Fusion Reactor Technology I
(459.760, 3 Credits)

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Contents

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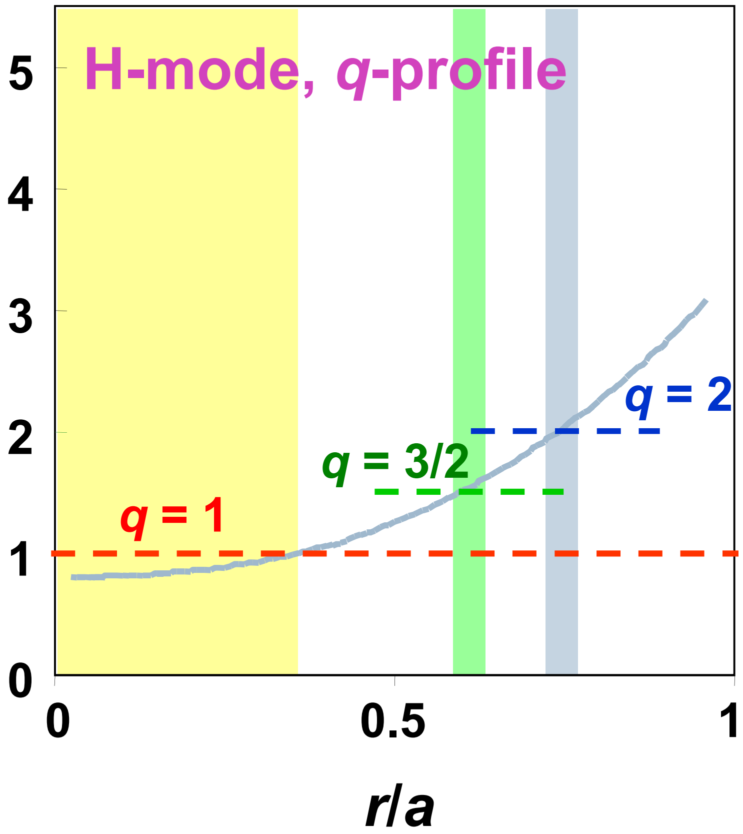
H–mode: Limitations

Stability of H-mode plasmas related safety factor profile: $q(r)$



$q_0 < 1$: Sawtooth instability, periodic flattening of the pressure in the core

$q = 3/2$ and $q = 2$:

Neoclassical Tearing Modes (NTMs):

- limit the achievable $\beta \equiv 2\mu_0 p/B^2$
- degrade confinement (+ disruptions)
- often triggered by sawteeth.

- ITER work point is chosen conservatively: $\beta_N \leq 1.8$!

Neoclassical Tearing Mode (NTM)

LETTERS

The purpose of this Letters section is to provide rapid dissemination of important new results in the fields regularly covered by The Physics of Fluids. Results of extended research should not be presented as a series of letters in place of comprehensive articles. Letters cannot exceed four printed pages in length, including space allowed for title, figures, tables, references and an abstract limited to about 100 words.

Island bootstrap current modification of the nonlinear dynamics of the tearing mode

R. Carrera, R. D. Hazeltine, and M. Kotschenreuther
Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712-1068

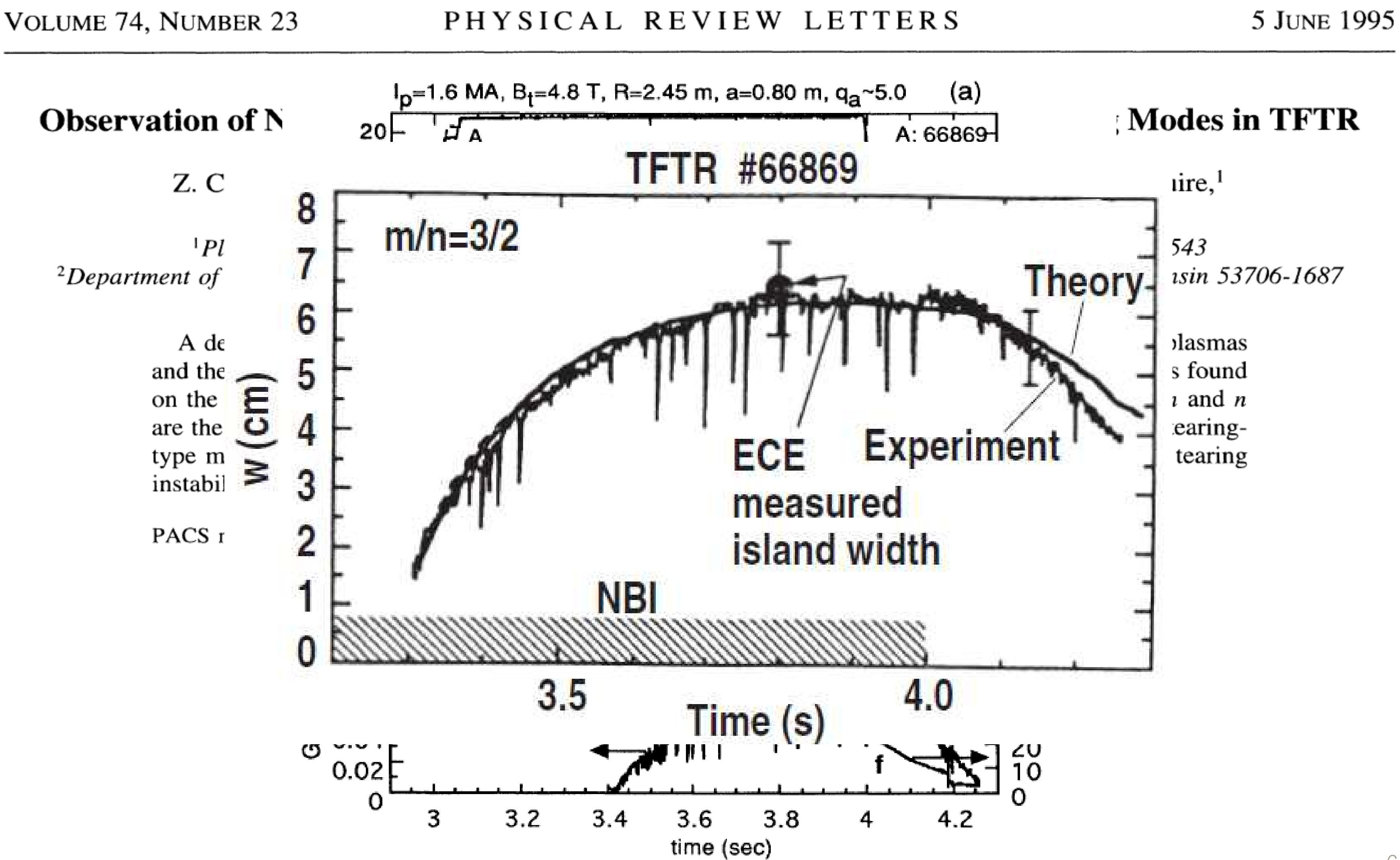
(Received 1 November 1985; accepted 13 January 1986)

A kinetic theory for the nonlinear evolution of a magnetic island in a collisionless plasma confined in a toroidal magnetic system is presented. An asymptotic analysis of a Grad–Shafranov equation including neoclassical effects such as island bootstrap current defines an equation for the time dependence of the island width. Initially, the island bootstrap current strongly influences the island evolution. As the island surpasses a certain critical width the effect of the island bootstrap current diminishes and the island grows at the Rutherford rate. For current profiles such that $\Delta' < 0$ the island bootstrap current saturates the island.

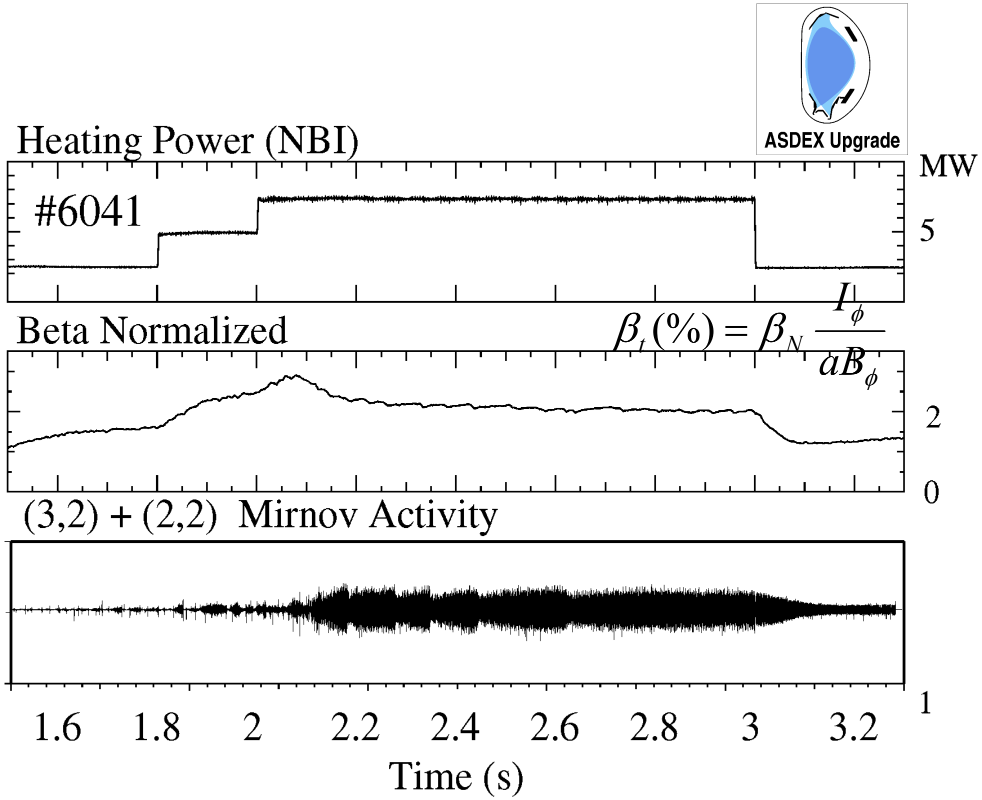
R. Carrera et al, Physics of Fluids ***29*** *899 (1986)*

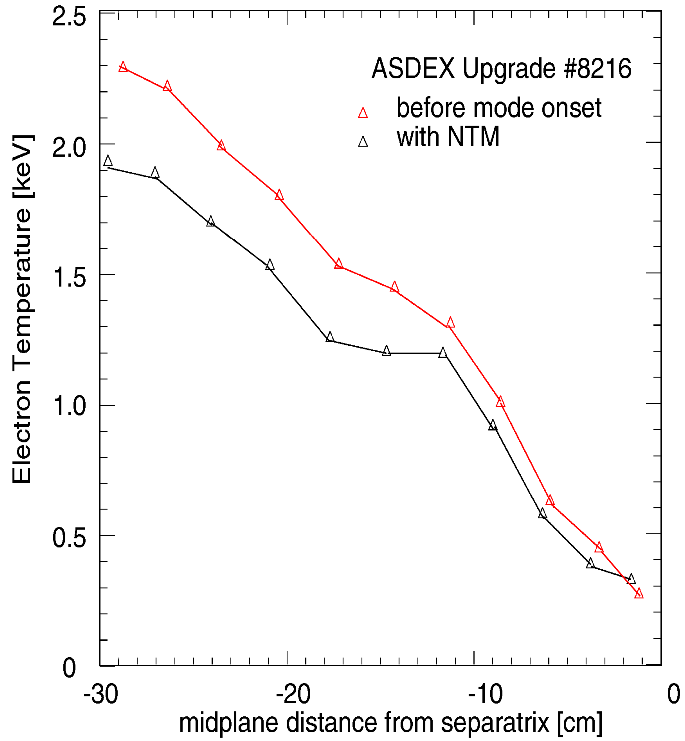
- One of the earliest theoretical paper

Neoclassical Tearing Mode (NTM)



Neoclassical Tearing Mode (NTM)



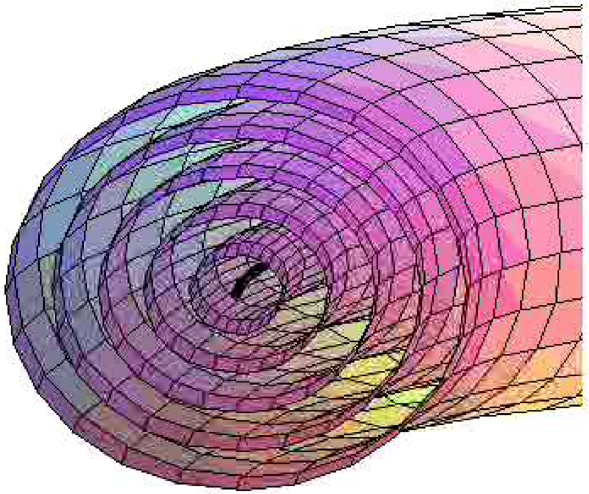


H. Zohm et al., *Plasma Phys. Contr. Fusion* 37 (1995)

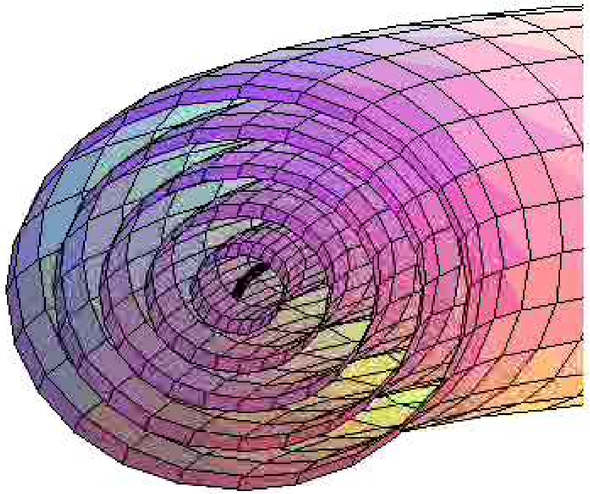
Neoclassical tearing modes can occur well below ideal limit
- ‚practical β-limit' in ITER standard scenario (ELMy H-mode)
- can also lead to disruptive temination (especially at low q)

Neoclassical Tearing Mode (NTM)

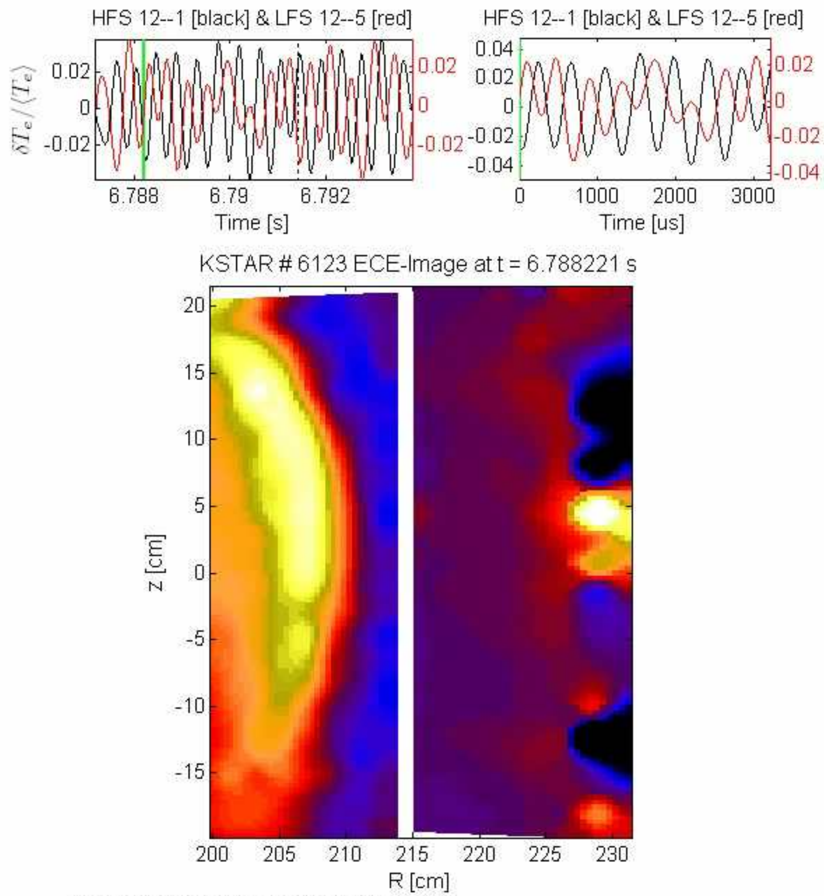
- Ideal MHD: $\eta = 0$



- Resistive MHD: $\eta \neq 0$

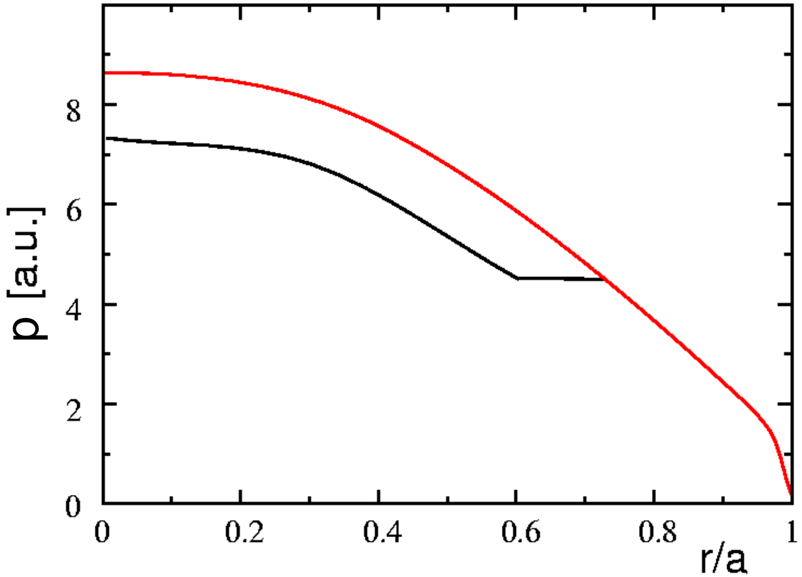


Neoclassical Tearing Mode (NTM)



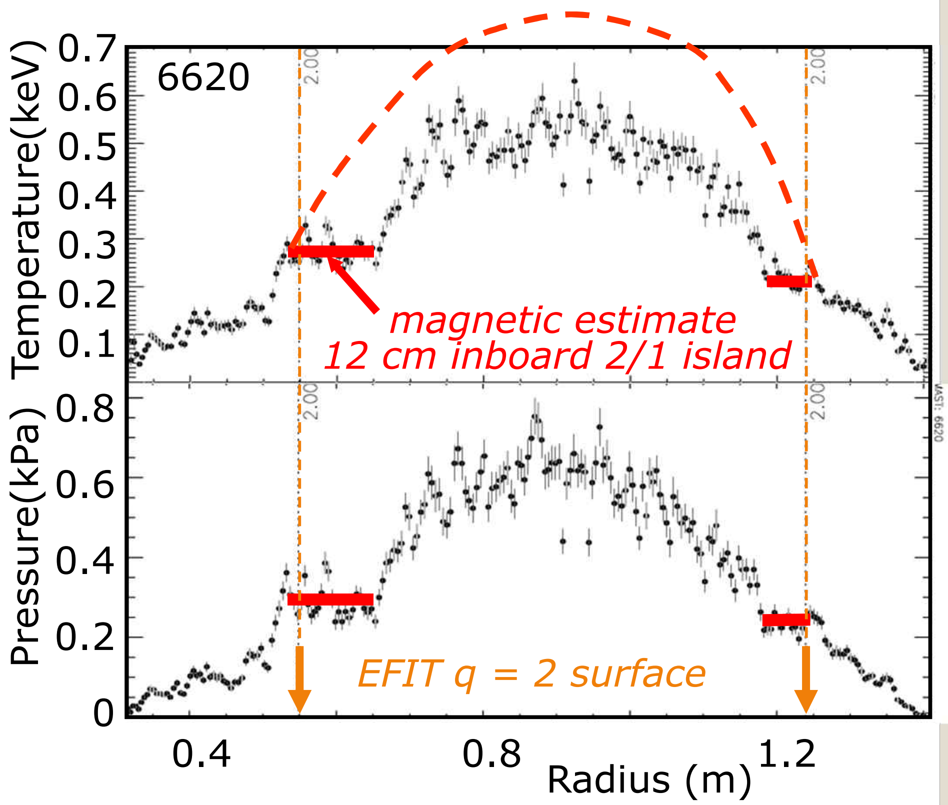
NOT Calibrated image. NOT for Publication

Neoclassical Tearing Mode (NTM)



- Pressure flattening across magnetic islands due to large transport coefficients along magnetic field lines

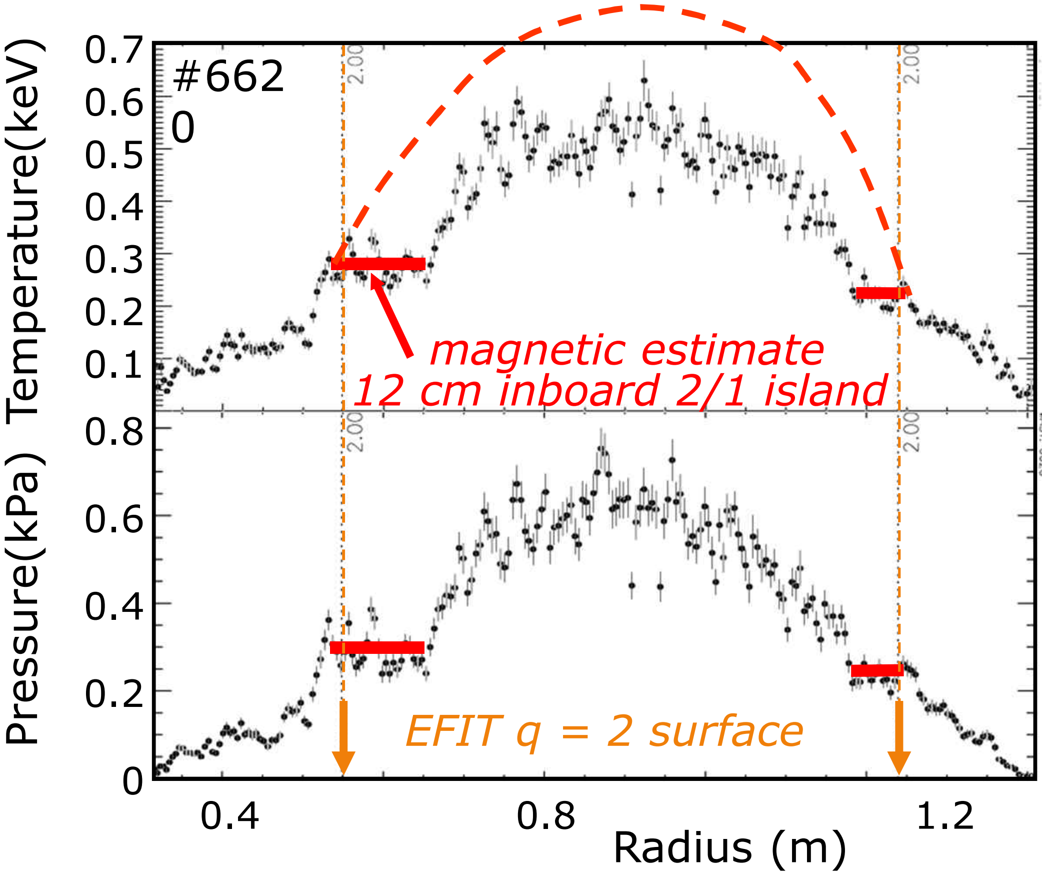
Neoclassical Tearing Mode (NTM)



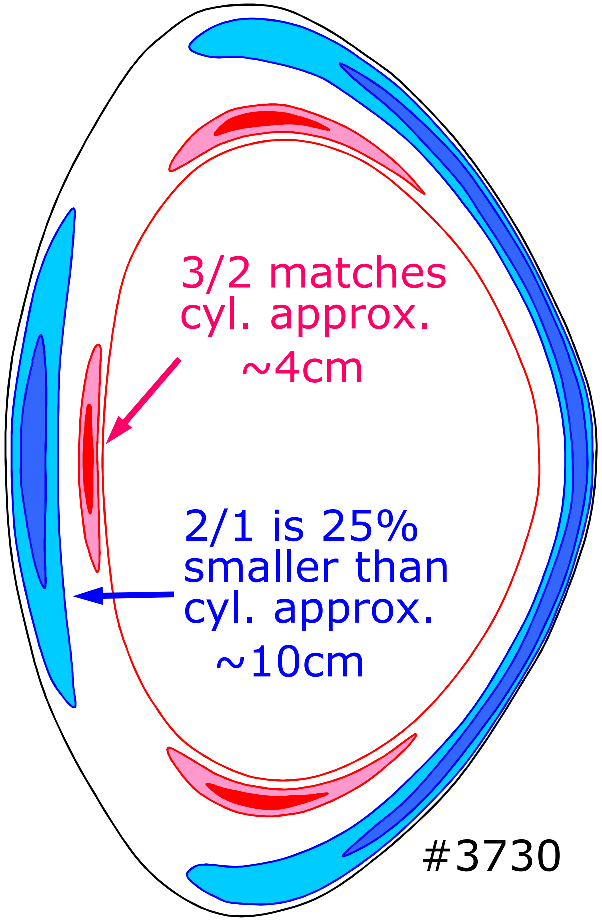
- Pressure flattening across magnetic islands due to large transport coefficients along magnetic field lines

Neoclassical Tearing Mode (NTM)

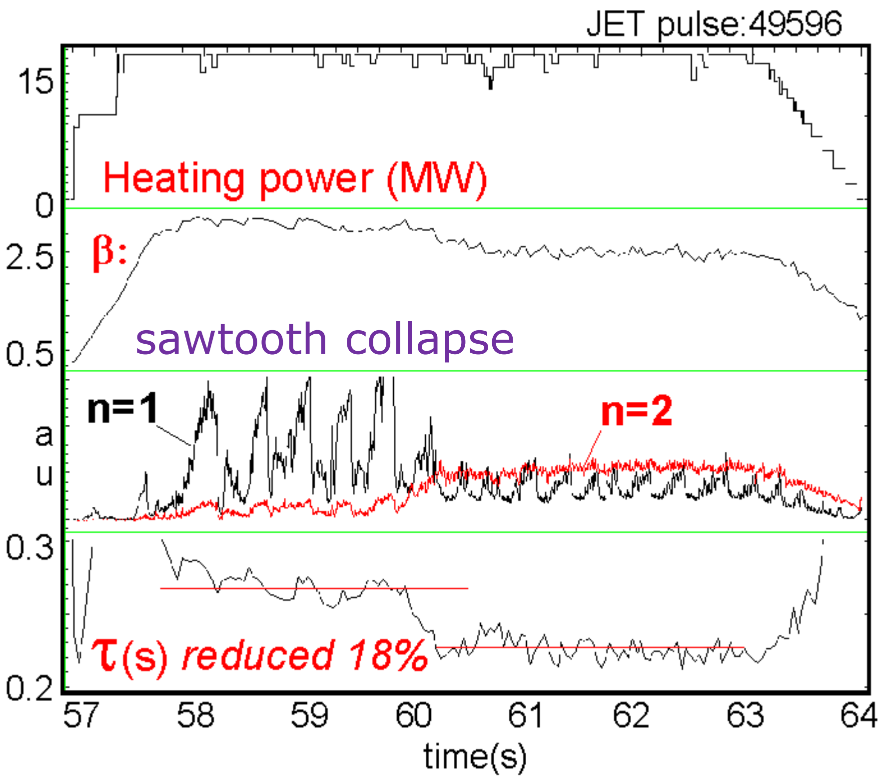
Pressure flattening due to island



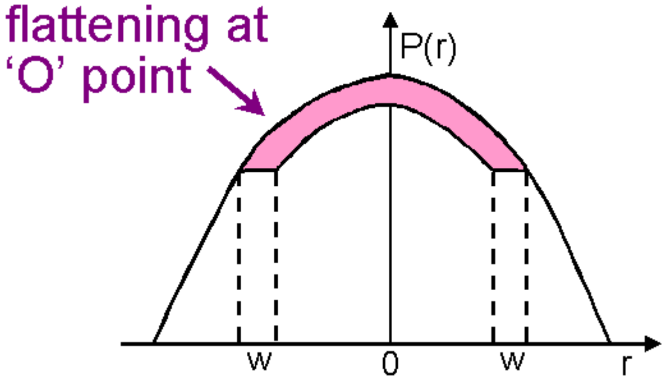
MAST (300 point Thomson scattering)



Neoclassical Tearing Mode (NTM)



- 3,2 NTM reduces β
- confinement reduced ~15-20%
- explained by 'Belt model':

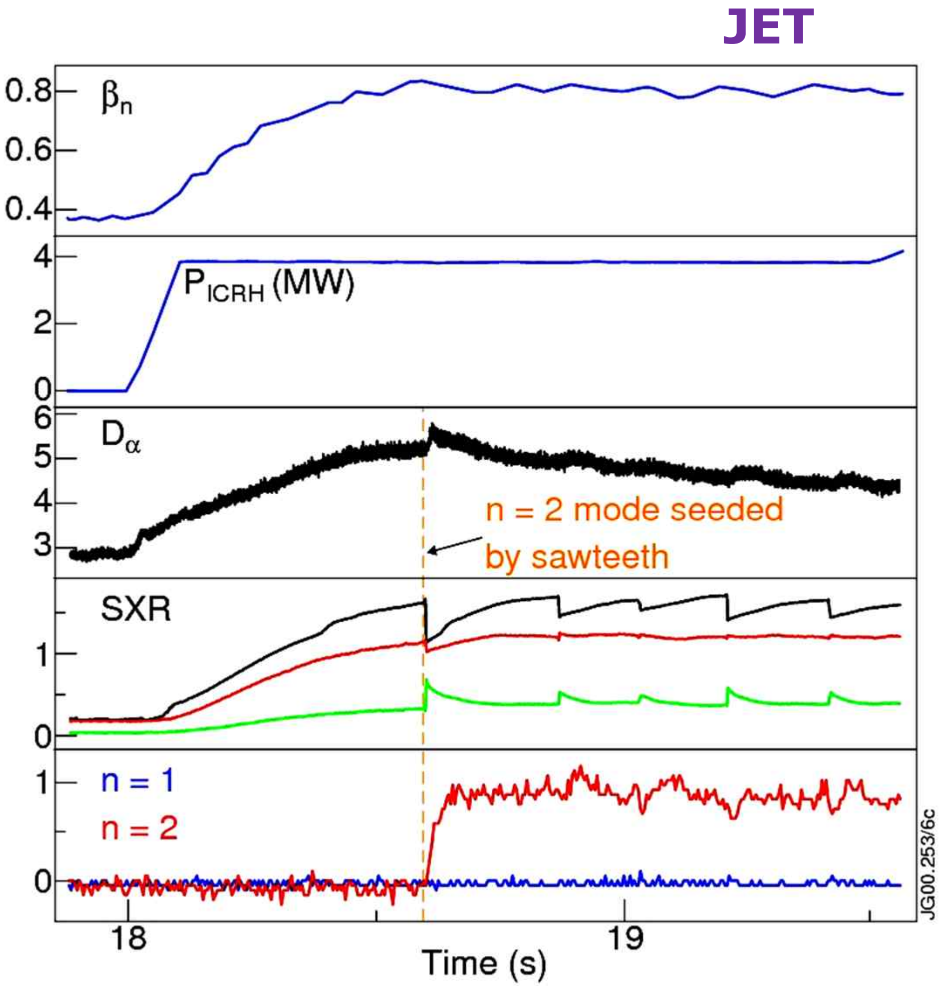


[Chang and Callen 1990]

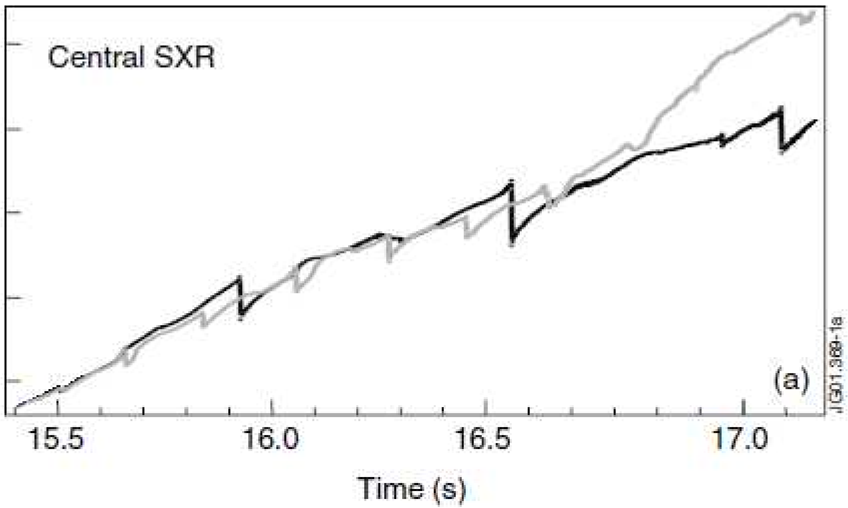
R. Buttery et al, Plasma Physics and Controlled Fusion (2000)

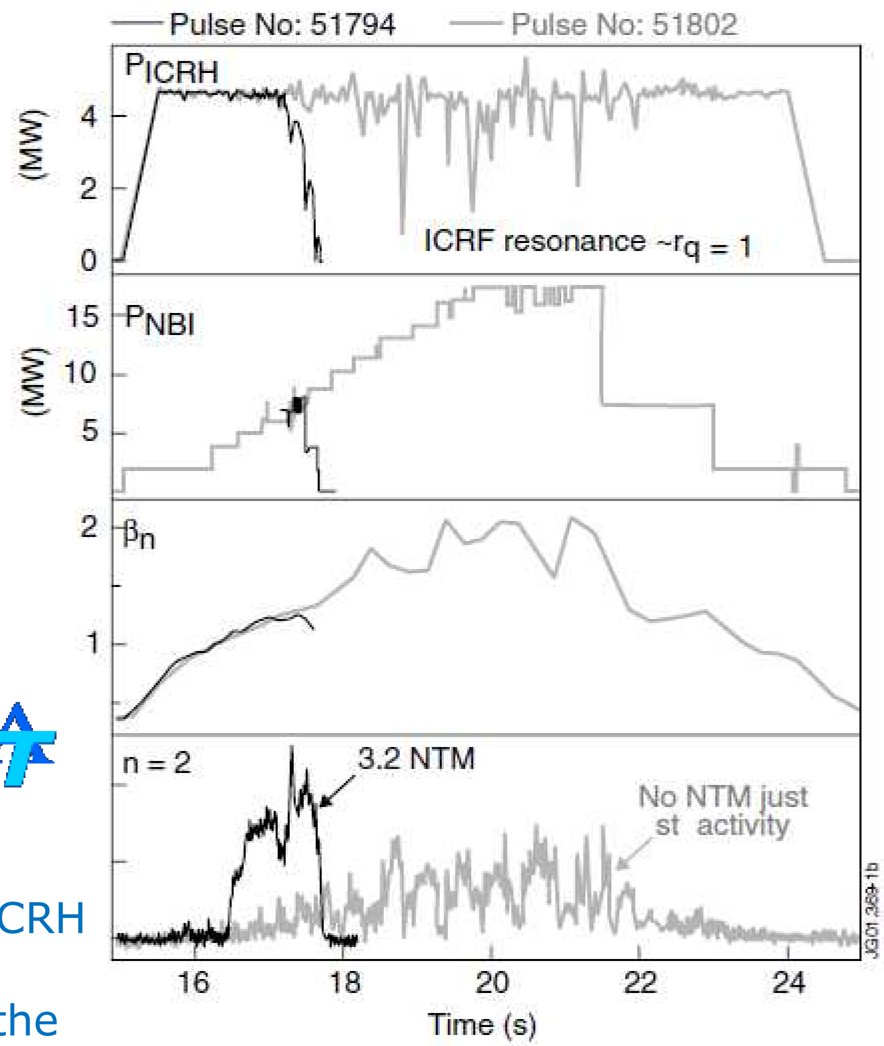
HW: What is the Belt model?

Neoclassical Tearing Mode (NTM)



Neoclassical Tearing Mode (NTM)

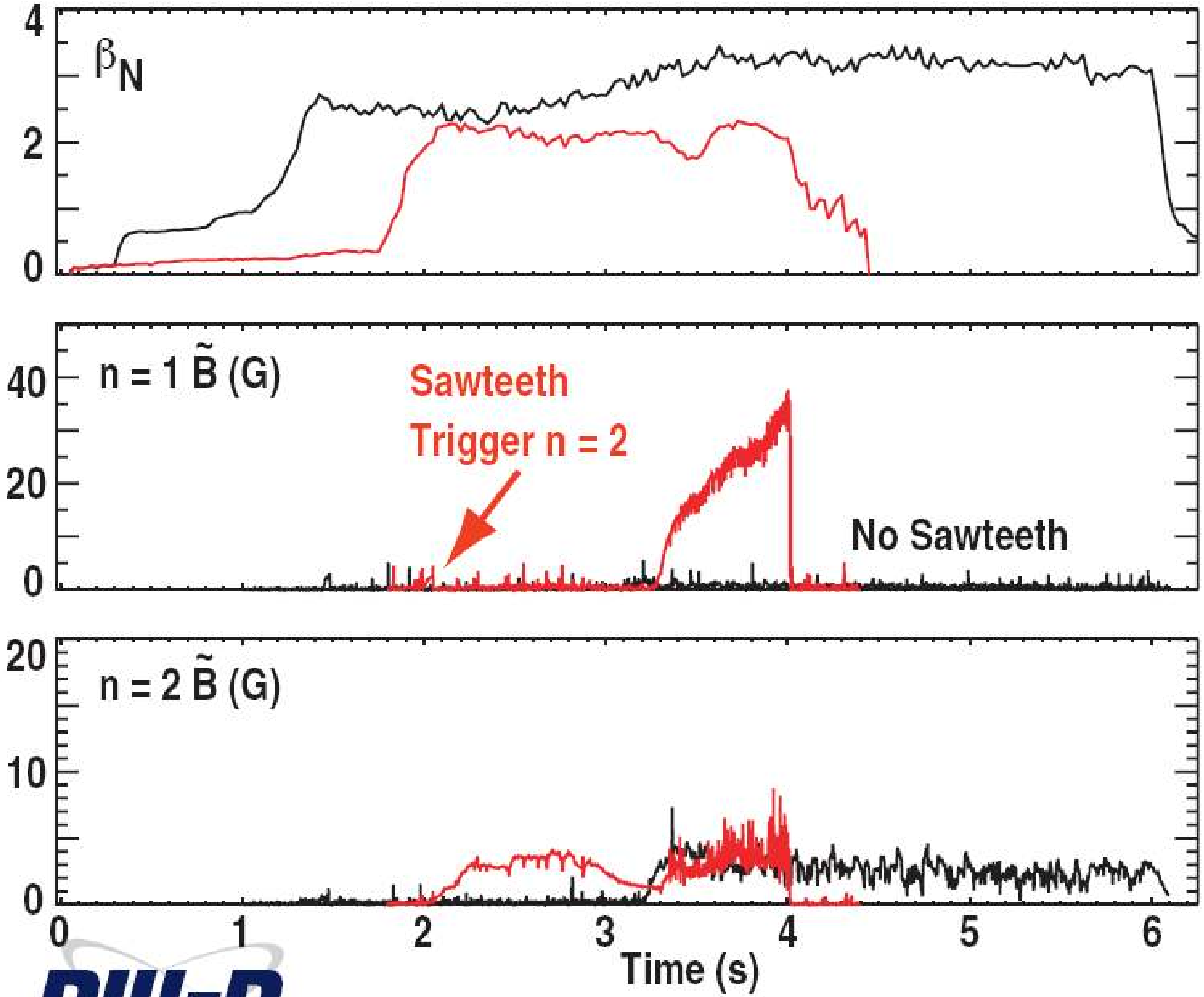




EFDA JET

- Increased sawtooth period due to stabilisation by fast ions produced by ICRH leads to the triggering of $n = 2$ NTM activity which causes a termination of the discharge.

Neoclassical Tearing Mode (NTM)



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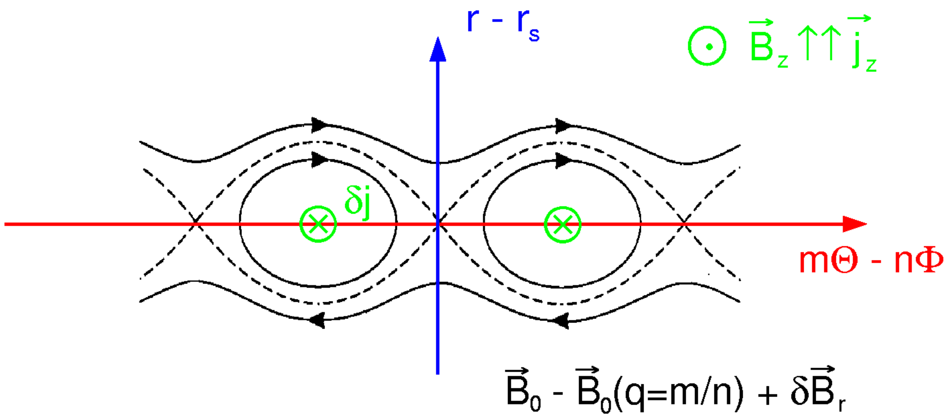
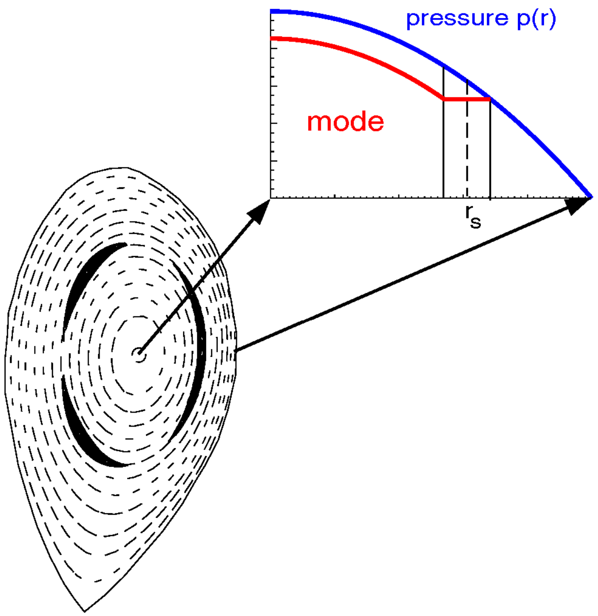
Neoclassical Tearing Mode (NTM)

- At high β_p, pressure gradient drives plasma current by thermo-electric effects (Bootstrap current):

$$j_{BS} \propto \nabla p$$

- Inside islands ∇p and thus j_{BS} vanish





- Loss of BS current inside magnetic islands (helical hole) acts as helical perturbation current driving the islands – so once seeded, island is sustained by lack of bootstrap current

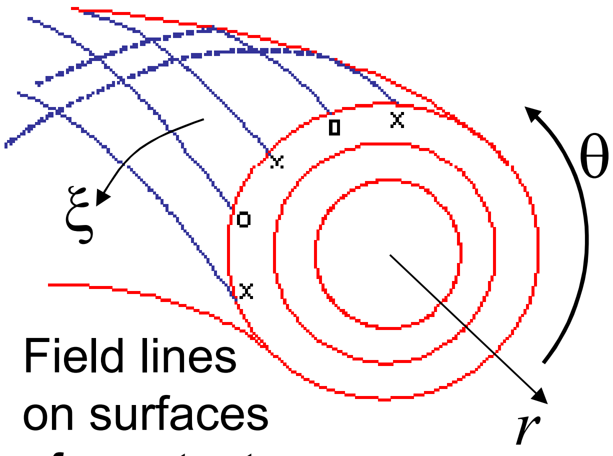
Background

- Tokamaks have good confinement because the flux surfaces lie on nested tori

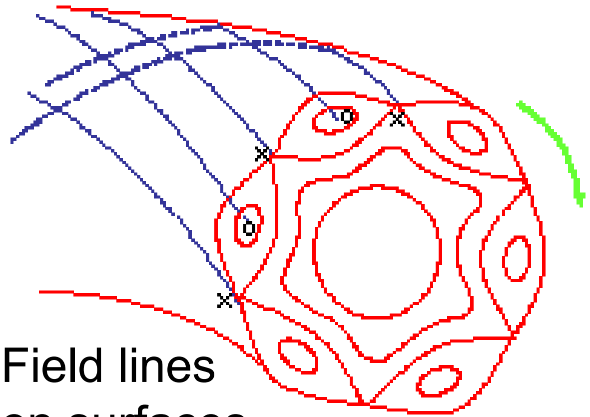
- If current flows preferentially along certain field lines, magnetic islands form

- The plasma is then ‘short-circuited’ across the island region

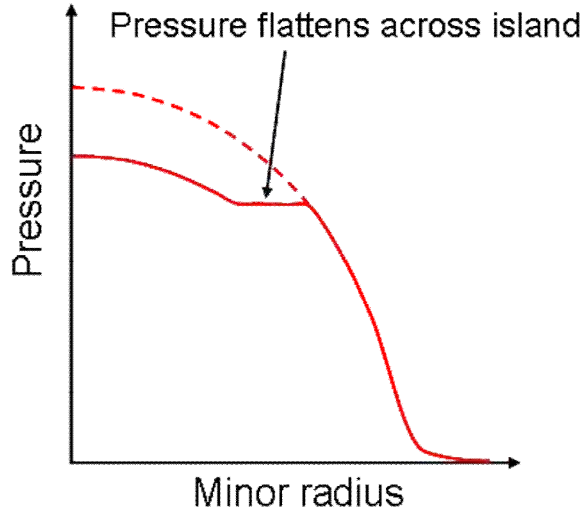
- As a result, the plasma pressure is flattened across the island region, and the confinement is degraded:



Field lines on surfaces of constant r



Field lines on surfaces of constant Ω

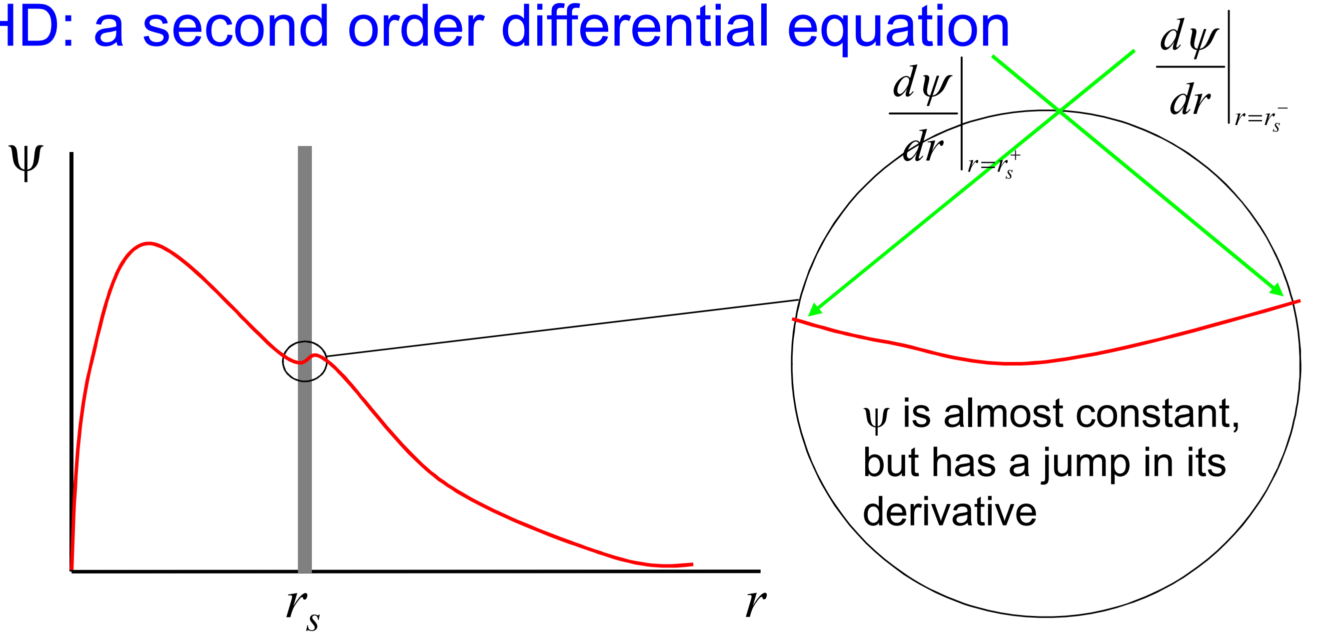


The concept of Δ'

- We begin by defining the perturbed flux:

$$\delta B = \nabla\phi \times \nabla\psi = B_r \sin m\xi \qquad \psi = \tilde{\psi}\cos m\xi \qquad B_r = \frac{m\tilde{\psi}}{rR}$$

- Away from the rational surface, ψ is determined by the equations of ideal MHD: a second order differential equation



– it predicts that ψ has a discontinuous derivative at $r=r_s$

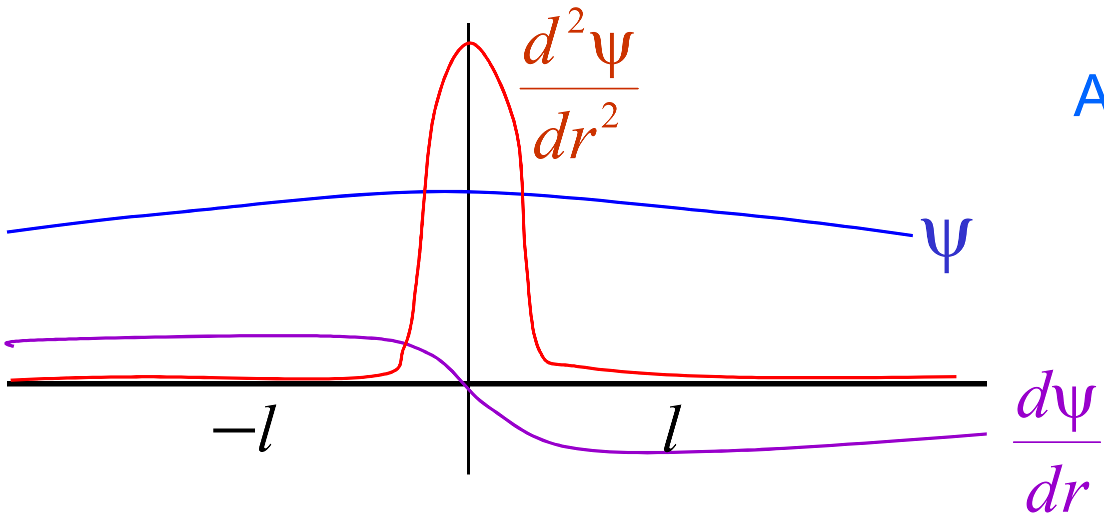
– this is conventionally parameterised by Δ':

$$\Delta' = \frac{1}{\psi}\left[\left.\frac{d\psi}{dr}\right|_{r=r_s^+} - \left.\frac{d\psi}{dr}\right|_{r=r_s^-}\right]$$

THE UNIVERSITY of York

Basic tearing mode equation

- The discontinuous derivative arises because of currents, localised around the rational surface, where ideal MHD breaks down



Ampere's law provides:

$$\frac{1}{R}\frac{d^2\psi}{dr^2} \approx \frac{\boldsymbol{B}\cdot(\nabla\times\delta\boldsymbol{B})}{B} = \mu_0 J_{\|}$$

- Integrate this over a period in ξ and out to a large distance, l, from the rational surface ($w \ll l \ll r_s$): basic tearing mode equation

$$\boxed{\Delta'\tilde{\psi} = 2\mu_0 R \int_{-\infty}^{\infty} dx \oint d\xi J_{\|} \cos m\xi} \qquad x = r - r_s$$

- The different models for non-linear tearing mode evolution employ different models for $J_{\|}$

Rutherford tearing mode equation

- Basic 'Rutherford' model: take a simple Ohm's law for $J_{||}$

$$\eta J_{||} = E_{||} = \frac{\partial \tilde{\psi}}{\partial t} \cos m\xi - \nabla_{||}\varphi$$

- In the absence of perpendicular drifts, perpendicular currents are zero, and so we have $\nabla \cdot \boldsymbol{J} = \nabla_{||} J_{||} = 0$, or $J_{||} = J_{||}(\Omega)$

- Thus, by averaging around flux surfaces <…>, we eliminate φ to derive

$$J_{||} = \frac{1}{\eta} \frac{\partial \tilde{\psi}}{\partial t} \left\langle \cos m\xi \right\rangle \qquad \Delta' \tilde{\psi} = 2\mu_0 R \int_{-\infty}^{\infty} dx \oint d\xi J_{||} \cos m\xi$$

- Relating ψ to the island width, w, we then arrive at Rutherford's eqn:

$$\boxed{0.82\tau_r \frac{dw}{dt} = r_s^2 \Delta'} \qquad \tau_r = \frac{\mu_0 r_s^2}{\eta} \qquad w = 2\left(\frac{q\tilde{\psi}}{RB_\theta dq/dr}\right)^{1/2}$$

Neoclassical tearing mode drive

- The bootstrap current in banana collisionality regime is approximately:

$$J_{bs} = -2.44 \frac{\sqrt{\varepsilon}}{B_\theta} \frac{dp}{dr}$$

- Recall that the pressure gradient is removed from inside the island:
  - there is a 'hole' in the bootstrap current around the island O-points
  - provides another contribution to $J_{||}$ perturbation to drive the tearing mode

- Using the above expression, we derive the neoclassical tearing mode equation:

$$\boxed{0.82 \frac{\tau_r}{r_s^2} \frac{dw}{dt} = \Delta' + a_2 \sqrt{\varepsilon} \frac{\beta_\theta}{w} \frac{L_q}{L_p}}$$

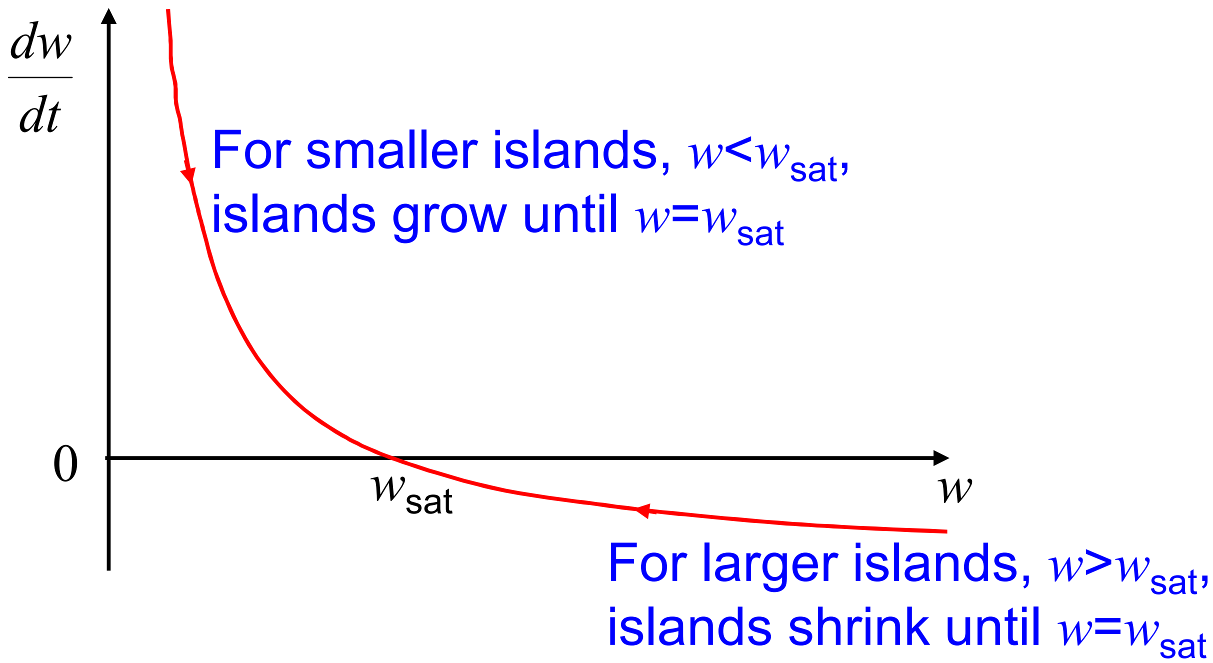
$$\beta_\theta = \frac{2\mu_0 p}{B_\theta^2} \qquad L_p^{-1} = -\frac{d \ln p}{dr} \qquad L_q^{-1} = \frac{d \ln q}{dr}$$

Neoclassical tearing mode: properties

$$0.82\frac{\tau_r}{r_s^2}\frac{dw}{dt}=\Delta'+a_2\sqrt{\varepsilon}\frac{\beta_\theta}{w}\frac{L_q}{L_p}$$

- For typical tokamak profiles bootstrap contribution *drives* island growth

- When Δ'<0 (Rutherford stable), there exists a stable, saturated island width solution:



Saturated island width

- The saturated island width is: $$\frac{w_{\text{sat}}}{r_s} = a_2\sqrt{\varepsilon}\,\frac{\beta_\theta}{(-r_s\Delta')}\frac{L_q}{L_p}$$
- The saturated island width increases with β_θ
  - as the confinement will deteriorate with increasing island size, this sets an effective β-limit in tokamaks
  - the saturated island width can become a large fraction of the plasma radius, and this can lead to disruption
- As Δ' becomes more negative for increasing poloidal mode number, m, it is the lowest m modes which are most dangerous.
- Nevertheless, the above model predicts magnetic islands at all rational surfaces:
  - so why does the tokamak work?
  - additional “threshold” physics is important at small island width

Threshold effects: small island width physics

- For sufficiently small island widths, the pressure is not completely flattened inside the island

  $\Longrightarrow$ the bootstrap current drive is not so effective for small islands

  $\Longrightarrow$ we refer to this as 'finite radial diffusion effects'

- The expression for the bootstrap current is based on an expansion in the ratio of the banana width to the equilibrium length scales

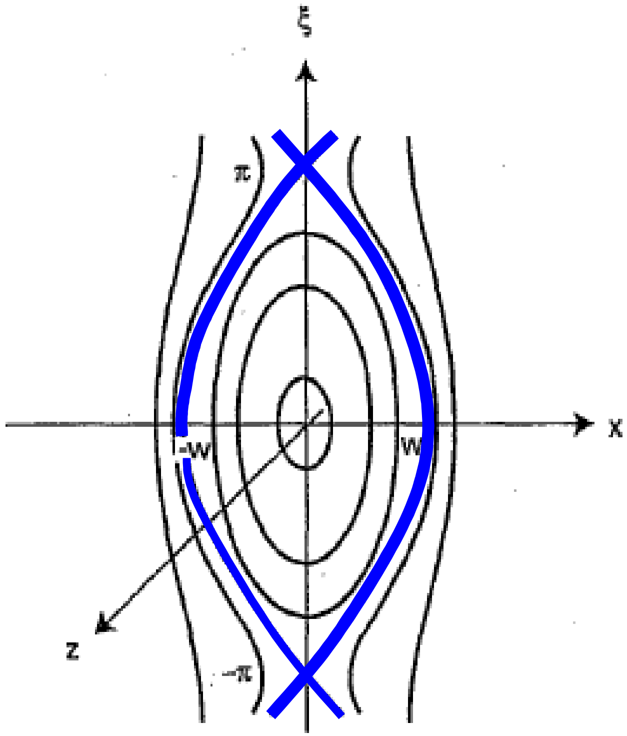
  $\Longrightarrow$ the theory must therefore be questioned for islands with a width comparable to the ion banana width

  $\Longrightarrow$ we refer to this as 'finite orbit width effects'

Fitzpatrick Model for Transport Threshold

- The connection length L_c is the distance along a field line from one side of the island to the other - i.e. the route for the enhanced transport that flattens the temperature. $L_c \sim 1/w$ so the **enhanced transport is reduced for small islands**.

- When w is close to a critical width w_c, both the flattening and hence the bootstrap drive are reduced, **giving rise to a threshold**.



Finite radial diffusion: (Kieran will discuss more detail)

- For a simple illustration, consider diffusive electron heat fluxes parallel and perpendicular to field lines: $Q_{\|}=-\chi_{\|}\nabla_{\|}T \qquad Q_{\perp}=-\chi_{\perp}\nabla_{\perp}T$

- In the absence of heat sources $\nabla\cdot\mathbf{Q}=0$, so that

$$\chi_{\|}\nabla_{\|}^{2}T+\chi_{\perp}\nabla_{\perp}^{2}T=0$$

- Now $\chi_{\|}\gg\chi_{\perp}$
  - $\Rightarrow$ generally radial diffusion can be ignored
  - $\Rightarrow$ $\nabla_{\|}T=0$, so that the temperature is flattened across the island

- However, the gradient operators depend on island size:
  - $\nabla_{\perp}\sim\partial/\partial r\sim 1/w$
  - $\nabla_{\|}=(\mathbf{B}\cdot\nabla)/B\sim mw/(RqL_q)$

- Balancing terms $\Rightarrow$ radial diffusion is important for $w<w_{\chi}$, where

$$w_{\chi}=\sqrt{\frac{RqL_q}{m}}\left(\frac{\chi_{\perp}}{\chi_{\|}}\right)^{1/4}$$

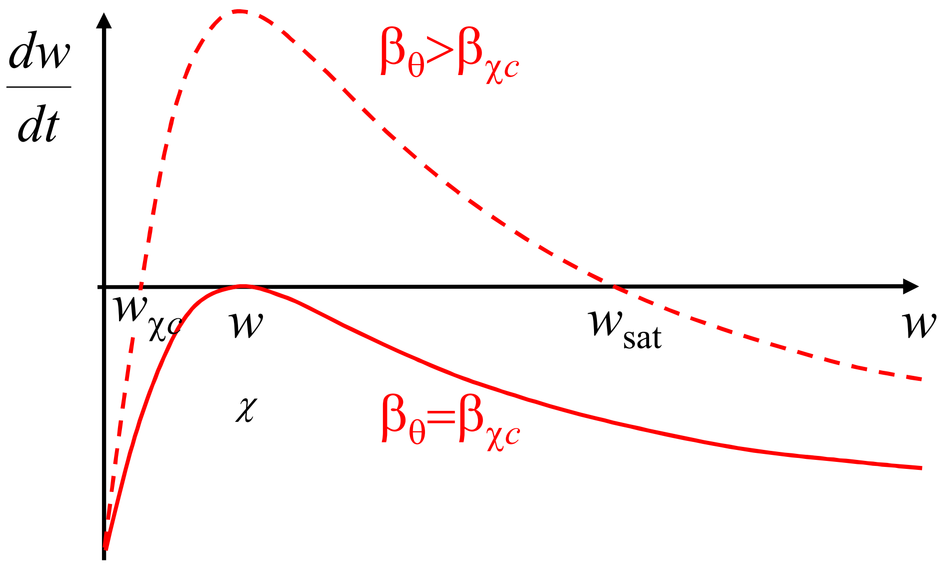
the island width for which perpendicular and parallel transport are equal.

- Needs much more care for ion thermal transport and particle transport

Finite radial diffusion: Threshold

- Thus, for sufficiently small islands, $w<w_\chi$, the temperature is not flattened across the island, and the bootstrap drive is weakened:

$$0.82\frac{\tau_r}{r_s^2}\frac{dw}{dt}=\Delta'+a_2\sqrt{\varepsilon}\frac{\beta_\theta}{w}\frac{L_q}{L_p}\left(\frac{w^2}{w^2+w_\chi^2}\right)$$



$$\boxed{\beta_{\chi c}=-\frac{2\Delta' w_\chi}{a_2\sqrt{\varepsilon}}\frac{L_p}{L_q} \qquad w_{\chi c}\approx\frac{w_\chi\beta_{\chi c}}{2\beta_\theta}}$$

- For $\beta_\theta<\beta_{\chi c}$, $dw/dt<0$ for all $w \Rightarrow$ all islands decay away

- For $\beta_\theta>\beta_{\chi c}$, an additional, unstable, root for $dw/dt=0$ at $w=w_{\chi c}$

  $\Rightarrow$ an island will only grow to its saturated state provided

$$\boxed{w>w_{\chi c} \qquad \text{AND} \qquad \beta_\theta>\beta_{\chi c}}$$

  $\Rightarrow$ a ‘seed’ island is required

Finite orbit width effects

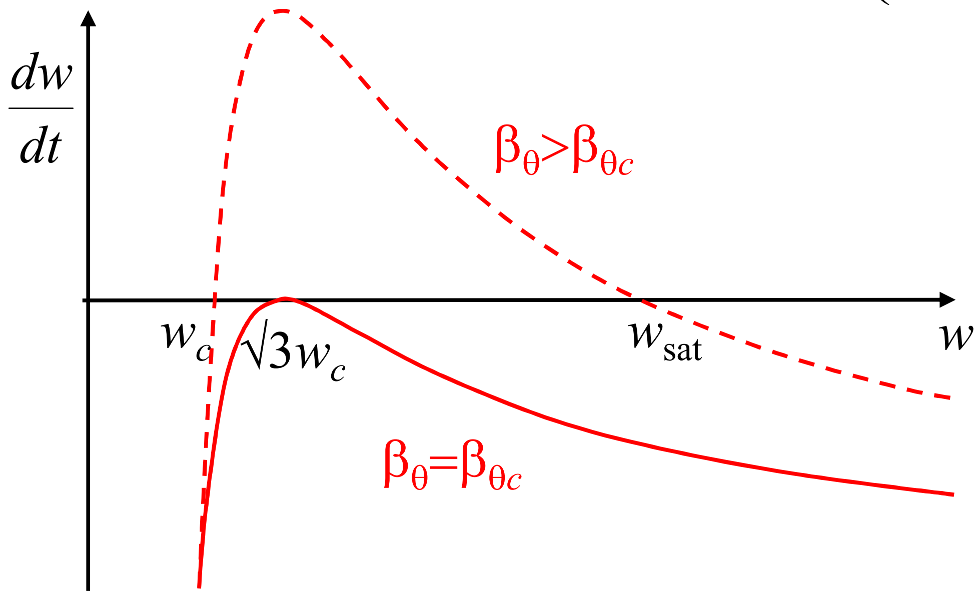
- For small islands of width comparable to the ion banana width, ions and electrons respond differently to the island:
  - an electrostatic potential is required to maintain quasi-neutrality in the vicinity of the island
  - ions and electrons experience $\mathbf{E}\times\mathbf{B}$ drifts
  - the ions experience an orbit averaged drift, which differs from the local drift experienced by the electrons for island width~ion banana width
  - a perpendicular current is generated; this is the polarisation current
  - the polarisation current is not divergence-free
  - sets up an electric field to drive a current parallel to field lines
  - this current can influence island evolution

- The theory is still under development
- Consider island width much greater that the ion banana width
  - led to the inclusion of the so-called ‘polarisation term’ in the modified Rutherford equation

Polarisation term

- Allowing for the polarisation term, the modified Rutherford equation is:

$$0.82\frac{\tau_r}{r_s^2}\frac{dw}{dt}=\Delta'+a_2\sqrt{\varepsilon}\frac{\beta_\theta}{w}\frac{L_q}{L_p}\left(\frac{w^2}{w^2+w_\chi^2}\right)-a_3 g(\varepsilon,\nu_i)\left(\frac{\rho_{bi}}{w}\right)^2\left(\frac{L_q}{L_p}\right)^2\frac{\beta_\theta}{w}$$



$$g(\varepsilon,\nu_i)=\begin{cases}1.64\varepsilon^{1/2} & \nu_i/\varepsilon\omega<<1\\ \varepsilon^{-1} & \nu_i/\varepsilon\omega>>1\end{cases}$$

Note:

$$w_c=\sqrt{\frac{a_3}{3a_2}\frac{\sqrt{g(\varepsilon,\nu_i)}}{\varepsilon^{1/4}}}\sqrt{\frac{L_p}{L_q}}\rho_{bi} \qquad \beta_{\theta c}=\frac{3\sqrt{3}}{2a_2\sqrt{\varepsilon}}\frac{L_p}{L_q}(-w_c\Delta')\propto\rho_*$$

- In general, the full story is more complicated
- The transport and polarisation terms interact (especially ion thermal and particle transport)

The challenge

- Both the transport model and the polarisation current provide a threshold island width comparable to the ion banana width:
  - Kinetic theory with full ion banana widths is essential
- This provides a rich, essentially unexplored vein of physics
- Gyrokinetic models are being developed to address this issue.

Tearing Modes – nonlinear growth

Consider various helical currents on resonant surface...

$$\boxed{B_\theta(r_s^+) - B_\theta(r_s^-) \propto \delta I = I_{Ohm} + I_{bs} + I_{extern}}$$

$$I_{Ohm} \propto j_{Ohm} W \propto \sigma W \, {}^{d\psi}\!/_{dt} \propto \sigma W^2 \, {}^{dW}\!/_{dt}$$ inductive

$$I_{bs} \propto j_{bs} W \propto -{}^{\nabla p}\!/_{B_\theta} \, W$$ pressure driven

$$I_{extern}$$ externally driven

...leads to the so-called Rutherford equation

$$\boxed{\tau_{res} \, {}^{dW}\!/_{dt} = a_1 \Delta' + a_2 \, {}^{\nabla p}\!/_{W} - a_3 \, {}^{I_{extern}}\!/_{W^2}}$$

where $\Delta' = (B_\theta(r_s^+) - B_\theta(r_s^-)) / \psi$

Interpretation of the different terms

$$\tau_{res} \, {}^{dW}\!/_{dt} = a_1 \Delta' + a_2 \, {}^{\nabla p}\!/_{W} - a_3 \, {}^{I_{extern}}\!/_{W^2}$$

- for small ∇p, current gradient (Δ') dominates

  ⇒ 'classical Tearing Mode', current driven

- for larger ∇p, pressure gradient dominates:

  ⇒ 'neoclassical Tearing Mode', pressure driven

- adding an externally driven helical current can stabilise

Neoclassical Tearing Mode (NTM)

- **The Modified Rutherford Equation (MRE)**

$$\frac{\tau_R}{r_s}\frac{d\omega}{dt} = \Delta'_0 r_s + \delta\Delta' r_s + a_2 \frac{j_{bs}}{j_\parallel}\frac{L_q}{\omega}\left[1 - \frac{\omega_{m\arg}^2}{3\omega^2} - K_1 \frac{j_{ec}}{j_{bs}}\right]$$

HW: derive Δ′

$$\Delta' \equiv \frac{1}{B_r}\left[\frac{\partial B_r}{\partial r}\right]_{r=r_s}$$

1st : Conventional tearing mode stability: assumed as $\Delta'_0 r_s \approx -m$ for m/n NTM

2nd: Tearing mode stab. enhancement by ECCD: Westerhof's model with no-island assumption

$\delta\Delta' r_s \approx -\frac{5\pi^{3/2}}{32} a_2 \frac{L_q}{\delta_{ec}} F(e) \frac{j_{ec}}{j_\parallel}\tau$, where the misalignment function $F(e) = 1 - 2.43e + 1.40e^2 - 0.23e^3$

3rd: Destabilization from perturbed bootstrap current:

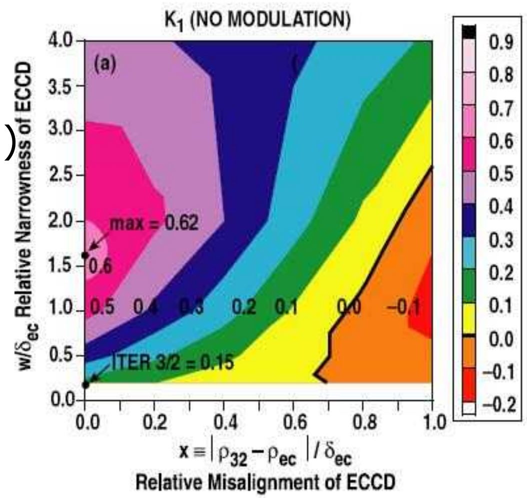
a_2 fitted by inferred size of saturated NTM island (e.g. ISLAND)

4th: Stabilization from small island & polarization threshold:

$w_{marg} \approx 2\varepsilon^{1/2}\rho_{\theta i}$ (= twice ion banana width)

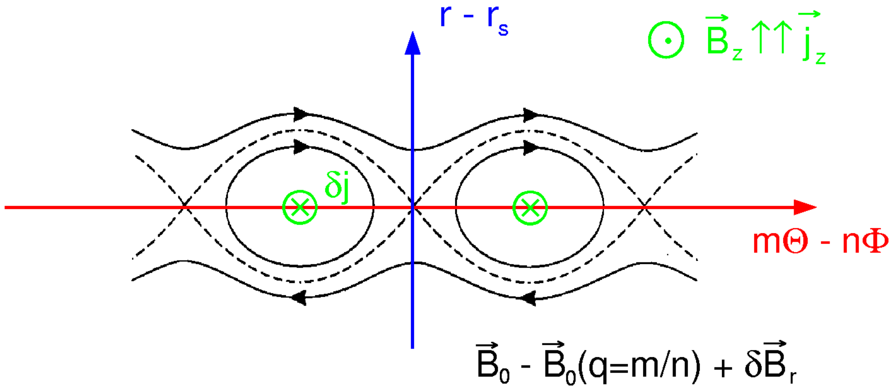
5th: Stabilization from replacing bootstrap current by ECCD:

K_1 calculated from improved Perkins' current drive model

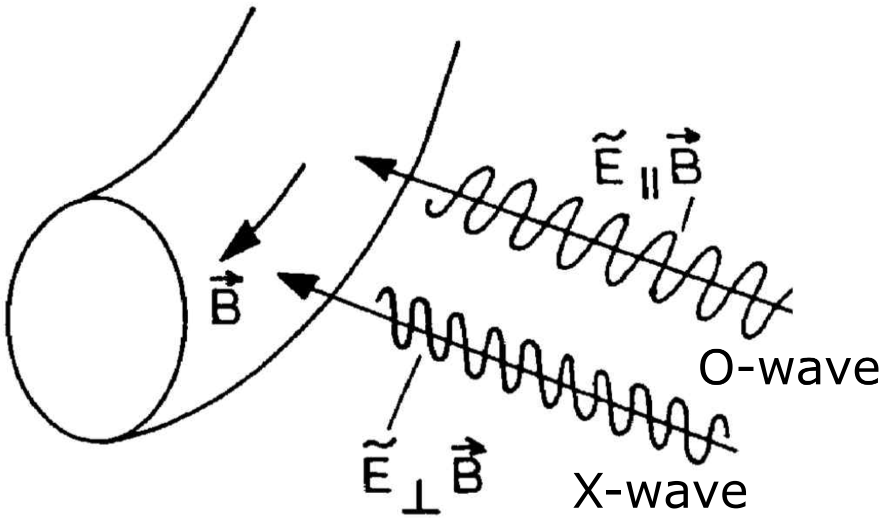


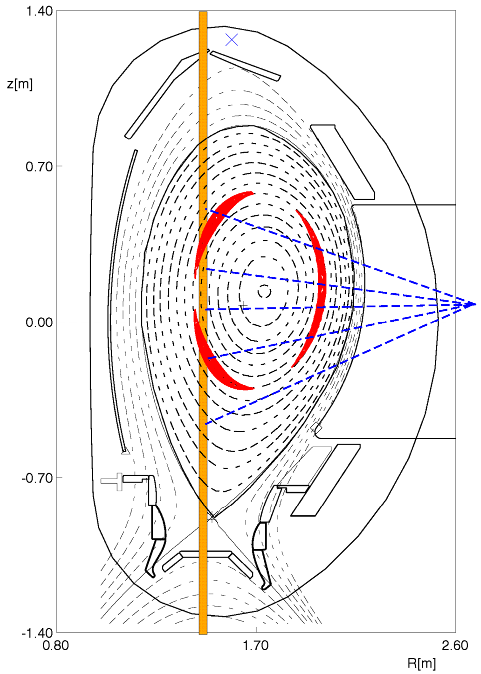
R. J. La Haye et al, Nuclear Fusion ***46*** *451 (2006)*

Neoclassical Tearing Mode (NTM)



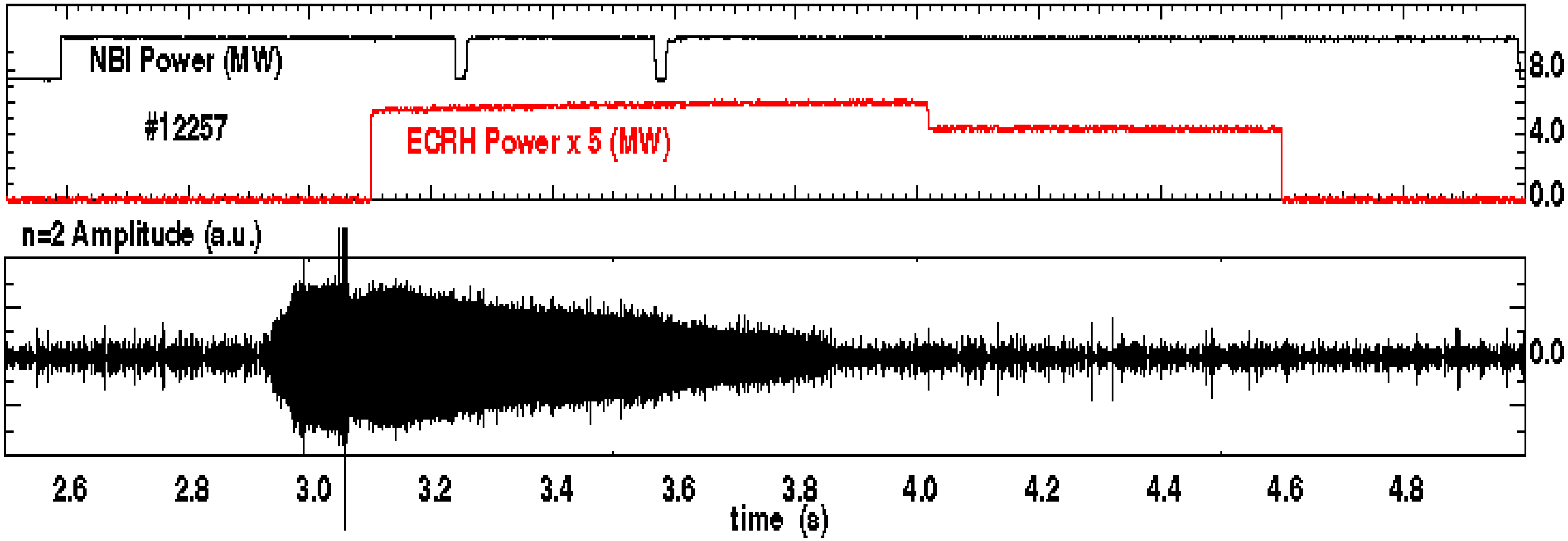
- Missing bootstrap current inside island can be replaced by localised external current drive.





Helical current can be driven by electron cyclotron resonance waves
Deposition controlled by local B-field ⇒ very good localisation
Feedback control of position possible via launch angle of ECCD beam

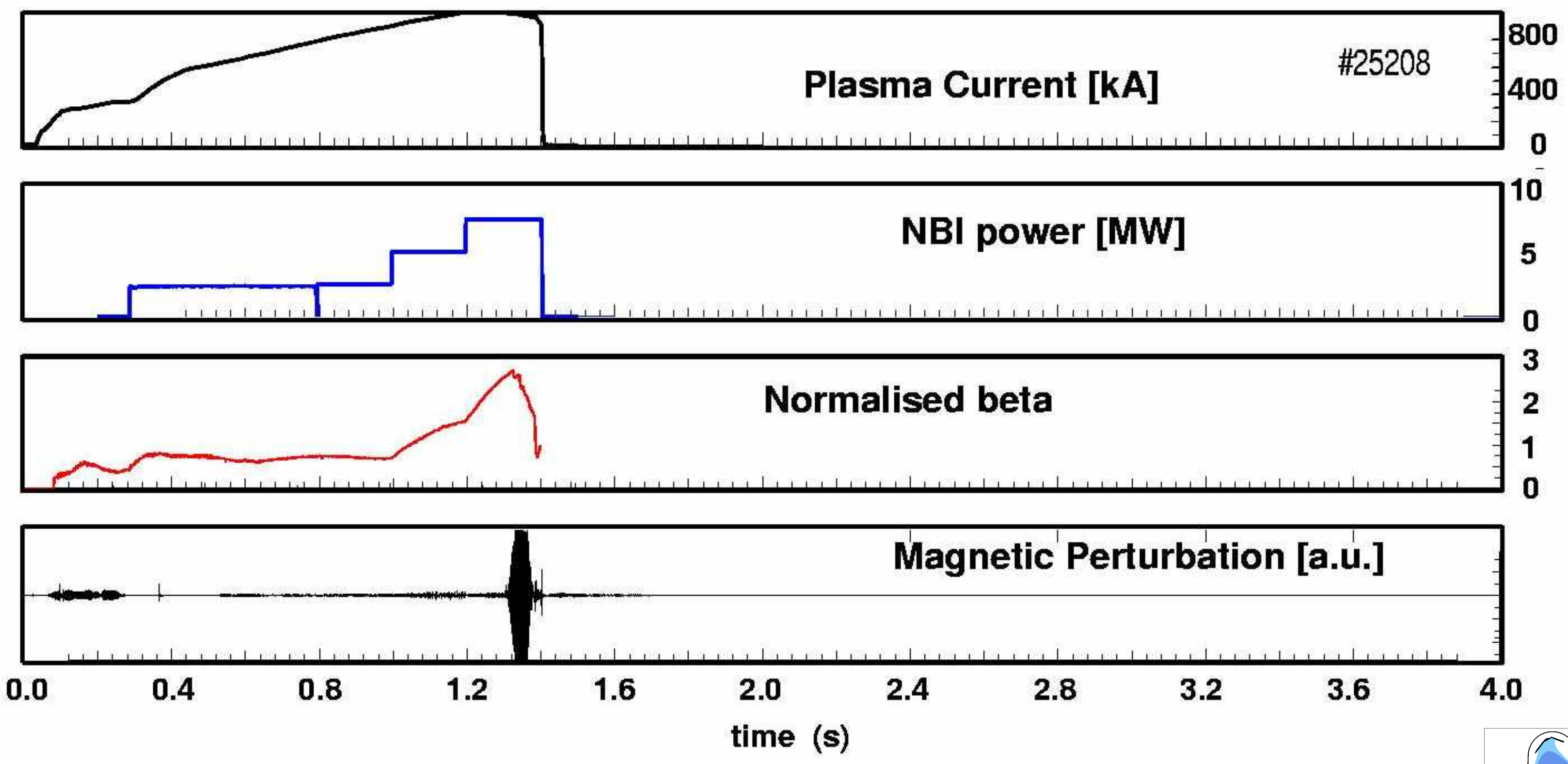
Neoclassical Tearing Mode (NTM)



- Complete stabilisation by searching the position of the magnetic island by scanning magnetic field in quantitative agreement with theory!

*1st Paper: G. Gantenbein et al, PRL **85** 1242 (2000)*

Neoclassical Tearing Mode (NTM)



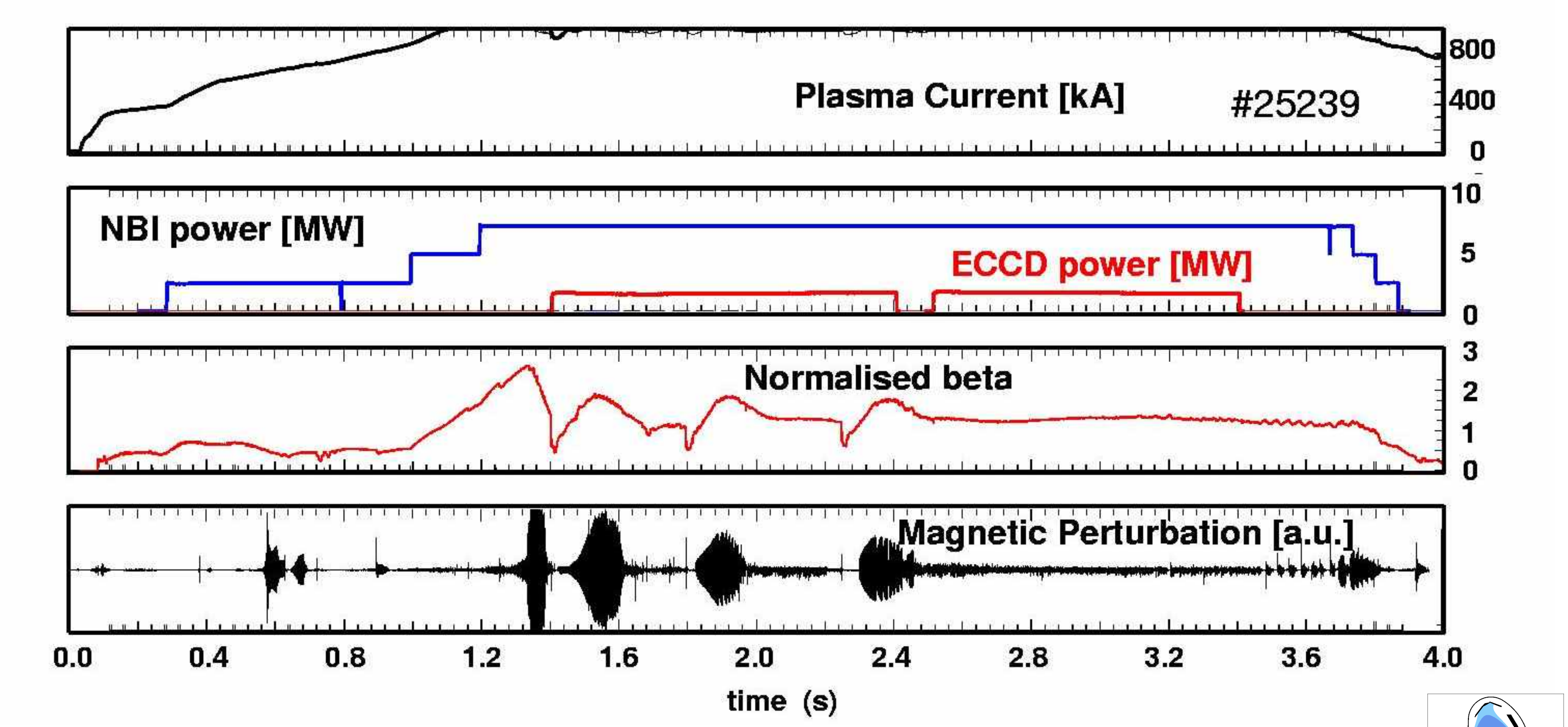
ASDEX Upgrade

- Disruption avoidance by ECRH

Target: a discharge that disrupts due to an early (2,1) NTM (q = 3.9, b_N=2.6)

B. Esposito et al, Nucl. Fusion (2011)

Neoclassical Tearing Mode (NTM)

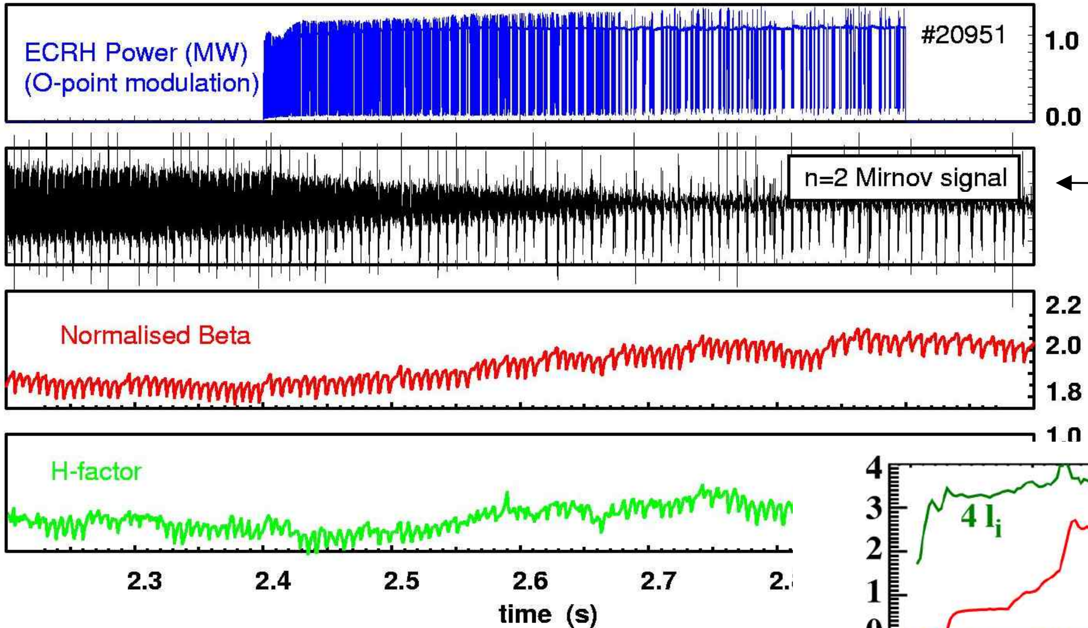


- Disruption avoidance by ECRH

1.5 MW of ECCD sufficient to avoid disruption, prepare safe landing
note: discharge never recovers performance – need to develop strategy
analysis of ‘scalability’ ongoing

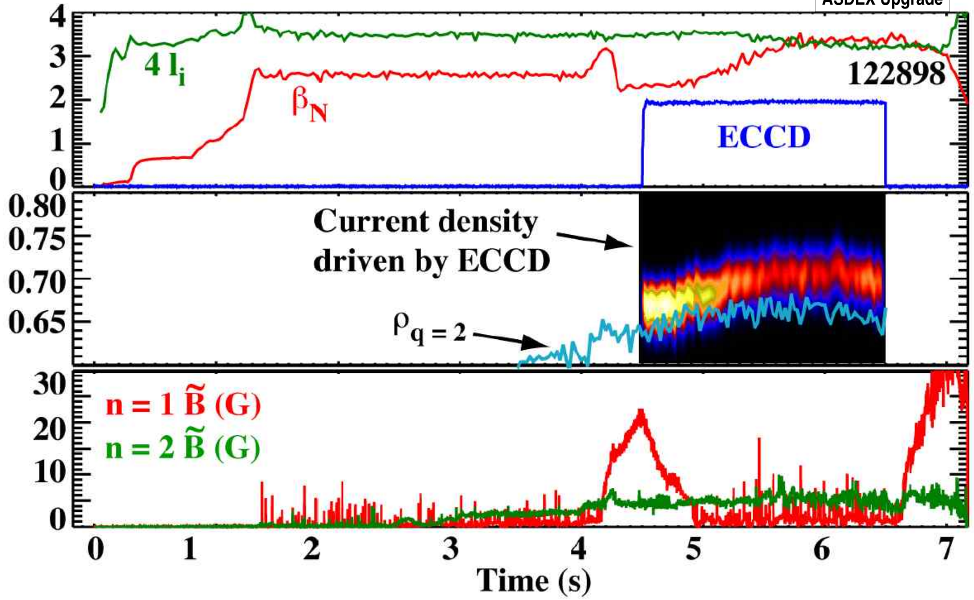
B. Esposito et al, Nucl. Fusion (2011)

NTM Stabilisation by ECCD



NTM stabilisation with ITER relevant broad deposition in ASDEX Upgrade

Feedback controlled Deposition in DIII-D

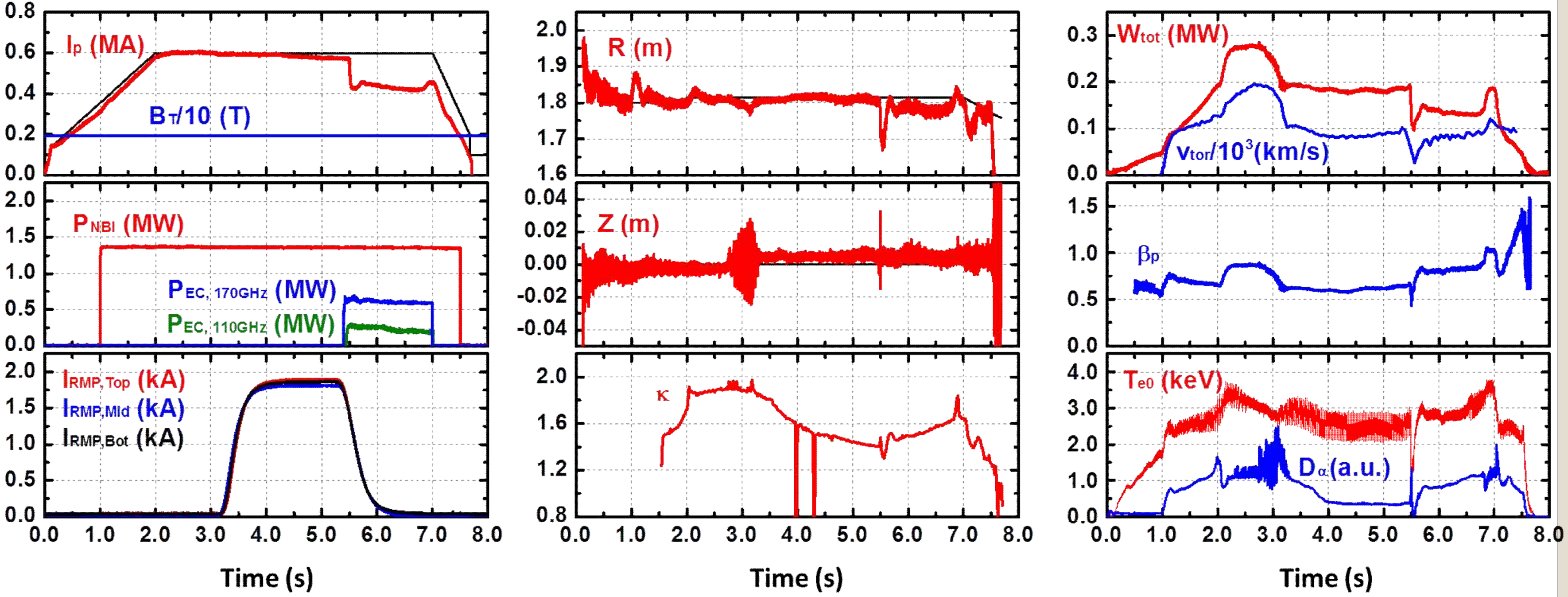


- Demonstration of individual elements as well as integrated feedback

NTM Stabilisation by ECCD

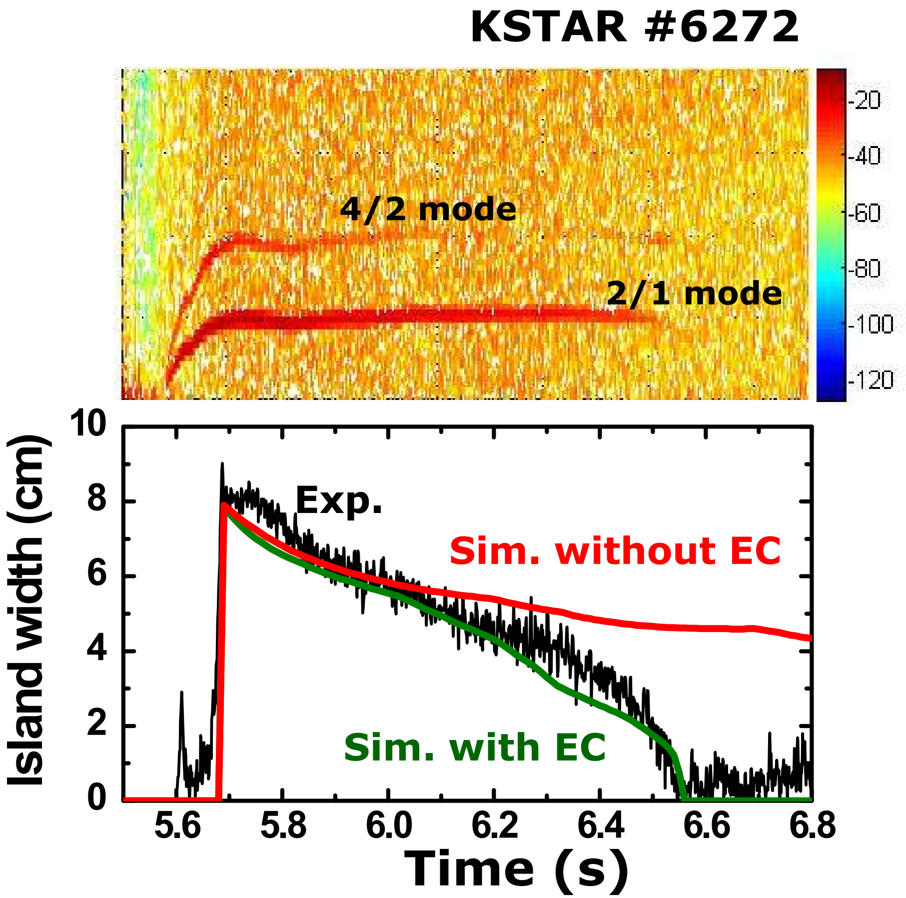
- **KSTAR**

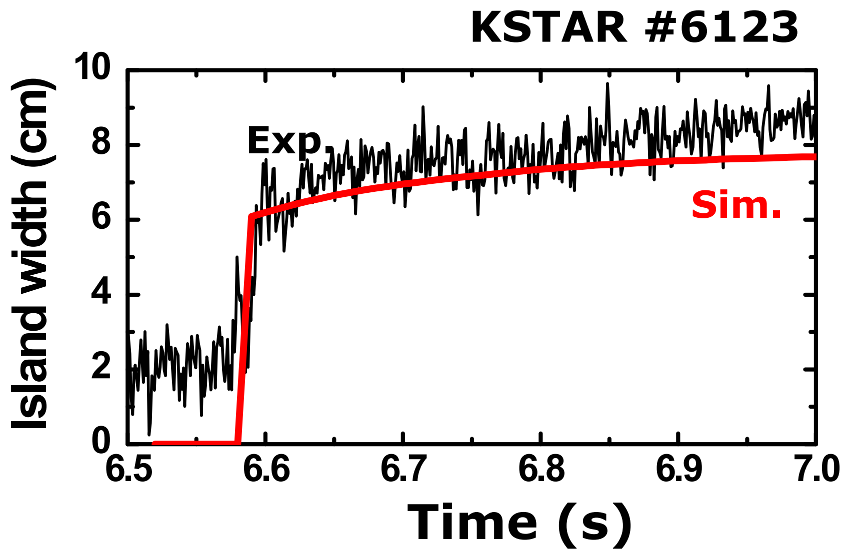
#6272



NTM Stabilisation by ECCD

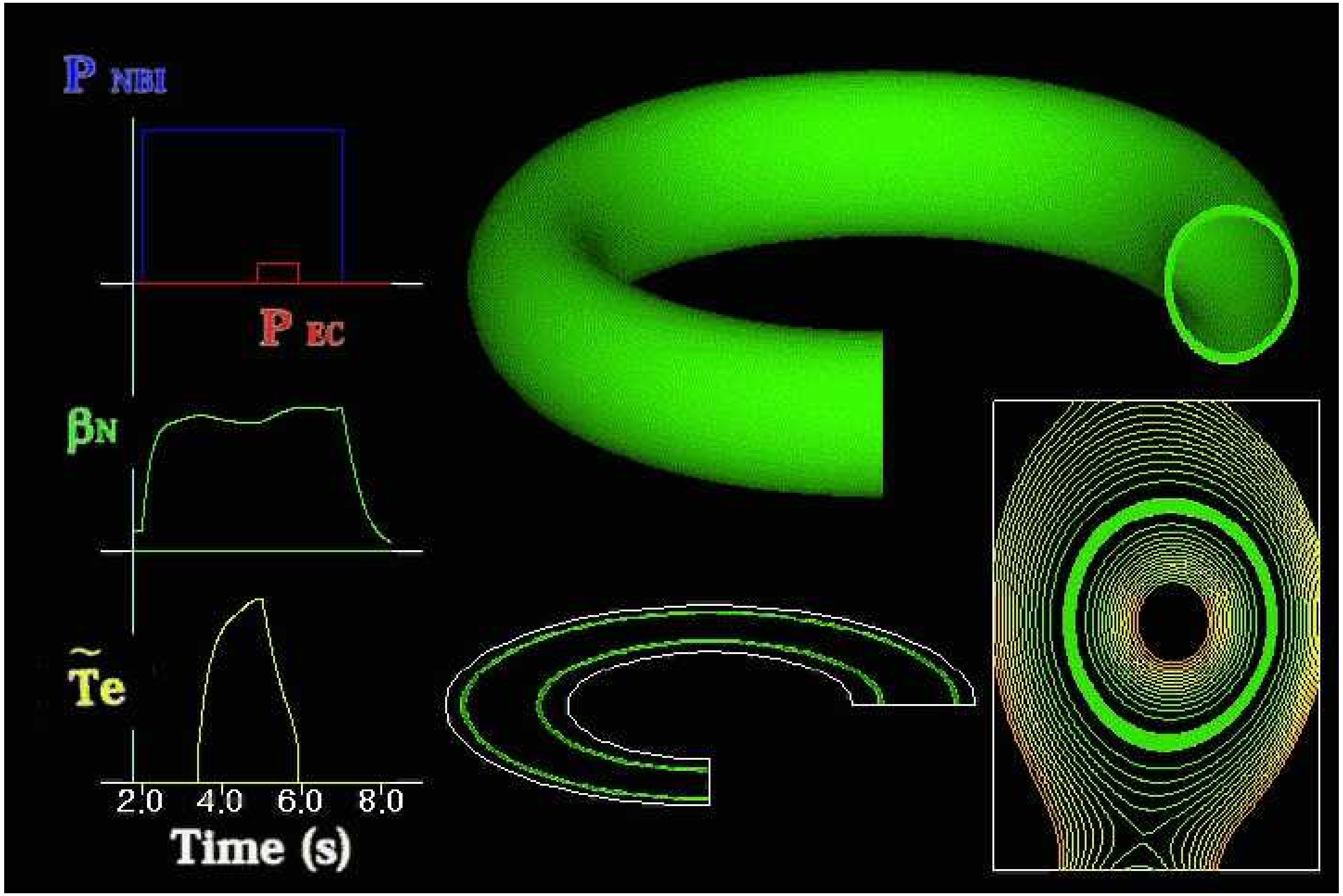
- **KSTAR**



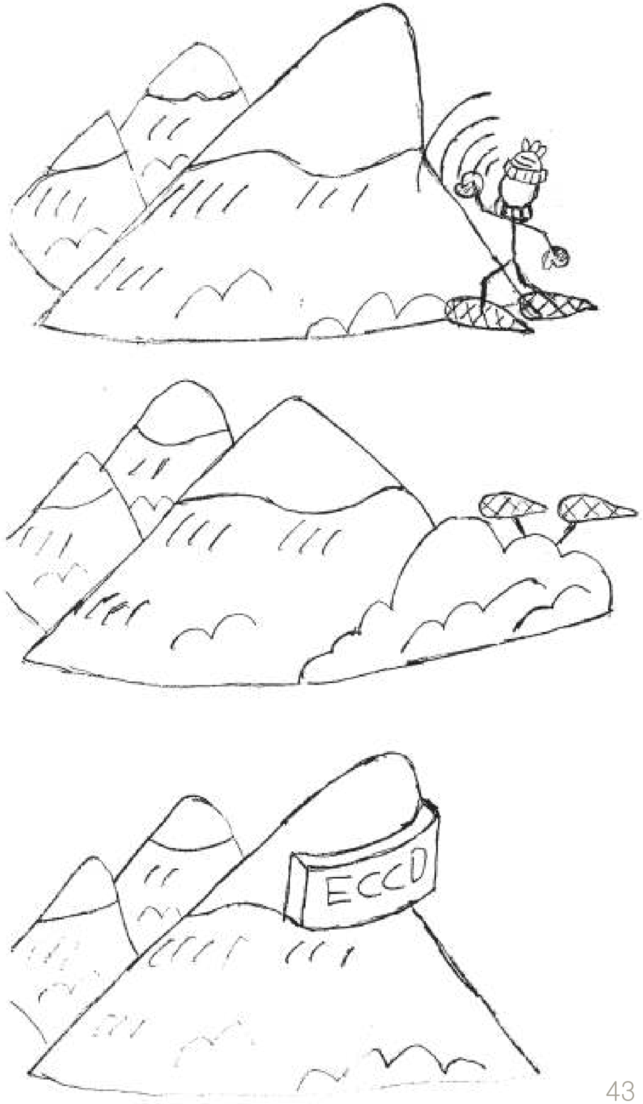


NTM Stabilisation by ECCD

- **JT-60U**



NTM Stabilisation by ECCD



Courtesy from R. J. La Haye, APS (2005)

References

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