Transmit Power and Bit Allocations for OFDM Systems in a Fading Channel

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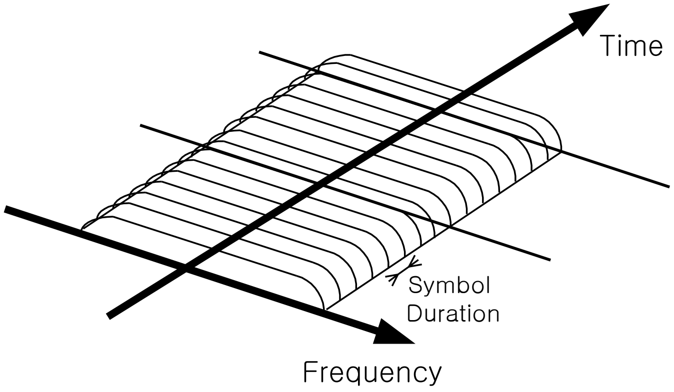
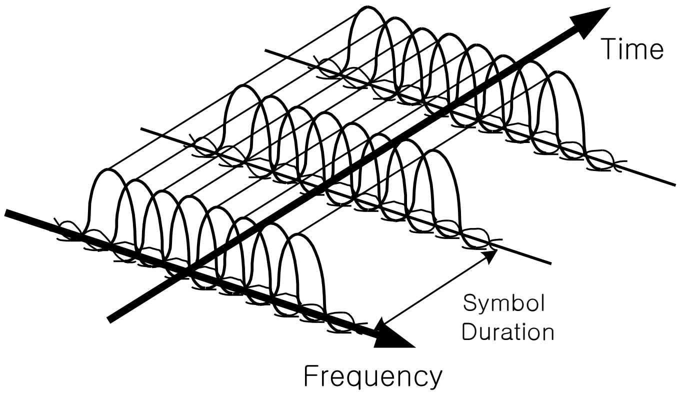
Introduction

- Orthogonal Frequency Division Multiplexing (OFDM)
  - Potential solution for high data rate system in the future
  - Parallel transmission over a number of subcarriers
  - Flat fading for each subcarrier
  - Robust to multipath fading
  - Wireless LANs, Broadband Wireless Access, DAB, DVB, etc.

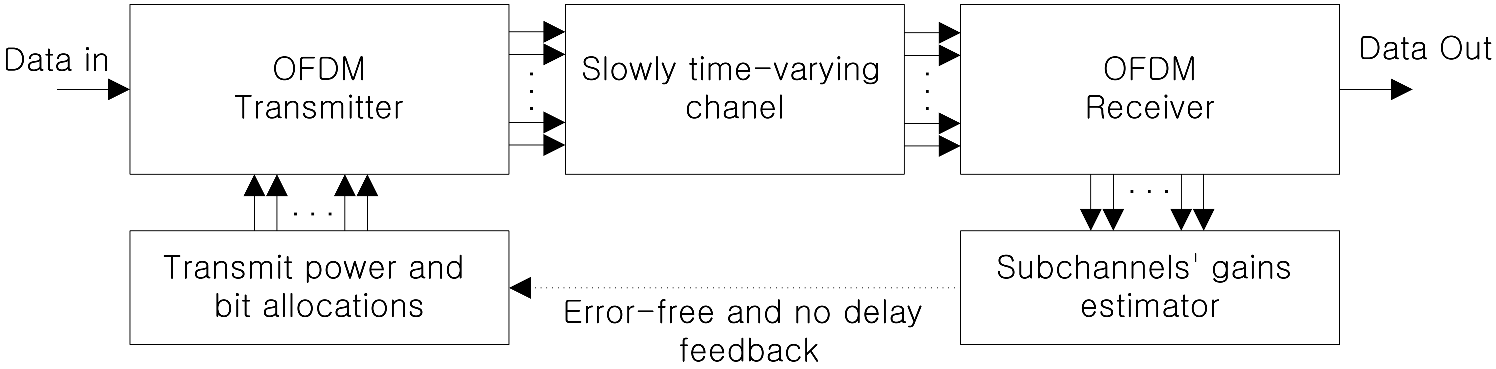


Single carrier system

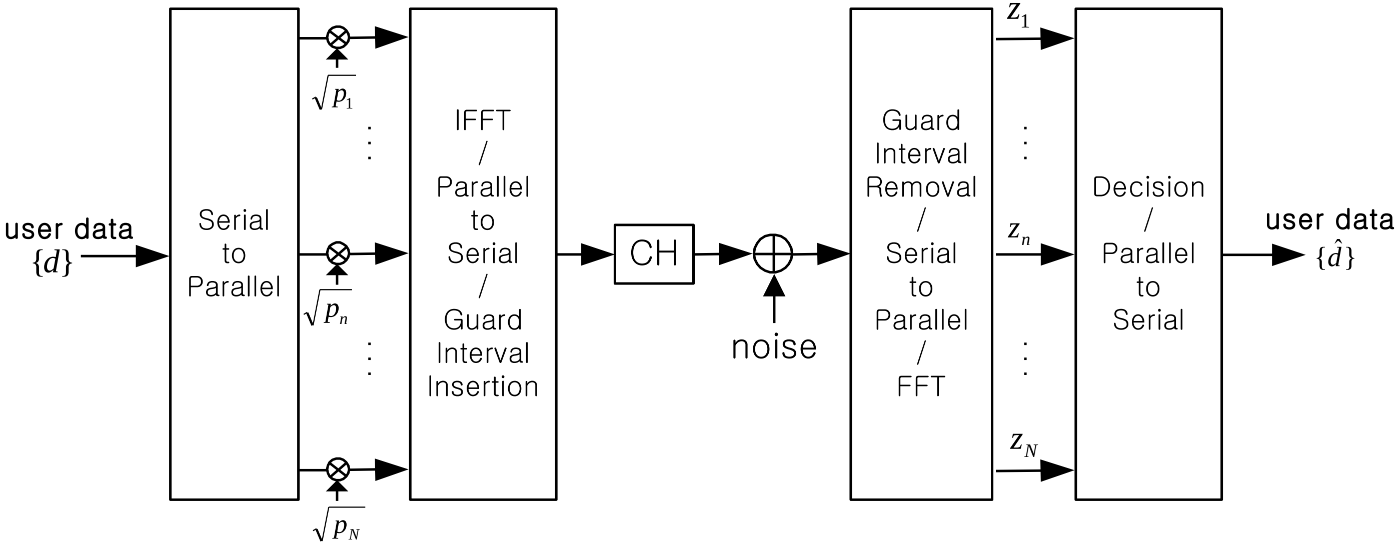


OFDM system

- Transmit power and bit allocations for OFDM
  - With the knowledge of the channel state information (CSI) at the transmitter
  - Transmit power and number of bits for each subcarrier are adaptively allocated according to the CSI
  - Can increase the data rate, can use the resources (power, time, frequency bandwidth) more efficiently



Single user OFDM systems

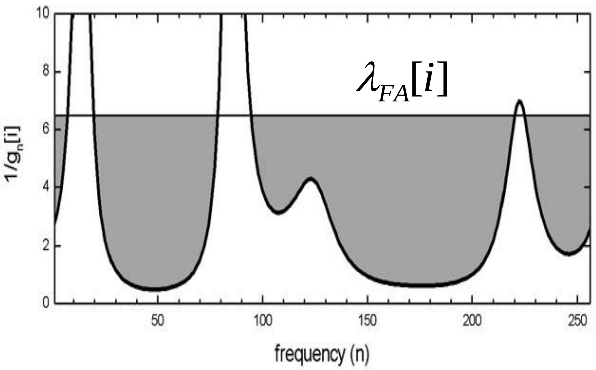


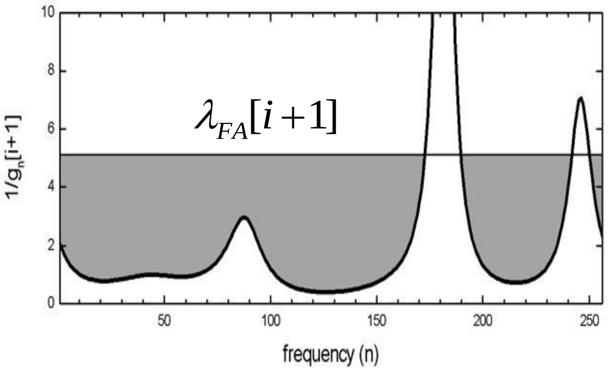
- Capacity : $C(\{g_n[i]\}) = \sum_{n=1}^{N} \frac{W}{N} \log_2\left(1 + \frac{p_n[i]g_n[i]}{N_0 W/N}\right)$

- Conventional power allocation schemes
  - Equal power allocation
    - When the CSI is not known at the transmitter
    - Transmit power: $p_n[i] = \frac{P_0}{N}$
  - Frequency domain power allocation
    - Usually in a fixed channel, may not fully exploit the time-varying nature of the fading channel
    - Transmit power: $p_n[i] = \frac{N_0 W}{N}\left[\lambda_{FA}[i] - \frac{1}{g_n[i]}\right]^+$
    - Examples:

constraint:

$$\sum_{n=1}^{N} p_n[i] = P_0$$



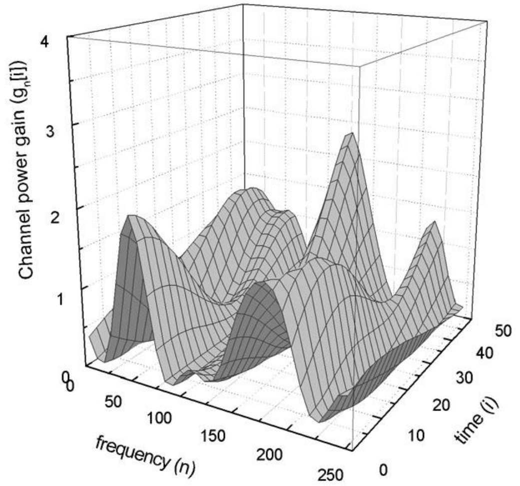


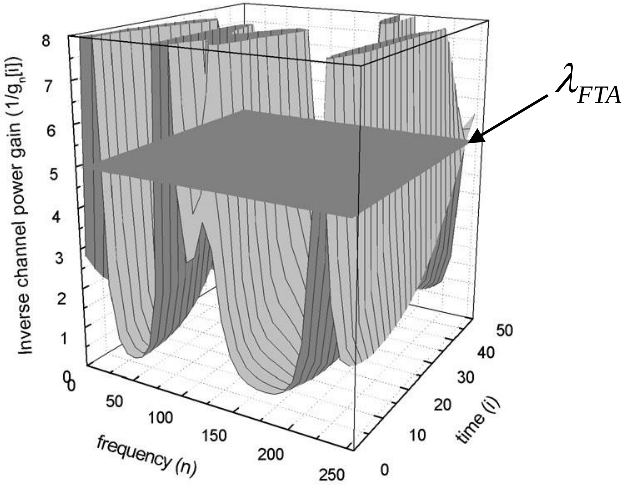
- Proposed frequency-time domain power allocation
  - Jointly optimized both in frequency and time domains
  - Exploitation of time variation of the fading channel
  - Transmit power : $p_n[i] = \frac{N_0 W}{N}\left[\lambda_{FTA} - \frac{1}{g_n[i]}\right]^+$

constraint:

$$\sum_{n=1}^{N}\int_0^\infty p_n[i]\,f(g_n)\,dg_n = P_0$$

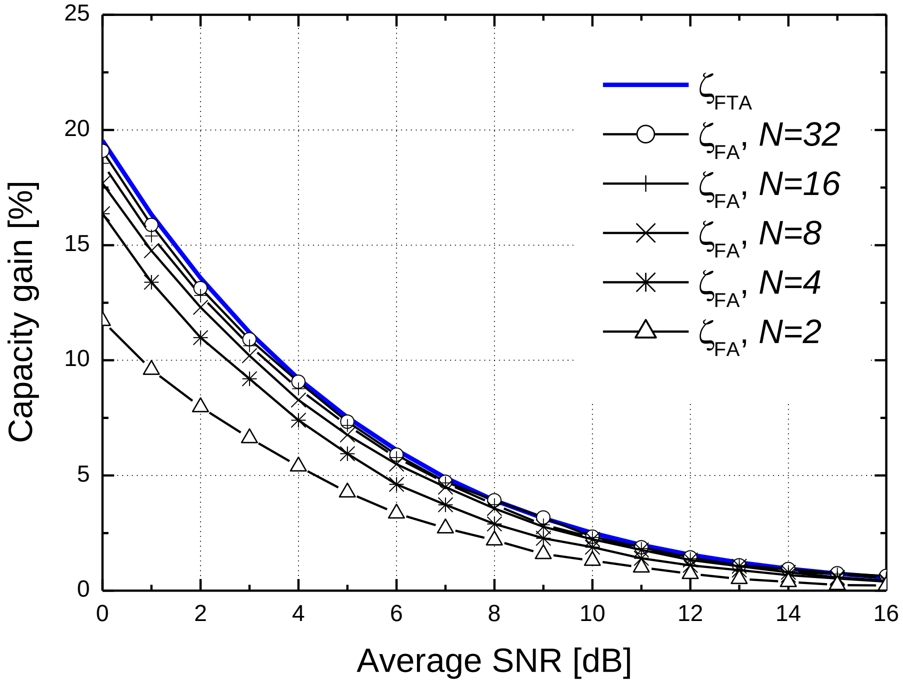
  - Examples :





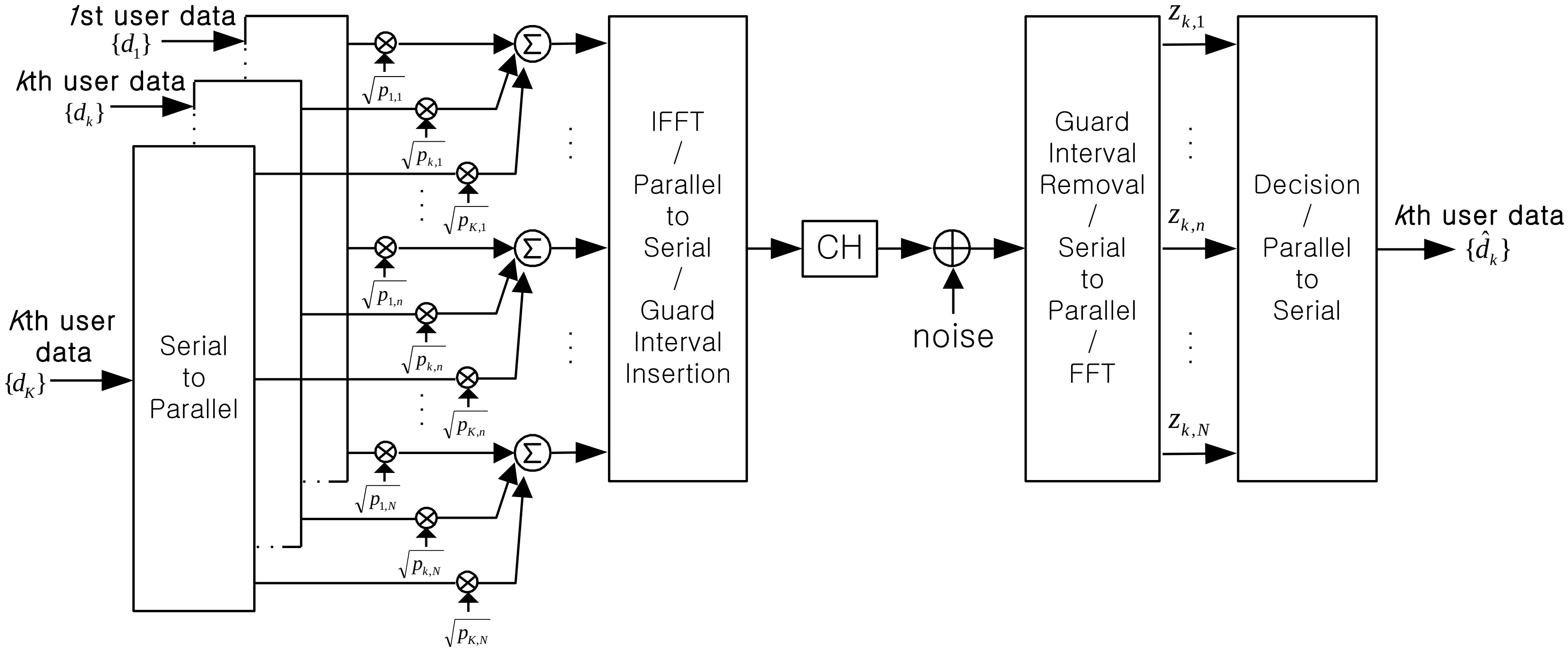
Results

- Capacity gains : $\varsigma_{FA} \overset{\Delta}{=} \left(\frac{C_{FA} - C_{EQ}}{C_{EQ}} \right) \cdot 100 \quad [\%], \quad \varsigma_{FTA} \overset{\Delta}{=} \left(\frac{C_{FTA} - C_{EQ}}{C_{EQ}} \right) \cdot 100 \ [\%]$



♣ Capacity gain for '<u>frequency-time</u> domain power allocation' is the upper bound of the capacity gain for '<u>frequency</u> domain power allocation'

Multiuser OFDM systems



♣ Generally formulated : multiple users can share a specific subcarrier simultaneously

- Total data rate

$$R_{TOTAL}(\{g_{k,n}[i]\}) = \sum_{k=1}^{K}\sum_{n=1}^{N}\frac{W}{N}\log_2\left(1+\frac{\gamma_{k,n}[i]}{\Gamma}\right)$$

$$BER \le \frac{1}{5}\exp\left(-1.5\frac{\gamma_{k,n}[i]}{2^{b_{k,n}[i]}-1}\right)$$

$$b_{k,n}[i] = \log_2\left(1+\frac{\gamma_{k,n}[i]}{\Gamma}\right)$$

$$\Gamma \overset{\Delta}{=} \frac{-\ln(5BER)}{1.5}$$

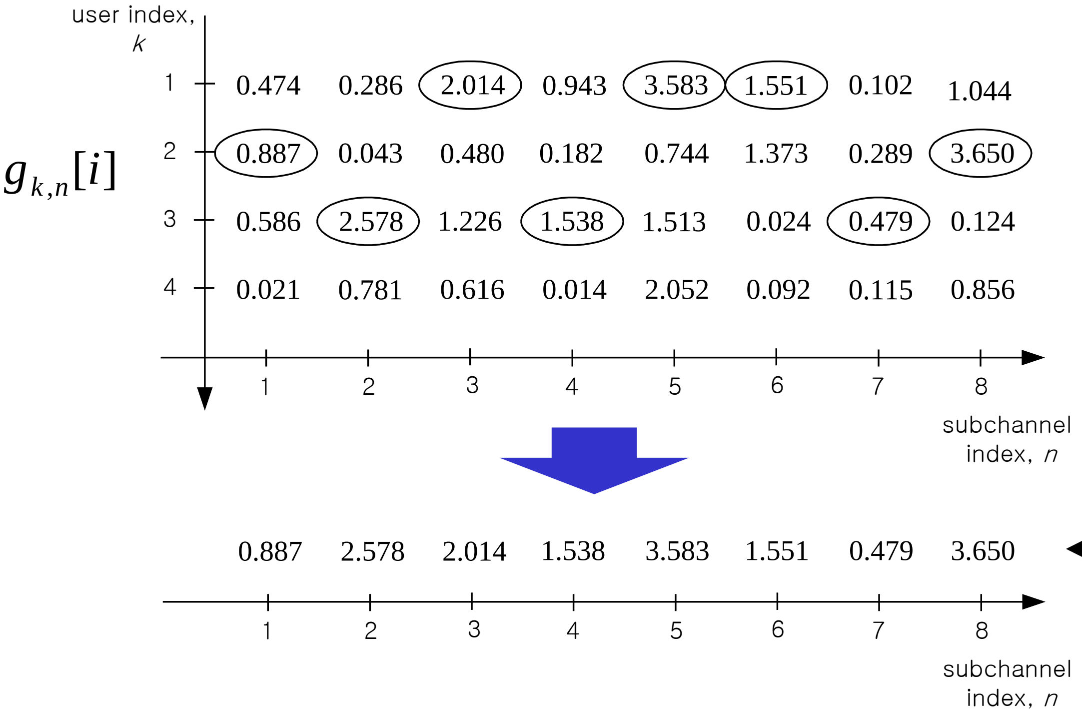
- SINR

$$\gamma_{k,n}[i] = \frac{p_{k,n}[i]g_{k,n}[i]}{\sum_{j=1,\ j\neq k}^{K} p_{j,n}[i]g_{k,n}[i] + N_0 W/N}$$

- Constraint

$$\sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n}[i] = P_0$$

- Derivation of the solution
  - Two-step approach
  - (1) Subcarrier assignment for users
    - Theorem: ***a subcarrier should be assigned to only one user who has the best channel gain for that subcarrier***



$g_{k,n}[i]$

k \ n	1	2	3	4	5	6	7	8
1	0.474	0.286	2.014	0.943	3.583	1.551	0.102	1.044
2	0.887	0.043	0.480	0.182	0.744	1.373	0.289	3.650
3	0.586	2.578	1.226	1.538	1.513	0.024	0.479	0.124
4	0.021	0.781	0.616	0.014	2.052	0.092	0.115	0.856

1	2	3	4	5	6	7	8
0.887	2.578	2.014	1.538	3.583	1.551	0.479	3.650

can be treated as a single user system

- Total data rate

different from the single user case

  - $R_{TOTAL}(\{G_n[i]\}) = \sum_{n=1}^{N} \frac{W}{N} \log_2\left(1 + \frac{p_n[i]\boxed{G_n[i]}}{N_0 W/N} \cdot \frac{1}{\Gamma}\right)$
  - $G_n[i] \overset{\Delta}{=} \max\left\{g_{1,n}[i], g_{2,n}[i], \cdots, g_{K,n}[i]\right\}$
  - constraint : $\sum_{n=1}^{N} p_n[i] = P_0$

– (2) Power allocation for subcarriers

♣ same as single user case except for the channel power gain

- Equal power allocation
  - $p_n[i] = \frac{P_0}{N}$
- Frequency domain power allocation
  - $p_n[i] = \frac{N_0 W \cdot \Gamma}{N}\left[\lambda_{FA}[i] - \frac{1}{G_n[i]}\right]^+$

constraint:

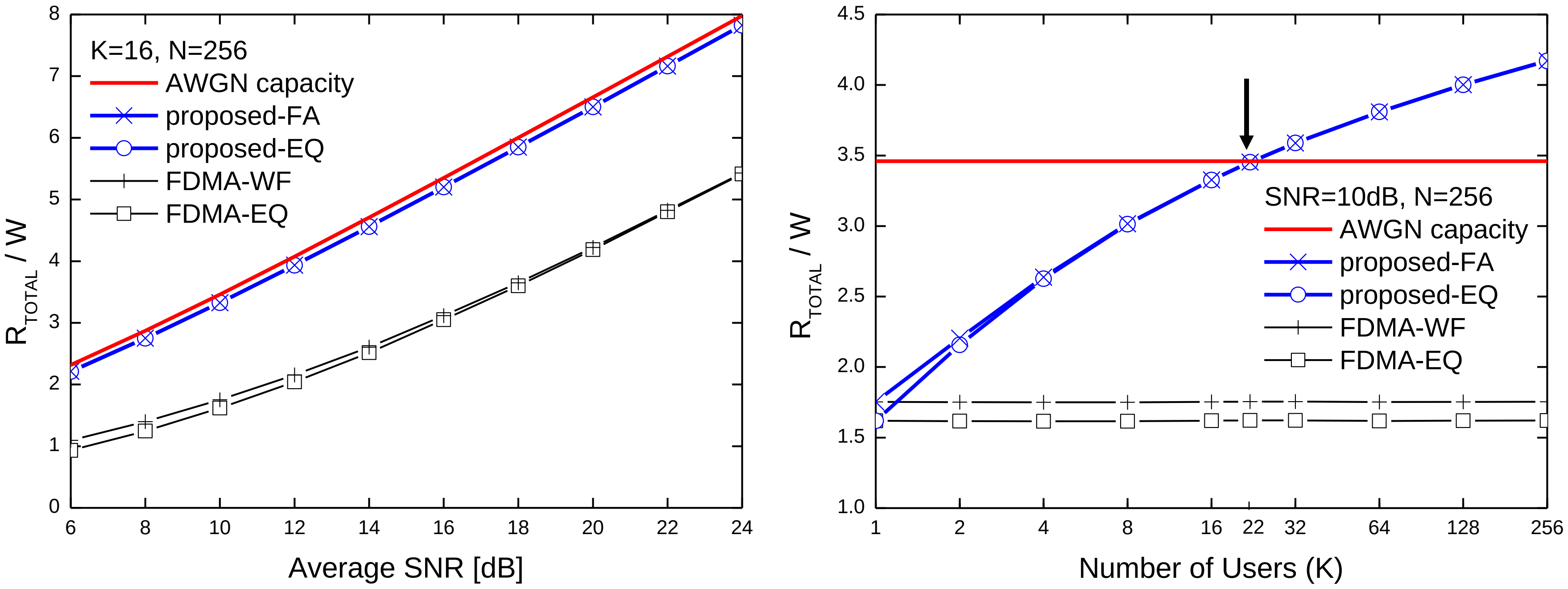
$$\sum_{n=1}^{N} p_n[i] = P_0$$

- Frequency-time domain power allocation
  - $p_n[i] = \frac{N_0 W \cdot \Gamma}{N}\left[\lambda_{FTA} - \frac{1}{G_n[i]}\right]^+$

constraint:

$$\sum_{n=1}^{N} \int_0^{\infty} p_n[i] f(G_n) dG_n = P_0$$

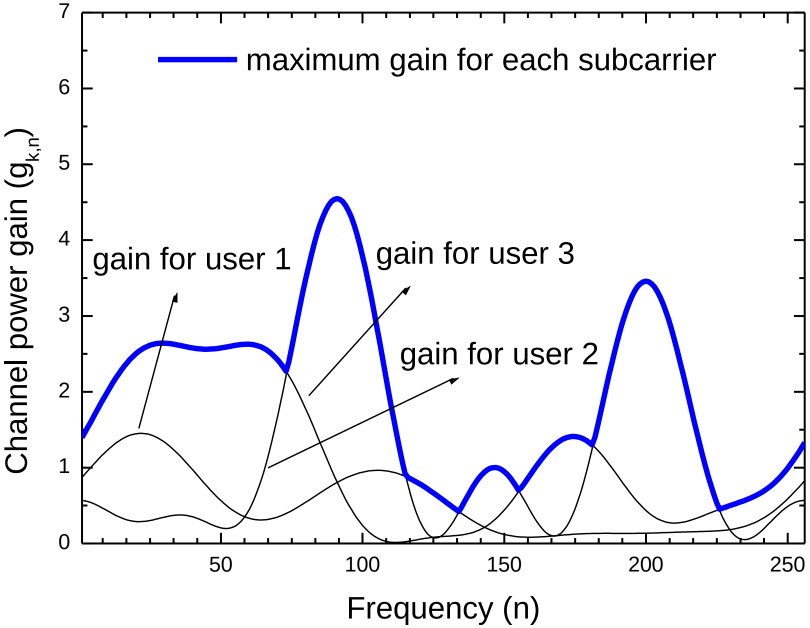
Results



♣data rates for the proposed methods increase as the number of users increases → multiuser diversity effects

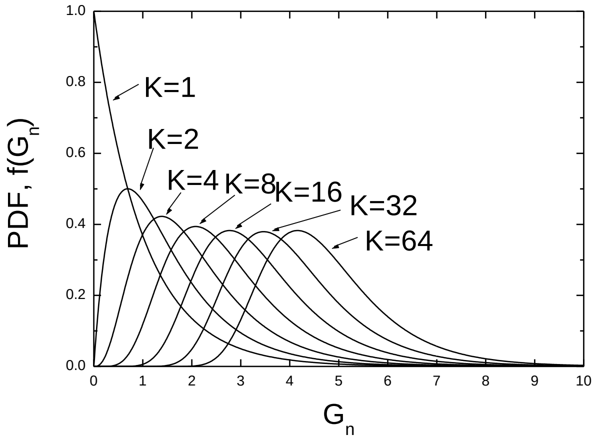
♣ data rates for the proposed methods are greater than AWGN capacity when K>22

- Multiuser diversity

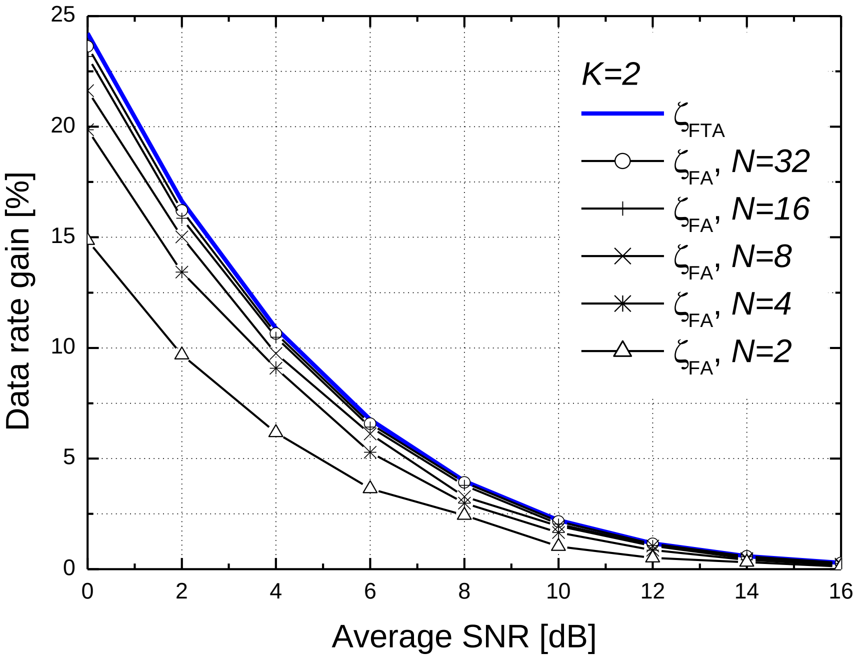


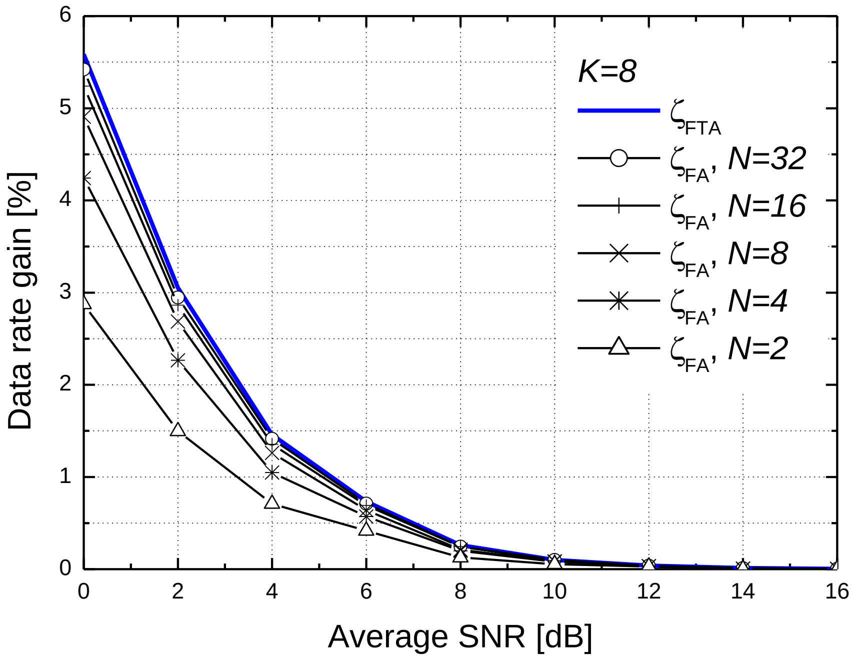
- Probability density functions

$$\begin{cases} f(g_{k,n}) = \exp(-g_{k,n}) \\ f(G_n) = K\left(1-\exp(-G_n)\right)^{K-1}\exp(-G_n) \end{cases}$$



- Data rate gain: $\varsigma_{FA} \overset{\Delta}{=} \left(\frac{R_{FA} - R_{EQ}}{R_{EQ}} \right) \cdot 100 \quad [\%], \quad \varsigma_{FTA} \overset{\Delta}{=} \left(\frac{R_{FTA} - R_{EQ}}{R_{EQ}} \right) \cdot 100 \ [\%]$



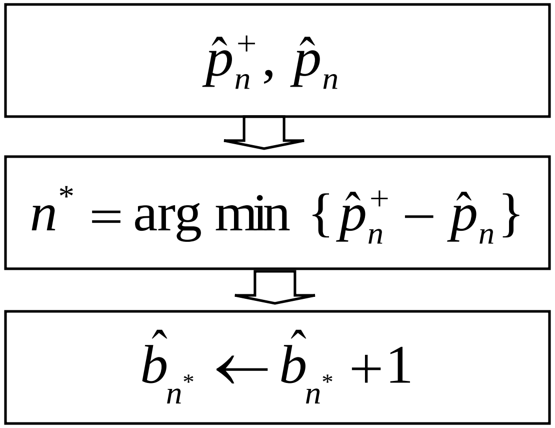


♣ as *K* and SNR increase, data rate gain decreases

Practical algorithm for transmit power and bit allocations

- Practical algorithm
  - Integer number of bits for each subcarrier
  - Practical modulation / demodulation
- Data rate maximization
  - Constraints : total transmit power and bit error rate
  - Single user case is considered
- Existing algorithms
  - Hughes-Hartogs : greedy search method, optimal
  - Chow : equal power allocation for subcarriers
  - **Leke** : water-filling power allocation for subcarriers
  - Krongold : table lookup method

- Hughes-Hartogs' algorithm
  - Greedy search method
  - One additional bit is allocated to the subchannel that requires the least incremental power at each step
  - Optimal performance
  - High complexity

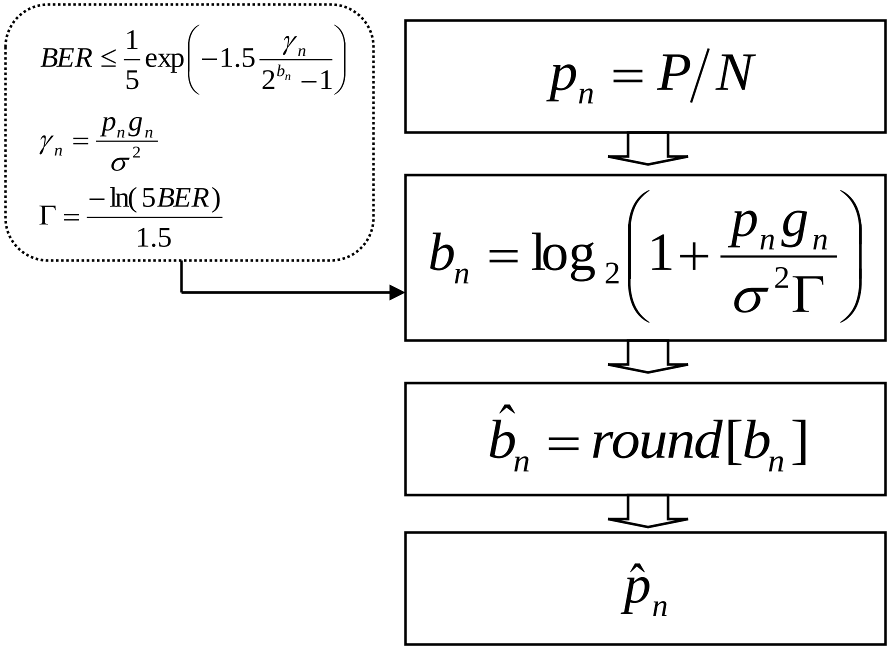


$$\hat{p}_n^+,\ \hat{p}_n$$

$$n^* = \arg\min\ \{\hat{p}_n^+ - \hat{p}_n\}$$

$$\hat{b}_{n^*} \leftarrow \hat{b}_{n^*} + 1$$

- Chow's algorithm
  - Total transmit power is equally distributed over all subcarriers
  - Low complexity



$$BER \le \frac{1}{5}\exp\left(-1.5\frac{\gamma_n}{2^{b_n}-1}\right)$$

$$\gamma_n = \frac{p_n g_n}{\sigma^2}$$

$$\Gamma = \frac{-\ln(5BER)}{1.5}$$

$$p_n = P/N$$

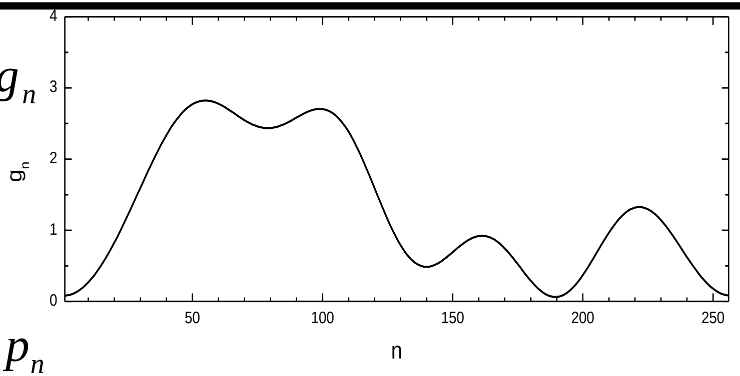
$$b_n = \log_2\left(1+\frac{p_n g_n}{\sigma^2\Gamma}\right)$$

$$\hat{b}_n = round[b_n]$$

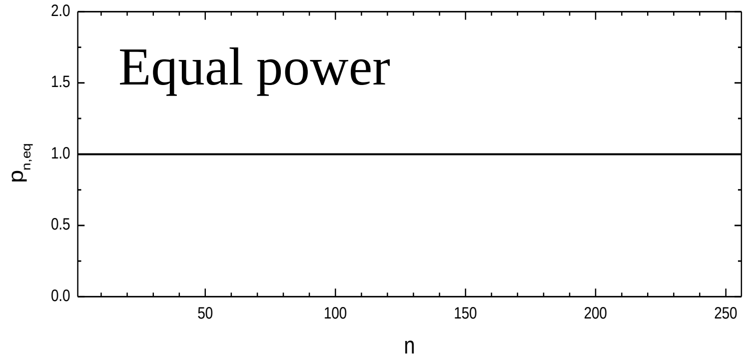
$$\hat{p}_n$$

- Examples:

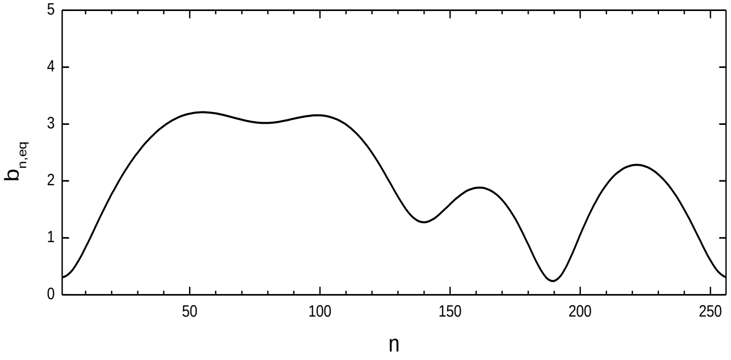
g_n



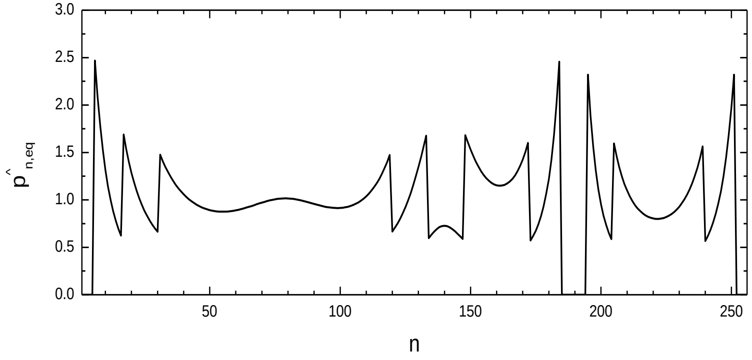
p_n



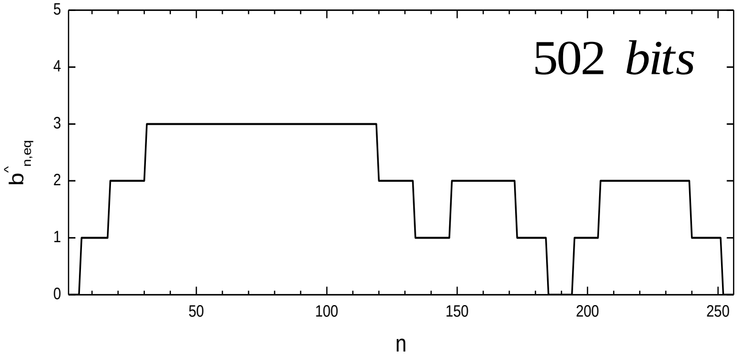
b_n



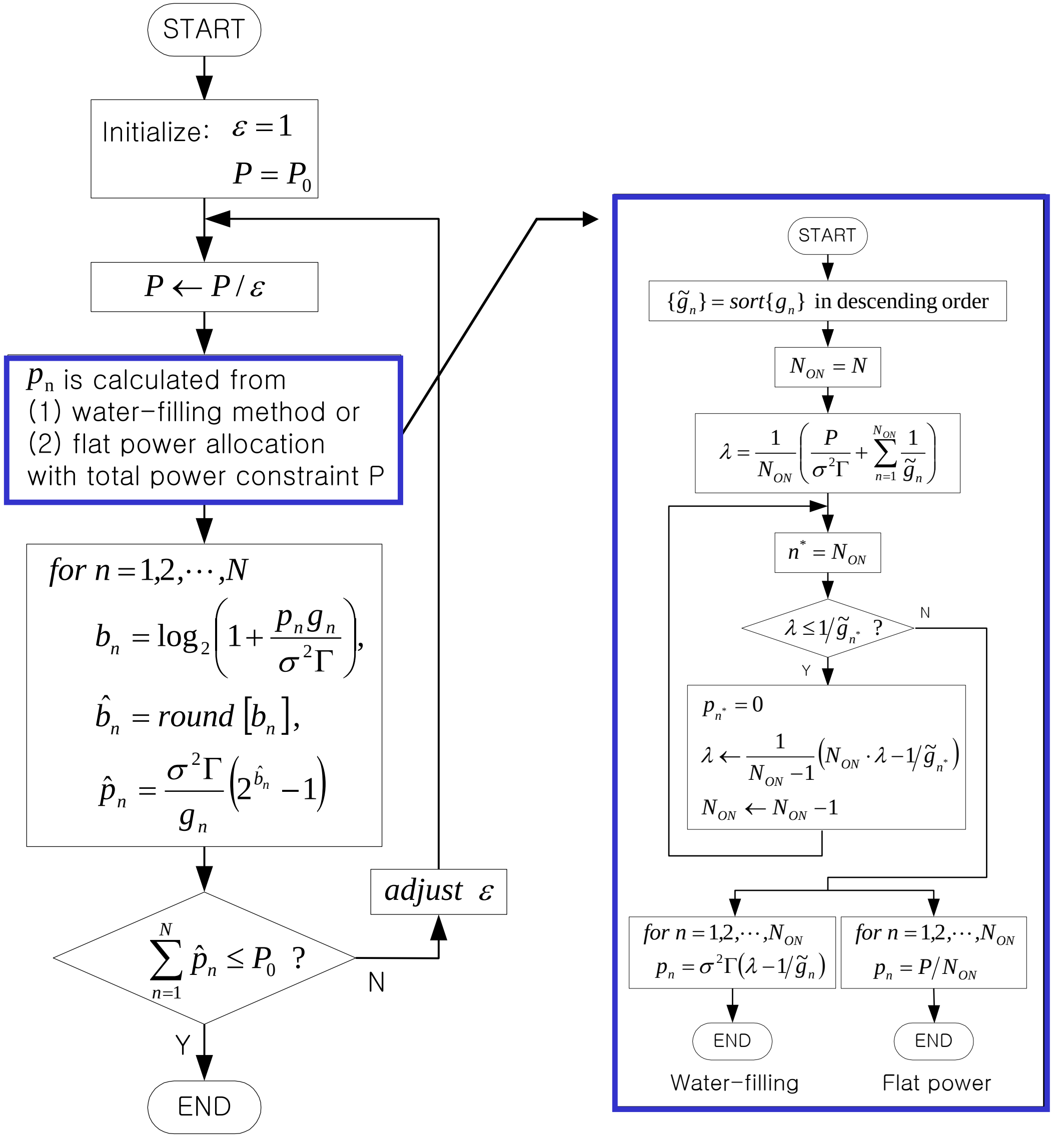
$\hat{p}_n$



$\hat{b}_n$



- Leke's algorithm
  - (1) <u>water-filling power allocation</u>
    (2) flat power allocation
  - High complexity



START

Initialize: $\varepsilon = 1$

$P = P_0$

$P \leftarrow P / \varepsilon$

p_n is calculated from
(1) water-filling method or
(2) flat power allocation
with total power constraint P

for $n = 1, 2, \cdots, N$

$$b_n = \log_2\left(1 + \frac{p_n g_n}{\sigma^2 \Gamma}\right),$$

$$\hat{b}_n = round\left[b_n\right],$$

$$\hat{p}_n = \frac{\sigma^2 \Gamma}{g_n}\left(2^{\hat{b}_n} - 1\right)$$

$\sum_{n=1}^{N} \hat{p}_n \le P_0$?

Y

N

adjust ε

END

START

$\{\tilde{g}_n\} = sort\{g_n\}$ in descending order

$N_{ON} = N$

$$\lambda = \frac{1}{N_{ON}}\left(\frac{P}{\sigma^2 \Gamma} + \sum_{n=1}^{N_{ON}} \frac{1}{\tilde{g}_n}\right)$$

$n^* = N_{ON}$

$\lambda \le 1/\tilde{g}_{n^*}$?

Y

N

$p_{n^*} = 0$

$$\lambda \leftarrow \frac{1}{N_{ON} - 1}\left(N_{ON} \cdot \lambda - 1/\tilde{g}_{n^*}\right)$$

$N_{ON} \leftarrow N_{ON} - 1$

for $n = 1, 2, \cdots, N_{ON}$
$p_n = \sigma^2 \Gamma \left(\lambda - 1/\tilde{g}_n\right)$

for $n = 1, 2, \cdots, N_{ON}$
$p_n = P / N_{ON}$

END

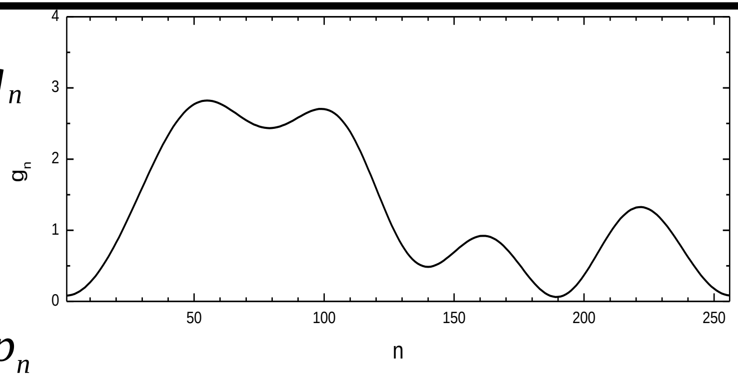
END

Water-filling

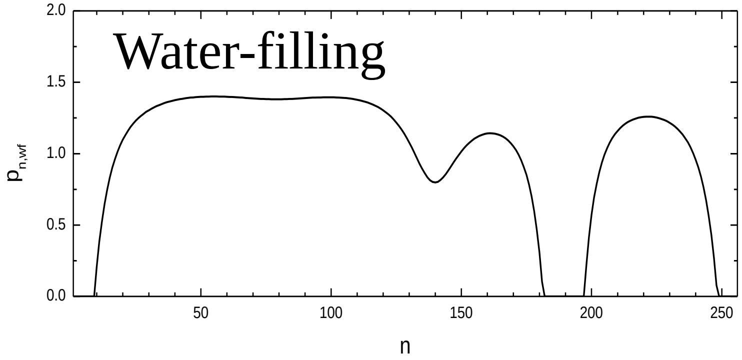
Flat power

- Examples:

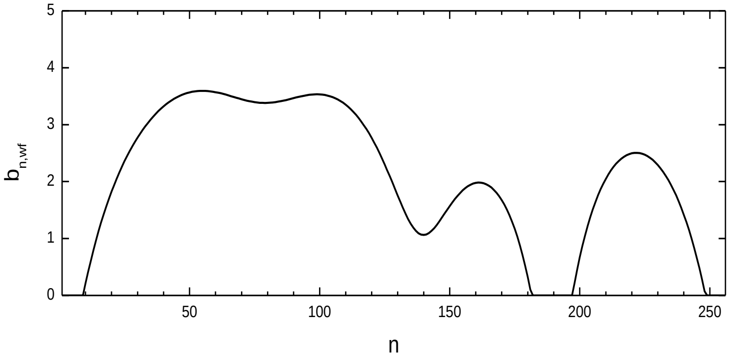
g_n



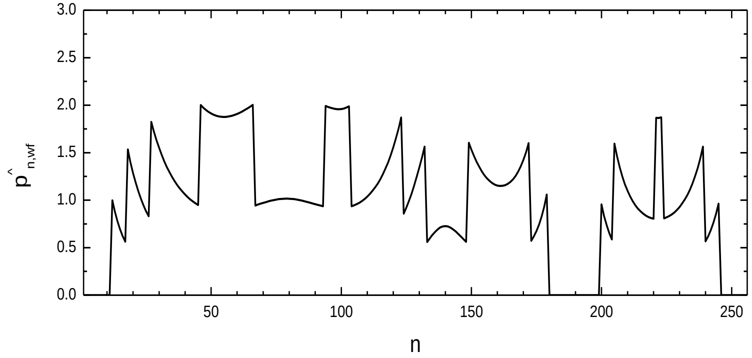
p_n


Water-filling

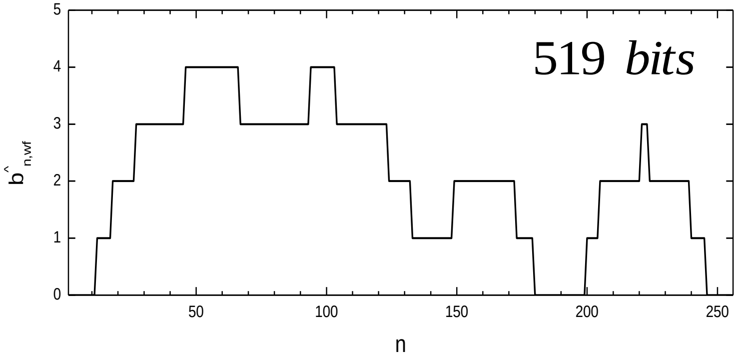
b_n



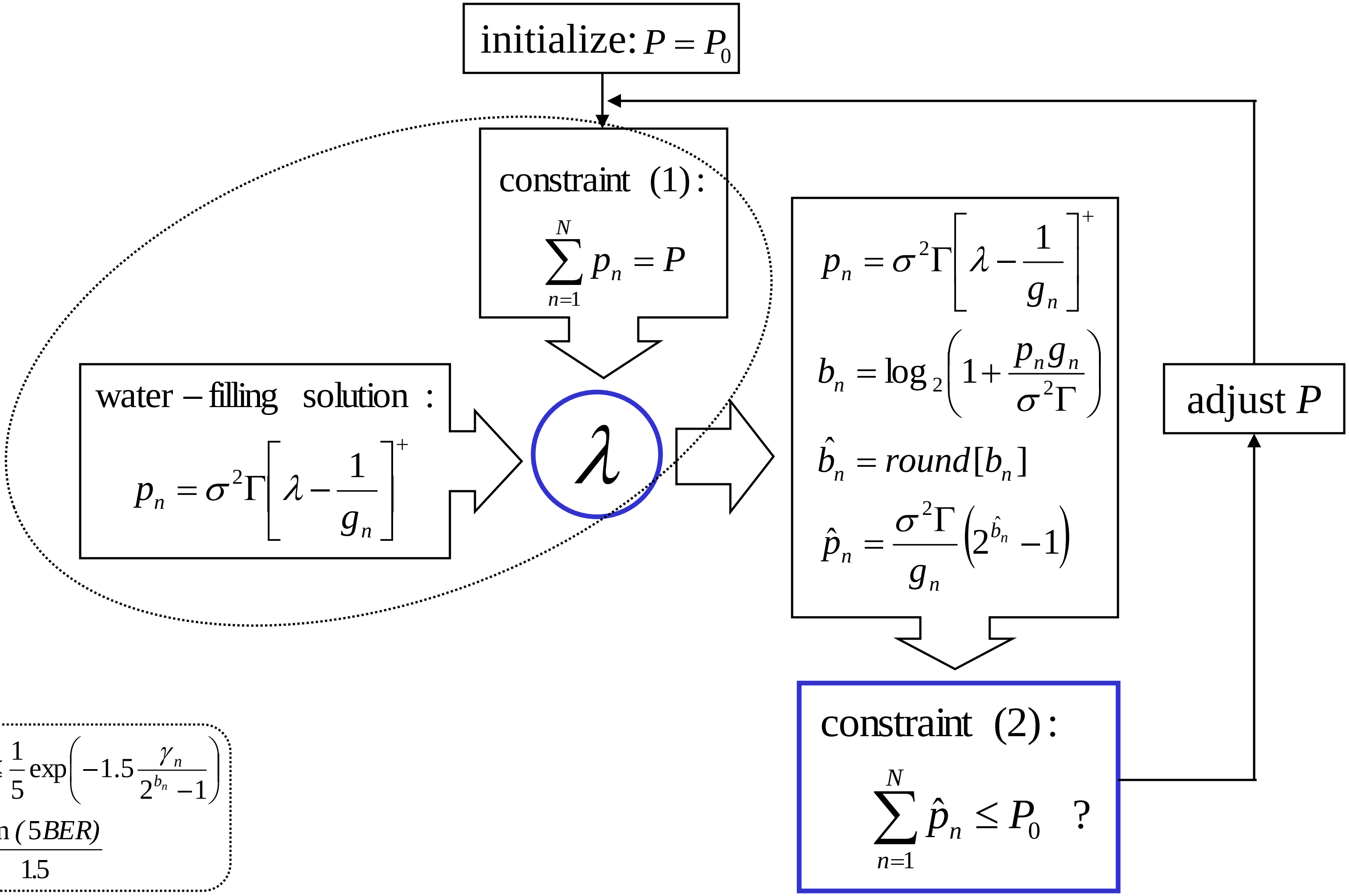
$\hat{p}_n$



$\hat{b}_n$


519 *bits*

- Leke's algorithm



initialize: $P = P_0$

constraint (1):

$$\sum_{n=1}^{N} p_n = P$$

water – filling solution :

$$p_n = \sigma^2\Gamma\left[\lambda - \frac{1}{g_n}\right]^+$$

$$\lambda$$

$$p_n = \sigma^2\Gamma\left[\lambda - \frac{1}{g_n}\right]^+$$

$$b_n = \log_2\left(1 + \frac{p_n g_n}{\sigma^2\Gamma}\right)$$

$$\hat{b}_n = round[b_n]$$

$$\hat{p}_n = \frac{\sigma^2\Gamma}{g_n}\left(2^{\hat{b}_n} - 1\right)$$

constraint (2):

$$\sum_{n=1}^{N} \hat{p}_n \le P_0 \quad ?$$

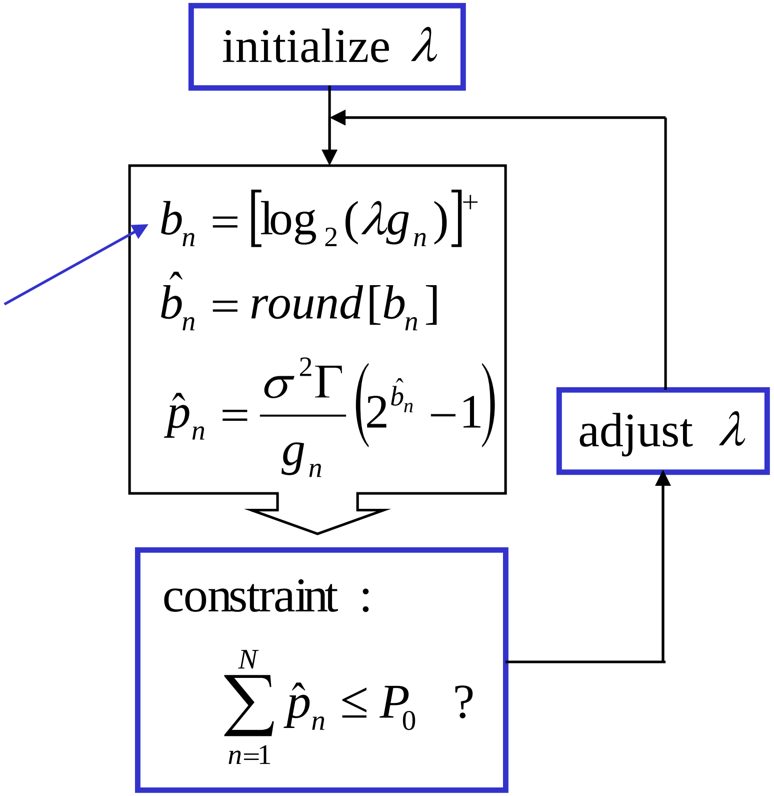
adjust P

$$BER \le \frac{1}{5}\exp\left(-1.5\frac{\gamma_n}{2^{b_n}-1}\right)$$

$$\Gamma \overset{\Delta}{=} \frac{-\ln(5BER)}{1.5}$$

- Basic idea for the proposed algorithm
  - relationship between water-filling level (λ) and transmit power

$$\left.\begin{aligned} p_n &= \sigma^2\Gamma\left[\lambda - \frac{1}{g_n}\right]^+ \\ b_n &= \log_2\left(1 + \frac{p_n g_n}{\sigma^2\Gamma}\right) \end{aligned}\right\}$$



initialize λ

$$b_n = \left[\log_2(\lambda g_n)\right]^+$$

$$\hat{b}_n = round[b_n]$$

$$\hat{p}_n = \frac{\sigma^2\Gamma}{g_n}\left(2^{\hat{b}_n} - 1\right)$$

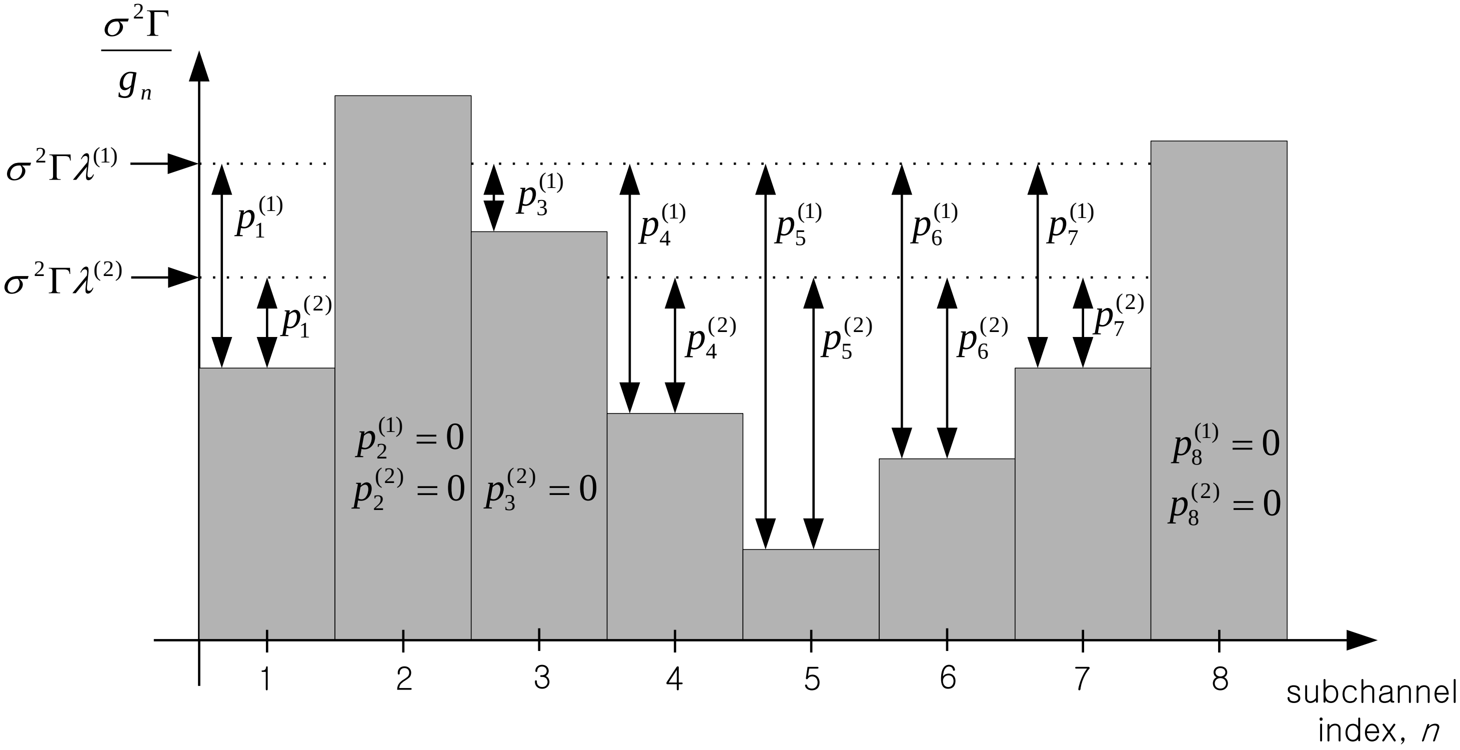
constraint :

$$\sum_{n=1}^{N} \hat{p}_n \le P_0 \quad ?$$

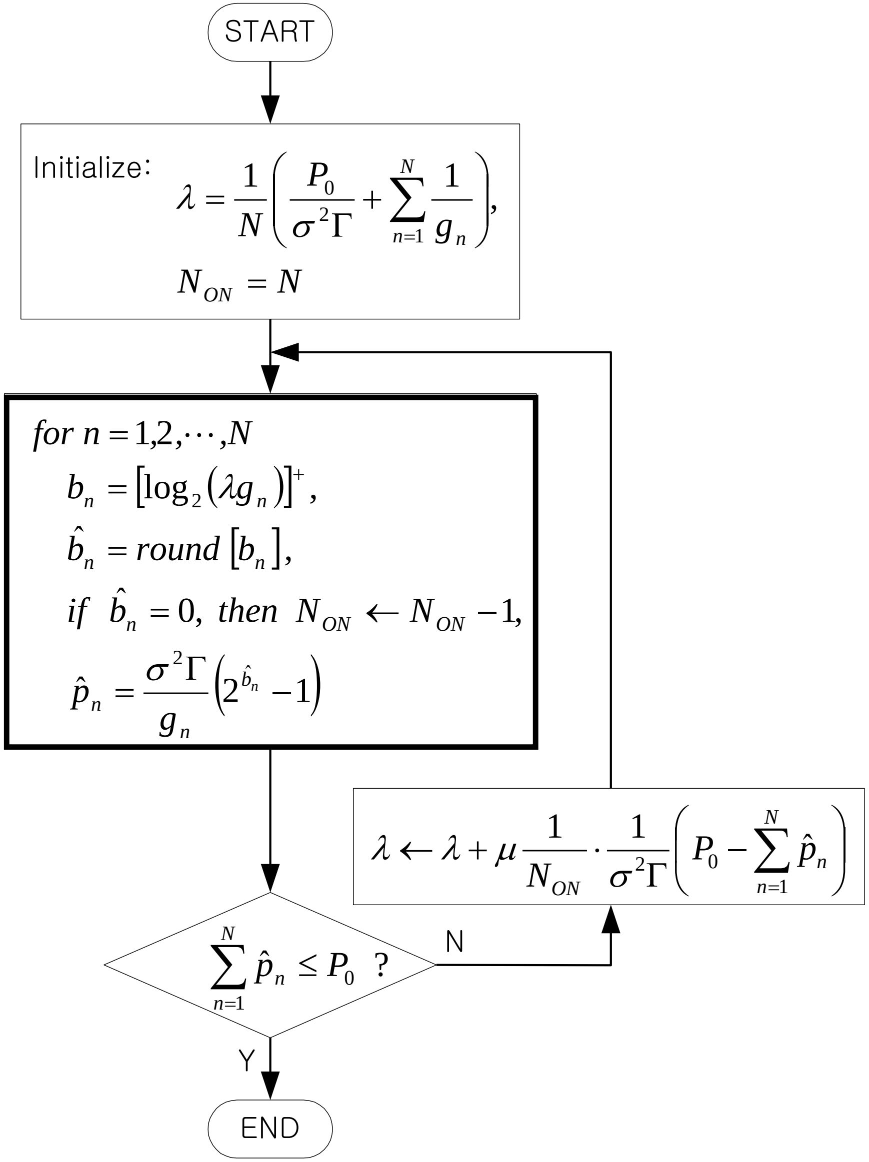
adjust λ

- Effects of λ on the transmit power and number of bits

if $\lambda^{(1)} > \lambda^{(2)}$, then $p_n^{(1)} \geq p_n^{(2)}$, $b_n^{(1)} \geq b_n^{(2)}$, $\hat{b}_n^{(1)} \geq \hat{b}_n^{(2)}$, and $\hat{p}_n^{(1)} \geq \hat{p}_n^{(2)}$



- Proposed algorithm
  - Based on water-filling approach
  - Only the water-filling level (λ) is adjusted
  - Low complexity



Initialize: $\lambda = \frac{1}{N}\left(\frac{P_0}{\sigma^2\Gamma} + \sum_{n=1}^{N}\frac{1}{g_n}\right),$

$N_{ON} = N$

$$\begin{aligned}
&\text{for } n = 1,2,\cdots,N \\
&\quad b_n = \left[\log_2(\lambda g_n)\right]^+, \\
&\quad \hat{b}_n = round\left[b_n\right], \\
&\quad if\ \ \hat{b}_n = 0,\ then\ \ N_{ON} \leftarrow N_{ON} - 1, \\
&\quad \hat{p}_n = \frac{\sigma^2\Gamma}{g_n}\left(2^{\hat{b}_n} - 1\right)
\end{aligned}$$

$\sum_{n=1}^{N}\hat{p}_n \le P_0\ \ ?$

$\lambda \leftarrow \lambda + \mu\frac{1}{N_{ON}}\cdot\frac{1}{\sigma^2\Gamma}\left(P_0 - \sum_{n=1}^{N}\hat{p}_n\right)$

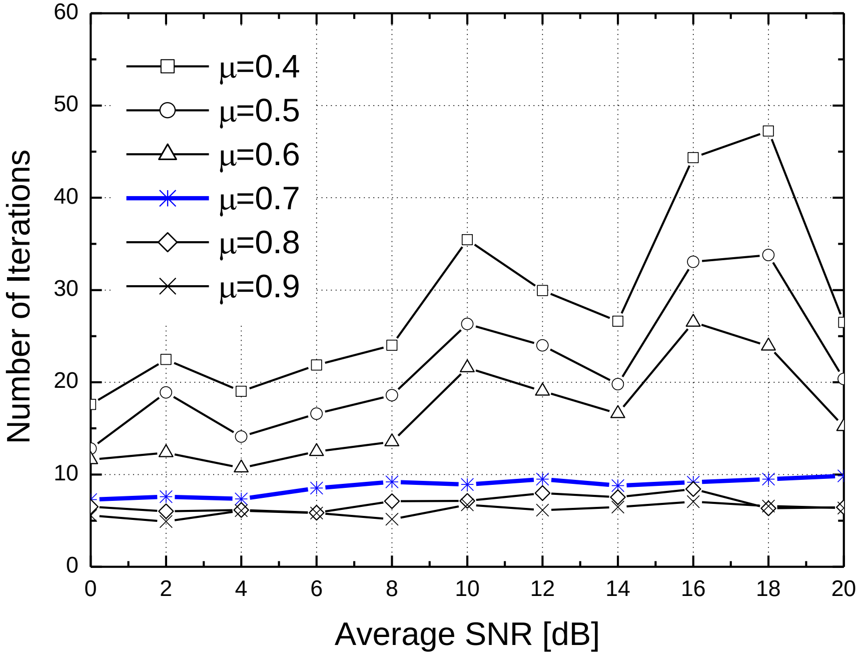
- Complexity comparisons

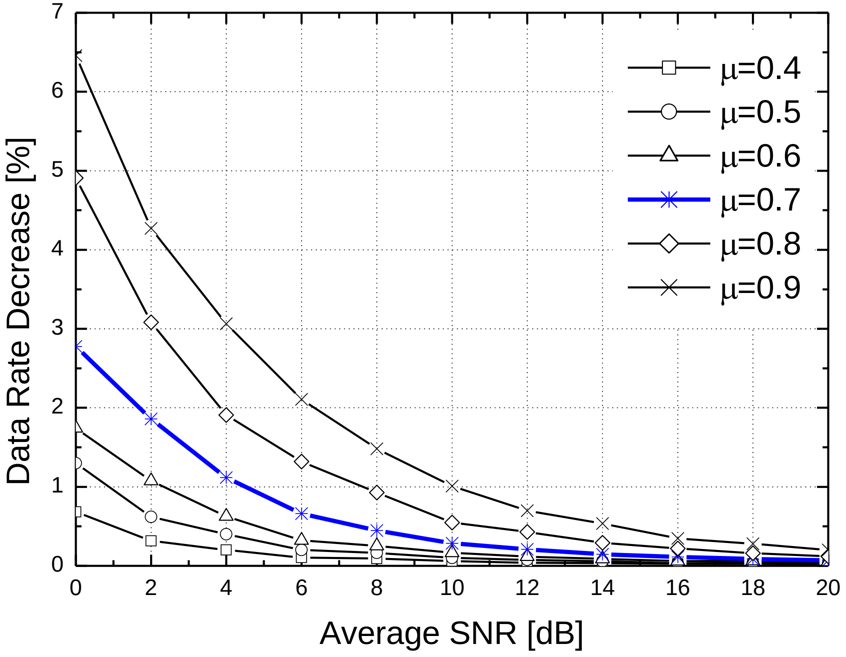
Loading algorithms	*Order of Operations*
Hughes-Hartogs (optimal)	$O(\hat{B}_{HH} \times N \times \log_2 N)$
Chow (equal power allocation)	$O(Iter \times N)$
Leke (water-filling, flat power)	$O(N \times \log_2 N + 2 \times Iter \times N)$
Krongold (lookup table)	$O(N \times M + Iter \times N \times M)$
Proposed	$O(Iter \times N)$

- $\hat{B}_{HH}$: total number of loaded bits when the Hughes-Hartogs' algorithm is used
- $Iter$: required number of iterations
- M : maximum number of bits mapped for a symbol in the constellation

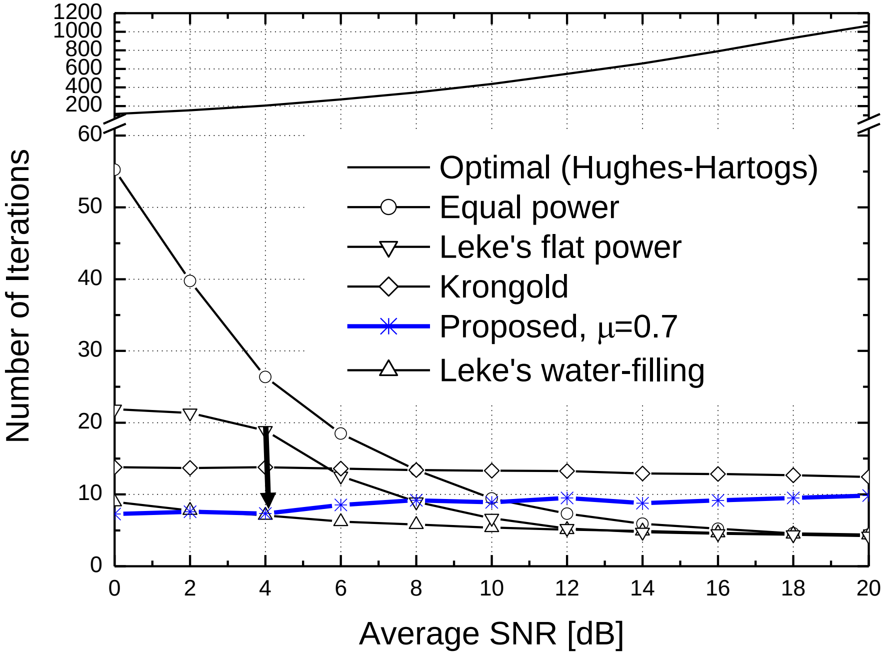
Results

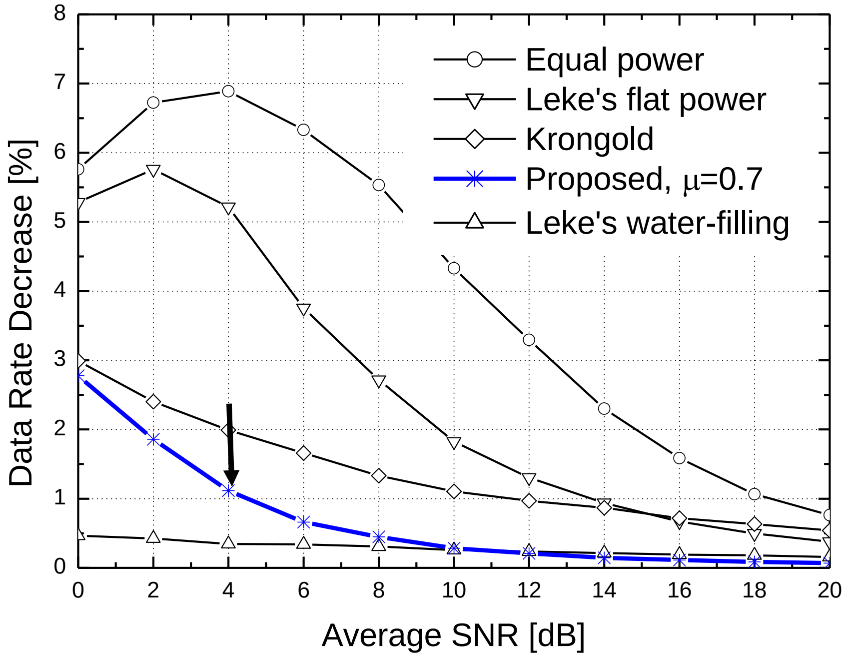
- $N = 256,\ BER_{target} = 10^{-3},\ Data\ \ Rate\ \ Decrease \overset{\Delta}{=} \frac{\hat{B}_{HH} - \hat{B}}{\hat{B}_{HH}} \times 100\ \ [\%]$
- Effects of step size, μ





- $N = 256,\ BER_{target} = 10^{-3},\ Data\ \ Rate\ \ Decrease \overset{\Delta}{=} \dfrac{\hat{B}_{HH} - \hat{B}}{\hat{B}_{HH}} \times 100\ \ [\%]$





♣ Complexity of the proposed is less than the Leke’s algorithm

(e.g.) at SNR=4 dB, O(8N) for the proposed and O(24N) for the Leke’s algorithm

♣ *Data Rate Decrease* for the proposed is less than 1% when SNR>4dB

지적 사항

- Practical considerations for transmit power and bit allocations
  - Transmit power level constraint (transmit spectrum mask)
  - Effects of imperfect channel information
- 학위 논문
  - Motivations 및 기존 연구 survey 보강
  - 참고문헌 보강
  - Conclusions 보강
  - 논문 쪽 수

Practical Considerations

- Transmit power level constraint
  - Transmit power level (transmit spectrum mask) is constrained by the regulations
  - Proposed algorithm needs to be modified
- Effects of imperfect channel information
  - Due to the feedback delay in time varying fading channels
  - Accurate channel estimation is assumed
  - Doppler frequency and feedback delay need to be considered

(1) Transmit power level constraint

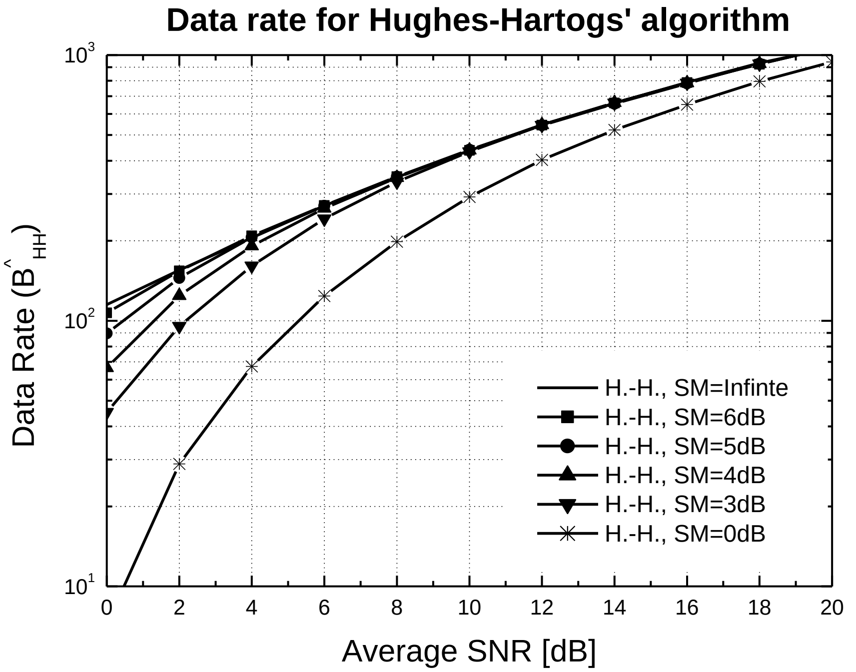
- Proposed algorithm needs to be modified

$$\hat{b}_n = \min\left\{ round\left[[\log_2(\lambda g_n)]^+\right],\ floor\left[\log_2\left(1+\frac{p_{mask}g_n}{\sigma^2\Gamma}\right)\right]\right\}$$

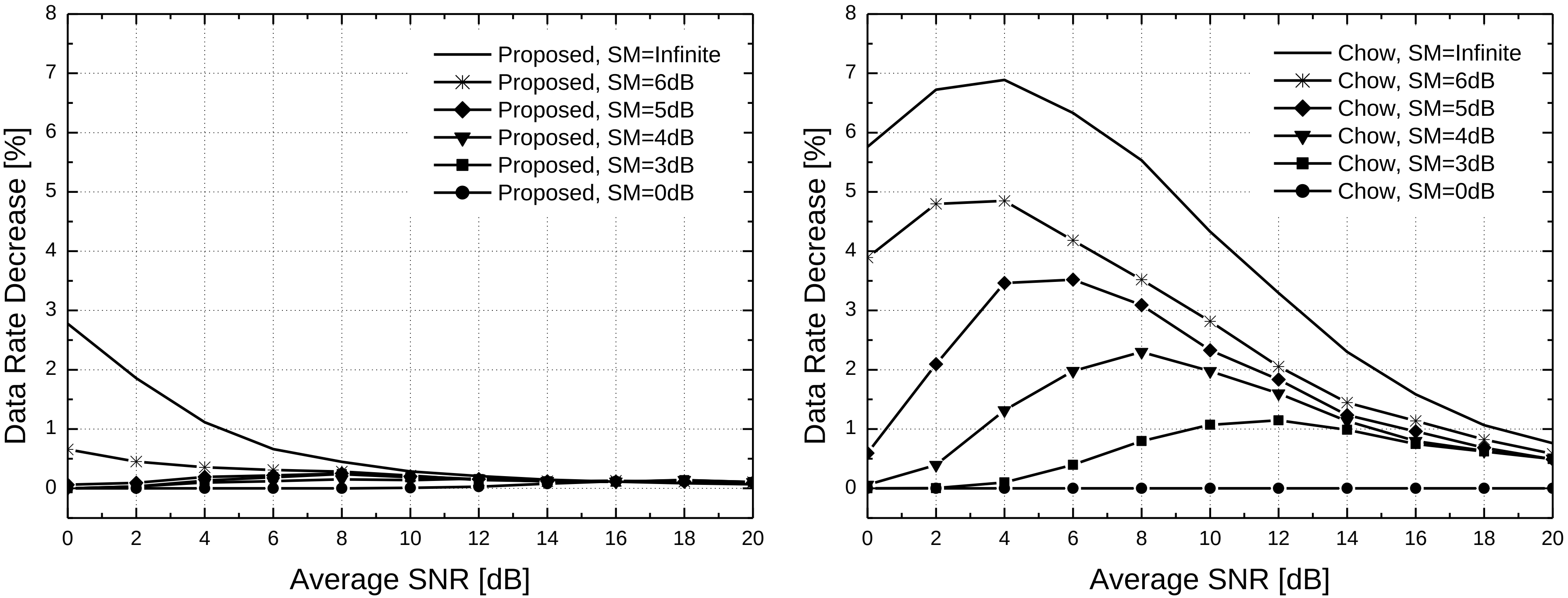
- Spectrum Margin

$$\mathrm{SM} \overset{\Delta}{=} 10\log_{10}\left(\frac{p_{mask}}{P_0/N}\right)$$

♣ Data rate for H.-H. algorithm decreases as SM decreases



- $N = 256,\ BER_{target} = 10^{-3},\ Data\ \ Rate\ \ Decrease \overset{\Delta}{=} \dfrac{\hat{B}_{HH} - \hat{B}}{\hat{B}_{HH}} \times 100\ \ [\%]$



♣As SM decreases, the difference of Data Rate Decrease between proposed algorithm and Chow’s algorithm decreases

(2) Effects of imperfect channel information

$$h[i,k]=\sum_{l=1}^{L}\beta_l[iT]s[kT-\tau_l]$$

$$\beta_i[iT]=\sum_{u=1}^{U}c_{l,u}\exp\Big(j(2\pi f_{D,\max}iT\cos(\theta_u)+\phi_{l,u})\Big)$$

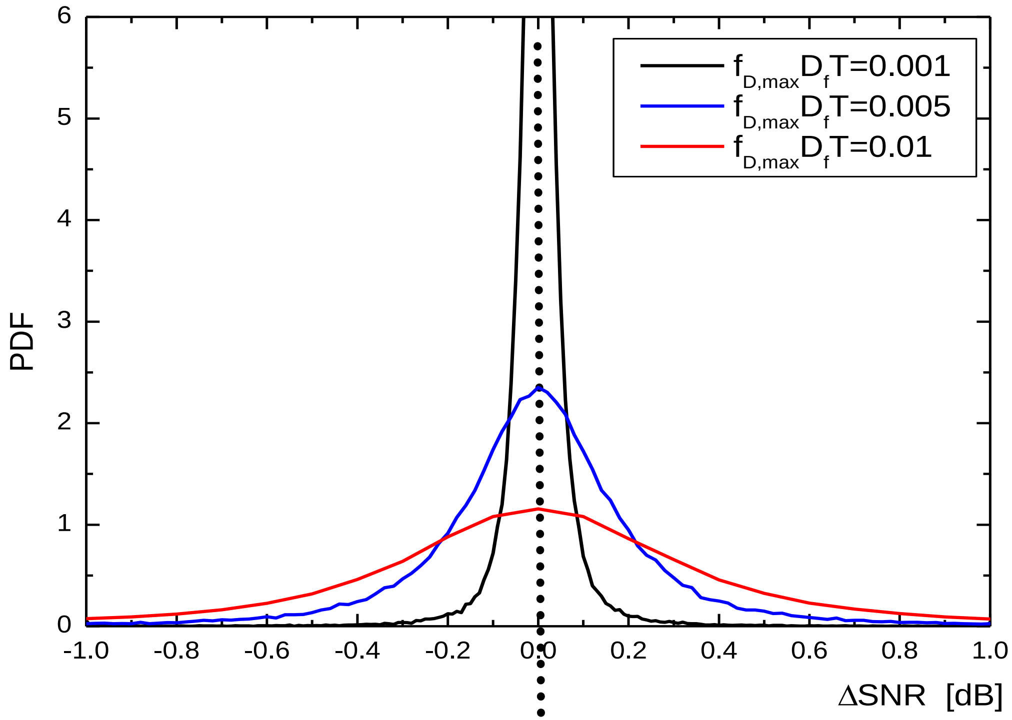
$$H(i,f)=\sum_{l=1}^{L}\beta_l[iT]\sum_{m=-\infty}^{\infty}S\left(f-\frac{m}{T}\right)\exp\left(-j2\pi\left(f-\frac{m}{T}\right)\tau_l\right)$$

$$\frac{SNR}{SNR_{actual}}\approx\frac{\left|\sum_{l=1}^{L}\sum_{u=1}^{U}c_{l,u}\right|^2}{\left|\sum_{l=1}^{L}\sum_{u=1}^{U}c_{l,u}e^{j\left(2\pi f_{D,\max}D_f T\cos(\theta_u)+\phi_{l,u}\right)}\right|^2}$$

- SNR mismatch error is a function of
  (Doppler frequency x feedback delay)

$$\Delta SNR\ (dB)\overset{\Delta}{=}SNR\ (dB)-SNR_{actual}\ (dB)$$

$$\approx 10\log_{10}\left(\frac{\left|\sum_{l=1}^{L}\sum_{u=1}^{U}c_{l,u}\right|^2}{\left|\sum_{l=1}^{L}\sum_{u=1}^{U}c_{l,u}e^{j\left(2\pi f_{D,\max}D_f T\cos(\theta_u)+\phi_{l,u}\right)}\right|^2}\right)$$

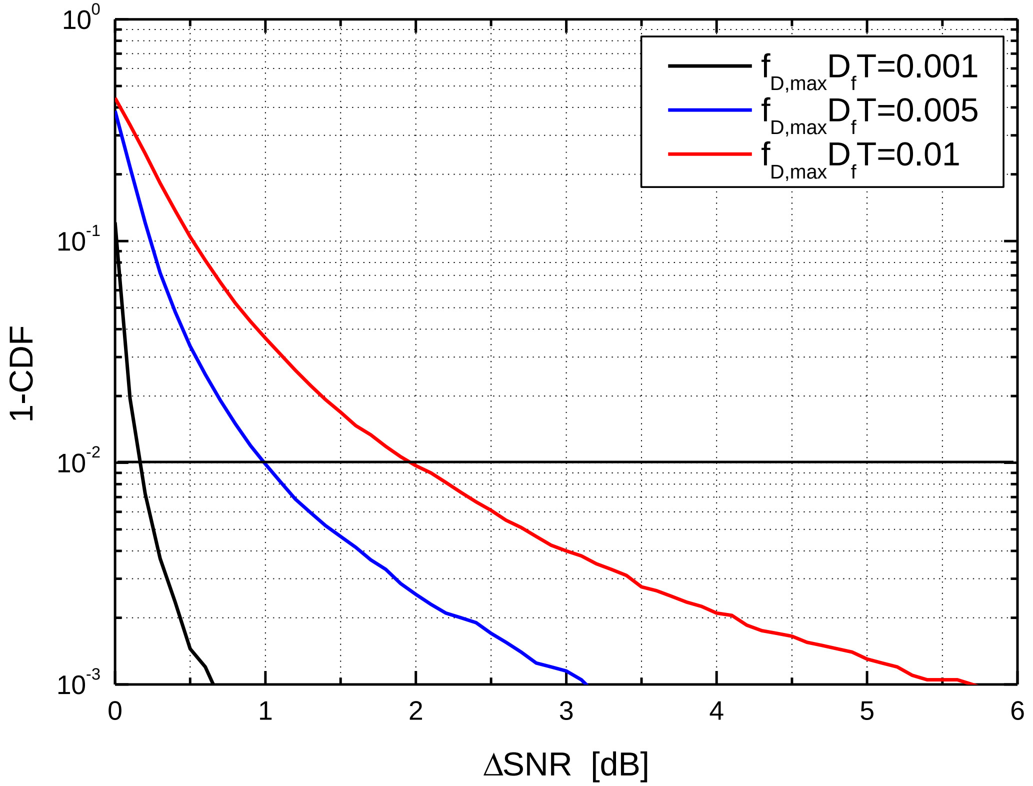


$SNR < SNR_{actual}$

$SNR > SNR_{actual}$

Under-loaded:
- **lower data rate**
- **better BER performance**

Over-loaded:
- **higher data rate**
- **worse BER performance**



- at 1% outage: 0.2dB SNR loss for $f_{D,\max} D_f T = 0.001$
  1.0dB SNR loss for $f_{D,\max} D_f T = 0.005$
  2.0dB SNR loss for $f_{D,\max} D_f T = 0.01$

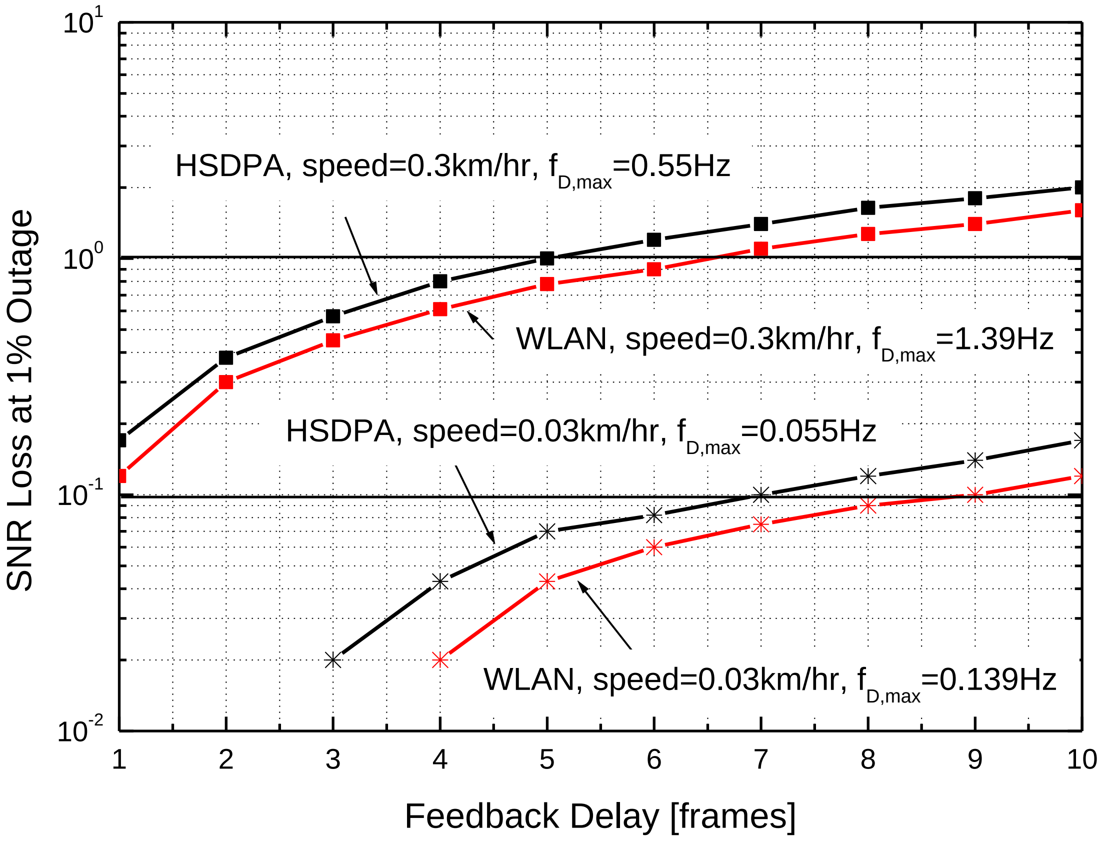
  → SNR margin is required in loading

- Examples

Parameters for High Speed Downlink Packet Access (HSDPA)		
Carrier frequency	2 GHz	
Speed	0.3 km/hr	0.03 km/hr
Doppler frequency	0.55 Hz	0.055 Hz
Frame size	2 msec	

Parameters for IEEE 802.11a Wireless LAN		
Carrier frequency	5 GHz	
Speed	0.3 km/hr	0.03 km/hr
Doppler frequency	1.39 Hz	0.139 Hz
Frame size	628 usec (54Mbps data rate, maximum length)	

- SNR Loss at 1% outage



Conclusions

- Data rate maximization in OFDM with limited power and bandwidth
  - With the knowledge of CSI at the transmitter
- Transmit power allocation in single user OFDM
  - Jointly optimized both in frequency and time domains
  - Water-filling solution in the frequency-time domain is derived
  - Theoretical limit of data rate in single user OFDM
- Transmit power allocation in multiuser OFDM
  - General formulation for multiuser case considering interference from other users
  - Two step approach: (1) subcarrier assignment for users
    (2) power allocation for subcarriers

- Subcarrier assignment for multiple users
  - Greedy policy
- Power allocation for subcarriers
  - Equal power allocation
  - Water-filling in the frequency domain
  - Water-filling in the frequency-time domain
- Theoretical limit of data rate in multiuser OFDM

- Practical algorithm for transmit power and bit allocations
  - With the constraint of integer number of bits for each subcarrier
  - Only the frequency domain is considered
  - Only the water-filling level is adjusted for the transmit power and bit allocations for each subcarrier
  - Low complexity and good performance
  - Suitable for the OFDM system in a time varying wireless channel

- Practical considerations for transmit power and bit allocations
  - Transmit power level constraint
    - The proposed algorithm is modified considering the transmit spectrum mask
  - Effects of imperfect channel information
    - SNR mismatch errors are a function of (Doppler frequency x feedback delay)
    - SNR loss is estimated by simulation

학위논문 수정 및 보완 사항

- Introduction 보강
  - Section 1.1, 1.2 보충 : motivation, 기존 연구 survey
- Practical Considerations 추가
  - Section 4.4 추가 : 4.4.1 Transmit power level constraint
    4.4.2 Effects of imperfect channel information
- Conclusions 보강
  - 5.2 Future Research 추가
- References 보강 : 26 개 → 89 개
- 논문 쪽수 : 85 pages → 115 pages