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# Multi-User Receiver

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# Multi-User Receiver

- A Conventional receiver is designed for a AWGN channel, not a co-channel interference limited environment
- In a multiuser environment,  
a conventional receiver is acceptable, if multiuser interference can be modeled as white Gaussian noise
  - Energies of received signals are not dissimilar
  - Cross correlation of signature waveforms is low

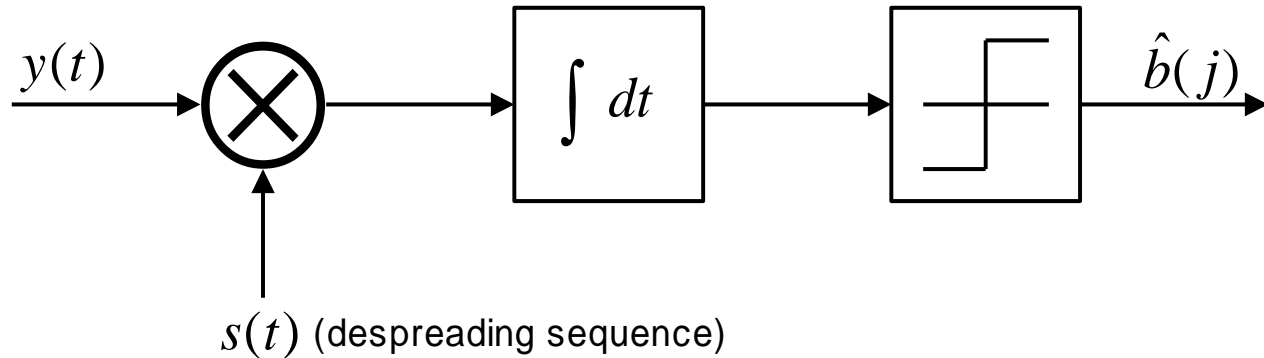
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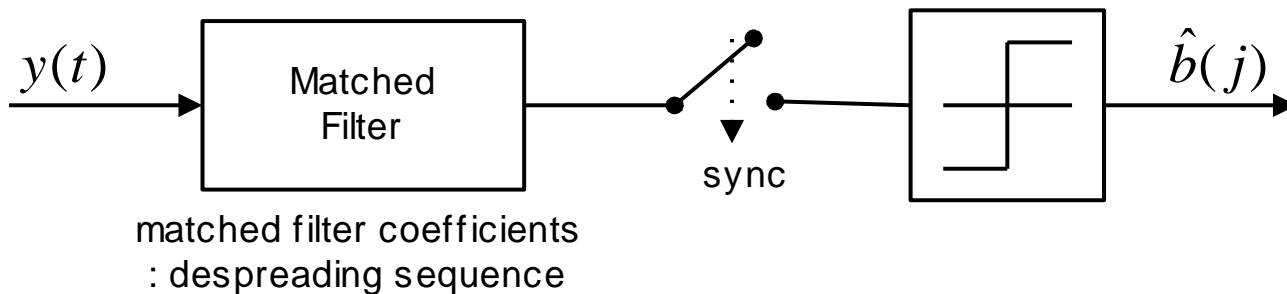
# Receiver Types

- Conventional receiver
- Multi-stage detection
- MMSE detection
- Optimum receiver
- Decorrelator

# Conventional Single User Receiver

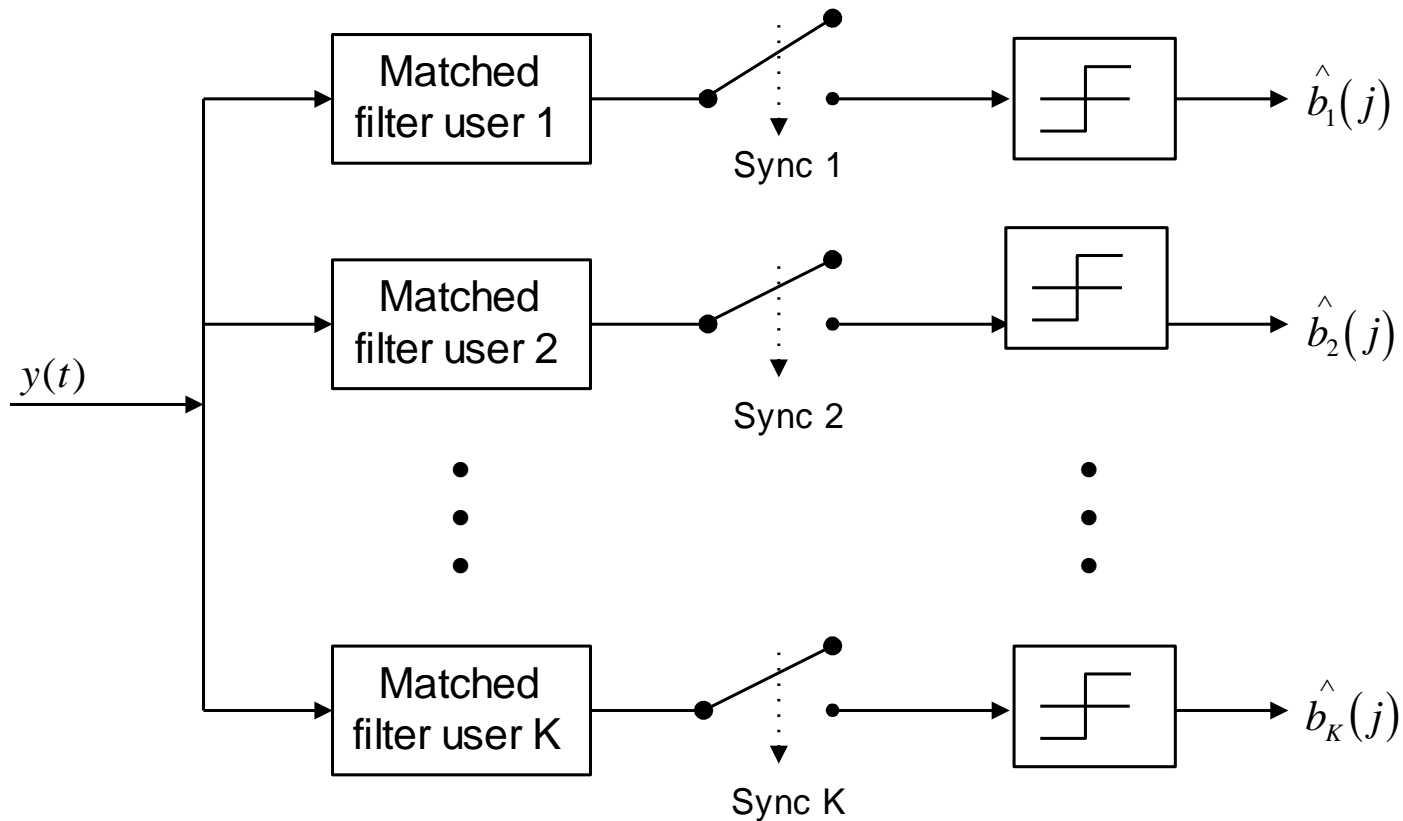


<Correlator type>



<Matched filter type>

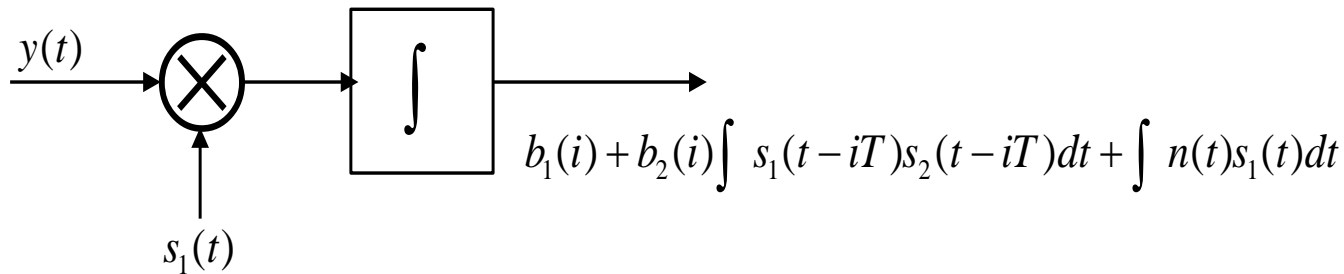
# Conventional Multi-User Receiver



# Conventional Receiver : Issues

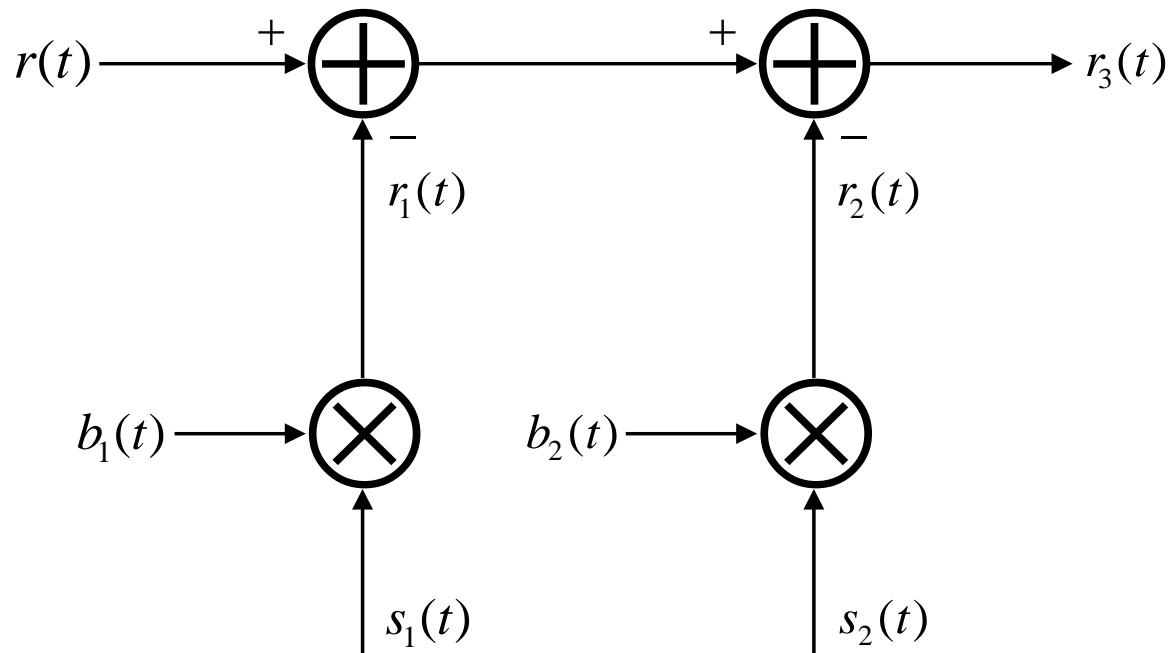
## (Two user case)

- Asynch. 
$$y(t) = \sum_{i=-M}^M b_1(i)s_1(t-iT - \tau_1) + \sum_{i=-M}^M b_2(i)s_2(t-iT - \tau_2) + n(t)$$
- Synch. 
$$y(t) = \sum_{i=-M}^M b_1(i)s_1(t-iT) + \sum_{i=-M}^M b_2(i)s_2(t-iT) + n(t)$$

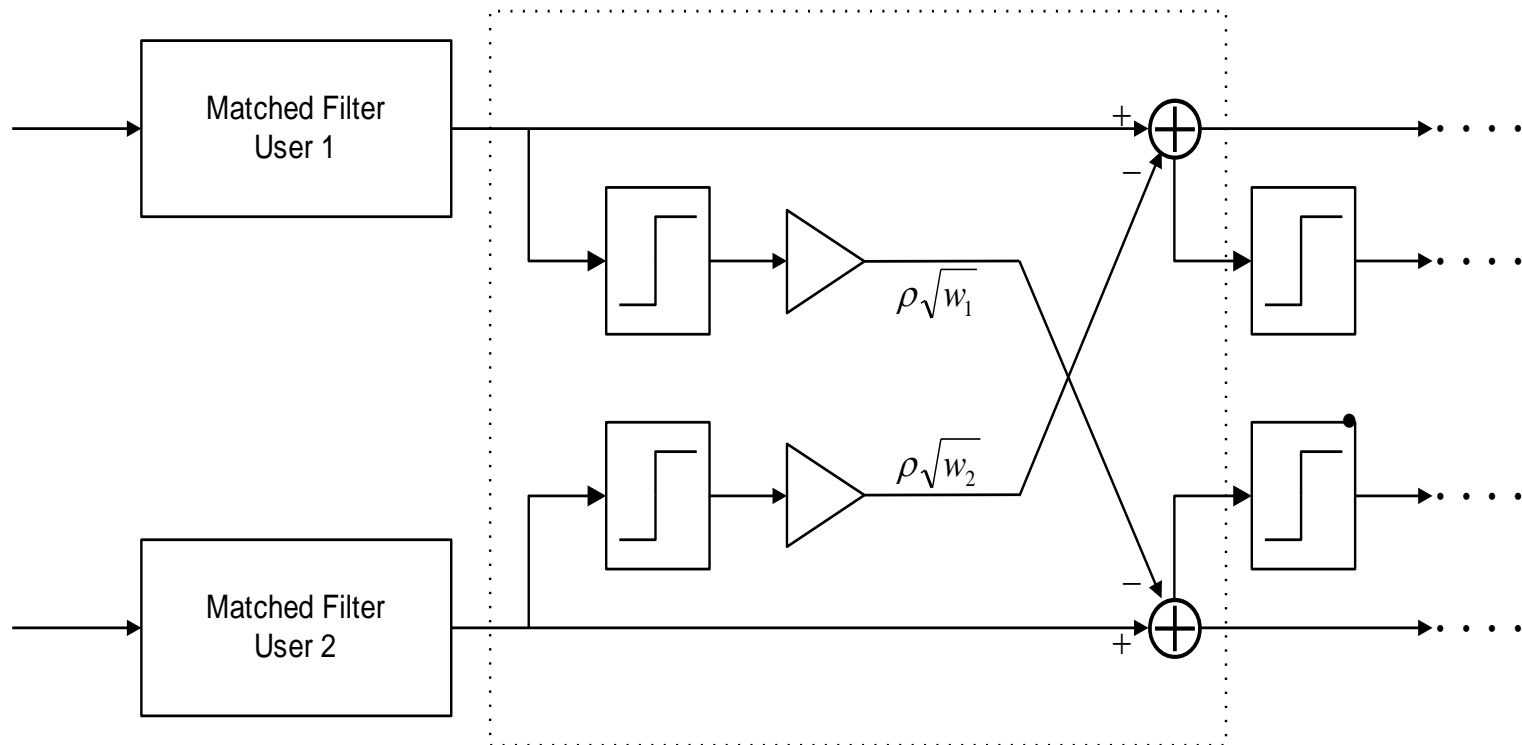


- Crosscorrelation is not zero
- IS-95 downlink, uplink
- power control
- Walsh sequence

# Multi-stage Receiver

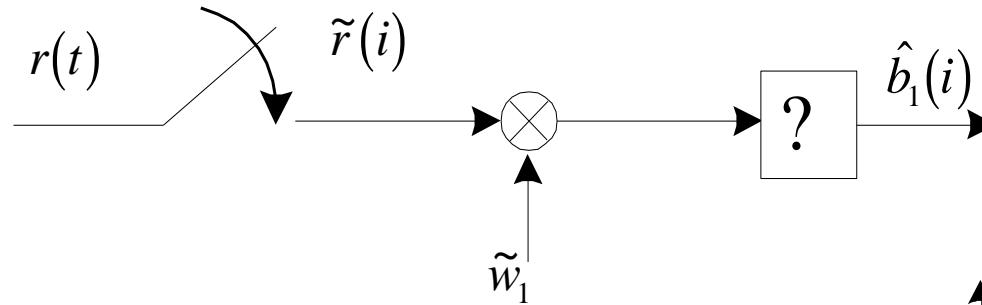


# Multi-stage Receiver (Cont'd)





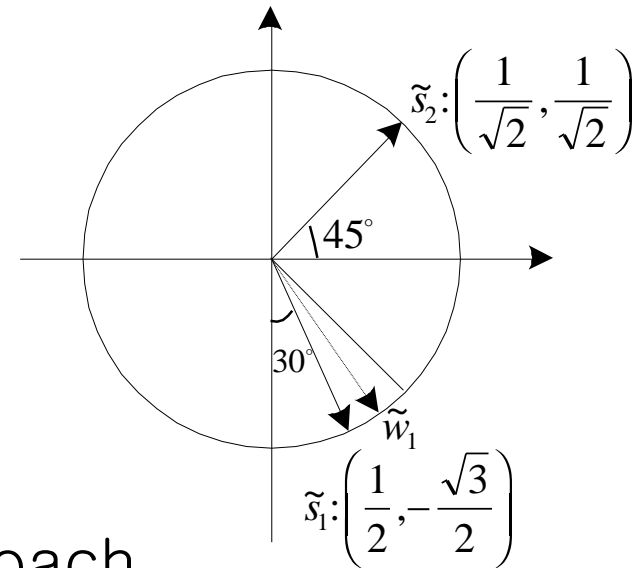
# MMSE Receiver



Find  $\tilde{w}_1$  minimizing

$$E\left[\left(b_1(i) - \hat{b}_1(i)\right)^2\right] = E\left[\left(b_1(i) - \tilde{r}^T(i)\tilde{w}_1\right)^2\right]$$

No a priori knowledge required  
Adaptive signal processing approach



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# ML receiver

$$\text{Prob}(r|d_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(r - d_i)^2\right)$$

Choose “ $d_i$ ” which maximizes  $\text{prob}(r|d_i)$

(Ex) 2users

Possible Tx Signals:  $(3, 1)$ ,  $(3, -1)$ ,  $(-3, 1)$ ,  $(-3, -3)$

Rx Signal      Likely Tx signal

4                     $(3, 1)$

2                     $(3, -1)$

-2                    $(-3, 1)$

-4                    $(-3, -1)$

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- Synchronous two-user case

Choose  $b_1(i)$  &  $b_2(i)$  which minimizes

$$\int_{iT}^{(i+1)T} (r(t) - b_1(i)s_1(t) - b_2(i)s_2(t))^2 dt$$

- Asynchronous two-user case

Find  $b_1(i)$  &  $b_2(i)$  for  $i=-M, \dots, M$  which minimizes

$$\int \left( r(t) - \sum_{k=1}^2 \sum_{i=-M}^M b_k(i) s_k(t - iT - \tau_k) \right)^2 dt$$

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# Decorrelating receiver

- Received signal for a two user case

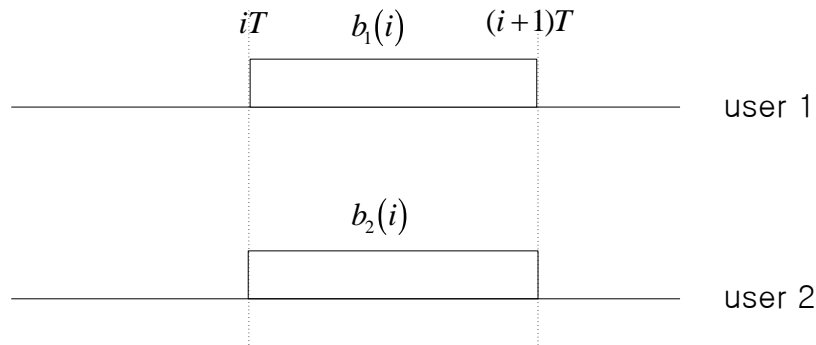
$$r(t) = \sum_{i=-M}^M b_1(i) s_1(t-iT) + \sum_{i=-M}^M b_2(i) s_2(t-iT) + n(t)$$

$s_1(t), s_2(t)$  : Spreading waveforms,  $\int_0^{T_b} s_1^2(t) dt = 1$

$b_1(i), b_2(i)$  : Symbol sequences

$\{-\sqrt{\omega_1}, \sqrt{\omega_1}\}, \{-\sqrt{\omega_2}, \sqrt{\omega_2}\}$ ,  $\omega_k$  : received energy per bit

$$\begin{aligned}
 y_1(i) &= \int_{iT}^{(i+1)T} r(t) s_1(t - iT) dt \\
 &= b_1(i) + \int_{iT}^{(i+1)T} \sum b_2(i) s_2(t - iT) s_1(t - iT) dt + \text{noise term}
 \end{aligned}$$



$$\rho = \int_0^T s_1(t) s_2(t) dt$$

$$y_1(i) = b_1(i) + \rho b_2(i) + \text{noise term}$$

$$y_2(i) = b_2(i) + \rho b_1(i) + \text{noise term}$$

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$$\begin{bmatrix} y_1(i) \\ y_2(i) \end{bmatrix} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} b_1(i) \\ b_2(i) \end{bmatrix} + \begin{bmatrix} n_1(i) \\ n_2(i) \end{bmatrix}$$

$$\tilde{H} \equiv \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \quad y(i) \equiv \begin{bmatrix} y_1(i) \\ y_2(i) \end{bmatrix}, \quad \tilde{b}(i) \equiv \begin{bmatrix} b_1(i) \\ b_2(i) \end{bmatrix}, \quad \tilde{n}(i) \equiv \begin{bmatrix} n_1(i) \\ n_2(i) \end{bmatrix}$$

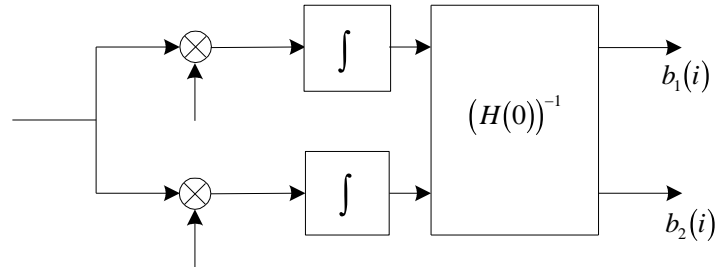
$$\tilde{y}(i) = \tilde{H} \cdot \tilde{b}(i) + \tilde{n}(i)$$

- Synchronous case

$$\tilde{y}(i) = \tilde{H}(0) \cdot \tilde{b}(i) + \tilde{n}(i)$$

Ignoring  $\tilde{n}(i)$ ,

$$\tilde{b}(i) = (\tilde{H}(0))^{-1} \tilde{y}(i)$$



$$(\tilde{H}(0))^{-1} = \frac{1}{1 - \rho_{12}^2} \begin{bmatrix} 1 & -\rho_{12} \\ -\rho_{12} & 1 \end{bmatrix}$$

$$\tilde{b}(i) = \frac{1}{1 - \rho_{12}^2} \begin{bmatrix} 1 & -\rho_{12} \\ -\rho_{12} & 1 \end{bmatrix} \begin{bmatrix} y_1(i) \\ y_2(i) \end{bmatrix} = \frac{1}{1 - \rho_{12}^2} \begin{bmatrix} y_1(i) - \rho_{12} y_2(i) \\ -\rho_{12} y_1(i) + y_2(i) \end{bmatrix}$$

$$\text{sgn}(\tilde{b}(i)) = \text{sgn}\left( (\tilde{H}(0))^{-1} \tilde{y}(i) \right)$$