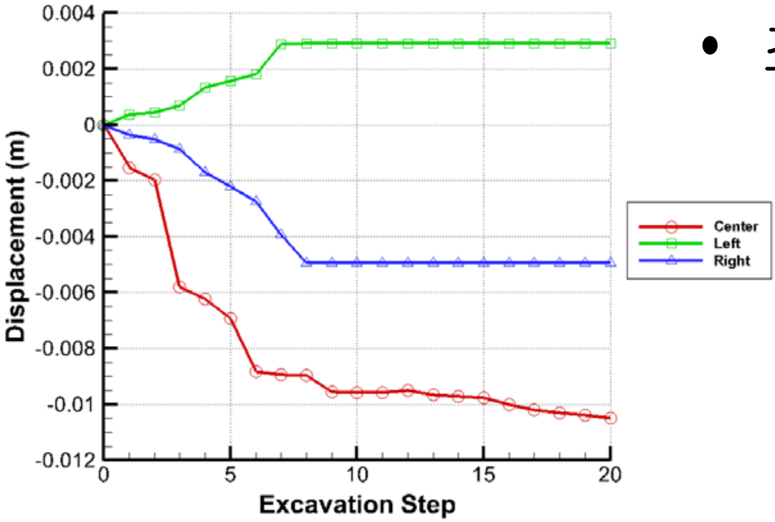
절리면 마찰각 **35**도

Min KB, Lee JW et al., 2011

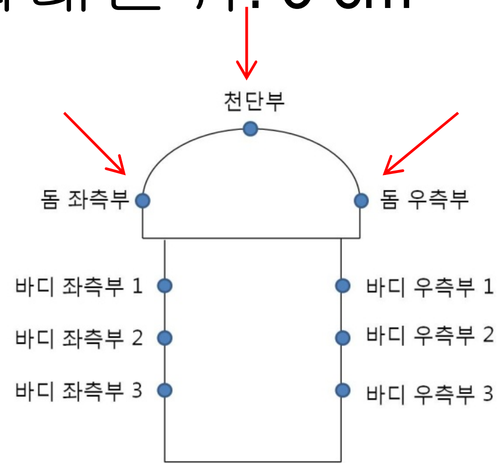
Numerical Approach in Rock Engineering
Examples (1) – underground construction







- 최대변위: 3 cm



Numerical Approach in Rock Engineering
Examples (2) – TM analysis

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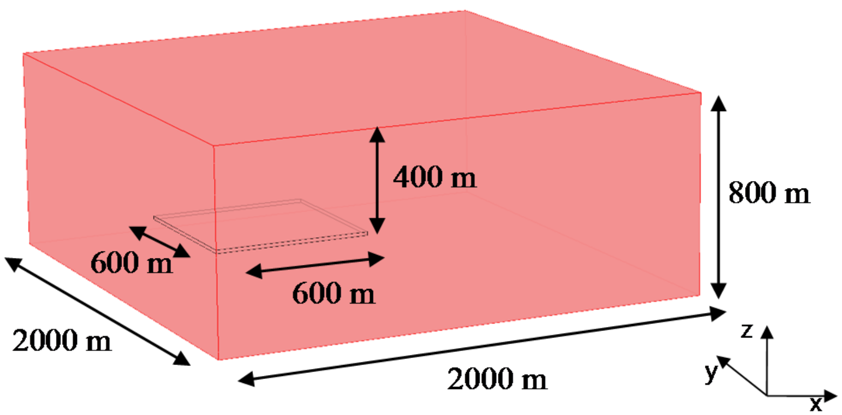
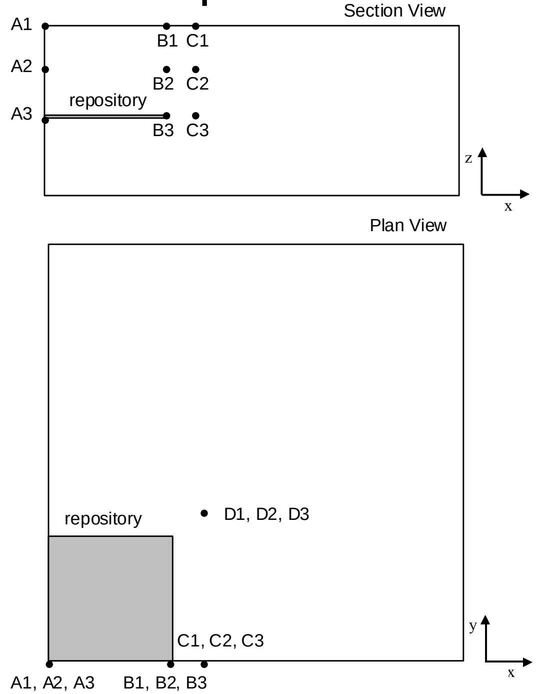
- High level nuclear waste repository in Sweden
  - What would be the stress, displacement and temperature around repository when ~6000 canisters are placed in the deposition holes





Locations of monitoring points

Min KB, Lee JW, Stephansson O, Implications of Thermally-Induced Fracture Slip and Permeability Change on the Long-term Performance of a Deep Geological Repository, Int J Rock Mech Min Sci, 2013;61:175-288.

Numerical Approach in Rock Engineering
Examples (2) – TM analysis

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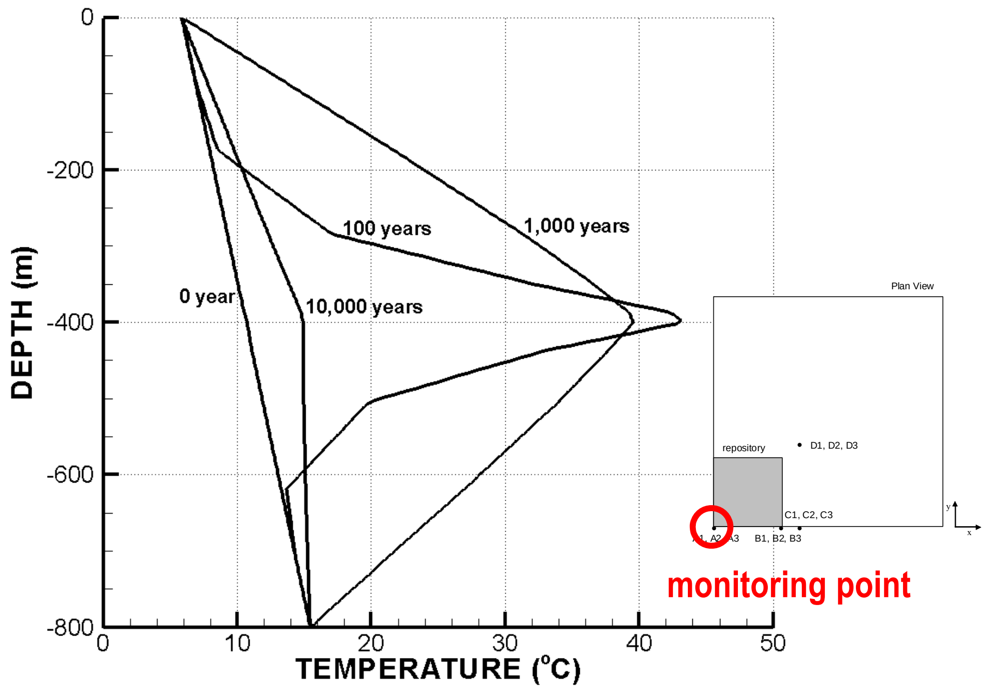
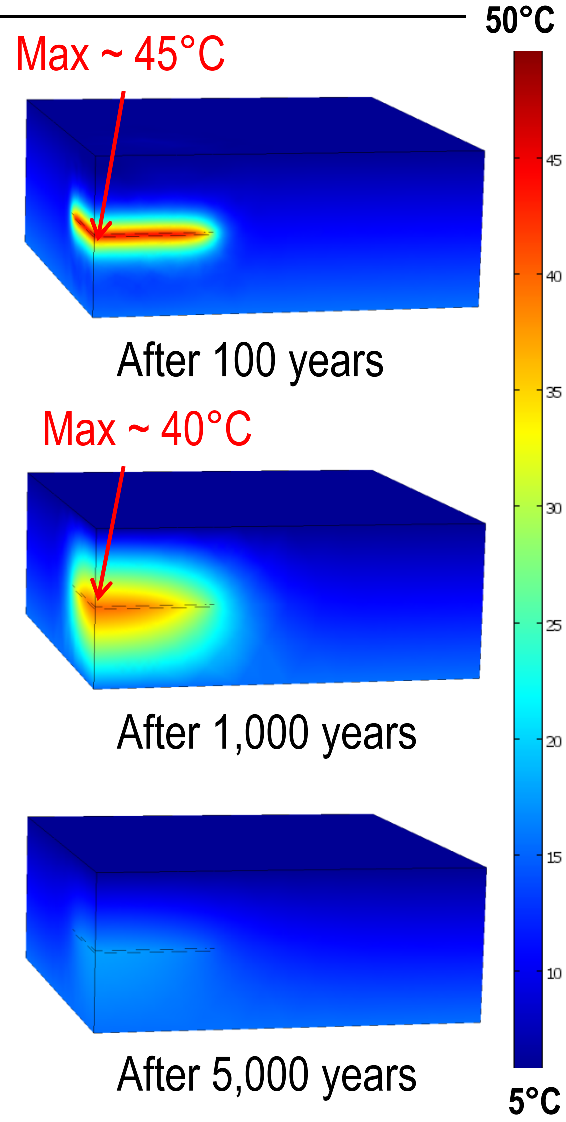
Maximum temperature around 45 °C.



Temperature profile along the center of the repository



Min KB, Lee JW, Stephansson O, Implications of Thermally-Induced Fracture Slip and Permeability Change on the Long-term Performance of a Deep Geological Repository, Int J Rock Mech Min Sci, 2013;61:175-288.

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Examples (2) – TM analysis

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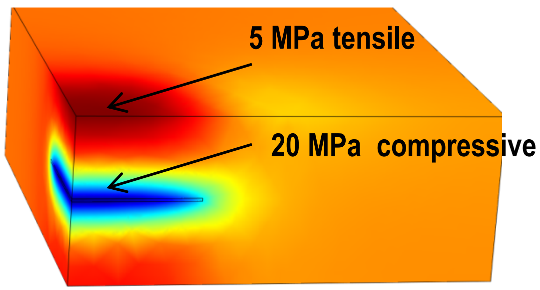
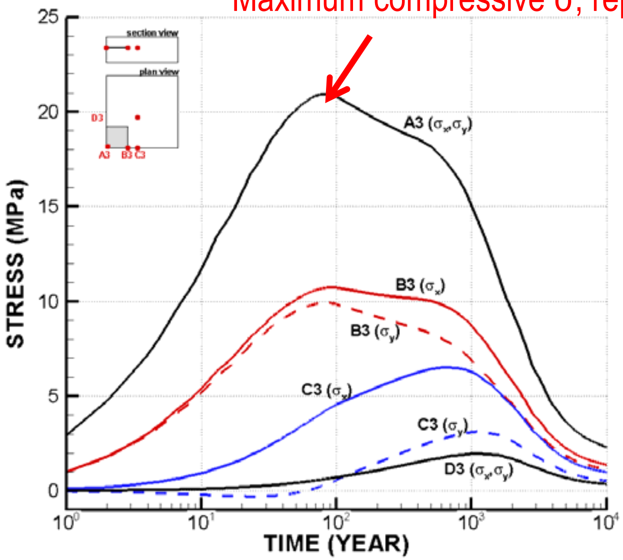
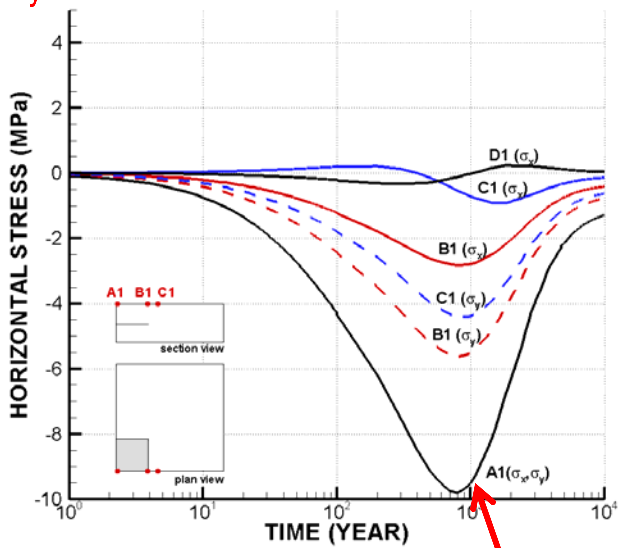


Horizontal stress at 100 years

Maximum compressive σ, repository level





Maximum tensile σ, surface

- Maximum compressive stress ~ 20 MPa near repository at ~ 100 years
- Maximum tensile stress ~10 MPa at surface at ~ 1,000 years

Min KB, Lee JW, Stephansson O, Implications of Thermally-Induced Fracture Slip and Permeability Change on the Long-term Performance of a Deep Geological Repository, Int J Rock Mech Min Sci, 2013;61:175-288.

Numerical Approach in Rock Engineering
Examples (3) – CO2 Geosequestration

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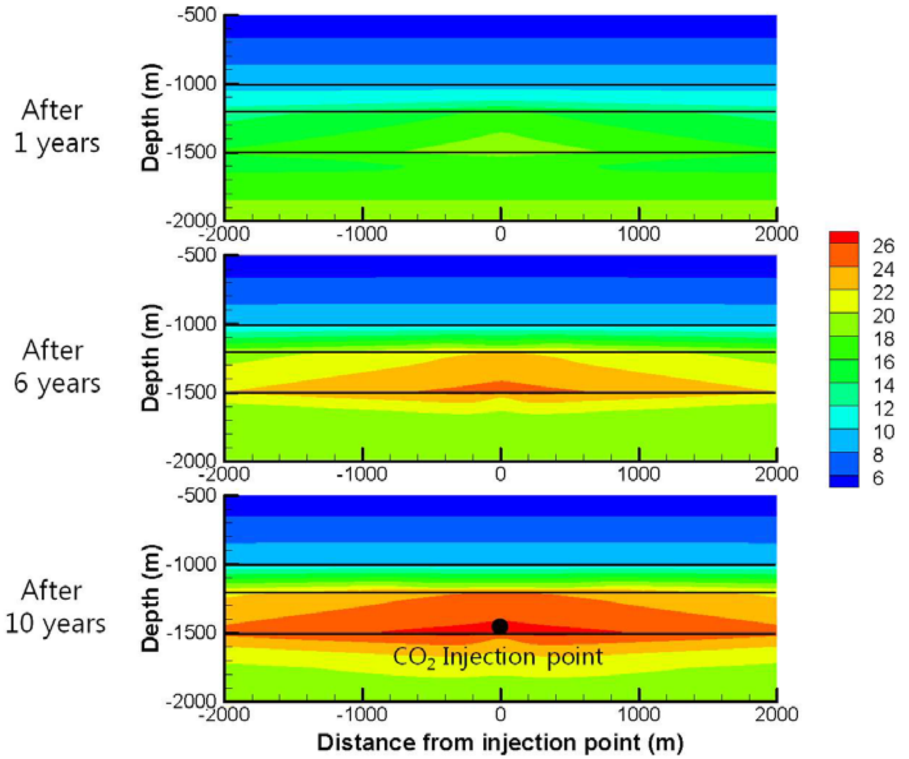
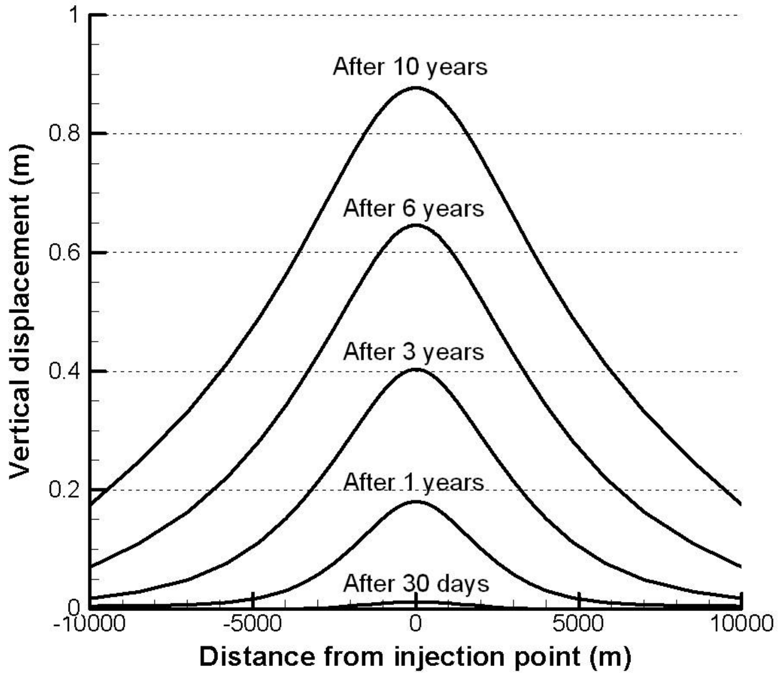
- How much heaving is expected after injecting xxx tons of CO2 at a given geological formation?

TOUGH-FLAC 해석 결과



주입공 주변에서의 간극 수압 변화 (단위: MPa)

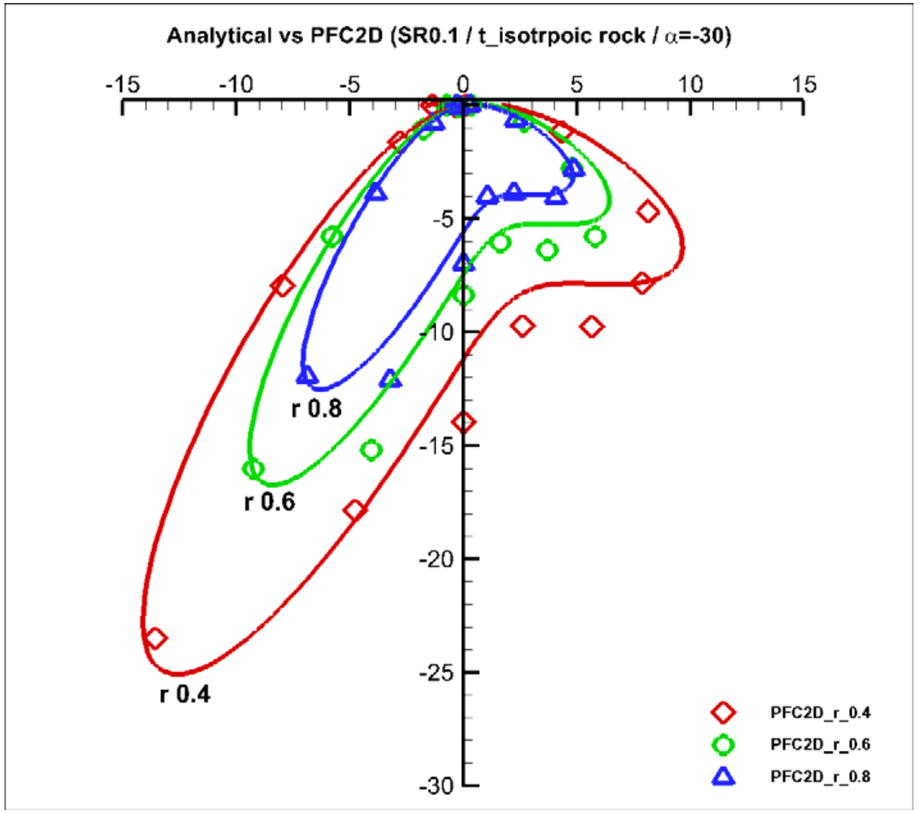


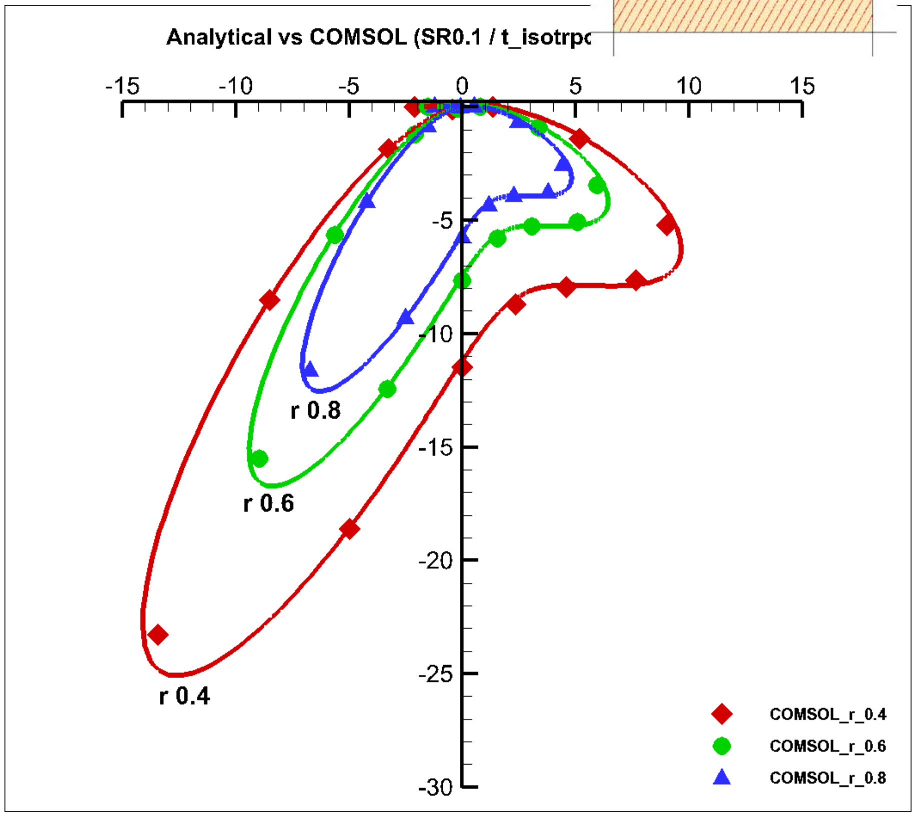
수직 변위 그래프

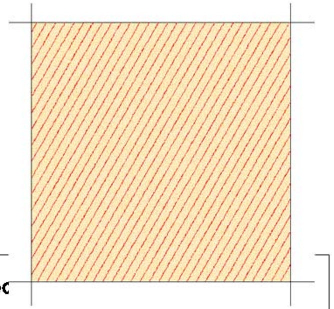
- **10년 후**
  - 간극 수압 : 약 **12 MPa**
  - 수직 변위 : 약 **0.87 m**

Lee JW, Min KB, Rutqvist J, Probabilistic Analysis of Fracture Reactivation Associated with Deep Underground CO_2 injection, Rock Mechanics and Rock engineering, 2013, 46(8):801-820.

Numerical Approach in Rock Engineering
Examples (4) – anisotropic discontinuum







Courtesy of Park B

Numerical Approach in Rock Engineering
Examples (4) – anisotropic discontinuum

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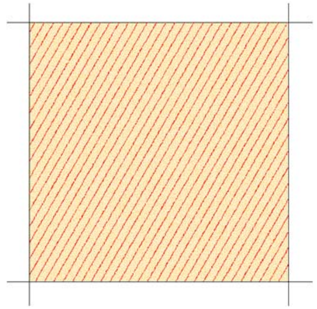

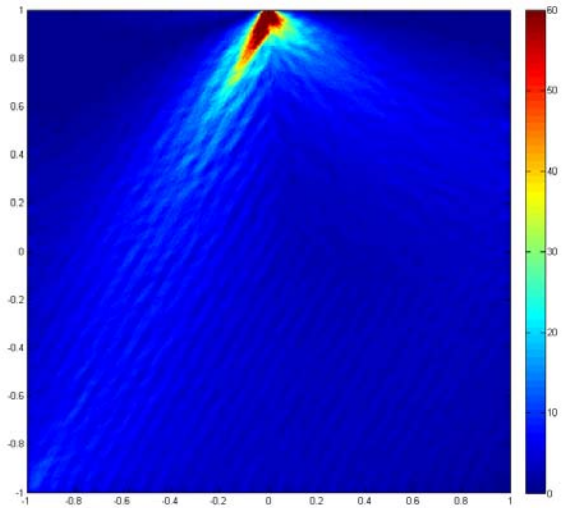

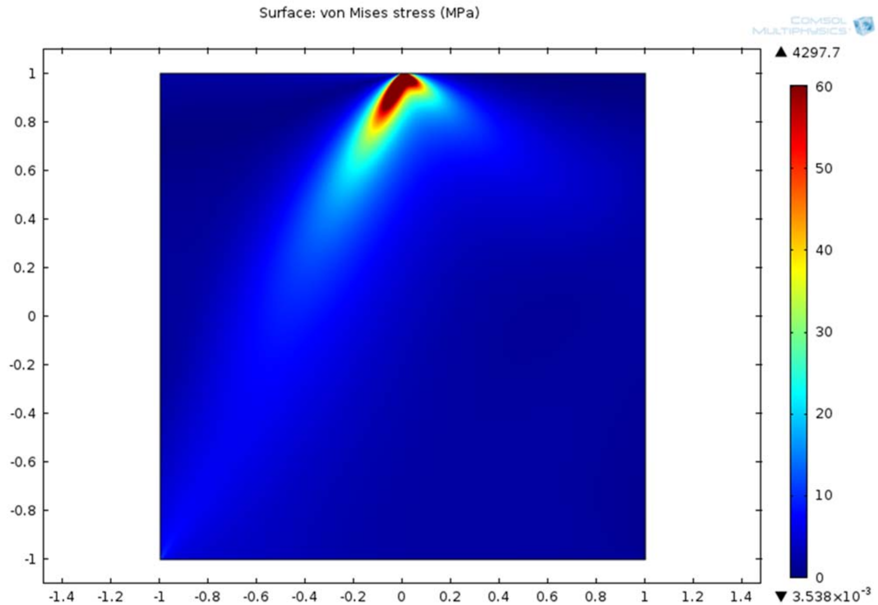

Courtesy of Park B

Numerical Approach in Rock Engineering
Examples (5) – TM analysis in discontinuous rock

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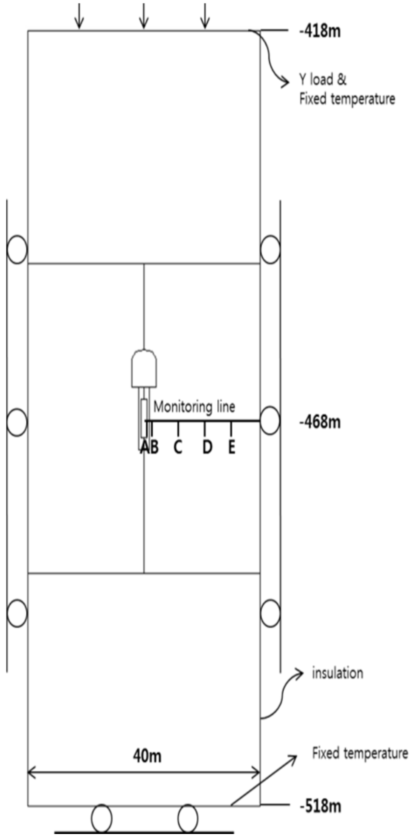
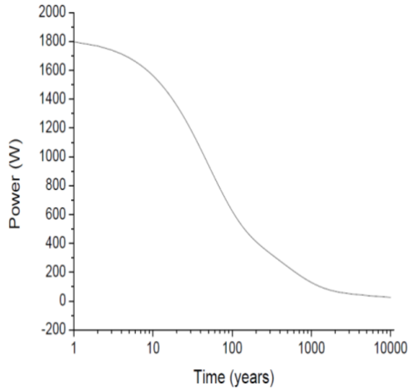
High-level nuclear repository in Forsmark, Sweden

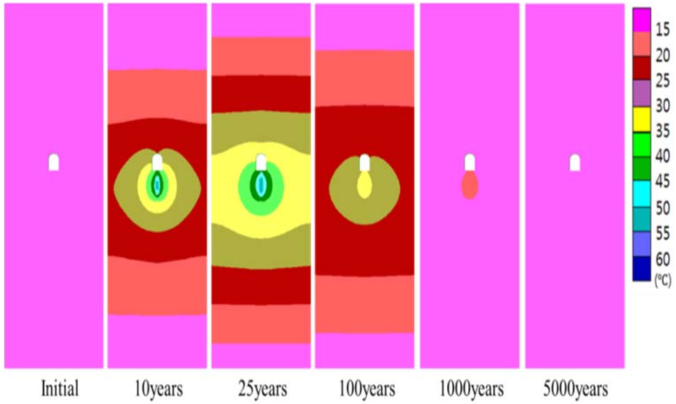
- Geometry and results of the TM modelling



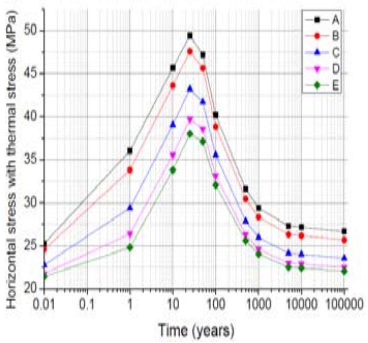
geometry



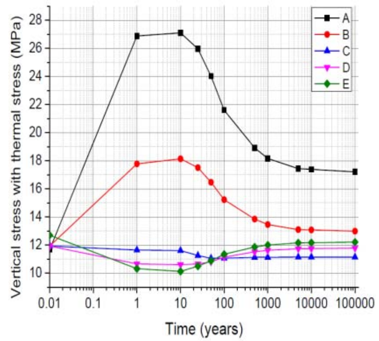
Decayed heat source



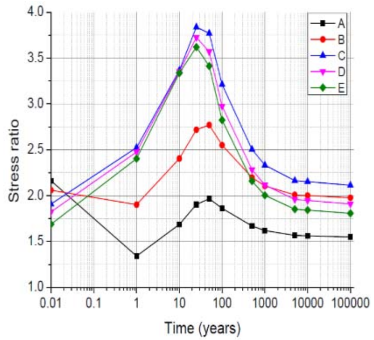
Temperature evolution



Sxx



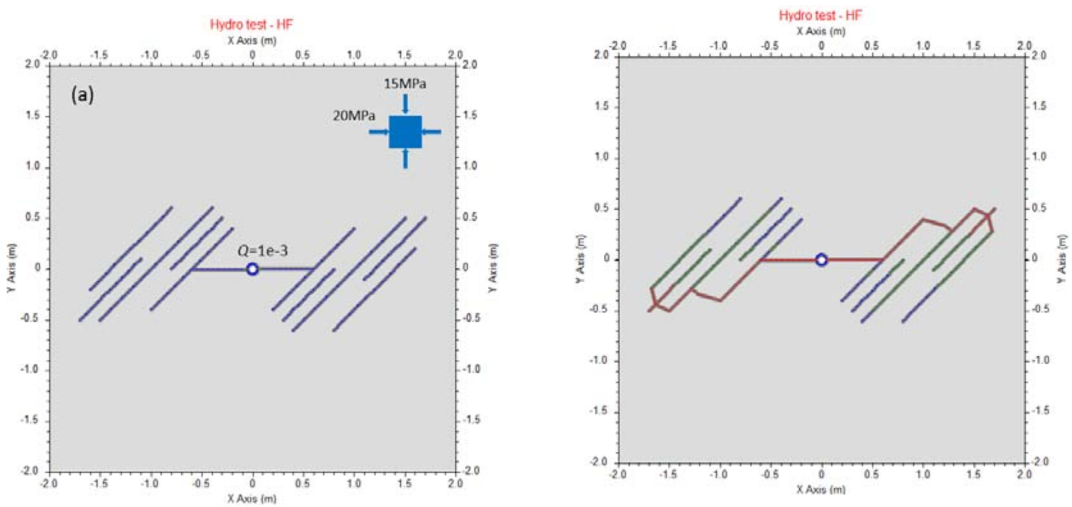
Syy



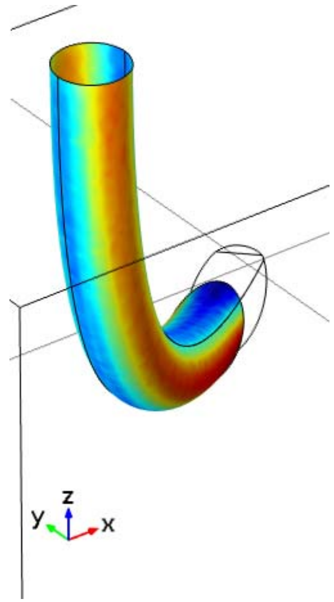
Stress ratio

Courtesy of Park J

Numerical Approach in Rock Engineering
Examples (6) – Hydraulic Fracturing/borehole Geomech



Hydraulic fracturing with pre-existing fractures



Tangential stress around inclined borehole

Xie LM, Min KB, Shen B, 2014, Displacement discontinuity method modelling of hydraulic fracturing with pre-existing fractures, 48th US Rock Mech/Geomech Symp, Paper No.14-7464

Numerical Approach in Rock Engineering*

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- Reasons for popularity in numerical modeling
  - Easy-to-access powerful PC
    - Positive /negative
    - Tool is a means to a solution.(not the solution itself!)
  - Dramatic increase in ability to include geological detail in models
    - More detail imply better model?
    - The art of modeling lies in determining what aspects of the geology are essential.
  - Predictive capability in physical process
  - Success of modeling in other branches of engineering
    - Similarity & differences with aerospace eng?

*Starfield, A.M. and P.A. Cundall, 1988, *TOWARDS A METHODOLOGY FOR ROCK MECHANICS MODELING.* Int J Rock Mech Min Sci & Geomech Abstr, 25(3): p. 99-106

Numerical Approach in Rock Engineering

- Problems in applying numerical approach
  - Misuse
    - Use in a wrong way:
    - Need to be familiar with the theory of the numerical methods
  - Abuse or overuse
    - Numerical tool is not a magic box
    - Appropriate modeling methodology needed

Rock Characterization Problem

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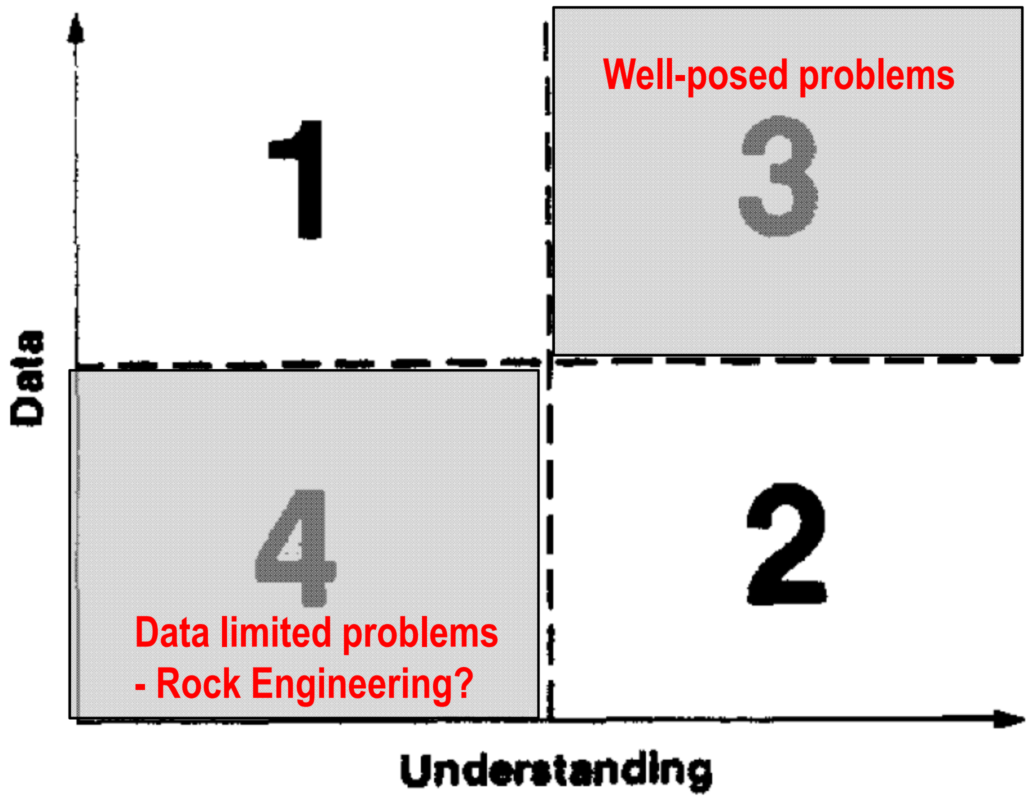
- Uncertainty in Geological Feature
- Uncertainty in Boundary Condition
  - In situ stress not easy to characterize
- Hard to obtain data in Rock/Fracture properties
  - Costly, unavailable
- Up-scaling issue
  - measure in the lab may not represent the values in large scale

Numerical Approach in Rock Engineering
Data limited problems



Recited from Starfield and Cundall (1988)

Numerical Approach in Rock Engineering
Data limited problems

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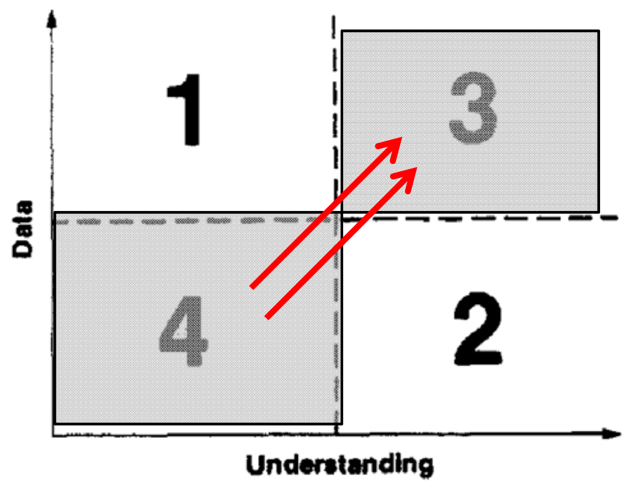
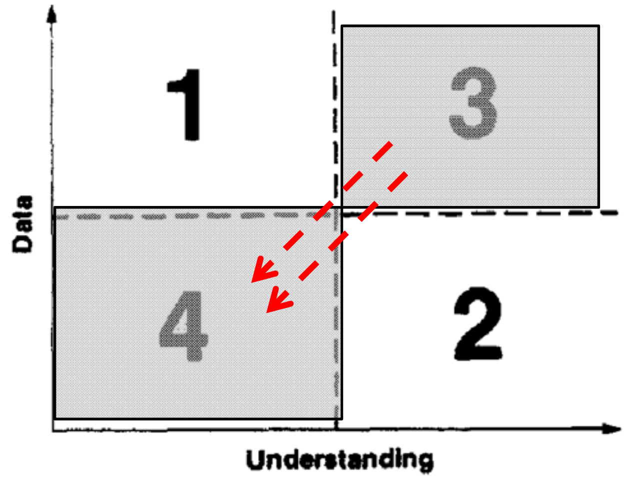
- Fitting rock engineering problem into region 3 (lots of data plus good understanding)
  - Impossible to have sufficient data
  - We loses control of intellectual control of the model



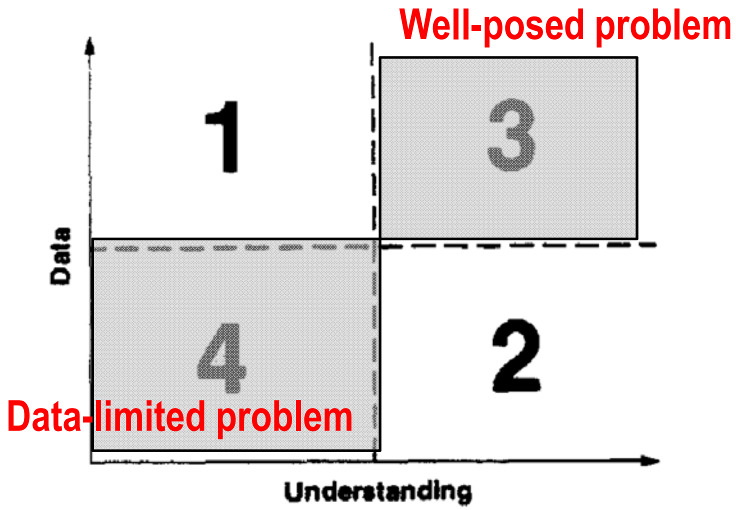
Numerical Approach in Rock Engineering
Data limited problems

- Apply the tools developed for region 3 to rock engineering problem
  - Numerical tool is a means to a solution!



Modeling guidelines

- Differences between well-posed and data-limited problems
  - Resolution
  - Validation
  - Once validated, can it be used routinely?

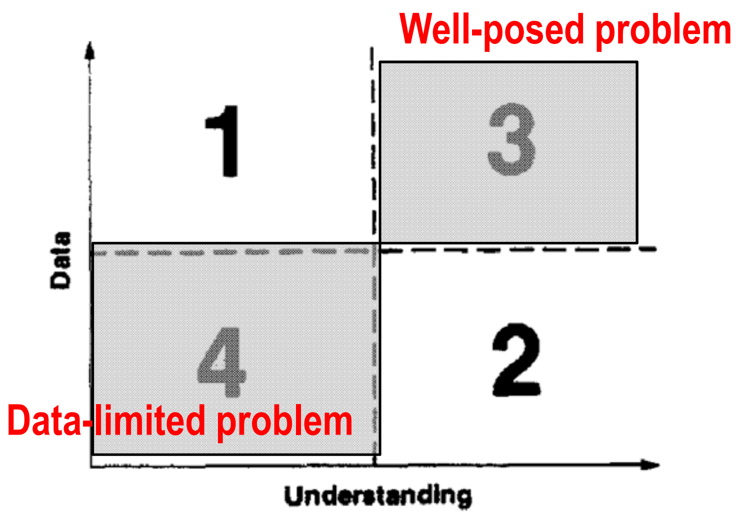


Modeling guidelines

- Data-limited problems
  - A model is a simplification of reality rather than an imitation of reality. A model is an intellectual tool.
  - The design of the model should be driven by the questions that the model is supposed to answer rather than the details of the system.
    → helps in simplify and control the model
  - More appropriate to build a few very simple models than one complex model.
  - Try to gain confidence in the model and modify it as one uses it. Approach to the model is that of a detective (not mathematician)
  - Pupose is to gain understanding and to explore potential trade-offs and alternatives. (not absolute predictions)

Modeling guidelines

- Data-limited problems
  - One progresses slowly from region 4 to region 3 ← from simple to complex model, suggest new data or new models. ← Adaptive modeling



Modeling guidelines

- Clear about why you are building a model and what questions you are trying to answer
- Use a model at the earliest possible stage in a project to generate both data and understanding.
  - Do not delay while waiting for field data. You need a conceptual model in place as soon as possible.
- Look at the mechanics of the problem.
  - Identify important mechanisms
- Try to visualize qualitatively what the answer of your modeling would be

Modeling guidelines

- Design the simplest model that will allow the important mechanisms to occur ← serve as a laboratory for the thoughts experiments
- Implement your simplest modeling – run it – and improve it.
  - Proceed to more complex modeling
  - Or, identify the weakness and remedy them before continuing
  - If your model has weakness that you cannot remedy → make a series of simulations that will bracket the true case.

Modeling guidelines

- Numerical modeling is very similar to laboratory work
- Visualizing and anticipating solutions *before* running a model is an important discipline.
- Modeling in a cautious way actually generate new knowledge

Numerical Approach in Rock Engineering
Case Studies (EGS hydraulic stimulation)

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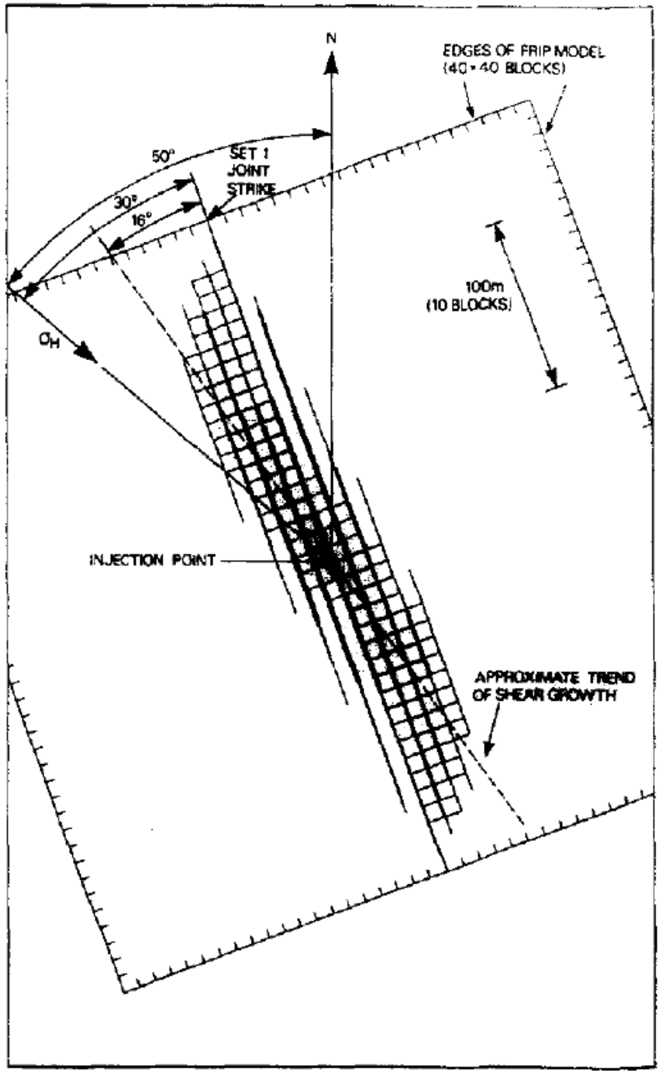
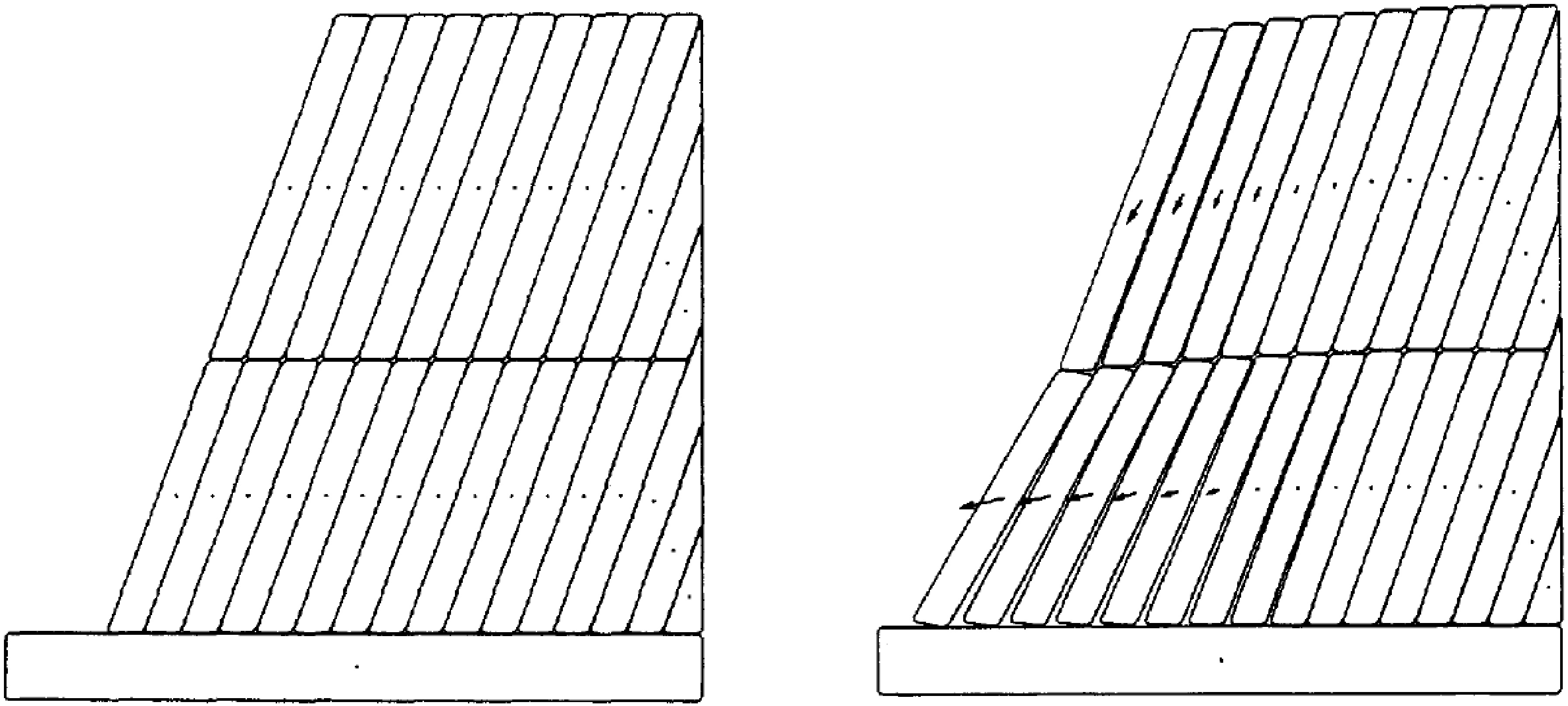
- Fluid flow enhancement due to hydraulic stimulation in a EGS project in Cornwall (Pine, 1985)



Recited from Starfield and Cundall (1988)

Numerical Approach in Rock Engineering
Case Studies (Rock Slope Stability)



- So obvious? Easy to say…detective novel…

Recited from Starfield and Cundall (1988)

Numerical Approach in Rock Engineering Good and bad examples

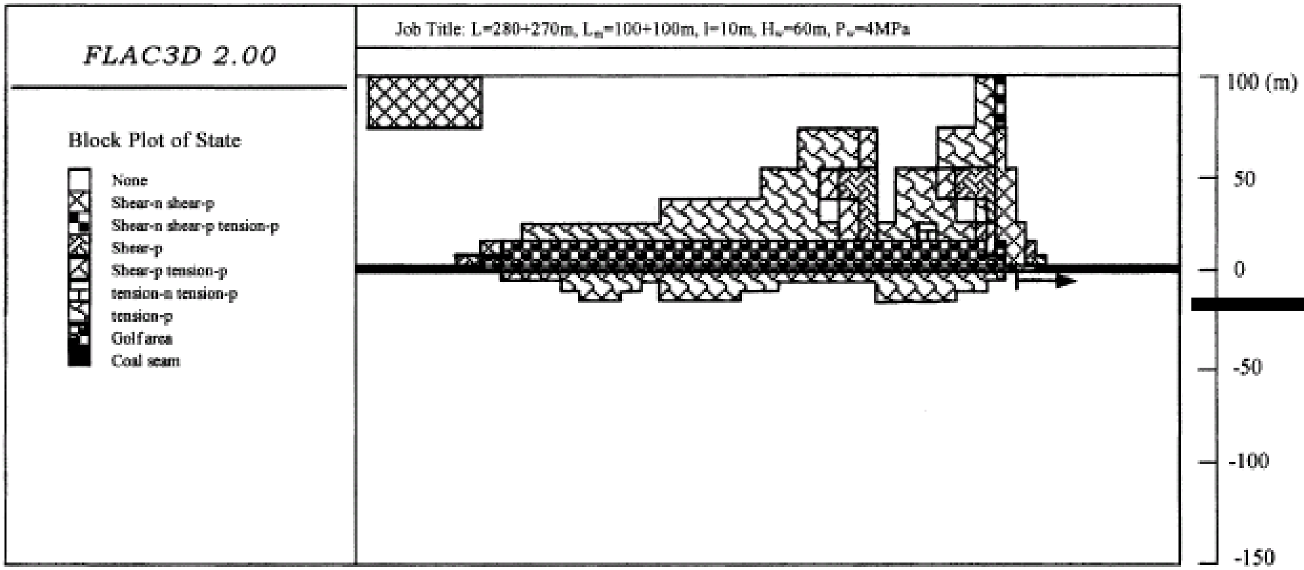


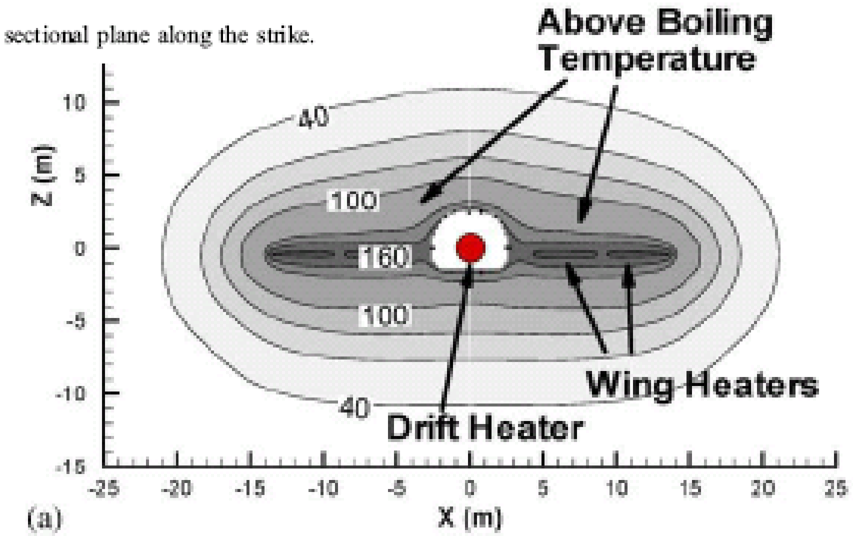
Fig. 7. Indication of the fracture zones in the floor and roof strata on a sectional plane along the strike.

unclear legend

unclear plot

Not easy to read

(xxx,2002)



Clear legend

Clear plot

Easy to read

(Rutqvist et al., 2005)

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Verification vs. Validation

- Verification: the provision of assurance that a code correctly performs the operations it specifies (e.g., PDE)[1].
  - A common method of verification is the comparison of a code's results with solutions obtained analytically (Kirsch solution, Boussinesq…)
- Validation: the determination that the code or model indeed reflects the behavior of the real world [2].
  - Validated model is the one that provides a good representation of the actual processes occurring a real system [3].

1. US Nuclear Regulatory Commission (NRC, NUREG-0865, 1990)
2. US Department of Energy (DOE/RE-0073, 1986)
3. IAEA, Radioactive waste management glossary (IAEA-TECDOC-264, 1982)

Verification vs. Validation

- *Verification:
  - Is the program doing what it claims to be doing?
- Are we getting the answers that we think we are getting?
- Validation
  - Are we getting the answers that we need?

DM Wood, Geotechnical modeling, 2004

Concluding remark

- Numerical method is a indispensable part of engineering analysis – needs a thorough understanding
- Numerical method has a unique role that other analytical or experimental methods cannot play.
- However, we must bear in mind that numerical methods is only a means not the answer itself.
  - Garbage in, garbage out - The results is only as good as the data
  - A model is an aid to thought, rather than a substitute for thinking
  - Plan the modeling exercise in the same way as you would plan a laboratory experiment

References

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