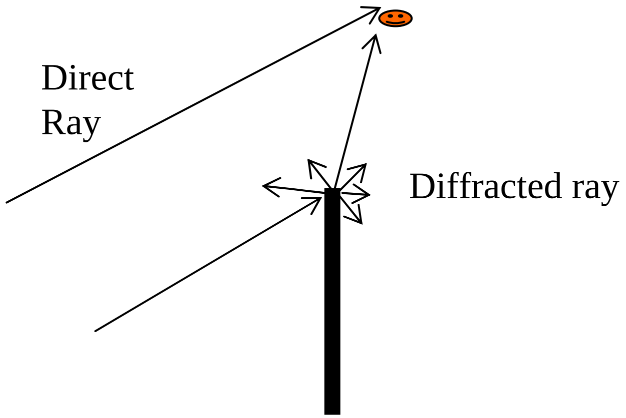
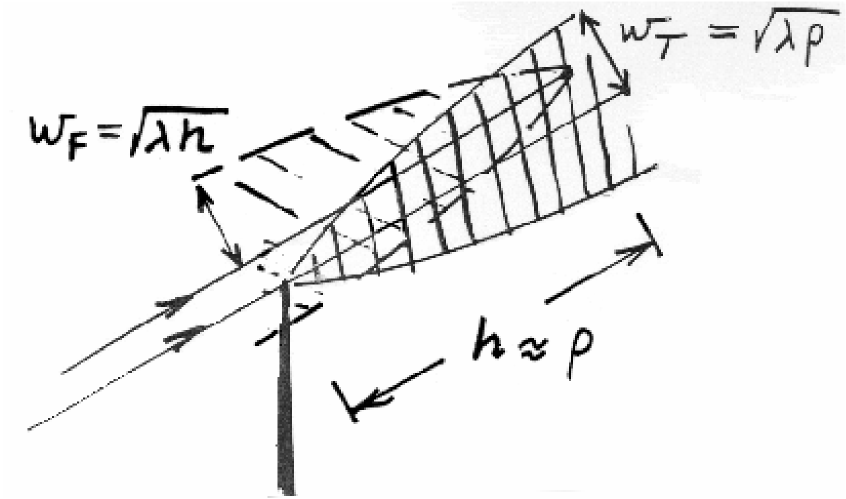
Relation between Transition Region and Fresnel Zone

- Away from transition Region

$$E^{total} = E^{GO} + E^{D}$$

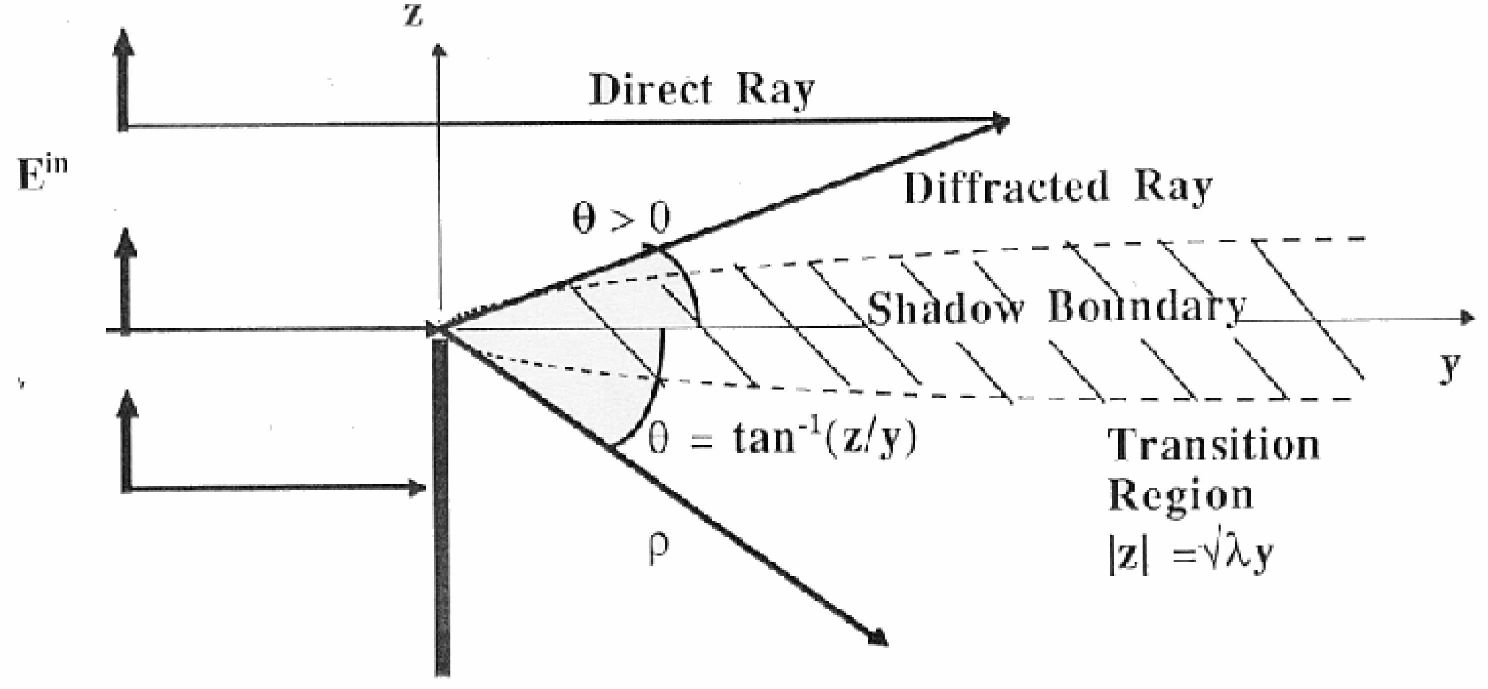


- Inside transition Region



Asymptotic Approximation of the Integration

(for y >> λ)



$$E(0,y,0) \approx E_0 e^{-jky} U(z) - E_0 D(\theta) e^{-j\pi/4} \frac{e^{-jk\rho}}{\sqrt{\rho}}$$

Diffraction Coefficient D(θ)

- Outside the transition region $|z| > \sqrt{\lambda y} \quad or \quad |\theta| > \sqrt{\lambda / y}$

$$D(\theta) = D_0(\theta) \equiv \frac{-1}{\sqrt{2\pi k}} \frac{1+\cos\theta}{2\sin\theta}$$

- Inside the transition region

$$D(\theta) = D_T(\theta) \equiv \frac{-j\sqrt{\rho}}{\sqrt{\pi}} e^{j2\xi^2} \left[\int_{\xi\sqrt{2}}^{\infty} e^{-jt^2} dt \right] \mathrm{sgn}(\theta)$$

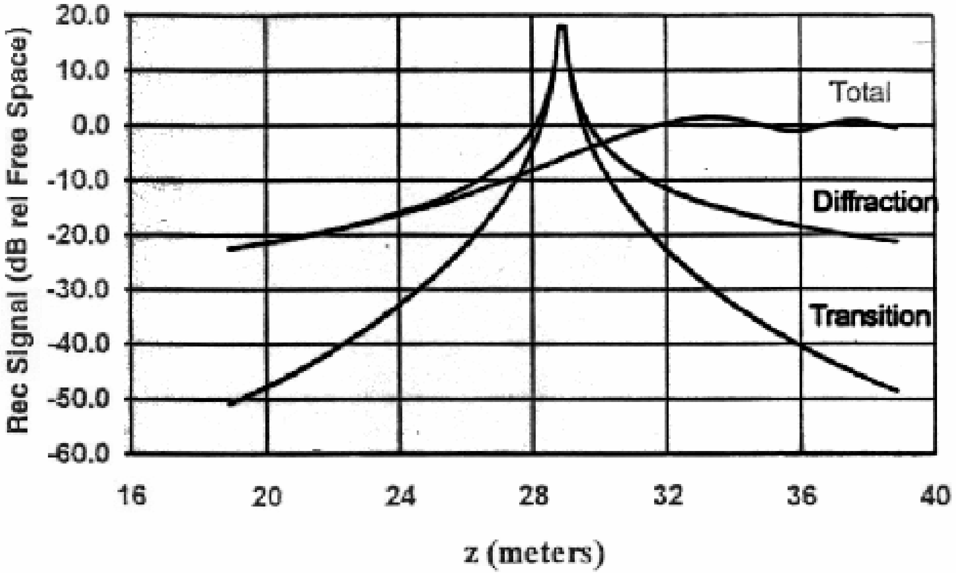
where $\xi = \sqrt{\frac{k}{y} \frac{|z|}{2}}$

- Rigorous theory for a absorbing screen gives identical results with

$$D_0(\theta) \equiv \frac{-1}{\sqrt{2\pi k}} \left[\frac{1}{\theta} + \frac{1}{2\pi - \theta} \right]$$

Contribution to the Total Diffracted Field

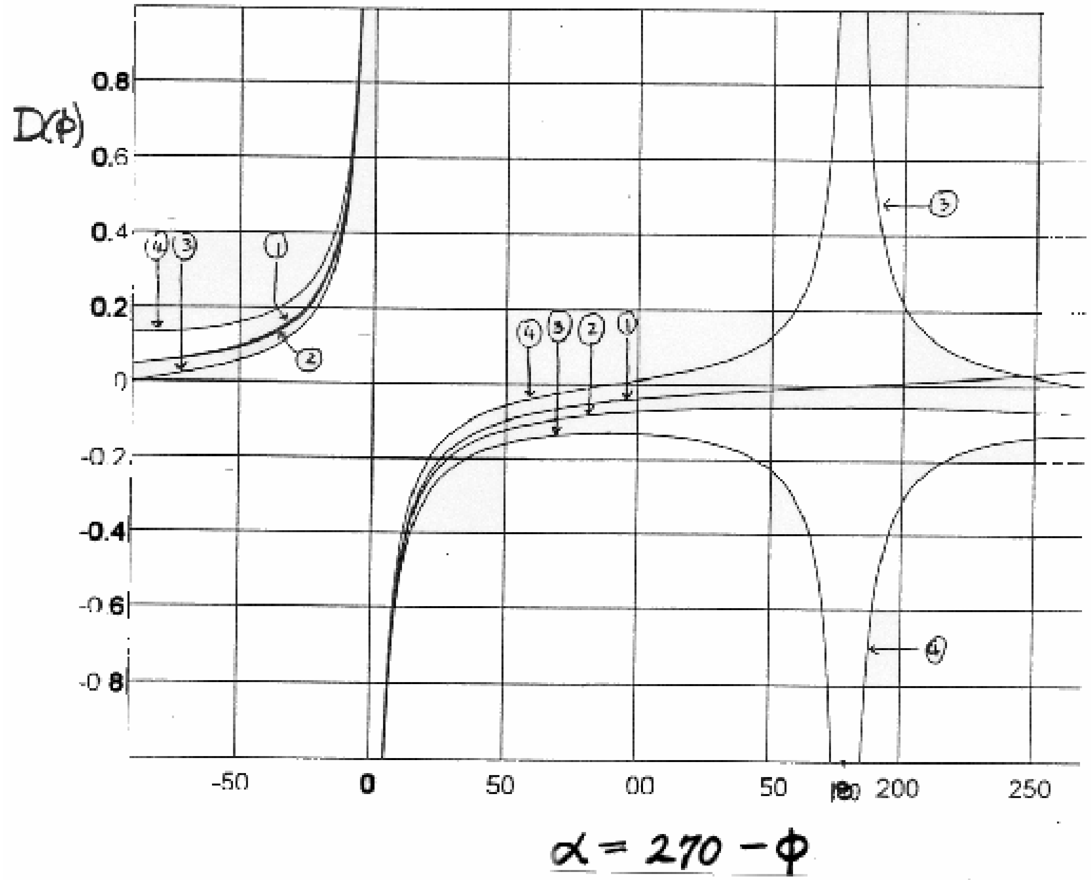
- Diffraction over single absorbing screen. Plane wave incidence, frequency 900 MHz, y=50 m and incidence angle ϕ’=60°.

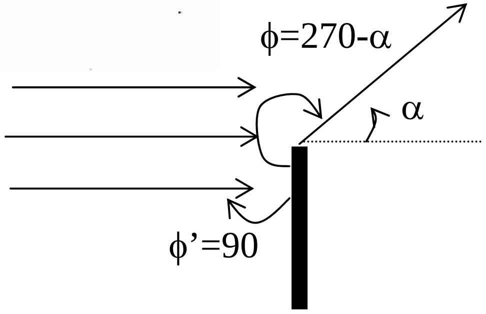


c.f. 1 : The shadow boundary $z_b = x / \tan\phi' = 50 / \tan(\pi/3) = 28.87m$

c.f. 2 : Transition curve represents contribution by $D_T(\alpha) + \frac{1}{\sqrt{2\pi k}} \frac{\mathrm{sgn}(z)}{|z| / \sqrt{\rho y}}$

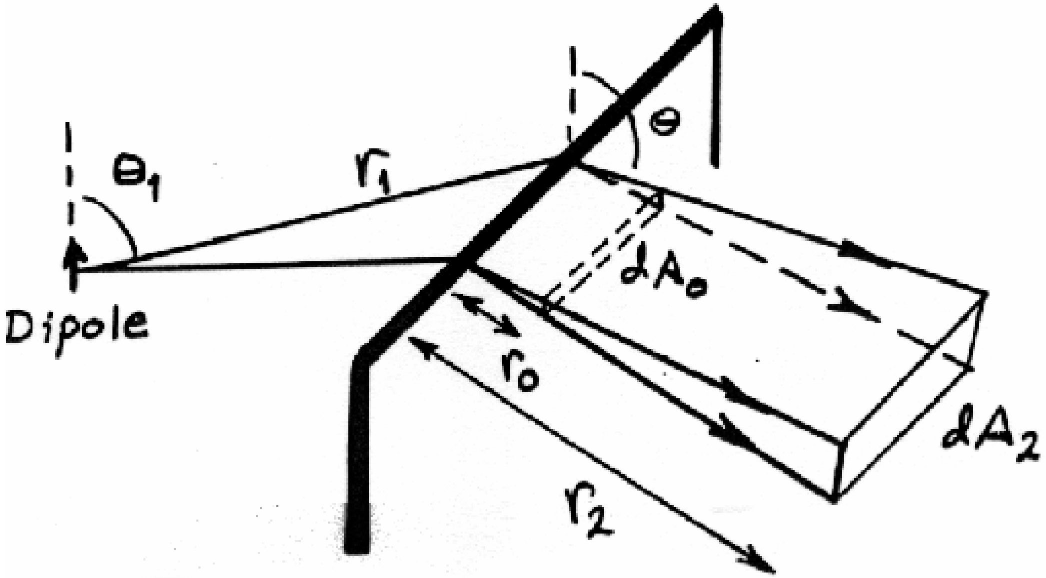
Comparison of Diffraction Coefficients for Normal Incidence (ϕ’=90)





1- Kirchhoff approximation
2- Absorbing screen
3- Conducting screen for TE (Horizontal)
4- Conducting screen for TM (Vertical)

Diffracted Fields Due to an Incident Spherucal Wave



- Near the edge ($\lambda << r_0 << r_1$)

$$E^D(r_0) = \underbrace{ZI\,\frac{e^{-jkr_1}}{r_1}\,f(\theta_1)}_{\text{Field incident on the edge}}\;\underbrace{\frac{e^{-jkr_0}e^{-j\pi/4}}{\sqrt{r_0}}\,D(\phi',\phi)}_{\text{Diffracted cyndrical wave}}$$

Diffracted Field Amplitude Must Conserve Power in a Ray Tube

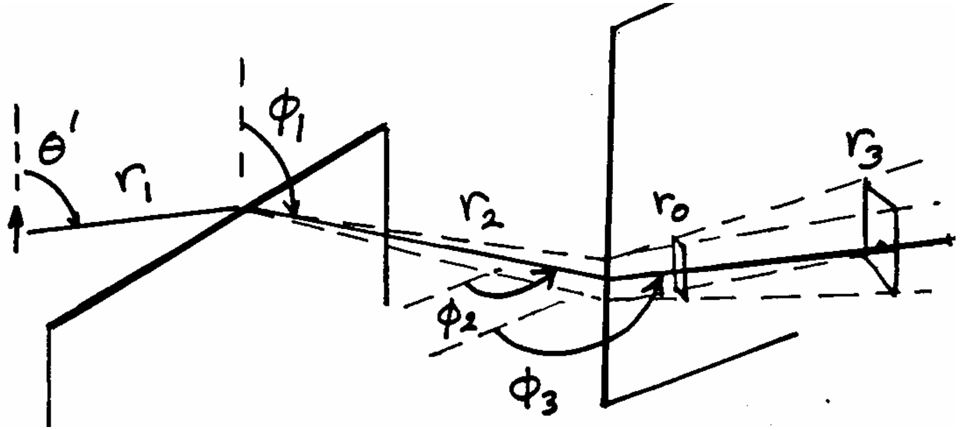
- Consider spreading of ray tube area

$$\frac{dA_0}{dA_2} = \underbrace{\boxed{\frac{r_0}{r_2}}}_{\text{Vertical spreading}} \underbrace{\boxed{\frac{r_1}{r_1 + r_2}}}_{\text{Horizontal spreading}}$$

- Accounting for phase change along the ray

$$E^D(r_2) = E^D(r_0) e^{-jk(r_2 - r_0)} \sqrt{\frac{dA_0}{dA_2}} = ZIf(\theta_1) e^{-j\pi/4} \frac{e^{-jk(r_1+r_2)}}{\sqrt{r_1 r_2 (r_1 + r_2)}} D(\phi', \phi)$$

Diffraction of Dipole Fields by Horizontal and Vertical Edges

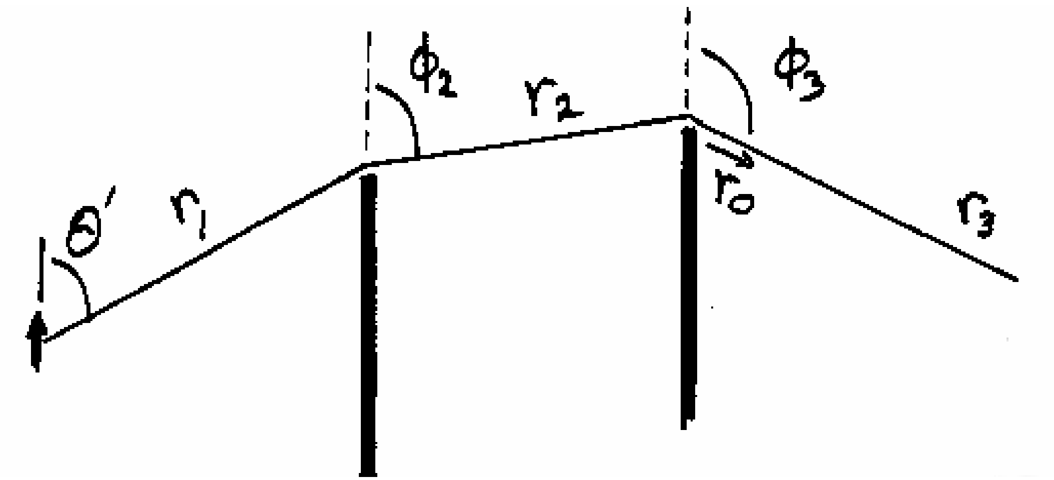


$$E(r_3)=E_{in}\underbrace{\frac{e^{-j\pi/4}e^{-jkr_0}}{\sqrt{r_0}}D(\phi_3,\phi_2)}_{\text{Cylindrical wave near edge}}\underbrace{\sqrt{\frac{r_0}{r_3}\frac{r_2}{r_2+r_3}}e^{-jk_0(r_3-r_0)}}_{dA(r_0)/dA(r_3)}$$

$$E_{in}=ZIf(\theta')\frac{e^{-j\pi/4}e^{-jk(r_1+r_2)}}{\sqrt{r_1r_2(r_1+r_2)}}D(\phi_1,\theta')$$

$$E(r_3)=ZIf(\theta')\frac{e^{-j\pi/2}e^{-jk(r_1+r_2+r_3)}}{\sqrt{r_1r_3(r_1+r_2)(r_2+r_3)}}D(\phi_1,\theta')D(\phi_3,\phi_2)$$

Diffraction of Vertical Dipole Fields by Two Successive Edges



$$E(r_3)=E_{in}\underbrace{\frac{e^{-j\pi/4}e^{-jkr_0}}{\sqrt{r_0}}D(\phi_3,\phi_2)}_{\text{Cylindrical wave near edge}}\underbrace{\sqrt{\frac{r_0}{r_3}\frac{r_1+r_2}{r_1+r_2+r_3}}e^{-jk_0(r_3-r_0)}}_{dA(r_0)/\,dA(r_3)}$$

$$E(r_3)=ZIf(\theta')\frac{e^{-j\pi/2}e^{-jk(r_1+r_2+r_3)}}{\sqrt{r_1r_2r_3(r_1+r_2+r_3)}}D(\phi_2,\theta')D(\phi_3,\phi_2)$$