Capacity of Fading Channel

(Ref : “Capacity of Fading Channel with Channel Side information,” Nov. 1997. T-Info Theory. Pp 1986~1992)

Contents

- System Model
- Capacity Analysis

System Model

- Discrete-time channel with stationary & ergodic time-varying gain $\sqrt{g[i]}$, $0 \le g[i]$
- AWGN $n[i]$: noise density N_0
- $g[i]$: independent of the channel input,
  expected value = 1
- S : average Tx power
- B : received signal BW

=> Instantaneous received SNR: $\gamma[i] = \frac{Sg[i]}{N_0 B}$

Expected value over all time: $\frac{S}{N_0 B}$

Capacity Analysis

- A time-invariant AWGN channel with average SNR γ has capacity,

$$C_\gamma = B\log(1+\gamma) \qquad \ldots(1)$$

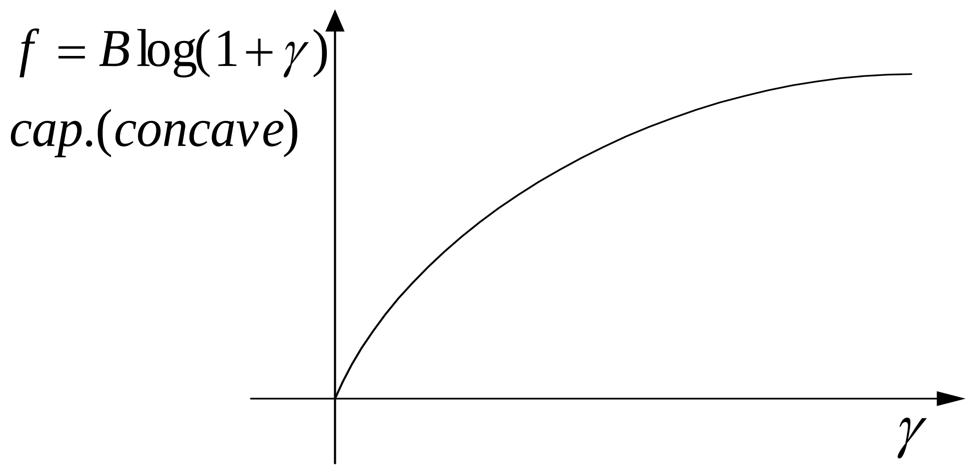
- $p(\gamma) = p(\gamma[i] = \gamma)$ denote the probability distribution of the received SNR γ

- Capacity of the fading channel,

$$C = \int_{\gamma} C_{\gamma}\, p(\gamma)\, d\gamma = \int_{\gamma} B \log(1+\gamma)\, p(\gamma)\, d\gamma \qquad \ldots(2)$$

- By Jensen's inequality,

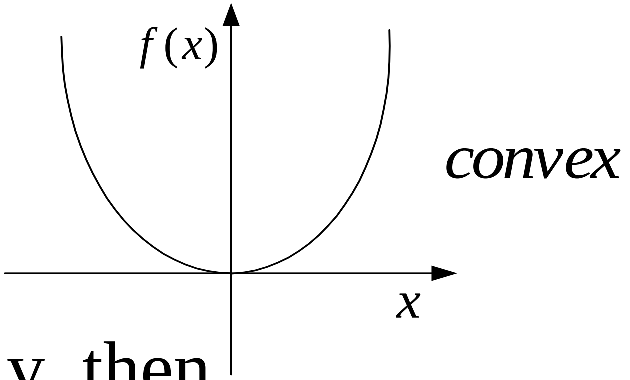
  (2) < capacity of an AWGN channel



Jensen's Inequality

- If f is a convex fn. & x is a r.v. then

$$E\{f(x)\} \geq f(E\{x\})$$



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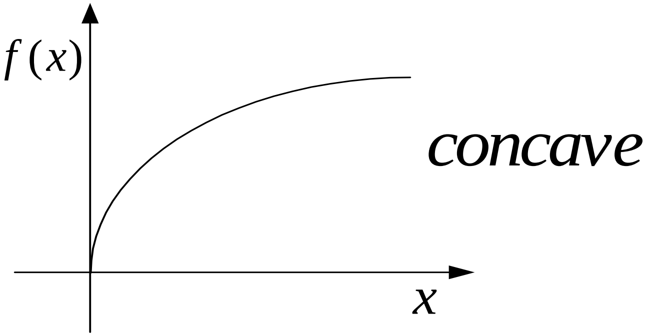

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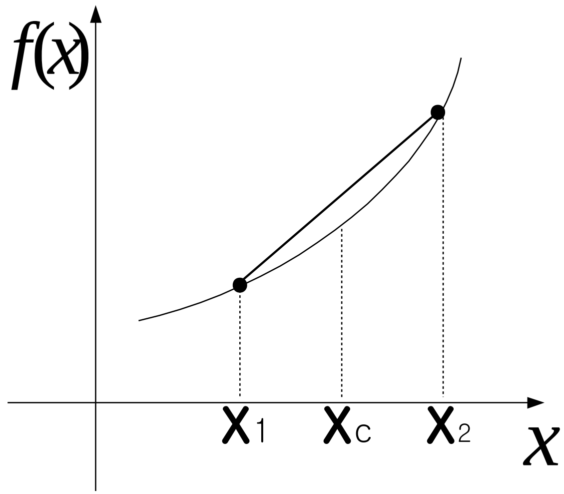

$RV : X$ (예) $x = x_1 \quad prob = \frac{1}{2}$

$x = x_2 \quad prob = \frac{1}{2}$

$x_c = (x_1 + x_2)/2$

$E\{f(x)\}$

$= p_1 f(x_1) + p_2 f(x_2)$

$= \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2)$

$f(E\{x\})$

$= f(p_1 x_1 + p_2 x_2)$

$= f(\frac{x_1}{2} + \frac{x_2}{2})$

$$E\{f(x)\} \geq f(E\{x\})$$

- We allow the Tx power $S(\gamma)$ to vary with $\gamma[i]$, subject to an average power constraint S

$$\int_{\gamma} S(\gamma)p(\gamma)d\gamma \le S \qquad \left(= \int_{\gamma} \frac{S(\gamma)}{S} p(\gamma)d\gamma \le 1\right) \quad \ldots(3)$$

- Given the average power constraint (3), the time-varying channel capacity,

$$C(S) = \max_{S(\gamma):\int S(\gamma)p(\gamma)d\gamma = S} \int_{\gamma} B\log\left(1 + \frac{S(\gamma)\gamma}{S}\right) p(\gamma)d\gamma \quad \ldots(4)$$

$$\left(\frac{S(\gamma)}{S} = \frac{inst.Tx\ pwr}{avg.Tx\ pwr},\ when S(\gamma) = S,\ \frac{S(\gamma)}{S} = 1 \right)$$

- The power adaptation which maximize (4) is

$$\frac{S(\gamma)}{S}=\begin{cases}\dfrac{1}{\gamma_0}-\dfrac{1}{\gamma} & , \quad \gamma \geq \gamma_0 \\ 0 & , \quad \gamma < \gamma_0\end{cases} \qquad \text{...(5)}$$

=> Water-pouring

- Determination of γ_0

$(5) \rightarrow (3)$

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p(\gamma)\, d\gamma = 1 \qquad \ldots(6)$$

$(5) \rightarrow (4)$

$$C(S) = \int_{\gamma_0}^{\infty} B \log\left(1 + \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right)\gamma\right) p(\gamma)\, d\gamma$$

$$= \int_{\gamma_0}^{\infty} B \log\left(\frac{\gamma}{\gamma_0}\right) p(\gamma)\, d\gamma \qquad \ldots(7)$$