Fusion Reactor Technology I
(459.760, 3 Credits)

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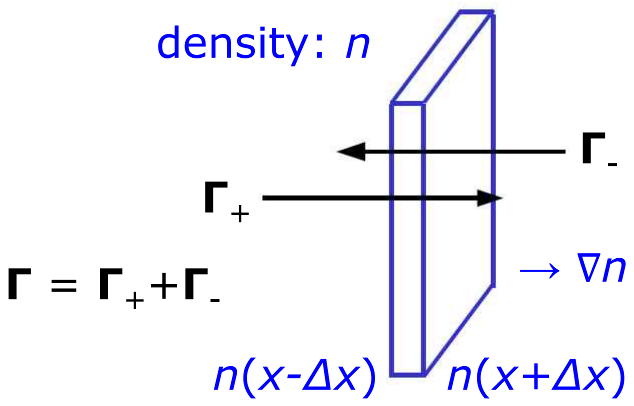
Plasma Transport

- **Classical Transport**
  - Particle transport



$$\Gamma_+ = \frac{1}{4} n(x-\Delta x) v$$

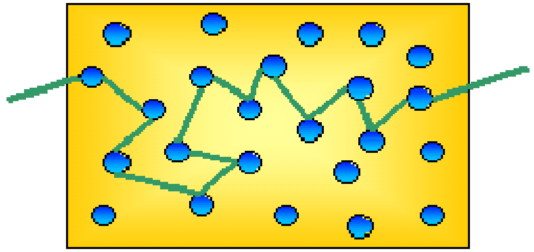
$$\vec{\Gamma}_- = \frac{1}{4} n(x+\Delta x) \vec{v}$$

$$\vec{\Gamma} = \vec{\Gamma}_+ - \vec{\Gamma}_- = \frac{1}{4}\left[n(x-\Delta x) - n(x+\Delta x)\right]\vec{v}$$

$$n(x \pm \Delta x) = n(x) \pm \Delta x \frac{\partial n}{\partial x}$$

$$= -\frac{\Delta x \vec{v}}{2}\frac{\partial n}{\partial x} = -\frac{(\Delta x)^2}{2\tau}\frac{\partial n}{\partial x}$$

$$= -D\frac{\partial n}{\partial x}$$: Particle flux- Fick's law



$$D = \frac{(\Delta x)^2}{2\tau}$$: diffusion coefficient (m^2/s)

The heat and momentum fluxes can be estimated in similar fashion.

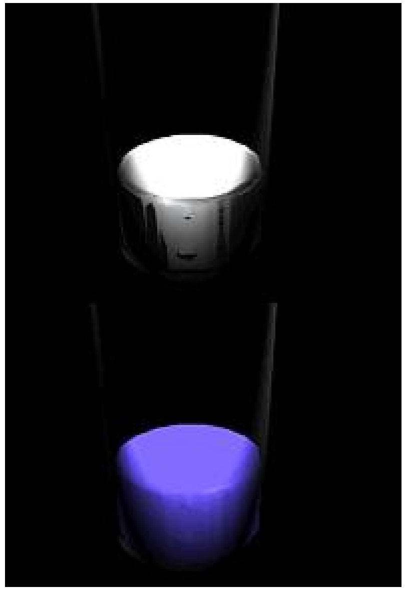
Plasma Transport

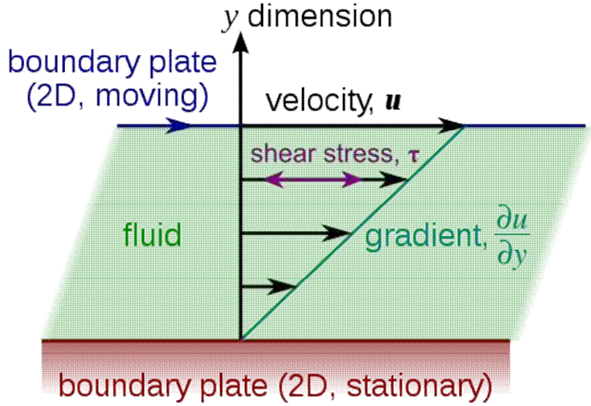
- **Classical Diffusion**
  - Momentum transport

Momentum flux

$$\pi_{\alpha\beta} = -\eta \frac{\partial v_y}{\partial x}$$

$$\eta \sim \frac{mn(\Delta x)^2}{\tau} \sim mnD \quad \text{: viscosity coefficient}$$





$$F = \mu A \frac{u}{y}, \quad \tau = \mu \frac{du_x}{dy}$$

  - Heat transport

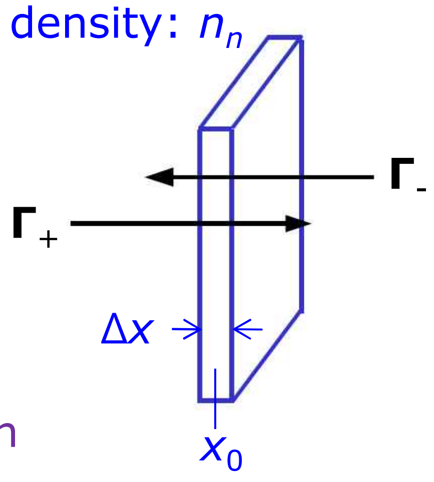
Heat flux

$$q = -\kappa \frac{\partial T}{\partial x} \quad \text{: Fourier's law}$$

$$\kappa \sim \frac{n(\Delta x)^2}{\tau} \sim nD \quad \text{: thermal conductivity}$$

Plasma Transport

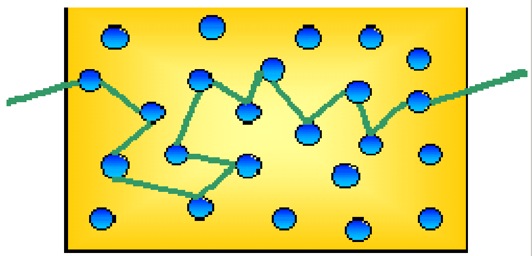
- **Classical Transport**
  - Particle transport in weakly ionised plasmas



$$D = \frac{(\Delta x)^2}{2\tau}$$

$$\Gamma = \Gamma_+ + \Gamma_-$$

Estimate transport coefficients: Δx from mean free path



$$\Delta x = \lambda_m = \frac{1}{n_n \sigma}$$

$$\frac{ct}{n_n \pi d^2 ct} = \frac{1}{n_n \pi d^2} = \frac{1}{n_n \sigma}$$: particle approach

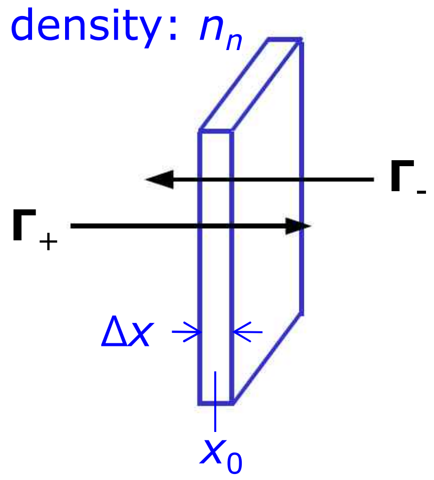
$$\Gamma = \Gamma_0 e^{-n_n \sigma x} \equiv \Gamma_0 e^{-x/\lambda_m}$$: fluid approach

Plasma Transport

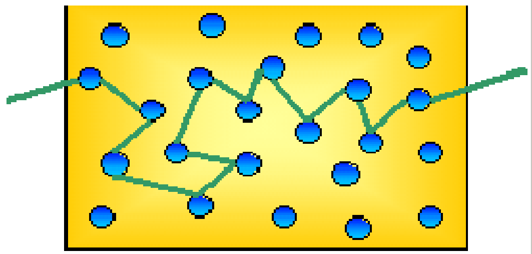
- **Classical Transport**
  - Particle transport in weakly ionised plasmas

$$D = \frac{(\Delta x)^2}{2\tau}$$

$$\Gamma = \Gamma_+ + \Gamma_-$$



Estimate transport coefficients:
τ from collision frequency with neutrals



Plasma Transport

- **Classical Transport**
  - Particle transport in weakly ionised plasmas

$$\vec{\Gamma}_j = n\vec{v}_j = \pm\mu_j n\vec{E} - D_j \nabla n$$

$$\mu \equiv \frac{|q|}{m\nu}$$: Mobility

$$D = \frac{kT}{m\nu} \sim v_{th}^2 \tau \sim \frac{\lambda_m^2}{\tau}$$: Diffusion coefficient

Ambipolar Diffusion

$$\vec{\Gamma} = -D_a \nabla n$$

$$D_a \equiv \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \sim D_i + \frac{T_e}{T_i} D_i$$

Plasma Transport

- **Classical Transport**
  - Particle transport in weakly ionised plasmas with magnetic field

$$\vec{\Gamma}_{\perp j} = n\vec{v}_{\perp j} = \pm\mu_{\perp j} n\vec{E} - D_{\perp j}\nabla n + \frac{n\left(\vec{v}_E + \vec{v}_D\right)}{1+\left(\nu^2/\omega_c^2\right)}$$

$$\mu_{\perp} \equiv \frac{\mu}{1+\omega_c^2\tau^2}$$: Mobility

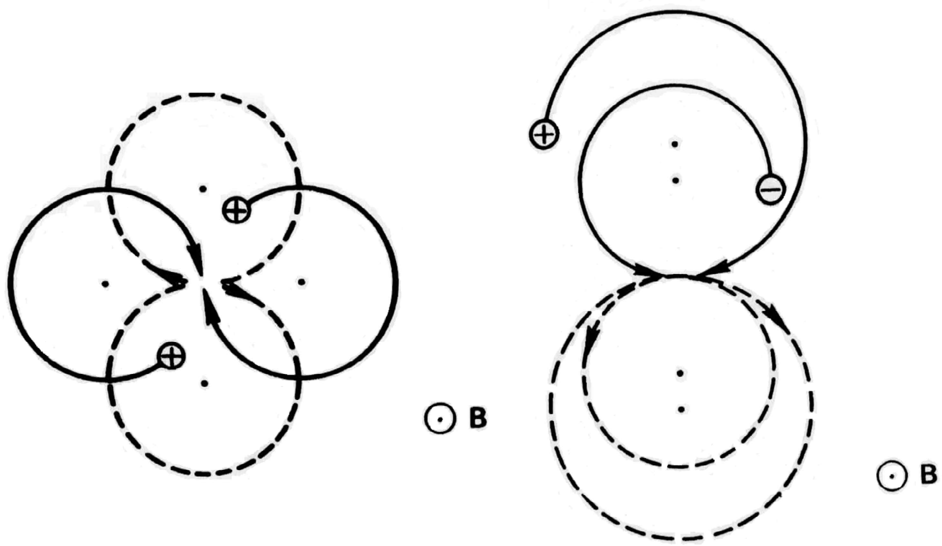
$$D_{\perp} = \frac{D}{1+\omega_c^2\tau^2} = \frac{kT\nu}{m\omega_c^2} \sim v_{th}^2 \frac{r_L^2}{v_{th}^2} \sim \frac{r_L^2}{\tau}$$: Diffusion coefficient

Plasma Transport

- **Classical Transport**
  - Particle transport in fully ionised plasmas with magnetic field

$$\vec{\Gamma}_{\perp} = n\vec{v}_{\perp} = -D_{\perp}\nabla n$$

$$D_{\perp} = \frac{\eta_{\perp} n \sum kT}{B^2}$$



τ from collision frequency

$$\nu_{ee} \approx \nu_{ei} \propto \frac{ne^4}{\sqrt{m_e} T_e^{3/2}}$$

$$\nu_{ie} = \left(\frac{m_e}{m_i}\right)\nu_{ee}$$

$$\nu_{ii} = \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_e}{Ti}\right)^{3/2}\nu_{ee}$$

Plasma Transport

- **Classical Transport**

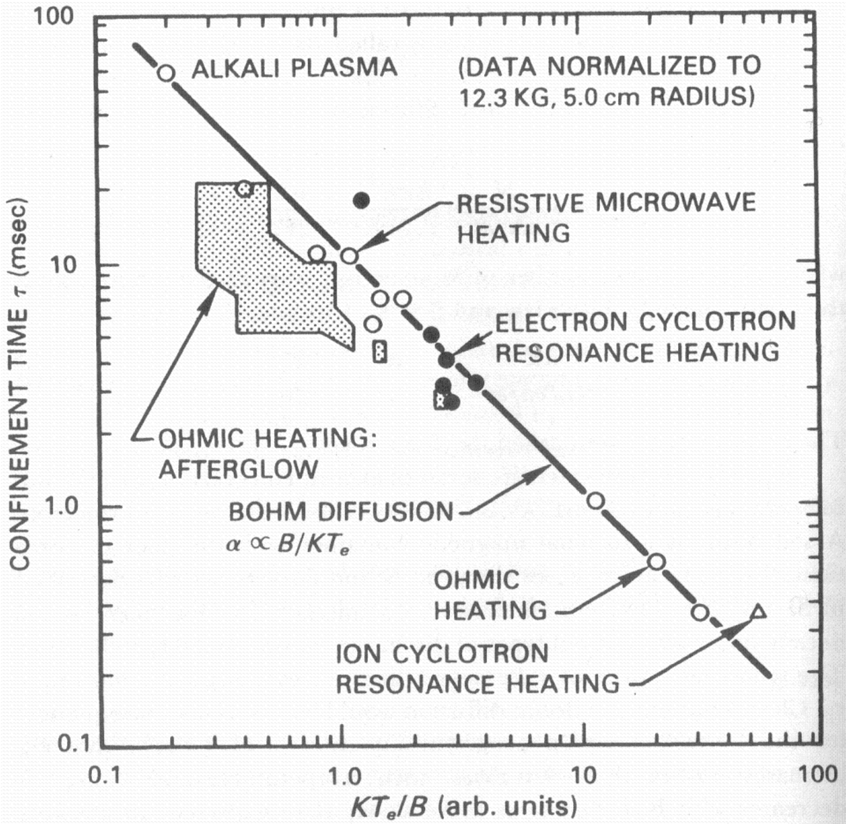
- Classical thermal conductivity (expectation): $\chi_i \sim 40\chi_e$
- Typical numbers expected: 10^{-4} m^2/s
- Experimentally found: 1 m^2/s, $\chi_i \sim \chi_e$

Bohm diffusion (1946): $$D_\perp = \frac{1}{16}\frac{kT_e}{eB}$$

Plasma Transport

- **Classical Transport**

Bohm diffusion: $$D_\perp = \frac{1}{16}\frac{kT_e}{eB}$$



τ_E in various types of discharges in the Model C Stellarator

F. F. Chen, "Introduction to Plasma Physics and Controlled Fusion" (2006)

Plasma Transport

- **Braginskii Equations**

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0$$

$$m_e n_e \frac{dv_{e\alpha}}{dt} = -\frac{\partial p_e}{\partial x_\alpha} - \frac{\partial \pi_{e\alpha\beta}}{\partial x_\beta} - en_e(E + [\mathbf{v}_e \times \mathbf{B}]_\alpha) + R_\alpha$$

$$m_i n_i \frac{dv_{i\alpha}}{dt} = -\frac{\partial p_i}{\partial x_\alpha} - \frac{\partial \pi_{i\alpha\beta}}{\partial x_\beta} - Zen_i(E + [\mathbf{v}_i \times \mathbf{B}]_\alpha) - R_\alpha$$

$$\frac{3}{2} n_e \frac{dT_e}{dt} + p_e \nabla \cdot \mathbf{v}_e = -\nabla \cdot \mathbf{q}_e - \pi_{e\alpha\beta} \frac{\partial v_{e\alpha}}{\partial x_\beta} + Q_e$$

$$\frac{3}{2} n_i \frac{dT_i}{dt} + p_i \nabla \cdot \mathbf{v}_i = -\nabla \cdot \mathbf{q}_i - \pi_{i\alpha\beta} \frac{\partial v_{i\alpha}}{\partial x_\beta} + Q_i$$

$$p_e = n_e T_e, \quad p_i = n_i T_i$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla)$$

$$n = n_e = Zn_i$$

assume $Z = 1$

R, $\pi_{\alpha\beta}$, q, Q?

Plasma Transport

- **Transport / Closure Theories**

	Braginskii	Neoclassical transport	Unified Closure (Ji)
Collisionality	High	High: PS Low: banana	General
Magnetic field strength	General	Strong	Strong
Magnetic geometry	General	Nested	General
Collision operator	Landau	Landau	Landau

Jeong-Young Ji, Lecture at SNU, 2012

Plasma Transport

- **Braginskii Equations**
  - Transfer of momentum from ions to electrons by collisions

$$\mathbf{R} = \mathbf{R}_u + \mathbf{R}_T$$

R$_u$: force of friction due to the existence of a relative velocity **u=v**$_e$**-v**$_i$

R$_T$: thermal force which arises by virtue of a gradient in the electron temperature

$$\mathbf{R}_u = -\frac{m_e n_e}{\tau_e}(0.51\mathbf{u}_{\|} + \mathbf{u}_{\perp}) = en\left(\frac{\mathbf{j}_{\|}}{\sigma_{\|}} + \frac{\mathbf{j}_{\perp}}{\sigma_{\perp}}\right)$$

$$\mathbf{R}_T = -0.71 n_e \nabla_{\|} T_e - \frac{3}{2}\frac{n_e}{\omega_e \tau_e}\left(\frac{\mathbf{B}}{B} \times \nabla T_e\right)$$

$$\sigma_{\perp} = \frac{e^2 n_e \tau_e}{m_e} = \sigma_1 T_e^{3/2}$$

$$\sigma_{\|} = 1.96\sigma_{\perp} = 1.96\sigma_1 T_e^{3/2}$$

$$\sigma_1 = \frac{0.9 \times 10^{13}}{(\lambda/10)Z} \quad \left[\mathrm{s}^{-1} \cdot \mathrm{eV}^{-3/2}\right]$$

Plasma Transport

- **Braginskii Equations**
  - Heat flux

$$D_\perp = \frac{kT\nu}{m\omega_c^2}$$

$$\kappa \sim nD$$

$$\mathbf{q}_e = \mathbf{q}_u^e + \mathbf{q}_T^e$$

$$\mathbf{q}_u^e = 0.71 n_e T_e \mathbf{u}_\parallel + \frac{3}{2}\frac{n_e T_e}{\omega_e \tau_e}\left(\frac{\mathbf{B}}{B}\times\mathbf{u}\right)$$

$$\mathbf{q}_T^e = -\kappa_\parallel^e \nabla_\parallel T_e - \kappa_\perp^e \nabla_\perp T_e - \frac{5}{2}\frac{n_e T_e}{eB}\left(\frac{\mathbf{B}}{B}\times\nabla T_e\right)$$

$$\kappa_\parallel^e = 3.16\frac{n_e T_e \tau_e}{m_e}, \quad \kappa_\perp^e = 4.66\frac{n_e T_e}{m_e \omega_e^2 \tau_e}$$

$$\mathbf{q}_i = -\kappa_\parallel^i \nabla_\parallel T_i - \kappa_\perp^i \nabla_\perp T_i + \frac{5}{2}\frac{n_i T_i}{ZeB}\left(\frac{\mathbf{B}}{B}\times\nabla T_i\right) \qquad \omega_i \tau_i >> 1$$

$$\kappa_\parallel^i = 3.9\frac{n_i T_i \tau_i}{m_i}, \quad \kappa_\perp^i = 2\frac{n_i T_i}{m_i \omega_i^2 \tau_i}$$

Plasma Transport

- **Braginskii Equations**
  - Heat generated as a consequence of collisions

$$Q_i = Q_\Delta = \frac{3m_e}{m_i}\frac{n_e}{\tau_e}(T_e - T_e)$$

$$Q_e = -\mathbf{Ru} - Q_\Delta = \frac{j_\parallel^2}{\sigma_\parallel} + \frac{j_\perp^2}{\sigma_\perp} + \frac{1}{en_e}\mathbf{jR}_T - \frac{3m_e}{m_i}\frac{n_e}{\tau_e}(T_e - T_e)$$

  - Stress tensor in the absence of a magnetic field

$$\pi_{\alpha\beta} = nm\left\langle v'_\alpha v'_\beta - (v'^2/3)\delta_{\alpha\beta}\right\rangle = -\underset{\substack{\downarrow \\ \text{viscosity coefficient}}}{\eta_0} W_{\alpha\beta}$$

Rate of strain tensor

$$W_{\alpha\beta} = \frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} - \frac{2}{3}\delta_{\alpha\beta}\nabla\cdot\mathbf{v}$$

Plasma Transport

$$D_\perp = \frac{kT\nu}{m\omega_c^2} \qquad \eta \sim mnD$$

- **Braginskii Equations**

In a strong magnetic field $\omega\tau >> 1$

$$\pi_{zz} = -\eta_0 W_{zz}$$

$$\pi_{xx} = -\eta_0 \frac{1}{2}\left(W_{xx} + W_{yy}\right) - \eta_1 \frac{1}{2}\left(W_{xx} - W_{yy}\right) - \eta_3 W_{xy}$$

$$\pi_{yy} = -\eta_0 \frac{1}{2}\left(W_{xx} + W_{yy}\right) - \eta_1 \frac{1}{2}\left(W_{yy} - W_{xx}\right) + \eta_3 W_{xy}$$

$$\pi_{xy} = \pi_{yx} = -\eta_1 W_{xy} + \eta_3 \frac{1}{2}\left(W_{xx} - W_{yy}\right)$$

$$\pi_{xz} = \pi_{zx} = -\eta_2 W_{xz} - \eta_4 W_{yz}$$

$$\pi_{yz} = \pi_{zy} = -\eta_2 W_{yz} + \eta_4 W_{xz}$$

$$\eta_0^i = 0.96 n_i T_i \tau_i$$

$$\eta_1^i = \frac{3}{10}\frac{n_i T_i}{\omega_i^2 \tau_i}, \quad \eta_2^i = 4\eta_1^i$$

$$\eta_3^i = \frac{1}{2}\frac{n_i T_i}{\omega_i}, \quad \eta_4^i = 2\eta_3^i$$

viscosity coefficients

$$\eta_0^e = 0.73 n_e T_e \tau_e$$

$$\eta_1^e = 0.51\frac{n_e T_e}{\omega_e^2 \tau_e}, \quad \eta_2^e = 4\eta_1^e$$

$$\eta_3^e = -\frac{1}{2}\frac{n_e T_e}{\omega_e}, \quad \eta_4^i = 2\eta_3^e$$

Plasma Transport

- **Braginskii Equations**
  - Heat generated as a result of viscosity

$$Q_{vis} = -\pi_{\alpha\beta}\frac{\partial v_\alpha}{\partial x_\beta} = -\frac{1}{2}\pi_{\alpha\beta}W_{\alpha\beta}$$

Individual Charge Trajectories

- **Invariant of Motion**

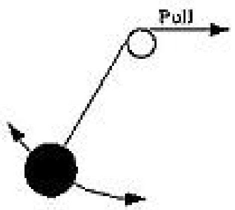
$$\frac{d}{dt}E_0 = \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = 0 \qquad \mu = \frac{mv_\perp^2/2}{B} \qquad m\frac{d\mathbf{v}_\parallel}{dt} = -\frac{\mu}{v_\parallel}\frac{d\mathbf{B}}{dt}$$

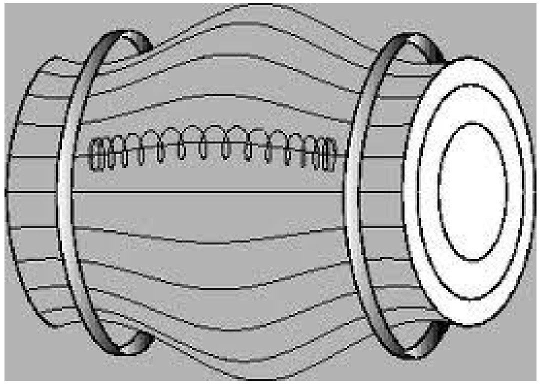
$$\mathbf{F}_\parallel = m\frac{d\mathbf{v}_\parallel}{dt} = -\mu\nabla_\parallel\mathbf{B} = -\mu\frac{\partial\mathbf{B}}{\partial s} = -\mu\frac{\partial\mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{1}{v_\parallel} = -\frac{\mu}{v_\parallel}\frac{d\mathbf{B}}{dt} \longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{dB}{dt}$$

$$\longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = \frac{d}{dt}(\mu B) + \left(-\mu\frac{dB}{dt}\right) = 0$$

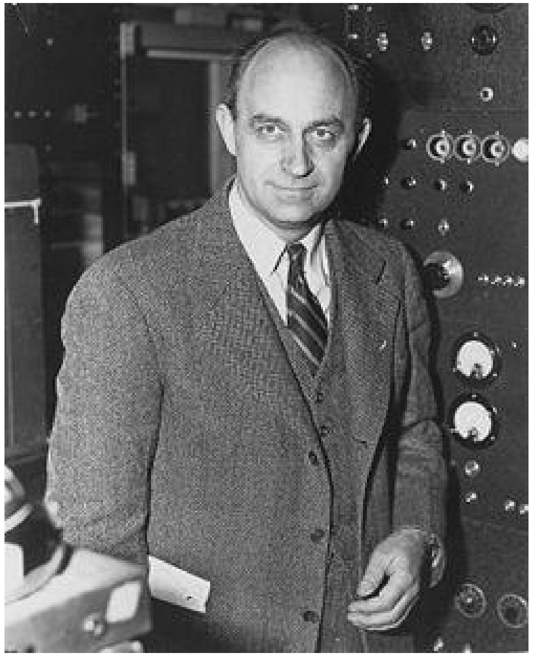
$\longrightarrow \dfrac{d}{dt}(\mu) = 0$: adiabatic invariant

- If B is constant
- $\dfrac{r_L}{B}\nabla_\parallel B << 1$
- $\dfrac{1}{\omega_c B}\dfrac{dB}{dt} << 1$



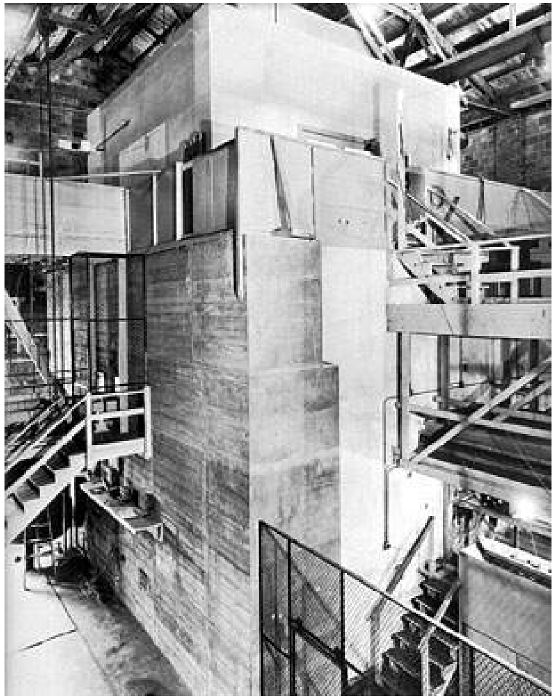


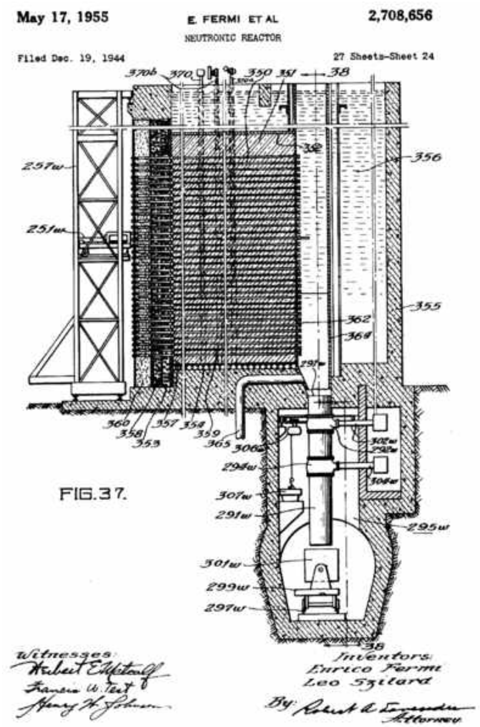
Magnetic Mirror



Enrico Fermi (1901-1954)

Nobel Laureate in physics in 1938
Cf. Marshall Rosenbluth (Doctoral student)





CP-1 (Chicago Pile-1, the world's first human-made nuclear reactor) and Drawings from the Fermi–Szilárd "neutronic reactor" patent

Magnetic Mirror

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

Magnetic Mirror

$$E_0 = \frac{1}{2}mv^2 = \frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2 = const.$$

$$\mu = \frac{\frac{1}{2}mv_\perp^2}{B} = \frac{\frac{1}{2}mv^2\sin^2\theta}{B} = const.$$

The path of a fast proton in an irregular magnetic field of the type that we have assumed will be represented very closely by a spiraling motion around a line of force. Since the radius of this spiral may be of the order of 10^{12} cm, and the irregularities in the field have dimensions of the order of 10^{18} cm, the cosmic ray will perform many turns on its spiraling path before encountering an appreciably different field intensity. One finds by an elementary discussion that as the particle approaches a region where the field intensity increases, the pitch of the spiral will decrease. One finds precisely that

$$\sin^2\vartheta/H \approx \text{constant}, \tag{12}$$

where ϑ is the angle between the direction of the line of force and the direction of the velocity of the particle, and H is the local field intensity. As the particle approaches a region where the field intensity is larger, one will expect, therefore, that the angle ϑ increases until $\sin\vartheta$ attains the maximum possible value of one. At this point the particle is reflected back along the same line of force and spirals backwards until the next region of high field intensity is encountered. This process will be called a "Type A" reflection. If the magnetic field were static, such a reflection would not produce any change in the kinetic energy of the particle. This is not so, however, if the magnetic field is slowly variable. It may happen that a region of high field intensity moves toward the cosmic-ray particle which collides against it. In this case, the particle will gain energy in the collision. Conversely, it may happen that the region of high field intensity moves away from the particle. Since the particle is much faster, it will overtake the irregularity of the field and be reflected backwards, in this case with loss of energy. The net result will be an average gain, primarily for the reason that head-on collisions are more frequent than overtaking collisions because the relative velocity is larger in the former case.

HW: Derive (13)

$$\frac{w'}{w} = \frac{1+2B\beta\cos\vartheta + B^2}{1-B^2}, \tag{13}$$

where βc is the velocity of the particle, ϑ is the angle of inclination of the spiral, and Bc is the velocity of the perturbation. It is assumed that the

Magnetic Mirror

- **Fermi as a genuine scientist**

spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

The present theory is incomplete because no satisfactory injection mechanism is proposed except for protons which apparently can be regenerated at least in part in the collision processes of the cosmic radiation itself with the diffuse interstellar matter. The most serious difficulty is in the injection process for the heavy nuclear component of the radiation. For these particles the injection energy is very high and the injection mechanism must be correspondingly efficient.

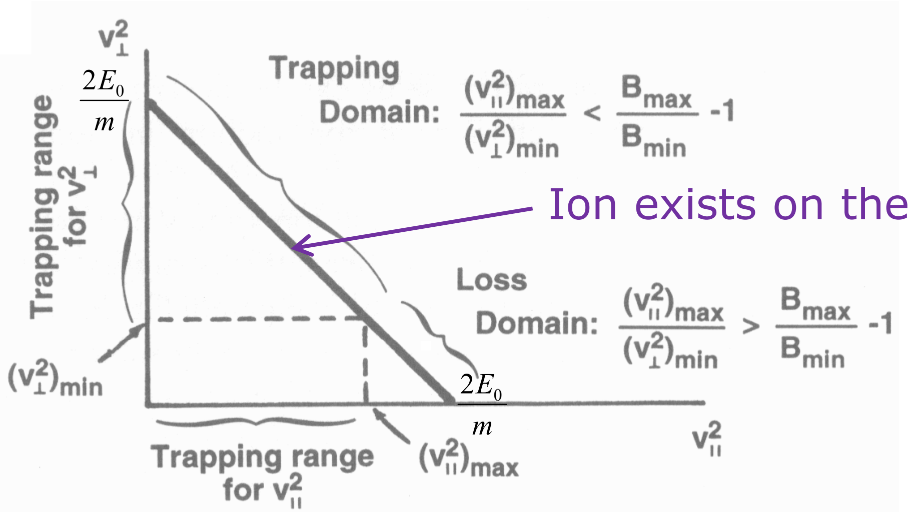
some equivalent mechanism. With respect to the injection of heavy nuclei I do not know a plausible answer to this point.

Magnetic Mirror

- **Condition for Trapping of Particles**

$$E_0 = \frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2 = \frac{1}{2}mv_\parallel^2 + \mu B = \frac{1}{2}m\left(v_\parallel^2\right)_{\max} + \mu B_{\min}$$

$$\mathbf{F}_\parallel = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_\parallel B = -\mu\nabla_\parallel B$$



Ion exists on the E_0=constant line.

$$v_\perp^2 = \frac{2E_0}{m} - v_\parallel^2$$

Condition for trapping of particles

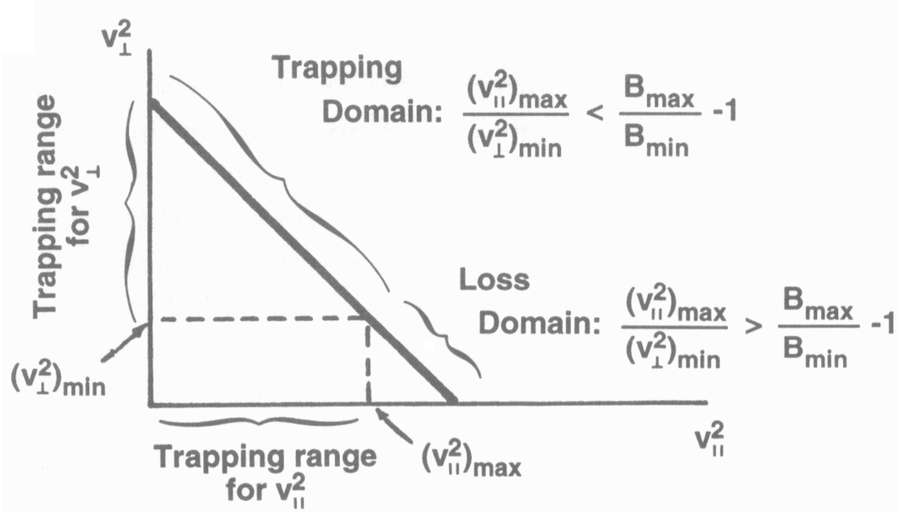
$$v_\parallel\Big|_{B \le B_{\max}} = 0 \longrightarrow E_0 = \frac{1}{2}m\left(v_\parallel^2\right)_{\max} + \mu B_{\min} \le 0 + \mu B_{\max}$$

Magnetic Mirror

- **Condition for Trapping of Particles**

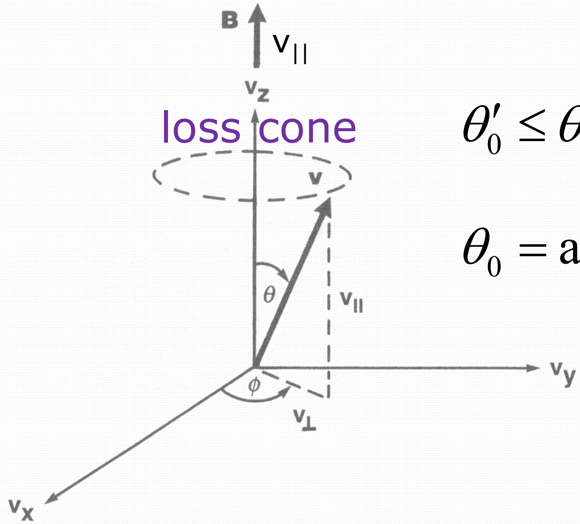
$$\frac{\frac{1}{2}m\left(v_{\parallel}^{2}\right)_{\max}}{\mu B_{\min}} \le \frac{B_{\max}}{B_{\min}} - 1$$

$$\frac{\left(v_{\parallel}^{2}\right)_{\max}}{\left(v_{\perp}^{2}\right)_{\min}} = \left(\frac{v_{\parallel}^{2}}{v_{\perp}^{2}}\right)_{mid-plane} \le \frac{B_{\max}}{B_{\min}} - 1$$



$$\frac{v_{\parallel}}{v_{\perp}} = \frac{\cos\theta}{\sin\theta} \qquad \sin^{2}\theta = \frac{1}{\dfrac{v_{\parallel}^{2}}{v_{\perp}^{2}} + 1}$$

$$\sin\theta \ge \sqrt{\frac{B_{\min}}{B_{\max}}}$$



$$\theta_0' \le \theta \le \pi - \theta_0'$$

$$\theta_0 = \arcsin\sqrt{\frac{B_{\min}}{B_{\max}}}$$

Magnetic Mirror

- **Mirror Ratio**

$$f_{loss} = \frac{\int_{\text{double cone}} f(\vec{v})d^3v}{\int_0^\infty f(\vec{v})d^3v} = \frac{\int_0^{2\pi} d\phi \left[\int_0^{\theta_0'} \sin\theta d\theta + \int_{\pi-\theta_0'}^{\pi} \sin\theta d\theta\right] \int_0^\infty \frac{f(v)}{4\pi v^2} v^2 dv}{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \int_0^\infty \frac{f(v)}{4\pi v^2} v^2 dv}$$

$$= 1 - \cos\theta_0'$$

$$f_{trap} = \cos\theta_0' = \sqrt{1 - \frac{B_{\min}}{B_{\max}}} \qquad \frac{B_{\max}}{B_{\min}} \equiv R_m$$

mirror ratio:
Determining the effectiveness of confinement

Why are particles reflected in the increased field of the mirrors?

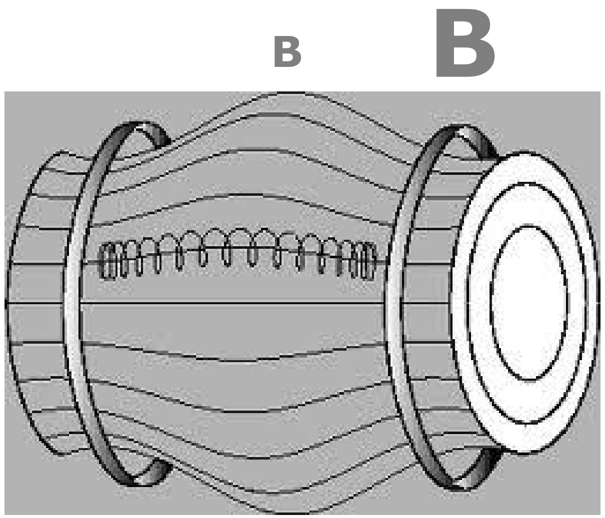
$$\mathbf{F}_{\|} = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_{\|}B = -\mu\nabla_{\|}B$$

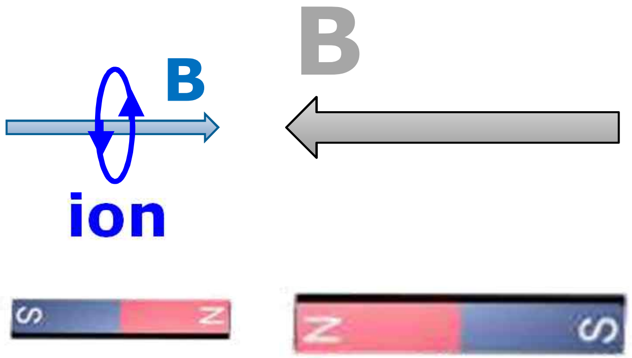
Magnetic Mirror

Why are particles reflected in the increased field of the mirrors?

Adiabatic invariant $\mu = \dfrac{mv_\perp^2/2}{B}$ $\dfrac{d}{dt}(\mu) = 0$

$$\mathbf{F}_\parallel = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_\parallel B = -\mu\nabla_\parallel B$$





Field generated by
ion's gyration

Tokamak Transport

- **Neoclassical theory of transport**

- A. A. Galeev and R. Z. Sagdeev
  "Transport phenomena in a collisionless plasma in a toroidal magnetic system", Zhurnal Experimentalnoi i Teoreticheskoi Fiziki ***53*** *348 (1967)*
- Major changes arise from toroidal effects characterized by inverse aspect ratio, $\varepsilon = a/R_0$

Tokamak Transport

- **Particle Trapping**

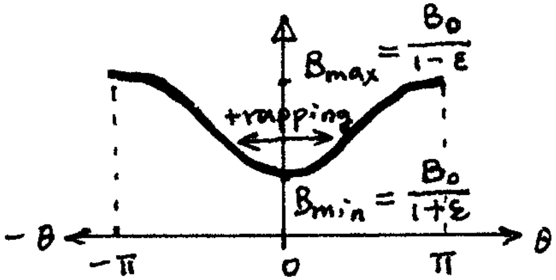
Inverse aspect ratio
$\varepsilon = a/R_0$

$$\nabla \cdot B = 0$$

$$\Rightarrow \quad \frac{1}{1+\varepsilon\cos\theta}\left\{\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{1}{r}\frac{\partial}{\partial\theta}\left[(1+\varepsilon\cos\theta)B_\theta\right] + \frac{1}{R_0}\frac{\partial B_\phi}{\partial\phi}\right\} = 0$$

$$\Rightarrow \quad B_\theta(r,\theta) = \frac{B_\theta^0(\theta=0)}{1+\varepsilon\cos\theta}$$

$$B(r,\theta) = B_\theta(r,\theta)\hat{\theta} + B_\phi(r,\theta)\hat{\phi} = \frac{B_0}{1+\varepsilon\cos\theta}$$



Condition for trapping of particles

$$\frac{\left(v_\parallel^2\right)_{\max}}{\left(v_\perp^2\right)_{\min}} = \left(\frac{v_\parallel^2}{v_\perp^2}\right)_{mid-plane} \le \frac{B_{\max}}{B_{\min}} - 1 = \frac{\frac{B_0}{1-\varepsilon}}{\frac{B_0}{1+\varepsilon}} - 1 = \frac{2\varepsilon}{1-\varepsilon} \sim 2\varepsilon$$

$$\Rightarrow \quad v_\parallel^2 \le 2\varepsilon v_\perp^2$$

Tokamak Transport

- **Particle Trapping**

  - Particle trapping by magnetic mirrors
    trapped particles with banana orbits
    untrapped (transit or passing) particles with circular orbits

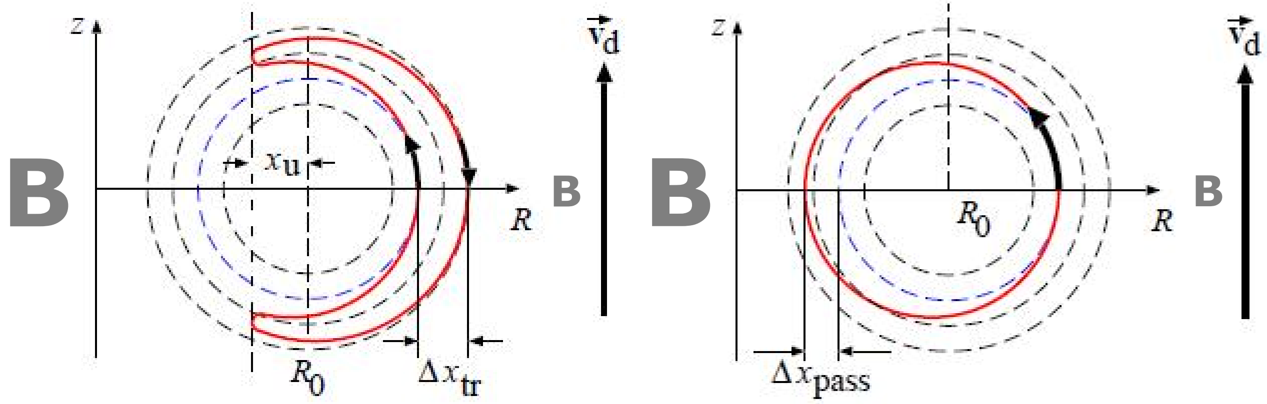
  - Trapped fraction: $f_{trap} = \sqrt{1 - \frac{1}{R_m}} = \sqrt{1 - \frac{B_{\min}}{B_{\max}}} = \sqrt{1 - \frac{1-\varepsilon}{1+\varepsilon}} = \sqrt{\frac{2\varepsilon}{1+\varepsilon}} \sim \sqrt{\varepsilon}$

  for a typical tokamak, $\varepsilon \sim 1/3 \rightarrow f_{trap} \sim 70\%$

Tokamak Transport

$$\mathbf{v}_d = \frac{m}{q}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)\frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2 B_0^2}$$

- **Particle Trapping**



trapped particles

Passing (transit) particles

Expelling force of diamagnetic Larmor motion

$$F_D = -\mu \nabla_\perp B = m v_\perp^2 / 2R$$

$$\mathbf{v}_{d,\nabla B} = \pm \frac{v_\perp^2}{2\omega_c}\frac{\mathbf{B}\times\nabla B}{B^2} = \pm \frac{1}{2} v_\perp r_L \frac{\mathbf{B}\times\nabla B}{B^2}$$

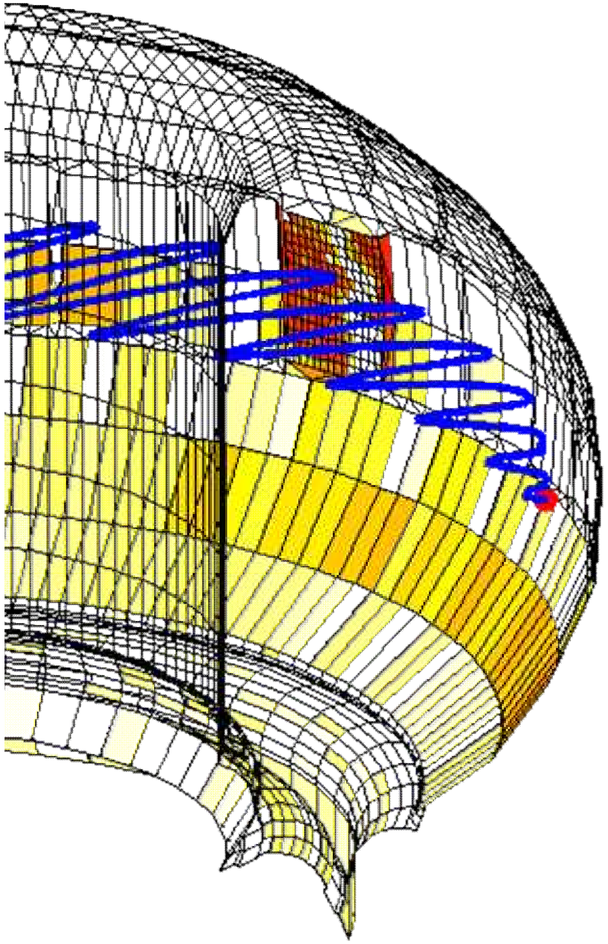
Centrifugal force

$$F_C = m v_\parallel^2 / R$$

$$\mathbf{v}_{d,R} = \frac{m v_\parallel^2}{q B_0^2}\frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2}$$

Tokamak Transport

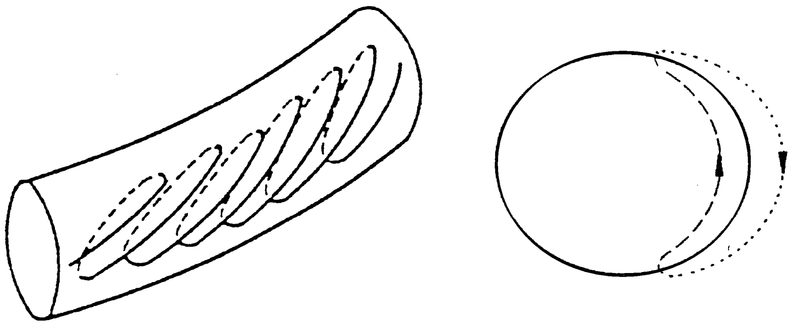
- **Particle Trapping**



HOMEWORK:

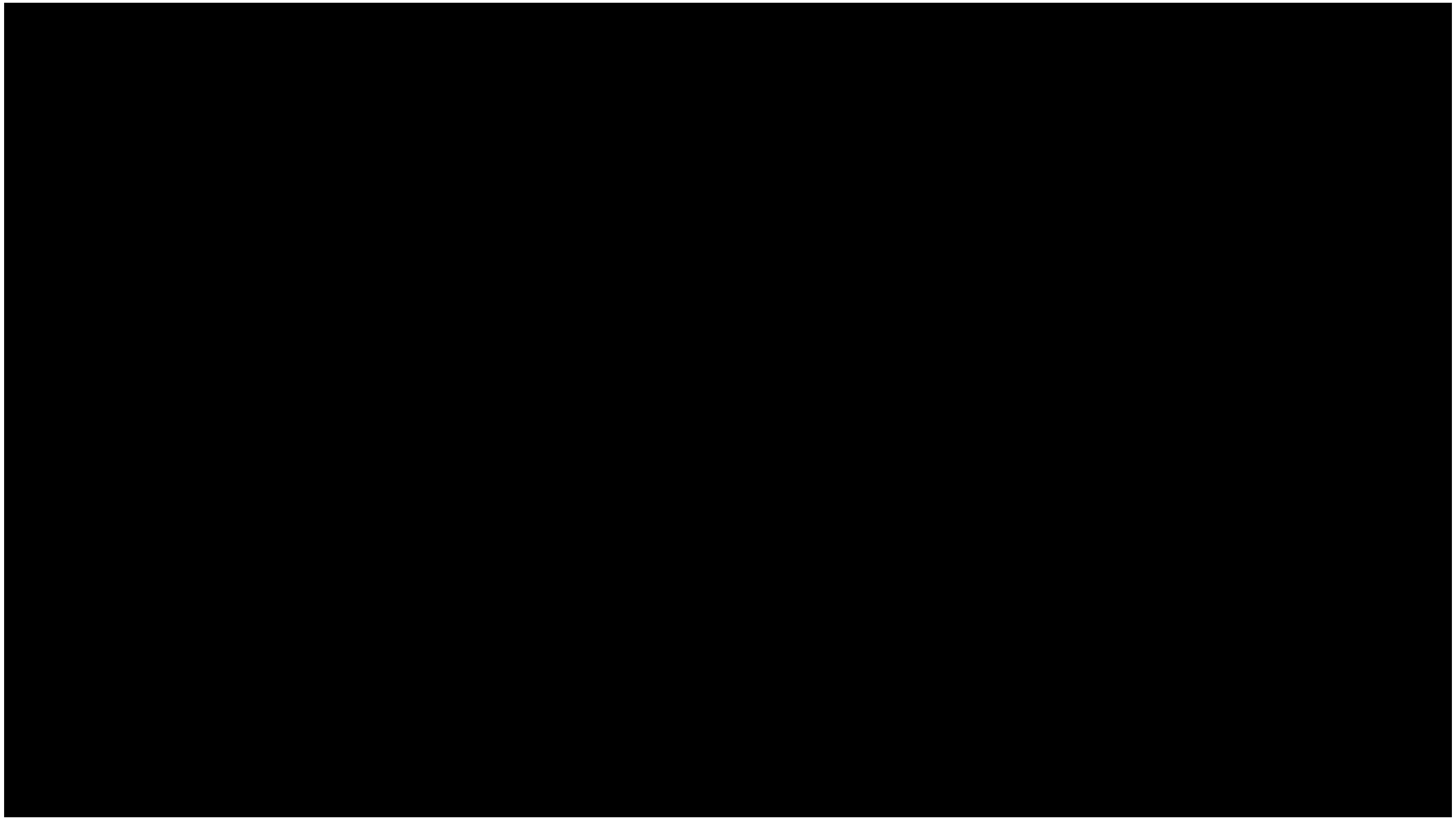
- The real particle trajectory is as shown. Why?

- In ST, B is small, what is the particle trajectory like?



Tokamak Transport

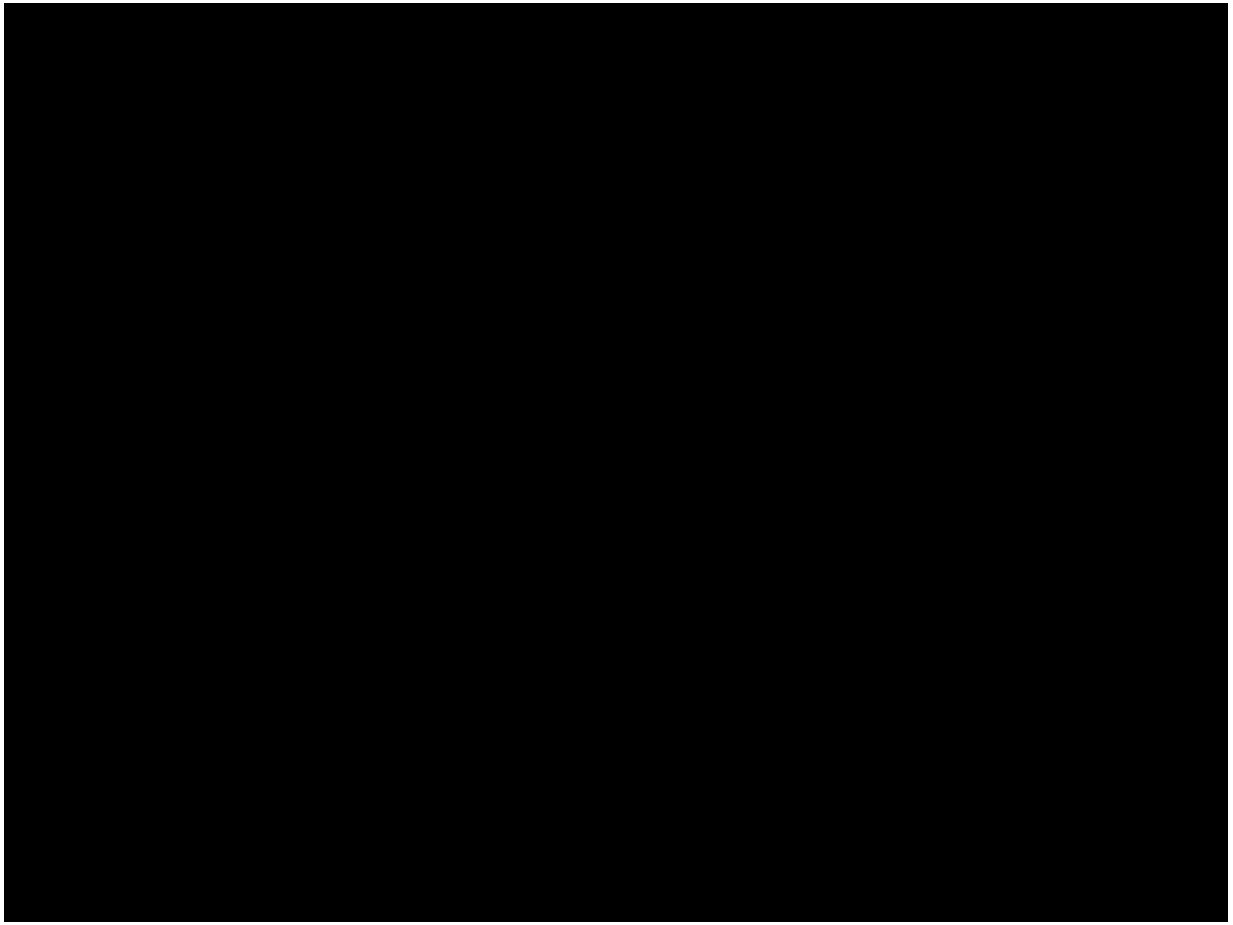
- **Particle Trapping**



*J. P. Graves et al, Nature Communications **3** 624 (2012)*

Tokamak Transport

- **Particle Trapping**



*J. P. Graves et al, Nature Communications **3** 624 (2012)*

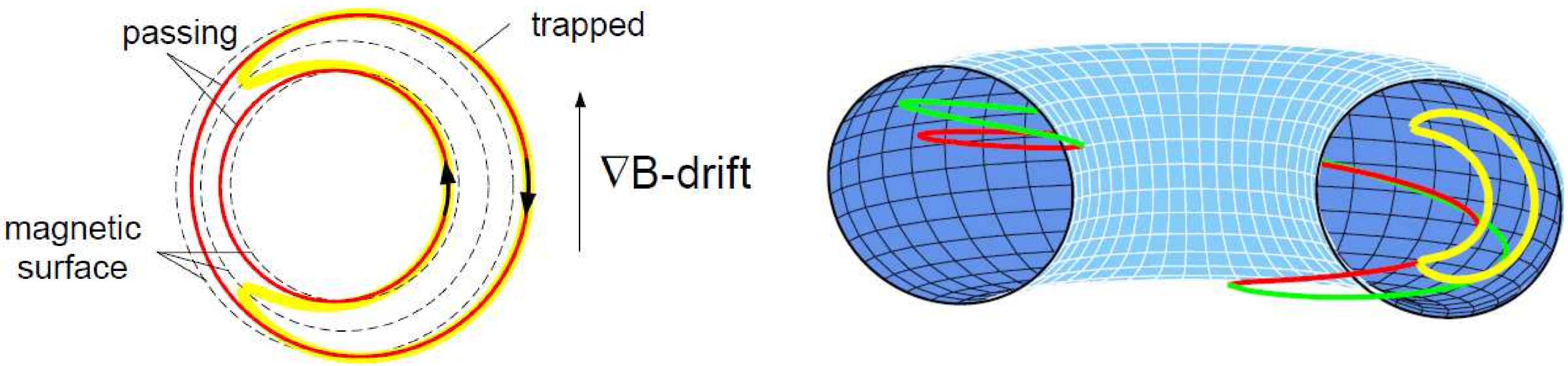
Tokamak Transport

- **Particle Trapping**

*J. P. Graves et al, Nature Communications **3** 624 (2012)*

Tokamak Transport

- **Particle Trapping**



- With known particle trajectories it is possible to find corresponding kinetic coefficients by solving the kinetic equations with Coulomb collisions.
- Rough estimation of transport coefficients: $\delta^2 \nu_{eff}$

  δ: particle displacement between collisions

  ν_{eff} : appropriate frequency of collisions

Tokamak Transport

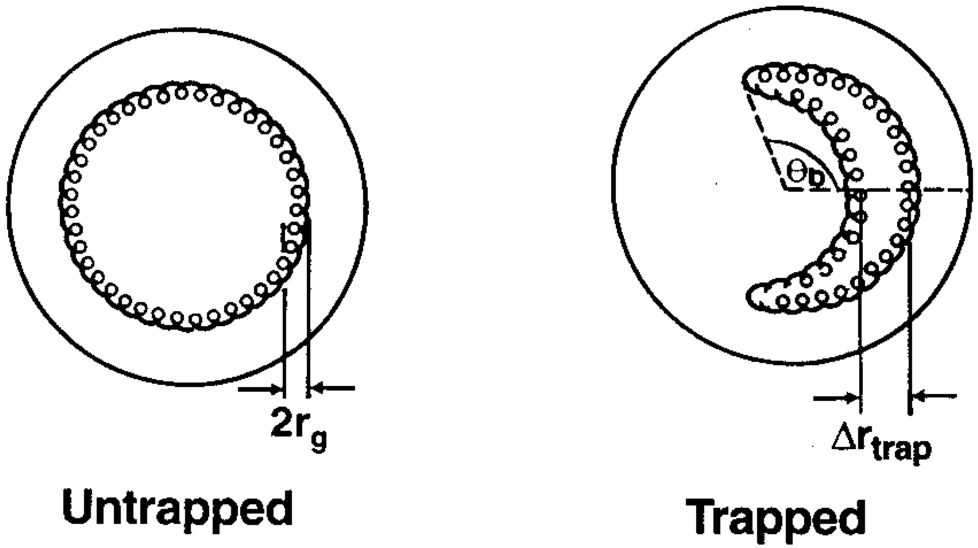
- **Particle Trapping**

- Collisional excursion across flux surfaces

  untrapped particles: $2r_g = 2r_{Li}$

  trapped particles: $\Delta r_{trap} >> 2r_g$

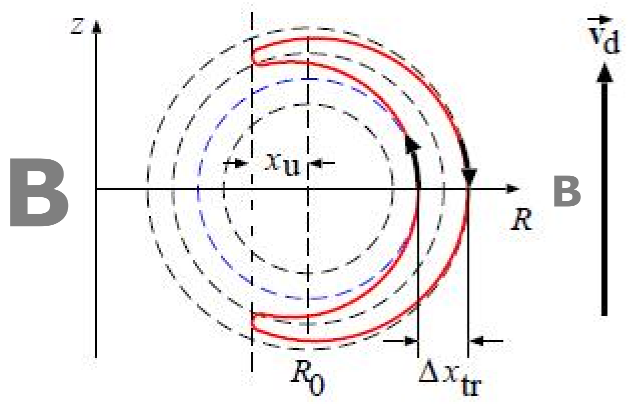
  – enhanced radial diffusion across the confining magnetic field



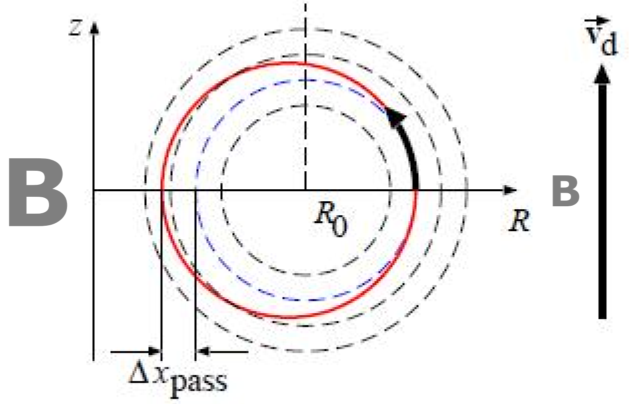
- If the fraction of trapped particle is large, this leakage enhancement constitutes a substantial problem in tokamak confinement.

Tokamak Transport

- **Particle Trapping**



trapped particles



Passing (transit) particles

Banana width: $\Delta x_{tr} \approx v_d t \approx q r_L / \sqrt{\varepsilon}$

$$v_d = \frac{v_\parallel^2 + v_\perp^2 / 2}{\omega_c R}, \quad v_\parallel \sim v_\perp \sqrt{\varepsilon}$$

t: transit time of one half of the banana

$$q = \frac{r B_T}{R B_\theta}, \quad r_L = \frac{v_T}{\omega_b}, \quad v_T = \sqrt{2T/m}$$

Displacement of transit particles:

$\Delta x_{pass} \approx q r_L / \sqrt{\varepsilon}$ for particles which have just become transit ones $v_\parallel \sim v_\perp \sqrt{\varepsilon}$

$\Delta x_{pass} \approx q r_L$ for a typical particle $v_\parallel \sim v_\perp$

References

- Francis F. Chen, "Introduction to Plasma Physics and Controlled Fusion", 2nd Edition, Plenum Press, New York (1984)

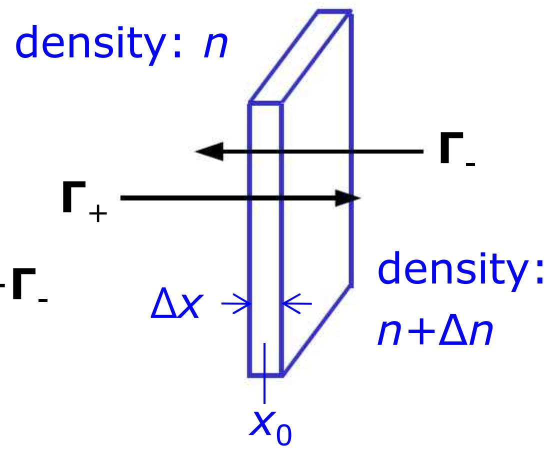
- Acad. M. A. Leontovich et al, "Reviews of Plasma Physics, Volume 1", Consultants Bureau, New York (1965)

- Jeffrey P. Freidberg, "Plasma Physics and Fusion Energy", Cambridge University Press (2007)

- Hartmut Zohm, "Tokamaks: Equilibrium, Stability and Transport", IPP Summer University on Plasma Physics, Garching, 18 September, 2001

Plasma Transport

- **Classical Transport**
  - Particle transport

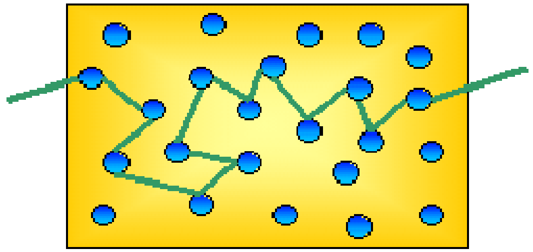


$$\Gamma = \Gamma_+ + \Gamma_-$$

$$n(x) = n(x_0) + \left.\frac{\partial n}{\partial x}\right|_{x=x_0} (x - x_0)$$

$$\Gamma_+ = \frac{1}{2}\int_{x_0-\Delta x/2}^{x_0} \frac{1}{\tau} n(x)dx = \frac{1}{4}\left[n(x_0) - \frac{\partial n}{\partial x}\Delta x\right]\frac{\Delta x}{\tau}$$

$$\Gamma_- = \frac{1}{2}\int_{x_0+\Delta x/2}^{x_0} \frac{1}{\tau} n(x)d(-x) = \frac{1}{4}\left[n(x_0) + \frac{\partial n}{\partial x}\Delta x\right]\frac{\Delta x}{\tau}$$



$$\Gamma = \Gamma_+ - \Gamma_- = -\frac{(\Delta x)^2}{2\tau}\frac{\partial n}{\partial x} = -D\frac{\partial n}{\partial x}$$: Particle flux- Fick's law

$$D = \frac{(\Delta x)^2}{2\tau}$$: diffusion coefficient (m^2/s)

The heat and momentum fluxes can be estimated in similar fashion.