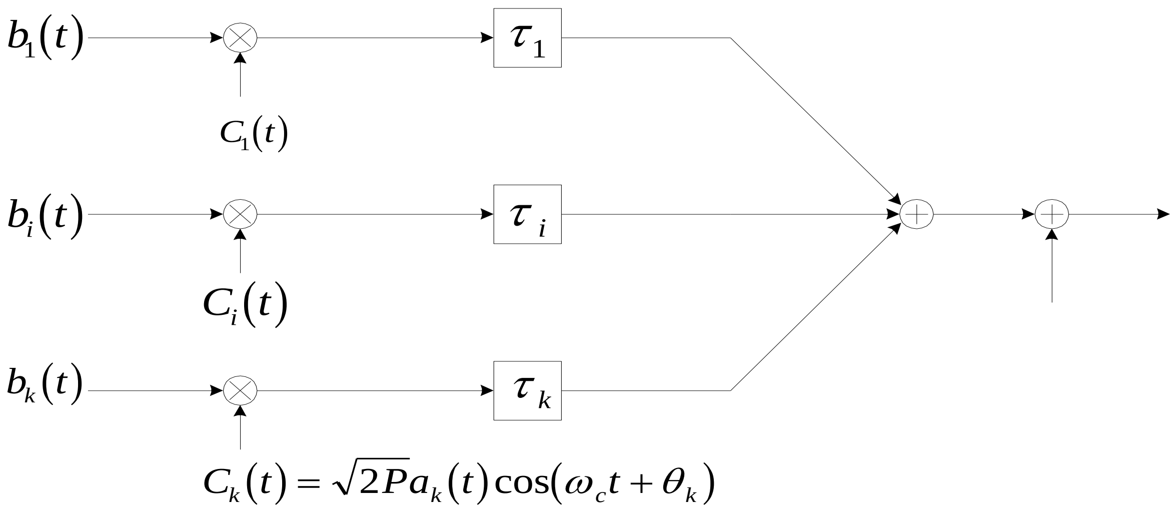
Performance of CDMA Systems

Contents

- System model
- Performance analysis
- Michael B. Pursley, “Performance Evaluation for Phase-coded Spread-Spectrum Multiple-Access Communication-Part 1 : System analysis,” *IEEE Trans. commun*., Vol.COM.25, pp.795-799, August 1977.

System model

- Transmitter



< Reverse link: Tx >

- kth user's data signal

$$b_k(t) = \sum_{l=-\infty}^{\infty} b_{k,l}\, p_T(t - lT), \quad b_{k_1 l} \in \{-1,1\}$$

- kth user's code waveform

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_j^{(k)} P_{T_c}(t - jT_c)$$

- The data signal is modulated on the phase-coded carrier.

$$C_k(t) = \sqrt{2p}\, a_k(t) \cos(\omega_c t + \Theta_k)$$

- kth user Tx signal

$$S_k(t) = \sqrt{2p}\, a_k(t) \cdot b_k(t) \cdot \cos(\omega_c t + \Theta_k)$$

- For asynchronous systems, the received signal

$$r(t) = \sum_{k=1}^{K} \sqrt{2p}\, a_k(t-\tau_k) b_k(t-\tau_k) \cos(\omega_c t + \phi_k) + n(t)$$

where $n(t)$: two-sided PSD = $\frac{N_0}{2}$

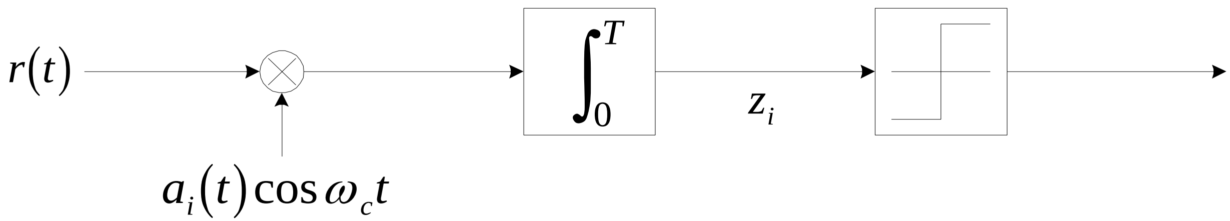
$$\phi_k = \theta_k - \omega_c \tau_k$$

* Assume the ith user

$$\tau_i = 0, \quad \phi_i = 0$$

* Assume: the received power from all users is equal.

- # Receiver



〈 Reverse link : Rx 〉

– At the receiver, the output is

$$z_i = \int_0^T r(t) a_i(t) \cos\omega_c t dt$$

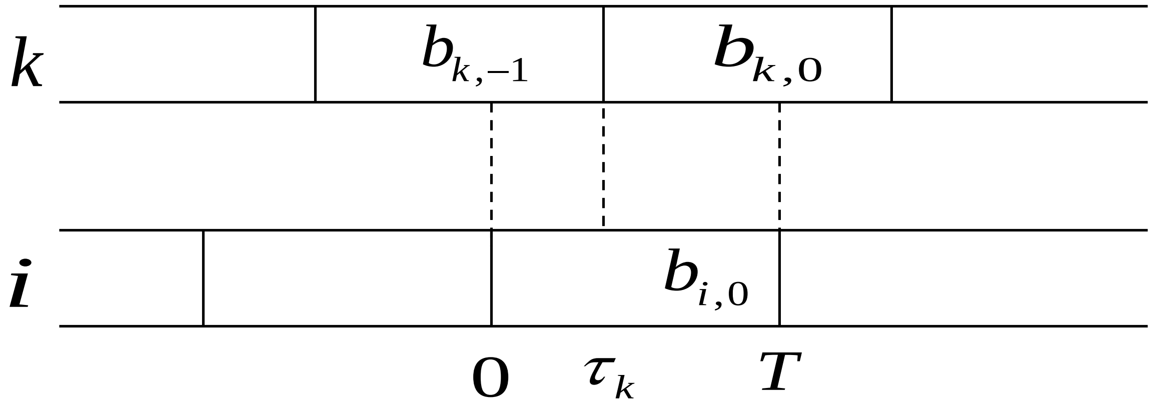
$$b_k(t) = \sum_{l=-\infty}^{\infty} b_{k,l}\, p_T(t - lT), \quad b_{k_1 l} \in \{-1, 1\}$$

$$z_i = \sqrt{\frac{p}{2}}\left\{ b_{i,0}T + \sum_{\substack{k=1 \\ k\neq i}}^{K}\left[b_{k,-1}R_{k,i}(\tau_k) + b_{k,0}\hat{R}_{k,i}(\tau_k)\right]\cdot \cos\phi_k \right\}$$

$$+ \int_0^T n(t)\cdot a_i(t)\cdot \cos\omega_c t dt$$

where $R_{k,i}(\tau_k) = \int_0^{\tau_k} a_k(t-\tau_k)a_i(t)dt$

$\hat{R}_{k,i}(\tau_k) = \int_{\tau_k}^{T} a_k(t-\tau_k)a_i(t)dt$



- Performance?

Performance analysis

– Aperiodic cross-correlation function

$$C_{k,i}(l)=\begin{cases}\sum_{j=0}^{N-1-l} a_j^{(k)} \cdot a_{j+l}^{(\mathrm{i})} & , \ 0 \le l \le N-1 \\ \sum_{j=0}^{N-1+l} a_{j-l}^{(k)} \cdot a_j^{(i)} & , \ 1-N \le l \le 0 \\ 0 & , \ |l| \ge N \end{cases}$$

where $a_j^{(k)}$ is *k*th user's *j*th chip.

– Periodic cross-correlation

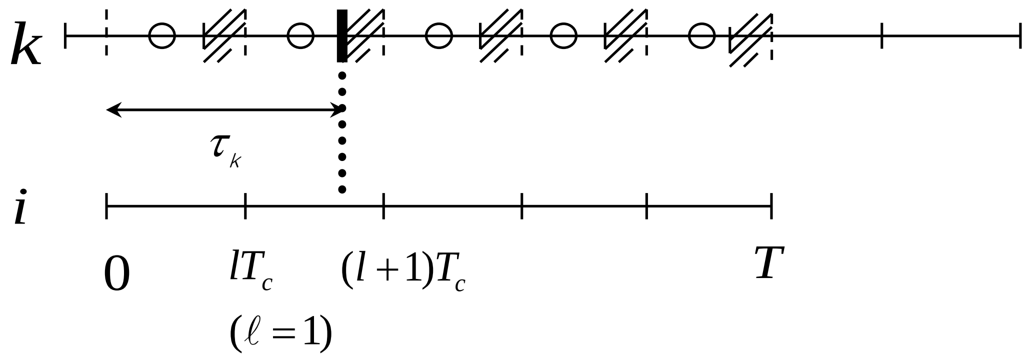
$$\theta_{k,i}(l) = \sum_{j=0}^{N-1} a_j^{(k)} \cdot a_{j+l}^{(i)} = C_{k,i}(l) + C_{k,i}(l-N)$$

– Odd cross-correlation

$$\hat{\theta}_{k,i}(l) = C_{k,i}(l) - C_{k,i}(l-N)$$

$$(\therefore \hat{\theta}_{k,i}(l) = \hat{\theta}_{i,k}(N-l))$$

– For $0 \le lT_c \le \tau_k \le (l+1)T_c \le T,$



$$R_{k,i}(\tau_k) = C_{k,i}(l-N)\cdot T_c + \left[C_{k,i}(l+1-N) - C_{k,i}(l-N)\right]\cdot(\tau_k - lT_c)$$

$$\hat{R}_{k,i}(\tau_k) = C_{k,i}(l)\cdot T_c + \left[C_{k,i}(l+1) - C_{k,i}(l)\right]\cdot(\tau_k - lT_c)$$

$$\mathrm{VAR}\left[Z_i\right] = E\left[(Z_i - \overline{Z}_i)^2\right]$$

$$= E\left[\frac{P}{2}\cdot\sum_{\substack{k=1\\k\neq i}}^{K}(b_{k,-1}R_{k,i}(\tau_k)+b_{k,0}\hat{R}_{k,i}(\tau_k))\cos\phi_k \cdot\sum_{\substack{h=1\\h\neq i}}^{K}(b_{h,-1}R_{h,i}(\tau_h)+b_{h,0}\hat{R}_{h,i}(\tau_h))\cos\phi_h\right]$$

$$+E\left[\int_0^T\int_0^T(n(t)a_i(t)\cos\omega_c t\cdot n(s)a_i(s)\cos\omega_c s\cdot dtds\right]$$

$$\mathrm{VAR}\left[\int_0^T n(t)a_i(t)\cos(w_c t)dt\right]$$

$$= E\left[\int_0^T n(t)a_i(t)\cos\omega_c tdt\cdot\int_0^T n(s)a_i(s)\cos\omega_c sds\right]$$

$$= E\left[\int_0^T\int_0^T n(t)n(s)a_i(t)a_i(s)\cos\omega_c t\cos\omega_c sdtds\right]$$

$$= \int_0^T\int_0^T E\left[n(t)n(s)\right]\cdot E\left[a_i(t)a_i(s)\right]\cos\omega_c t\cos\omega_c sdtds$$

$$= \int_0^T\frac{N_0}{2}a_i{}^2(t)\cos^2\omega_c tdt = \frac{N_0T}{4}$$

$$\text{VAR}\left[Z_i\right] = E\left[(Z_i - \overline{Z}_i)\right]^2 \quad \text{(Random VAR : } \tau_k,\ \phi_k\text{)}$$

$$= \frac{P}{2} \cdot E\left[\sum_{\substack{k=1 \\ k\neq i}}^{K} \left[b^2{}_{k,-1} R^2{}_{k,i}(\tau_k) + b^2{}_{k,0} \hat{R}^2{}_{k,i}(\tau_k) \right] \cdot \cos^2 \phi_k \right] + noise \quad term$$

$$= \frac{P}{2} \cdot E\left[\sum_{\substack{k=1 \\ k\neq i}}^{K} \left[R^2{}_{k,i}(\tau_k) + \hat{R}^2{}_{k,i}(\tau_k) \right] \right] \cdot E\left[\cos^2 \phi_k\right] + noise \quad term$$

$$= \frac{P}{4} \cdot E\left[\sum_{\substack{k=1 \\ k\neq i}}^{K} (R^2{}_{k,i}(\tau_k) + \hat{R}^2{}_{k,i}(\tau_k)) \right] + noise \quad term$$

$$= \frac{P}{4T} \cdot \sum_{\substack{k=1 \\ k\neq i}}^{K} \int_0^T (R^2{}_{k,i}(\tau_k) + \hat{R}^2{}_{k,i}(\tau_k)) d\tau_k + noise \quad term$$

$$= \frac{P}{4T} \sum_{\substack{k=1 \\ k\neq i}}^{K} \sum_{l=0}^{N-1} \int_{lT_c}^{(l+1)T_c} (R^2{}_{k,i}(\tau_k) + \hat{R}^2{}_{k,i}(\tau_k)) d\tau_k + \frac{N_o T}{4}$$

$$\mathrm{VAR}\left[Z_i\right] = \frac{PT^2}{12N^3}\left(\sum_{\substack{k=1\\k\neq i}}^{K} r_{k,i}\right) + \frac{N_0 T}{4}$$

where $$r_{k,i} = \sum_{l=0}^{N-1} C^2{}_{k,i}(l-N) + C_{k,i}(l-N)\cdot C_{k,i}(l-N+1) + C^2{}_{k,i}(l-N+1)$$
$$+ C^2{}_{k,i}(l) + C_{k,i}(l)\cdot C_{k,i}(l+1) + C^2{}_{k,i}(l+1)$$

Define $$\mu_k(n) \equiv \sum_{l=1-N}^{N-1} C_{k,i}(l)\cdot C_{k,i}(l+n)$$

then $$r_{k,i} = 2\mu_{k,i}(0) + \mu_{k,i}(1)$$

$$SNR = \frac{\sqrt{\frac{p}{2}}\cdot T}{\sqrt{Var[Z_i]}} = \frac{\sqrt{\frac{p}{2}}\cdot T}{\left(\frac{PT^2}{12N^3}\sum_{\substack{k=1\\k\neq i}}^{K}(2\mu_{k,i}(0)+\mu_{k,i}(1))+\frac{N_0T}{4}\right)^{1/2}}$$

$$= \frac{1}{\left(\frac{1}{6N^3}\sum_{\substack{k=1\\k\neq i}}^{K}(2\mu_{k,i}(0)+\mu_{k,i}(1))+\frac{N_0T}{4}\cdot\frac{2}{PT^2}\right)^{1/2}} = \frac{1}{\left(\frac{1}{6N^3}\sum_{\substack{k=1\\k\neq i}}^{K}\left(2\mu_{k,i}(0)+\mu_{k,i}(1)\right)+\frac{N_0}{2E}\right)^{1/2}}$$

$$* \quad K=1, \qquad SNR = \sqrt{\frac{2E}{N_0}}$$

$$\mu_{k,i}(0) = \sum_{l=1-N}^{N-1} C^2{}_{k,i}(l)$$

$$E[\mu_{k,i}(0)] = \sum_{l=1-N}^{N-1} E[C^2{}_{k,i}(l)]$$

$$= \sum_{l=1-N}^{N-1}(N-|l|) = N^2$$

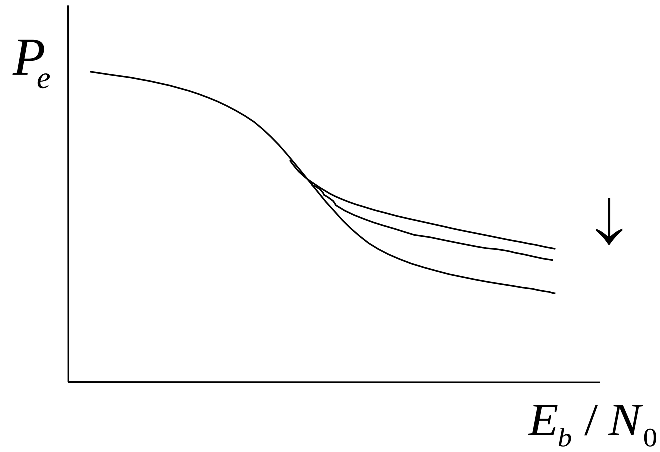
$$E[\mu_{k,i}(1)] = 0$$

$$SNR = \frac{1}{\left(\frac{K-1}{3N} + \frac{N_o}{2E}\right)^{1/2}} \quad \left(\frac{signal\ part(\neq power)}{noise\ part(\neq power)}\right)$$

If $\frac{2E}{N_0}$ is large, $SNR \simeq \sqrt{\frac{3N}{K-1}}$

Prob of Error : $P_e \approx Q\left(1/\sqrt{\frac{K-1}{3N} + \frac{N_0}{2E_b}}\right)$ $(here: Q(SNR));\ \ convention: Q(\sqrt{SNR}))$

When $\frac{E_b}{N_0}$ is very large , $P_e \approx Q\left(\sqrt{\frac{3N}{K-1}}\right)$



↓ as N increase for a fixed K.

(interference limited)

System Capacity

$$P_e \approx Q\left(\sqrt{\frac{3N}{K-1}}\right)$$

Prob of error	$3N/\left(Q^{-1}(P_e)\right)^2$
0.01	0.567N
0.001	0.312N
0.0001	0.219N

$$Q^{-1}(P_e) \approx \sqrt{\frac{3N}{K-1}}$$

$$K \approx 1 + \frac{3N}{(Q^{-1}(P_e))^2}$$: no of max users for given P_e

example

$$P_e = 0.01, \qquad K \approx 1 + 0.567N$$