Wireless Channel Model

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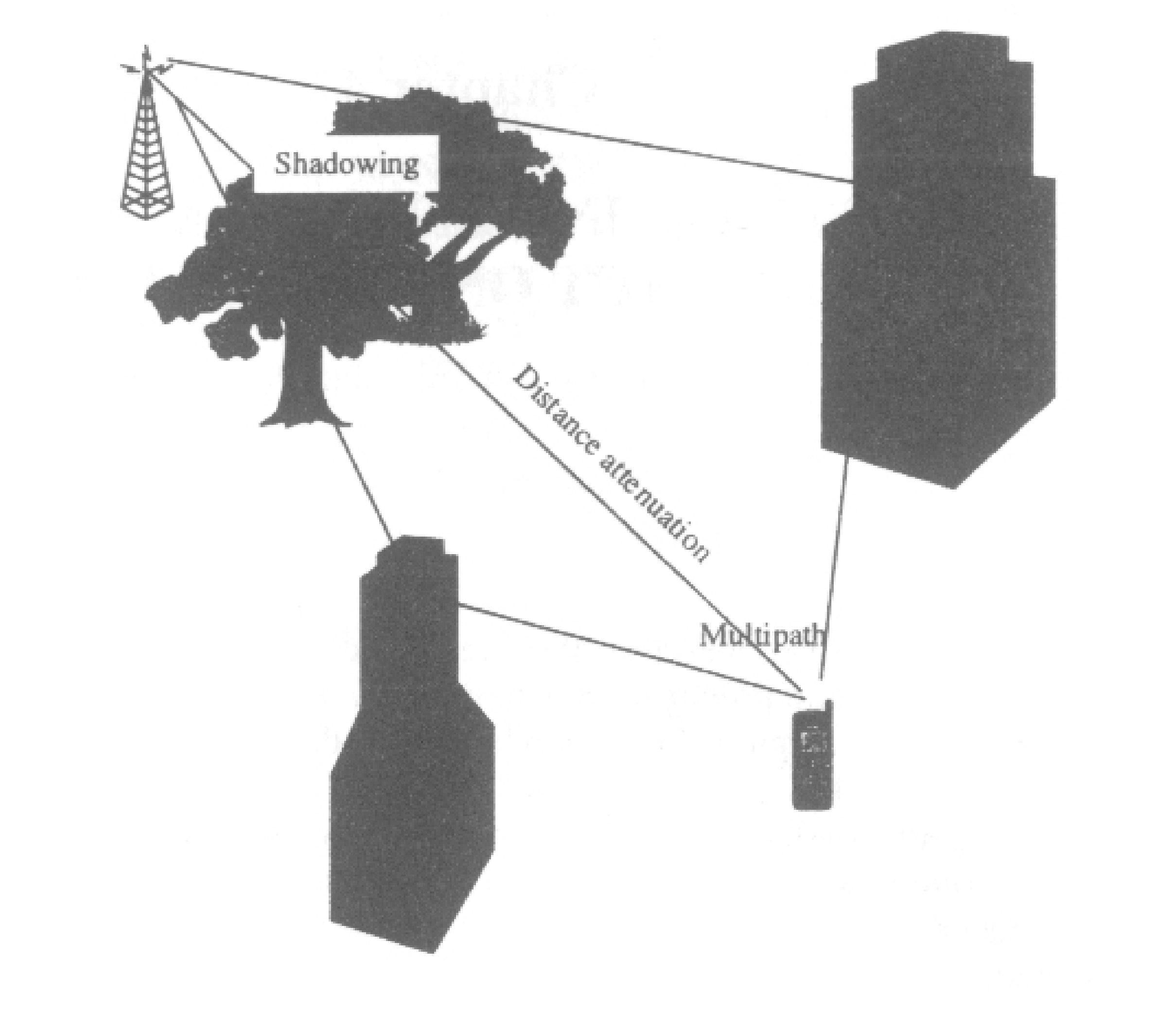
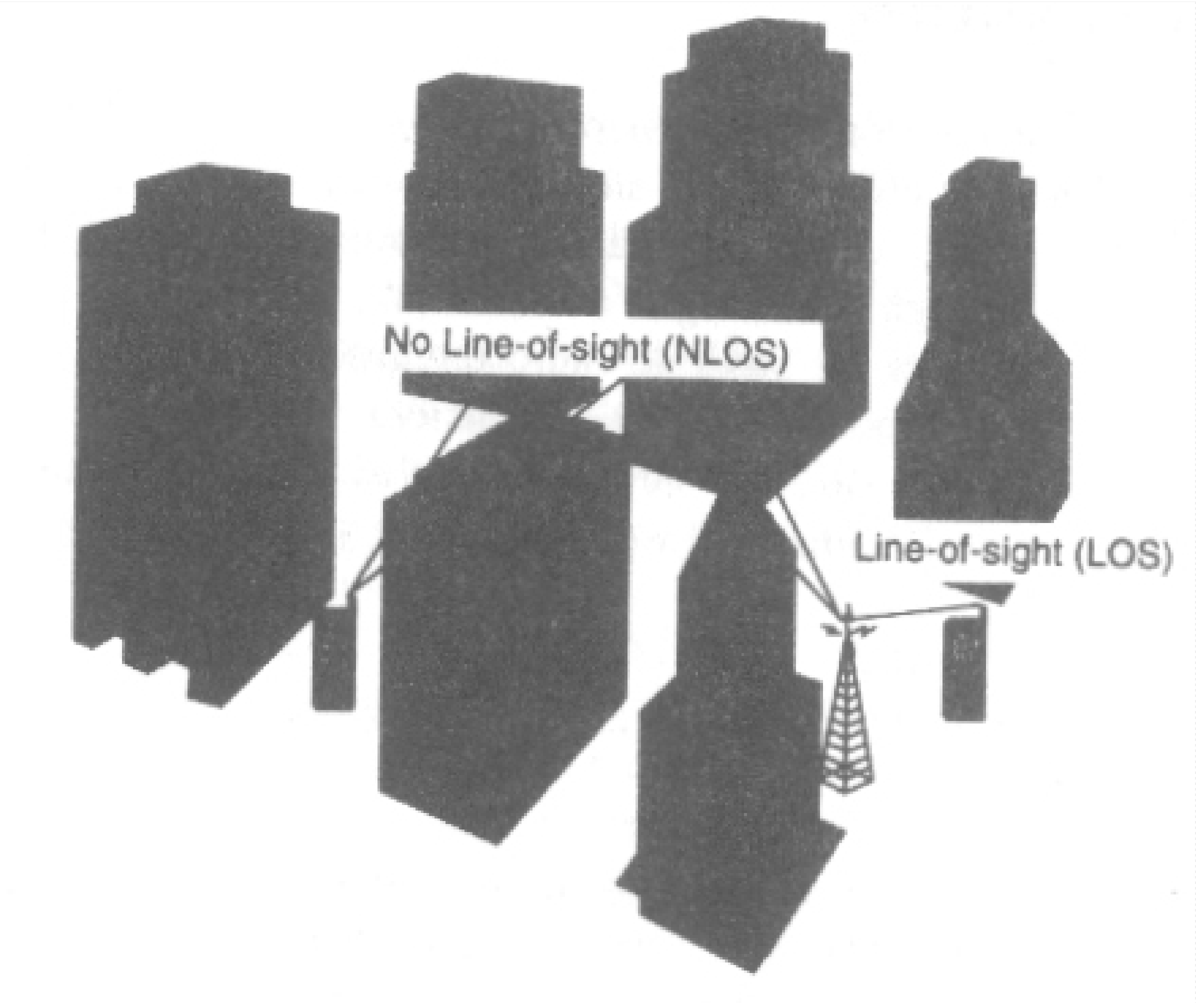
Radio Channel (1)

- Signal Fading
  - large-scale component: Path Loss
  - medium-scale slow varying component: Shadowing
  - small-scale fast varying component: Multipath fading

Radio Channel (2)



LOS Path vs. NLOS Path

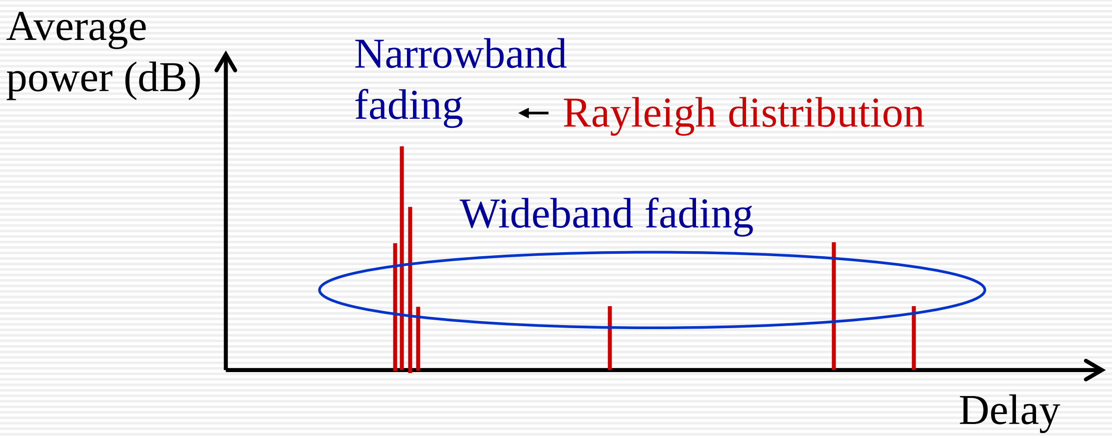


Path Loss & Shadowing

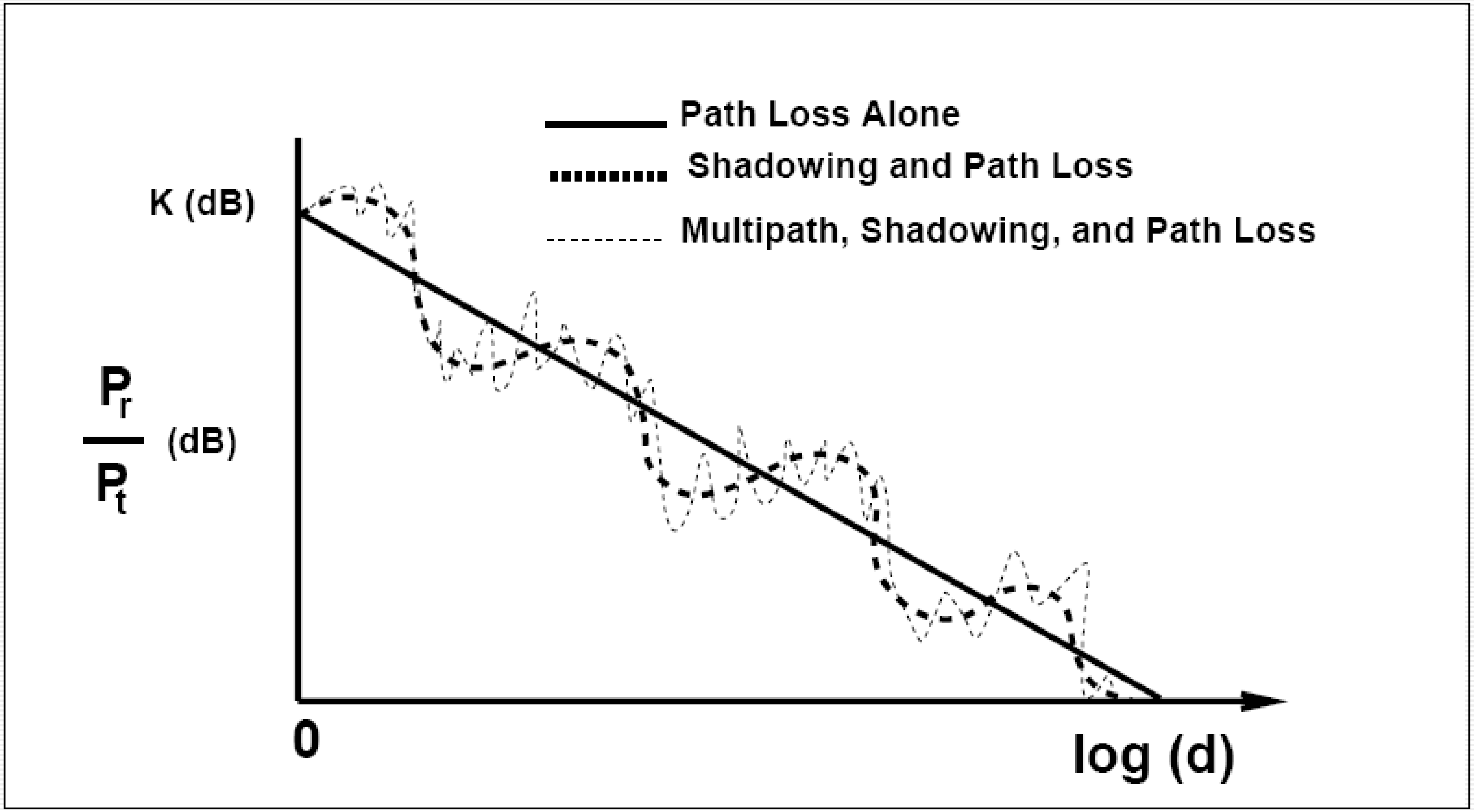
- Path Loss
  - caused by dissipation of the power radiated by the transmitter
  - depends on the distance between transmitter and receiver
- Shadowing
  - caused by obstacles between the transmitter and receiver that absorb power.

Multipath fading

- short-term fluctuation of the received signal caused by multipath propagation
- when mobile is moving
- Fading becomes fast as a mobile moves faster
- delay-power profile (delay spread)



Channel Model



Simplified Pass Loss Model

- Path loss as a function of distance: $P_r = P_t K \left[\frac{d_0}{d} \right]^{\gamma}$
  - sometimes simple model can captures the essence of signal propagation without resorting to complicated path loss models.
  - Free-space, two-ray, Hata, COST extension to Hata are all of the same form
  - K : constant which depends on antenna characteristics and the average channel attenuation
    - Free space path gain at distance d_0 assuming omni-directional antennas
    - Empirical measurements at d_0
  - d_0: reference distance
    - generally valid only at $d > d_0$
    - d_0 : 1-10m (indoor), 10-100m (outdoor)
  - γ : path loss exponent
    - at higher frequencies tend to be higher and at higher antenna heights tend to be lower

Empirical Path Loss Models

- Piecewise Linear Model

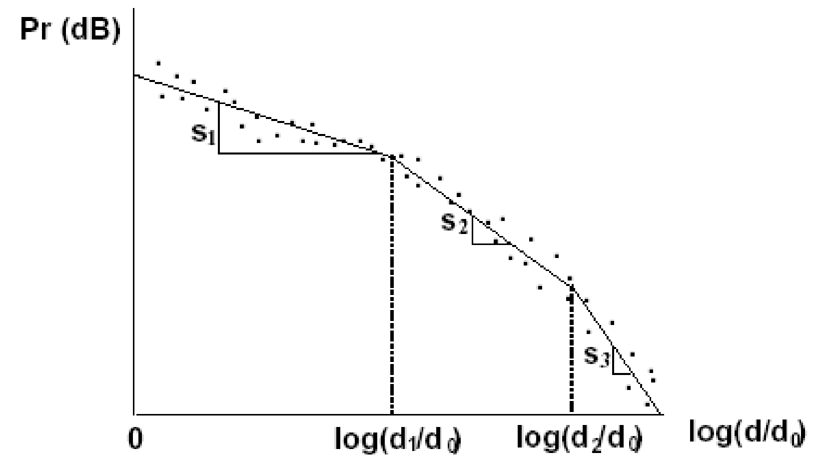


Figure 2.9: Piecewise Linear Model for Path Loss.

- Indoor Attenuation Factors
  - partition loss

Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

  - floor loss
  - the building penetration loss
  - It is difficult to find generic models

Shadowing (1)

- Statistical models
  - The transmitted signal experiences random variation due to blockage from objects in the signal path and changes in reflecting surfaces and scattering objects.
- Log-normal shadowing
  - Ratio of transmit-to-receive power $\psi = P_r / P_t$ is a random variable with a log-normal distribution

$$p(\psi) = \frac{10/\ln 10}{\sqrt{2\pi}\sigma_{\psi_{dB}}\psi} \exp\left[-\frac{(10\log_{10}\psi - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}}\right]$$

  - Distribution of ψ_{dB} (the dB value of ψ) is Gaussian with mean $\mu_{\psi_{dB}}$ and standard deviation $\sigma_{\psi_{dB}}$

$$\boxed{p(\psi_{dB}) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{dB}}} \exp\left[-\frac{(\psi_{dB} - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}}\right]}$$

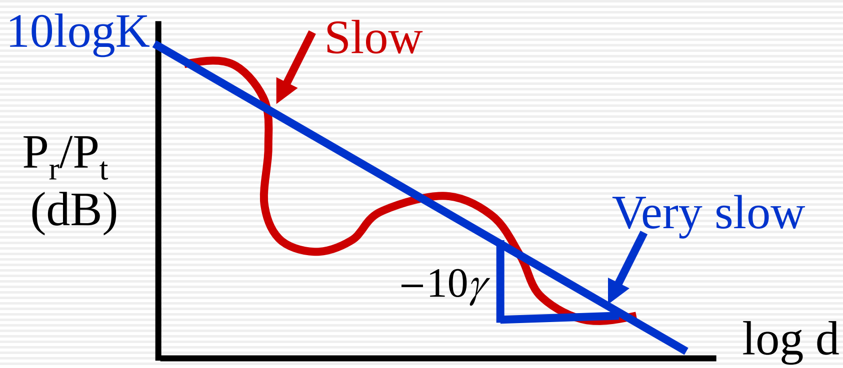
Shadowing (2)

- Justification for the Gaussian model as the distribution of ψ_{dB}
  - when shadowing is dominated by the attenuation from blocking object
  - attenuation of a signal as traveling through an obstacle with depth *d*
    - $s(d) = e^{-\alpha d}$, where α is an attenuation constant.
  - attenuation of a signal as it propagates through the region $d_t = \sum d_i$
    - $s(d_t) = e^{-\alpha \sum d_i} = e^{-\alpha d_t}$
  - d_t : Gussian r.v. (by the Central Limit Theorem)
  - $\log_{10} s(d_t)$: Gussian r.v.

- Decorrelation distance:
  - the distance at which autocovariance equals 1/e of its maximum value
  - on the order of the size of the blocking objects or clusters of objects

Combined Path Loss and Shadowing

- Combined model
  - average dB path loss: the path loss model
  - shadow fading with mean of 0 dB: variations about the path loss
- Simplified path loss with log-normal shadowing
  - $$\boxed{\frac{P_r}{P_t}(dB) = 10\log_{10} K - 10\gamma \log_{10}\frac{d}{d_0} + \psi_{dB}}$$
  - ψ_{dB}: a Guassian r.v. with mean zero and variance $\sigma_{\psi_{dB}}$



Outage Probability

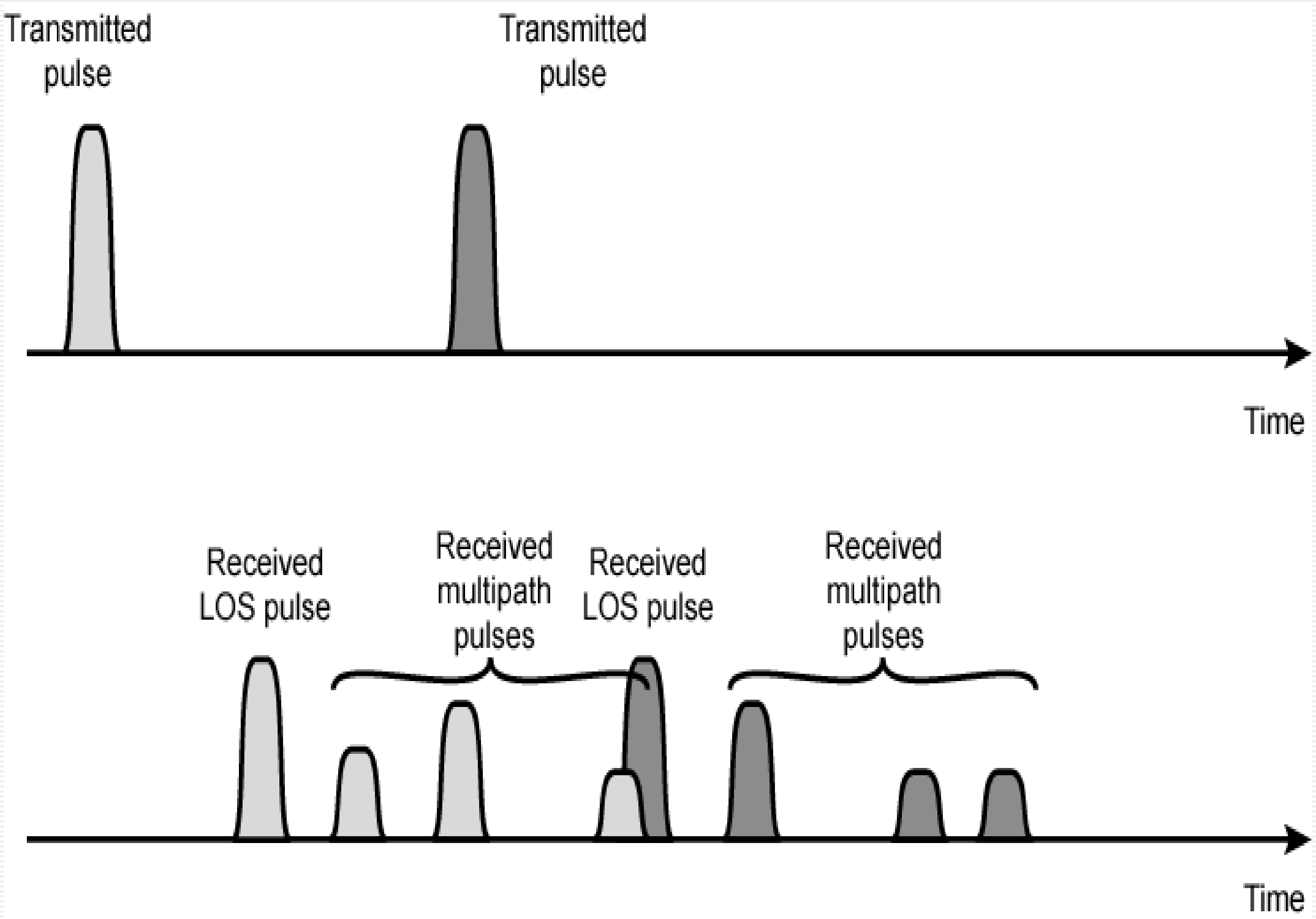
- under the path loss and shadowing
  - $P_{\min}$: the minimum received power level
  - outage probability: $p_{out}(P_{\min}, d)$
    - the probability that the received power at a given distance d falls below $P_{\min}$
    - $p_{out}(P_{\min}, d) = p(P_r(d) < P_{\min})$
    - for the combined path loss and shadowing (at slide 23)

$$p(P_r(d) < P_{\min}) = 1 - Q\left(\frac{P_{\min} - (P_t + 10\log_{10} K - 10\gamma \log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right)$$

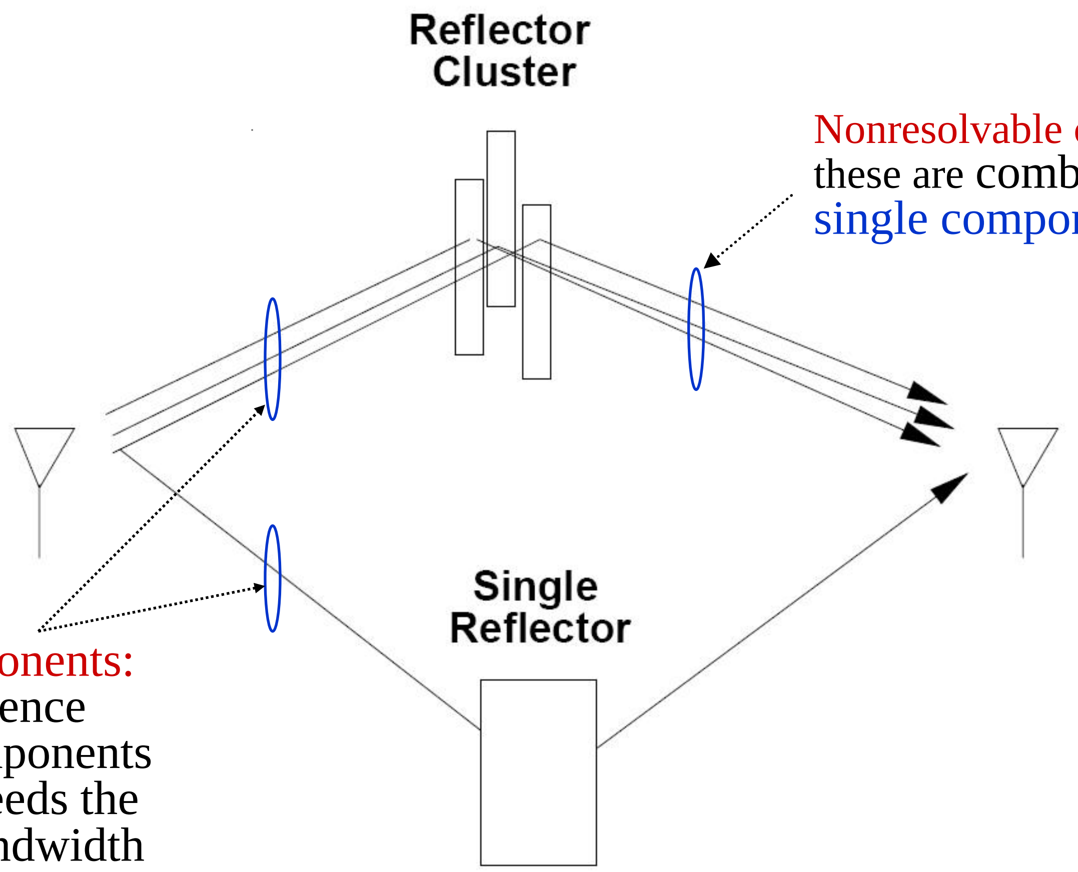
Multipath

- Multipath fading
  - Constructive and destructive addition of different multipath components introduced by a channel
- Time-varying channel impulse response
  - If a single pulse is transmitted over a multipath channel, the received signal appears as a pulse train
  - A multipath channel is modeled by a channel impulse response.
- Characteristic of the multipath channel
  - time delay spread
    - Time delay between the arrivals of the first received signal component and the last received component associated with a single transmitted pulse
  - time-varying nature due to moving
- Narrowband fading model
- Wideband fading model

Example of Multipath



Multipath Component (1)



Nonresolvable components: these are combined into a single component.

Resolvable components: if the delay difference between two components significantly exceeds the inverse signal bandwidth

Figure 3.1: A Single Reflector and A Reflector Cluster.

Multipath Component (2)

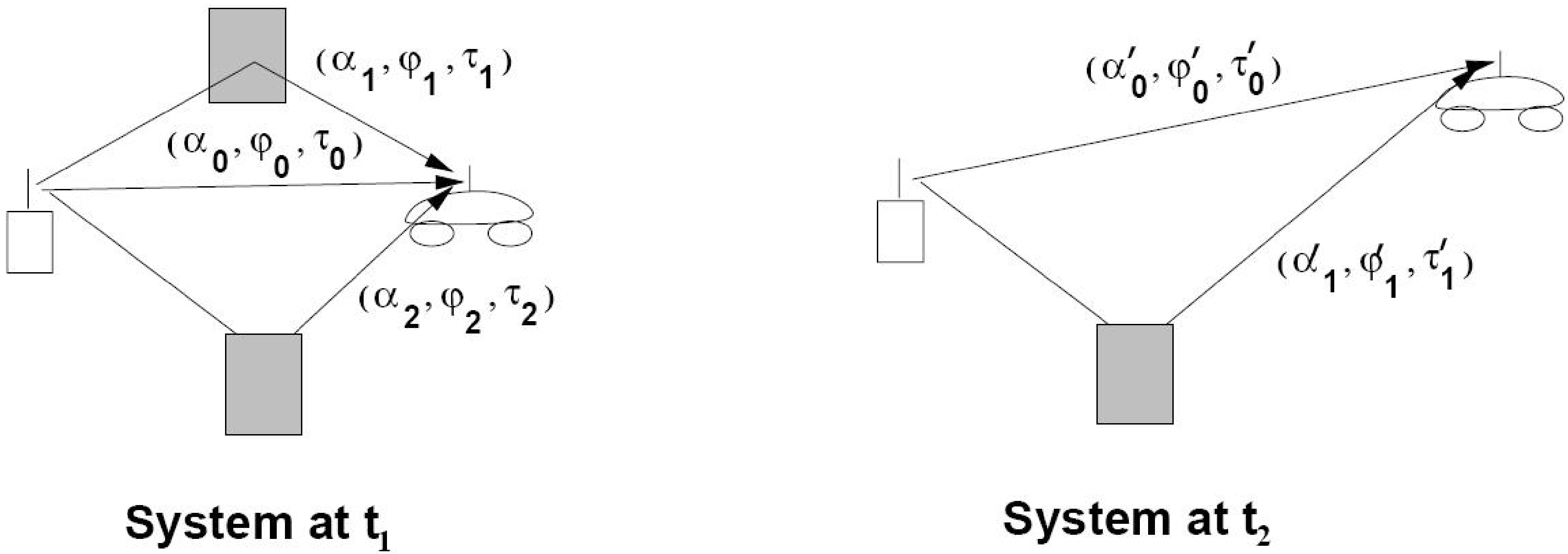


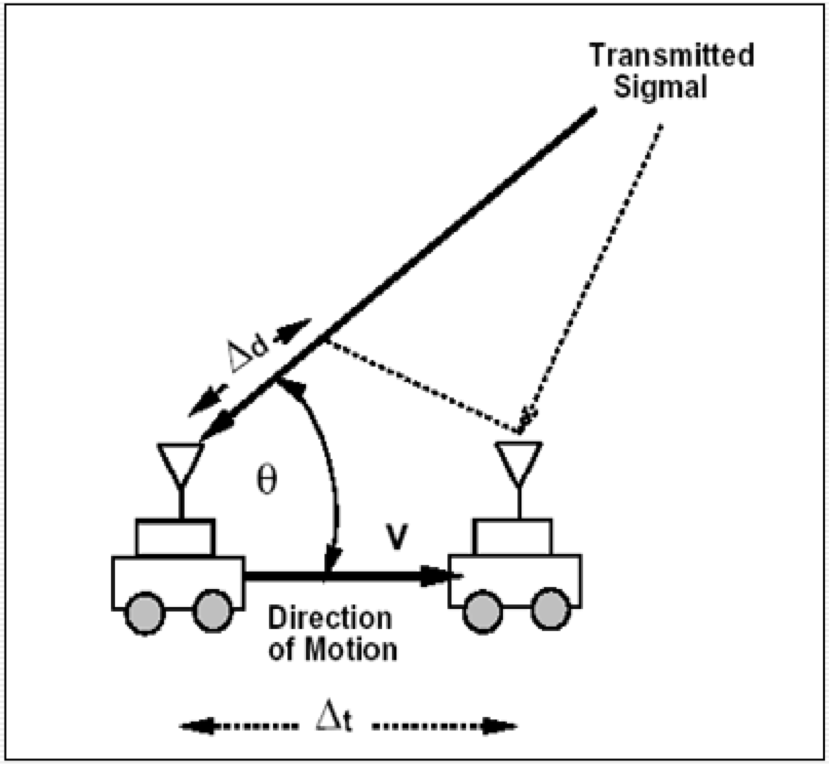
Figure 3.2: System Multipath at Two Different Measurement Times.

Doppler Shift

- When the transmitter is moving, the received signal has a Doppler shift

- $$f_D = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$$
  $$\phi_D = 2\pi f_D$$

- Doppler effect is on the order of 100 Hz for typical vehicle speed (75km/hr) and frequencies (about 1GHz)



Time-varying Channel Impulse Response

- Equivalent lowpass time-varying channel impulse response at time *t* to an impulse at time $t-\tau$: $c(\tau,t)=\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau-\tau_n(t))$
  - $N(t)$: the number of multipath components
  - For the *n*th component
    - $\tau_n(t)$: delay
    - $\phi_n(t)$: phase shift
      - $\phi_n(t)=2\pi f_c \tau_n(t)-\phi_{D_n}$
    - ϕ_{D_n} : Doppler phase shift
    - $\alpha_n(t)$: amplitude
- Time-invariant channel:

$$c(\tau)=\sum_{n=0}^{N} \alpha_n e^{-j\phi_n} \delta(\tau-\tau_n)$$

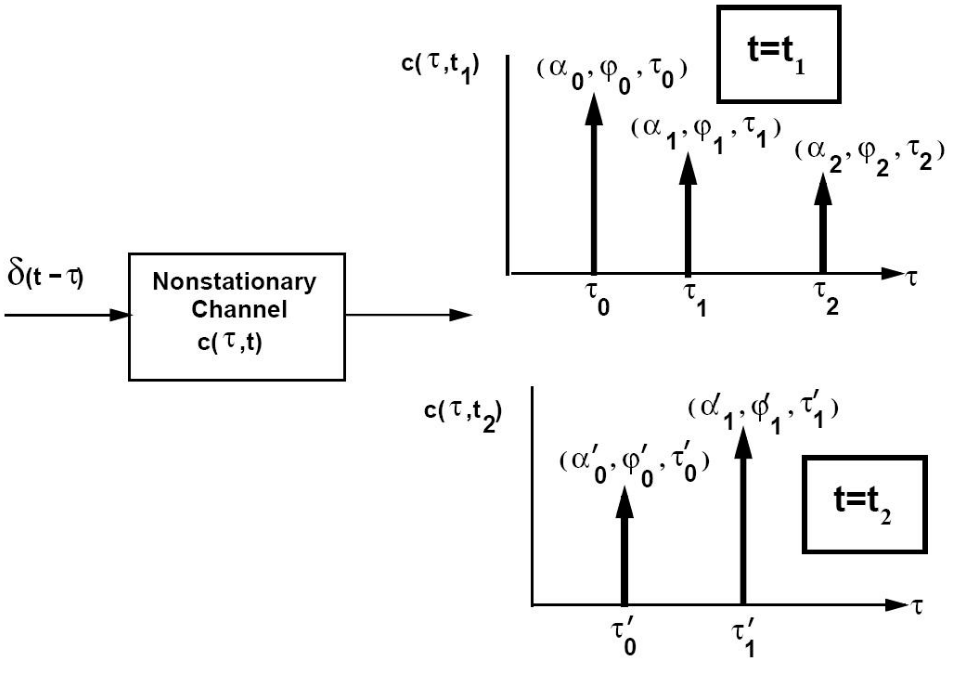


Figure 3.3: Response of Nonstationary Channel.

Transmit & Receive Signal Models (1)

- Complex baseband representation
  - The transmitted or received signals are actually real sinusoids
  - The complex representations are used to facilitate analysis
- Transmitted signal
  - $s(t) = \mathrm{Re}\left\{u(t)e^{j2\pi f_c t}\right\} = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$
  - $u(t) = s_I(t) + js_Q(t)$
    - complex baseband signal with in-phase component $s_I(t)$ and quadrature component $s_Q(t)$

Transmit & Receive Signal Models (2)

- Transmitted signal
  - $s(t) = \mathrm{Re}\left\{u(t)e^{j2\pi f_c t}\right\}$
- Received signal
  - $$r(t) = \mathrm{Re}\left\{\left(\int_{-\infty}^{\infty} c(\tau,t)u(t-\tau)\,d\tau\right)e^{j2\pi f_c t}\right\}$$
    $$= \mathrm{Re}\left\{\left(\int_{-\infty}^{\infty}\sum_{n=0}^{N(t)}\alpha_n(t)\,e^{-j\phi_n(t)}\,\delta(\tau-\tau_n(t))u(t-\tau)\,d\tau\right)e^{j2\pi f_c t}\right\}$$
    $$= \mathrm{Re}\left\{\left(\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\left(\int_{-\infty}^{\infty}\delta(\tau-\tau_n(t))u(t-\tau)\,d\tau\right)\right)e^{j2\pi f_c t}\right\}$$
    $$= \mathrm{Re}\left\{\left(\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\,u(t-\tau_n(t))\right)e^{j2\pi f_c t}\right\}$$
  - $$r(t) = \mathrm{Re}\left\{\left(\sum_{n=0}^{N(t)}\alpha_n(t)\,e^{-j\phi_n(t)}u(t-\tau_n(t))\right)e^{j2\pi f_c t}\right\}$$

Narrowband Fading Model

Narrowband Fading Model (1)

- Narrowband fading assumption
  - Delay spread: $T_m << 1/B$
  - The LOS and all multipath components are typically nonresolvable.
- Received Signal
  - $u(t-\tau_i) \approx u(t)$ for all τ_i
  - $$r(t) = \operatorname{Re}\left\{ u(t) e^{j2\pi f_c t} \left(\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \right) \right\}$$

channel characteristic

- In order to characterize the random scale factor caused by the multipath, $u(t)$=1
  - Transmitted signal: $s(t) = \operatorname{Re}\left\{ e^{j2\pi f_c t} \right\} = \cos 2\pi f_c t$
  - Received signal:

$$r(t) = \operatorname{Re}\left\{ \left(\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \right) e^{j2\pi f_c t} \right\} = r_I(t) \cos 2\pi f_c t + r_Q(t) \sin 2\pi f_c t$$

Narrowband Fading Model (2)

- Received Signal
  - $r(t) = r_I(t)\cos 2\pi f_c t + r_Q(t)\sin 2\pi f_c t$
  - in-phase component: $r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\cos\phi_n(t)$
  - quadrature component: $r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\sin\phi_n(t)$

- $r_I(t)$ and $r_Q(t)$ can be approximated as Gaussian random processes
  - if $N(t)$ is large, and $\alpha_n(t)$ and $\phi_n(t)$ are independent for different components.
  - when for small $N(t)$ each ray has a Rayleigh distributed envelope and uniform phase
  - $r_I(t)$ and $r_Q(t)$ are independent Gaussian process with the same autocorrelation , a mean of zero, and a crosscorrelation of zero

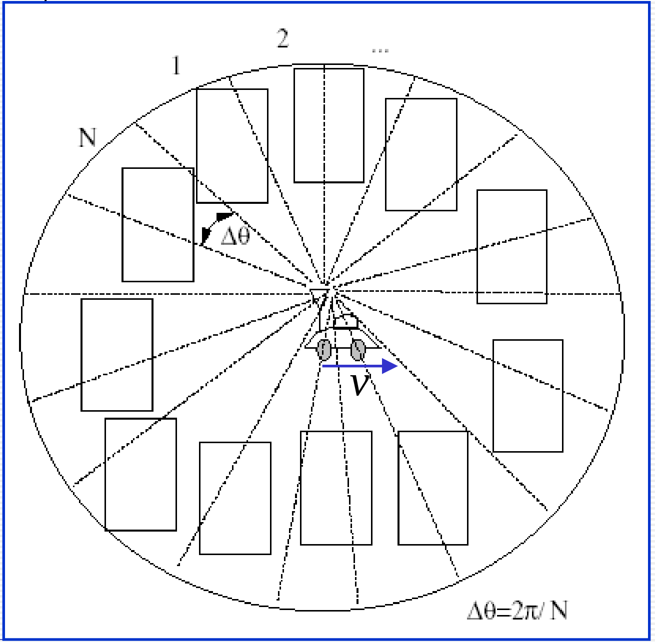
Autocorrelation and Cross Correlation (1)

- Assumption
  - There is no dominant LOS component
  - The amplitude, multipath delay and Doppler frequency shift are changing slowly enough to be considered constant over the time interval of interest
    - $\alpha_n(t) \approx \alpha_n,\ \tau_n(t) \approx \tau_n,\ f_{D_n}(t) \approx f_{D_n}$
    - $\phi_n(t) = 2\pi f_c \tau_n - 2\pi f_{D_n} t$
  - $\phi_n(t)$ is uniform distributed on [-π, π]
    - this is reasonable because f_c is large and hence can go through a 360° rotation for a small change τ_n
- The received signal $r(t)$ is zero-mean Gaussian process
  - $E[r_I(t)] = \sum_n E[\alpha_n] E[\cos\phi_n(t)] = 0$
  - $E[r_Q(t)] = \sum_n E[\alpha_n] E[\sin\phi_n(t)] = 0$
  - $E[r(t)] = 0$

$(E[r_I(t) r_Q(t)] = 0)$

Autocorrelation and Cross Correlation (2)

- Autocorrelation
  - $A_r(\Delta t) = \mathrm{E}[r(t)r(t+\Delta t)] = A_{r_I}(\Delta t)\cos(2\pi f_c \Delta t) - A_{r_I,r_Q}(\Delta t)\sin(2\pi f_c \Delta t)$
    - $A_{r_I}(\Delta t) = \mathrm{E}[r_I(t)r_I(t+\Delta t)] = 0.5\sum_n \mathrm{E}[\alpha_n^2]\cos\left(\frac{2\pi v \Delta t}{\lambda}\cos\theta_n\right)$
    - $A_{r_I,r_Q}(\Delta t) = \mathrm{E}[r_I(t)r_Q(t+\Delta t)] = -0.5\sum_n \mathrm{E}[\alpha_n^2]\sin\left(\frac{2\pi v \Delta t}{\lambda}\cos\theta_n\right)$
  - A uniform scattering environment assumption
    - The channel consists of many scatters densely packed with respect to angle
    - $\theta_n = n\Delta\theta = 2\pi n/N$
  - Each component has the same received power: $\mathrm{E}[\alpha_n^2] = 2P_r/N$
  - $A_{r_I}(\Delta t) = \frac{P_r}{2\pi}\sum_{n=1}^{N}\cos\left(\frac{2\pi v \Delta t}{\lambda}\cos n\Delta\theta\right)\Delta\theta$



Autocorrelation and Cross Correlation (3)

- $N \to \infty,\ \Delta\theta \to 0$

$$A_{r_I,r_Q}(\Delta t) = \frac{P_r}{2\pi}\int_0^{2\pi} \sin\left(\frac{2\pi v \Delta t}{\lambda}\cos\theta\right) d\theta$$
$$= 0$$

$$A_{r_I}(\Delta t) = \frac{P_r}{2\pi}\int_0^{2\pi} \cos\left(\frac{2\pi v \Delta t}{\lambda}\cos\theta\right) d\theta$$
$$= P_r J_0(2\pi f_D \Delta t)$$

$$J_n(x) = \frac{1}{2\pi j^n}\int_0^{2\pi} e^{jx\cos\theta} e^{jn\theta} d\theta$$

$$\boxed{A_r(\Delta t) = A_{r_I}(\Delta t)\cos(2\pi f_c \Delta t)}$$

Antenna spacing of 0.4λ: the signal decorrelates over a distance of 0.4λ

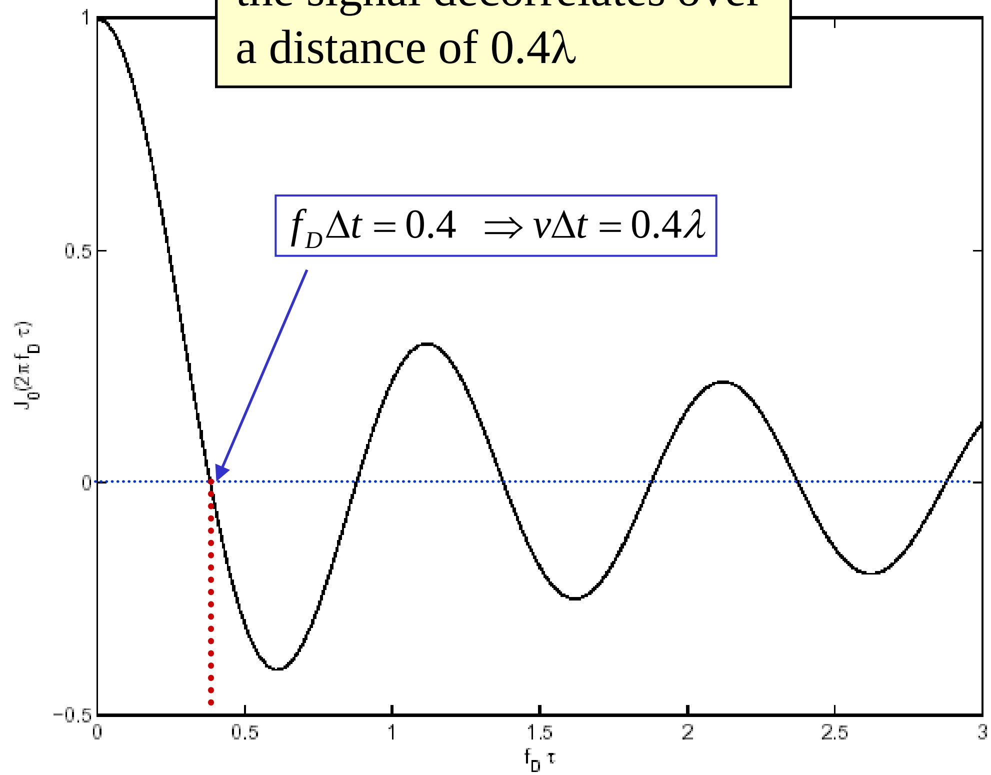
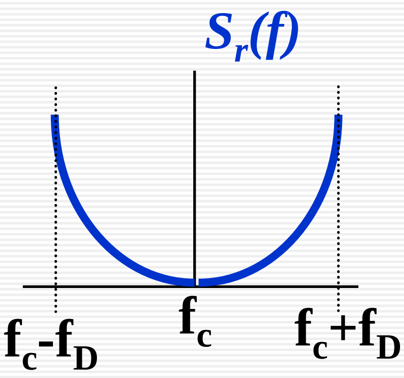


Figure 3.5: Bessel Function versus $f_d\tau$

Power Spectral Density

$$S_r(f) = .25[S_{r_I}(f - f_c) + S_{r_I}(f + f_c)]$$

$$S_{r_I}(f) = \mathcal{F}[PJ_0(2\pi f_D \tau)]$$



Envelope and Power Distributions without LOS

- Signal envelope
  - $$z(t) = |r(t)| = \sqrt{r_I^{\,2}(t) + r_Q^{\,2}(t)}$$
  - r_I and r_Q are Gaussian random variables with mean zero and variance σ^2
  - z(t) is Rayleigh distributed
    - $$p_Z(z) = \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}}, \quad z \ge 0$$
- Power
  - $z^2(t) = |r(t)|^2$
  - The received signal power is exponentially distributed with mean $2\sigma^2$
    - $$p_{Z^2}(x) = \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}}, \quad \mathrm{x} \ge 0$$

Signal Envelope over Channel having a LOS

- Signal envelope
  - The received signal equals the superposition of a LOS component and a complex Gaussian component
  - The signal envelope z(t) has a Rician distribution.

$$p_Z(z) = \frac{z}{\sigma^2} e^{-\frac{(z^2+s^2)}{2\sigma^2}} I_0\left(\frac{zs}{\sigma^2}\right), \quad z \ge 0$$

* I_0: the modified Bessel function

- Average received power
  - $\overline{P_r} = \int_0^\infty x^2 p_Z(x)dx = s^2 + 2\sigma^2$

LOS component power

NLOS component power

  - Fading parameter : $K = \frac{s^2}{2\sigma^2}$
    - K = 0: Rayleigh fading
    - K = ∞: no fading (only a LOS component)
    - the smaller K, the severer fading

Nakagami Fading Channel

- Nakagami fading distribution
  - More general fading distribution whose parameters (m) can be adjusted to fit a variety of empirical measurements

$$p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)\overline{P_r}^m} e^{-\frac{mz^2}{\overline{P_r}}}, \quad m \geq 0.5$$

  - $m = 1$: Rayleigh fading
  - $m = \infty$: no fading
  - $m = (K+1)^2 / (2K+1)$: approximately Rician fading

Wideband Fading

Intersymbol Interference

- Two multipath components with delay τ_1 and τ_2 are resolvable if $|\tau_1 - \tau_2| >> 1/B_u$

- Narrowband fading:
  delay spread $T_m << T$
  - There is little interference with a subsequently transmitted pulse.

- Wideband fading: $T_m >> T$
  - The resolvable multipath components interfere with subsequently transmitted pulses:
    intersymbol interference (ISI)

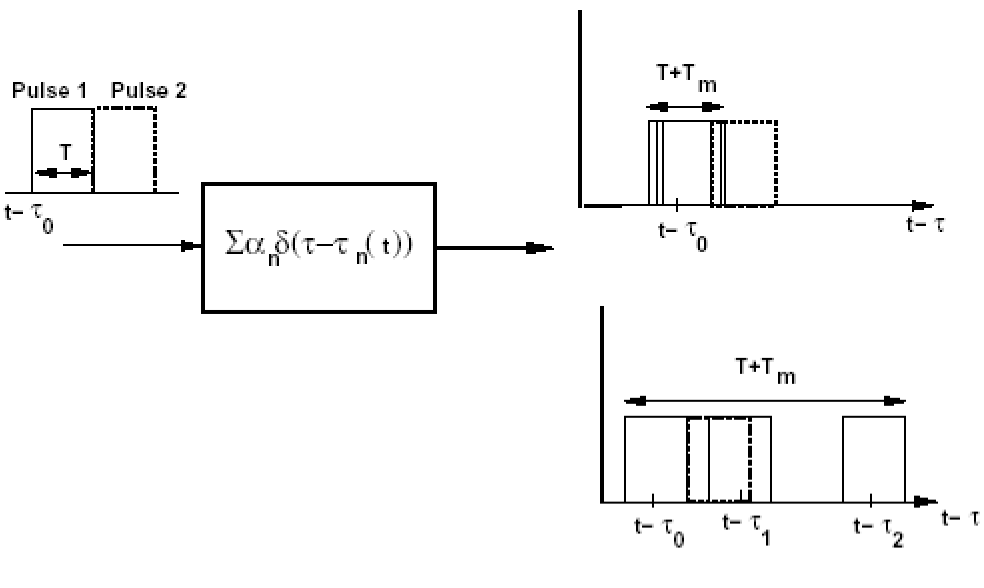


Figure 3.11: Multipath Resolution.

Autocorrelation: Time Domain

- Equivalent lowpass time-varying channel impulse response at time *t* to an impulse at time $t-\tau$
  - $c(\tau,t)=\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\delta(\tau-\tau_n(t))$: a complex zero-mean Gaussian random process
- The statistical characterization of $c(\tau,t)$ is determined by its autocorrelation function $A_c(\tau_1,\ \tau_2,\ t,\Delta t)=\mathrm{E}[c^*(\tau_1,t)\,c(\tau_2,t+\Delta t)]$
- For the WSS channel, the autocorrelation is independent of *t*.
  - $A_c(\tau_1,\ \tau_2,\ \Delta t)=\mathrm{E}[c^*(\tau_1,t)\,c(\tau_2,t+\Delta t)]$
- For the WSS channel with uncorrelated scattering
  - The channel response associated with a multipath component of delay τ_1 is uncorrelated with the response associated with a component of a different delay $\tau_2\neq\tau_1$ because they are caused by different scatters.
  - $\mathrm{E}[c^*(\tau_1,t)\,c(\tau_2,t+\Delta t)]=A_c(\tau_1,\ \Delta t)\,\delta(\tau_1-\tau_2)=\underline{A_c(\tau,\ \Delta t)}$ $(\tau=\tau_2=\tau_1)$

Autocorrelation: Frequency Domain

- Characteristics of the time-varying multipath channel at time t
  - in time domain: $c(\tau, t)$
  - in frequency domain: $C(f,t) = \int_{-\infty}^{\infty} c(\tau,t) e^{-j2\pi f\tau} d\tau$
    - A complex zero-mean Gaussian process
    - $C(f,t)$ is completely characterized by its autocorrelation.
- Autocorrelation of $C(f,t)$
  - $A_C(f_1,\ f_2,\ t, \Delta t) = \mathrm{E}[C^*(f_1,t)C(f_2,t+\Delta t)]$
  - the WSS channel: $A_C(f_1,\ f_2,\ \Delta t) = \mathrm{E}[C^*(f_1,t)C(f_2,t+\Delta t)]$
  - $$A_C(f_1,\ f_2,\ \Delta t) = \mathrm{E}\left[\int_{-\infty}^{\infty} c^*(\tau_1,t)e^{j2\pi f_1\tau_1} d\tau_1 \int_{-\infty}^{\infty} c(\tau_2,t+\Delta t)e^{-j2\pi f_2\tau_2} d\tau_2\right]$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left[c^*(\tau_1,t)c(\tau_2,t+\Delta t)e^{-j2\pi f_1\tau_1}e^{-j2\pi f_2\tau_2} d\tau_1 d\tau_2\right]$$
$$= \int_{-\infty}^{\infty} A_c(\tau,\Delta t)\, e^{-j2\pi(f_2-f_1)\tau} d\tau$$
$$\boxed{= \int_{-\infty}^{\infty} A_c(\tau,\Delta t)\, e^{-j2\pi\Delta f\tau} d\tau = A_C(\Delta f,\ \Delta t)}$$
$$\xrightarrow{\Delta t = 0} \quad \boxed{A_C(\Delta f) = \int_{-\infty}^{\infty} A_c(\tau)\, e^{-j2\pi\Delta f\,\tau} d\tau}$$

Power Delay Profile

- Power delay profile
  - $A_c(\tau) = \mathrm{E}[c^*(\tau,t)c(\tau,t)]$ $\quad\quad (A_c(\tau,\Delta t)$ when $\Delta t = 0)$
  - Average power associated with a given multipath delay τ
- Delay spread
  - Average delay spread: $\mu_{T_m} = \dfrac{\int_0^\infty \tau A_c(\tau)\,d\tau}{\int_0^\infty A_c(\tau)\,d\tau}$ Relative power
  - rms delay spread:

$$\sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau - \mu_{T_m})^2 A_c(\tau)\,d\tau}{\int_0^\infty A_c(\tau)\,d\tau}}$$

  - the delay associated with a given multipath component is weighted by its relative power
- The delay spread of the channel is roughly by the time delay T where $A_c(\tau) \approx 0$ for $\tau \geq T$
  - $T_s << \sigma_{T_m}$ $\quad (T_s < \sigma_{T_m}/10)$: the system experiences significant ISI
  - $T_s >> \sigma_{T_m}$ $\quad (T_s > 10\sigma_{T_m})$: ISI can be negligible

symbol period

Coherence Bandwidth (1)

- $A_C(\Delta f) = \mathrm{E}[C^*(f,t)C(f+\Delta f,t)]$
  - describes the autocorrelation of the time-varying multipath channel in frequency domain (coherence)
- Coherence bandwidth of the channel
  - B_c such that $A_C(\Delta f) \approx 0$ for all $\Delta f \geq B_c$
  - By the Fourier transform relationship between $A_C(\Delta f)$ and $A_c(\tau)$

$$\boxed{\text{if } A_c(\tau) \approx 0 \text{ for } \tau > T, \text{ then } A_C(\Delta f) \approx 0 \text{ for } \Delta f > 1/T} \Rightarrow \boxed{B_c \approx 1/T = k/\sigma_{T_m}}$$

- Flat fading and frequency selective fading
  - signal bandwidth: B
  - Flat fading: $B << B_c$
    - $T_s \approx 1/B >> 1/B_c \approx \sigma_{T_m} \quad \Rightarrow$ negligible ISI
  - Frequency selective fading: $B >> B_c$
    - $T_s \approx 1/B << 1/B_c \approx \sigma_{T_m} \quad \Rightarrow$ significant ISI

Coherence Bandwidth (2)

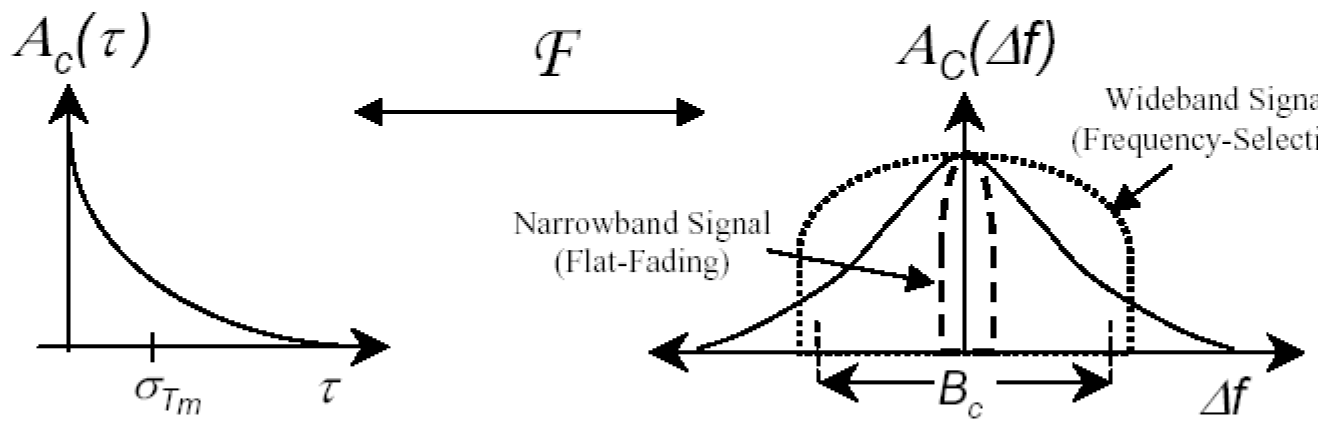


Figure 3.13: Power Delay Profile, RMS Delay Spread, and Coherence Bandwidth.

Coherence Time and Doppler Power Spectrum (1)

- Time variation of the channel which arise from transmitter or receiver motion cause a Doppler shift
- $A_C(\Delta t) = \mathrm{E}[C^*(f,t)C(f,t+\Delta t)]$
  - describe the autocorrelation of the channel at a frequency over the time (channel coherence over the time)
- Coherence time
  - T_c such that $A_C(\Delta t) \approx 0$ for $\Delta t \geq T_c$
- Doppler power spectrum
  - $S_C(\rho) = \int_{-\infty}^{\infty} A_C(\Delta t) e^{-j2\pi\rho\Delta t} d\Delta t$
  - ρ : Doppler shift
  - $S_c(\rho)$: the PSD of the received signal as a function of ρ
- Doppler Spread: B_D
  - the range of ρ such that $|S_C(\rho)| > 0$

Coherence Time and Doppler Power Spectrum (2)

- Relationship between the coherence time and the Doppler spread
  - By the Fourier transform relationship between $A_C(\Delta t)$ and $S_C(\rho)$

$$\boxed{\text{if } A_C(\Delta t) \approx 0 \text{ for } \Delta t > T_c, \text{ then } S_C(\rho) \approx 0 \text{ for } \rho > 1/T_c} \Rightarrow \boxed{B_D \approx 1/T_c}$$

- If the transmitter and reflector are stationary and the receiver is moving with velocity v
  - $B_D \le v/\lambda = f_D$ Maximum Doppler shift
  - Remind that in the narrowband fading model the signal decorrelates over the time of $0.4/f_D$
  - In general, $B_D \approx k/T_c$, where k depends on the shape of $S_C(\rho)$.
- In summary,

$$\boxed{B_c \approx 1/\sigma_{T_m}, \qquad T_c \approx 1/B_D}$$