

RF waves in Fusion Plasmas

Seminar at SNU

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The role of RF Heating and CD

□ Increase of temperature

- Nuclear fusion requires high temperature more than 10 keV.
- Ohmic heating is limited by the low resistance in high temperature.
- Alternatives : NB heating / RF heating
- NB heating is effective but requires high technology to increase the beam energy up to 1 MeV. (negative ion generation/acceleration/cooling)
- RF wave can heat up selectively ion and electrons and is deposited locally or globally depending on the driving schemes (magnetic field/driving frequency/plasma density)
- But, there are coupling problems related with ICRF and LHFR and power transmission and power source limitations regarding ECRF power.

- ICRF : Ion heating
- LHFR : Current drive
- ECRF : local current drive and MHD control / pre-ionization and start-up

The role of RF waves

□ Non inductive current drive

- Tokamak requires current drive to confine the plasmas. Otherwise, the particles is lost outward by EXB drift due to charge separation of non-uniform magnetic field.
- Most efficient current drive is Ohmic inductive current drive. However, it is limited by Ohmic swing flux.
- Therefore, the **non-inductive current drive is an indispensable element** for the success of fusion reactor.

- NB current drive/RF current drive/Helicity injection
- **LHRF current drive is proven to be most efficient non-inductive current drive scheme** ever tried and experimentally, 2 hours 20 kA in TRIAM and 2 minute 0.8 MA in Tore supra. 3.6 MA and 3 MA in JT-60U and JET are achieved respectively.
- However, there is a coupling problem.

RF waves in plasmas

- ❑ To utilize RF waves for the heating and current drive of tokamak plasmas, we should answer the two questions?
- ❑ What kind of RF waves can exist in plasmas? (Identity of RF plasma waves)
- ❑ How do RF waves propagate and are mode converted, and absorbed in plasmas? (Characteristics of RF plasma waves)

RF waves in plasmas

□ What kind of RF waves can exist in plasmas? (Identity of RF plasma waves)

- Wave Equation in vacuum?

- Governing Equation: Maxwell equation with vacuum medium property.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

- Wave Equation in Plasmas?

- Governing Equation: Maxwell equation with plasma medium property.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}_{rf} \right) = \mu_0 \left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \vec{\sigma} \mathbf{E} \right) = \mu_0 \epsilon_0 \vec{\epsilon}_r \frac{\partial \mathbf{E}}{\partial t}, \quad \vec{\epsilon}_r \equiv \vec{I} + \frac{i\vec{\sigma}}{\epsilon_0 \omega} (\chi_s)$$

- The plasma waves can be described by above Maxwell equation. One can obtain information of linear plasma waves from this governing equation.

- The remaining problem is how to obtain the conductivity or dielectric tensor.

RF waves in plasmas

□ How to obtain the dielectric tensor?

- Governing Equation: Vlasov equation : Equation of evolution of particle distribution in phase space

$$\frac{df_s}{dt} = \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \mathbf{a} \cdot \nabla_{\mathbf{v}} f_s = 0$$

$$\mathbf{a} = \frac{eZ_s}{m_s} \left[\mathbf{E} + \mathbf{v} \times (\mathbf{B} + \mathbf{B}_0) \right]$$

- By linearization, one can obtain linearized Vlasov equation. $f_s = F_s(\mathbf{r}, \mathbf{v}) + \tilde{f}_s(\mathbf{r}, \mathbf{v}, t)$

$$\frac{d\tilde{f}_s}{dt} = \frac{\partial \tilde{f}_s}{\partial t} + \mathbf{v} \cdot \nabla \tilde{f}_s + \frac{eZ_s}{m_s} [\mathbf{v} \times \mathbf{B}_0] \cdot \nabla_{\mathbf{v}} \tilde{f}_s = -\frac{eZ_s}{m_s} [\mathbf{E} + \mathbf{v} \times \mathbf{B}_0] \cdot \nabla_{\mathbf{v}} F_s$$

- The solution is as follows.

$$\tilde{f}_s = -\frac{eZ_s}{m_s} \int_{-\infty}^t \left[\mathbf{E}(\mathbf{r}', t') + \mathbf{v}' \times \mathbf{B}(\mathbf{r}', t') \right] \cdot \nabla_{\mathbf{v}'} F_s dt'$$

$$\mathbf{J}_{rf} = \sum_s n_s eZ_s \int_{\mathbf{v}} \tilde{f}_s \mathbf{v} d\mathbf{v}$$

RF waves in plasmas

□ Dielectric(Conductivity) tensor

$$\vec{\epsilon}_r = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}, \quad \vec{\sigma} = \frac{\epsilon_0 \omega}{i} (\vec{\epsilon}_r - \vec{I}) = -i\epsilon_0 \omega \chi_s$$

- The detailed expression of dielectric tensor elements for F_s of Maxwellian distribution function are as follows.

$$\epsilon_{xx} = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{\infty} \frac{n^2}{\lambda_s} I_n(\lambda_s) e^{-\lambda_s} [-\zeta_{0s} Z(\zeta_{ns})]$$

$$\epsilon_{xy} = -i \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{\infty} n [I'_n(\lambda_s) - I_n(\lambda_s)] e^{-\lambda_s} [-\zeta_{0s} Z(\zeta_{ns})]$$

$$\epsilon_{xz} = -\frac{1}{2} N_{\perp} N_{\parallel} \sum_s \frac{\omega_{ps}^2}{\omega \Omega_{cs}} \frac{v_{ths}^2}{c^2} \sum_{n=-\infty}^{\infty} \frac{n}{\lambda_s} I_n(\lambda_s) e^{-\lambda_s} [\zeta_{0s}^2 Z'(\zeta_{ns})]$$

$$\epsilon_{yy} = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{\infty} \left[\frac{n^2}{\lambda_s} I_n(\lambda_s) - 2\lambda_s [I'_n(\lambda_s) - I_n(\lambda_s)] \right] e^{-\lambda_s} [-\zeta_{0s} Z(\zeta_{ns})]$$

$$\epsilon_{yz} = \frac{i}{2} N_{\perp} N_{\parallel} \sum_s \frac{\omega_{ps}^2}{\omega \Omega_{cs}} \frac{v_{ths}^2}{c^2} \sum_{n=-\infty}^{\infty} [I'_n(\lambda_s) - I_n(\lambda_s)] e^{-\lambda_s} [\zeta_{0s}^2 Z'(\zeta_{ns})]$$

$$\epsilon_{zz} = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{\infty} I_n(\lambda_s) e^{-\lambda_s} [-\zeta_{0s} \zeta_{ns} Z'(\zeta_{ns})]$$

$$\lambda_s = \frac{k_{\perp}^2 v_{ths}^2}{2\Omega_{cs}^2}$$

$$\zeta_{ns} = \frac{\omega - n\Omega_{cs}}{k_{\parallel} v_{ths}}$$

$$\epsilon_{yx} = -\epsilon_{xy}$$

$$\epsilon_{zx} = \epsilon_{xz}$$

$$\epsilon_{zy} = -\epsilon_{yz}$$

RF waves in plasmas

□ Cold dielectric tensor

$$\lim_{\nu_{ths} \rightarrow 0} \overleftrightarrow{\epsilon}_r = \lim_{\nu_{ths} \rightarrow 0} \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}$$

- The detailed expression of cold dielectric tensor elements are as follows.

$$S = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_{cs}^2}$$

$$D = \sum_s \frac{\Omega_{cs}}{\omega} \frac{\omega_{ps}^2}{\omega^2 - \Omega_{cs}^2}$$

$$P = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}$$

- If the plasma density goes to zero, the cold dielectric tensor becomes unity tensor.
- It is a vacuum relative permittivity.

RF waves in plasmas

- It is easier to approach from cold plasma dielectric tensor for RF wave exploration.

$$\begin{aligned} \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times B &= \mu_0 \epsilon_0 \vec{\epsilon}_c \frac{\partial E}{\partial t} \end{aligned} \quad \vec{\epsilon}_{cold} = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}$$

- By manipulating the Maxwell equation with cold plasma dielectric response, one can obtain the wave equation.

$$\nabla \times \nabla \times E = -\mu_0 \epsilon_0 \vec{\epsilon}_c \frac{\partial^2 E}{\partial t^2}$$

- For spatially uniform plasmas

$$\vec{N} \times \vec{N} \times E_0 = \vec{\epsilon}_c E_0, \quad E = E_0 e^{i(k_0 \vec{N} \cdot \vec{r} - \omega t)}$$

$$(\vec{N}^2 - \vec{\epsilon}_c) E_0 = 0$$

$$\vec{N}^2 = \begin{bmatrix} N_{\parallel}^2 & 0 & -N_{\parallel} N_{\perp} \\ 0 & N^2 & 0 \\ -N_{\parallel} N_{\perp} & 0 & N_{\perp}^2 \end{bmatrix}$$

$$\det(\vec{N}^2 - \vec{\epsilon}_c) = 0 : \text{dispersion relation}$$

RF waves in plasmas

- Dispersion relation

$$H \equiv \det(\vec{N}^2 - \vec{\epsilon}_c) = 0 : \text{dispersion relation}$$

- Several forms of dispersion relations

$$AN^4 + BN^2 + C = 0$$

$$A = S \sin^2 \theta + P \cos^2 \theta, \quad \theta = \angle(\vec{B}, \vec{N})$$

$$B = -RL \sin^2 \theta - SP(1 + \cos^2 \theta), \quad R = S + D, L = S - D$$

$$C = PRL$$

$$AN_{\perp}^4 + BN_{\perp}^2 + C = 0$$

$$A = S$$

$$B = N_{\parallel}^2(S + P) - (SP + RL)$$

$$C = P(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)$$

$$AN_{\parallel}^4 + BN_{\parallel}^2 + C = 0$$

$$A = P$$

$$B = N_{\perp}^2(S + P) - 2SP$$

$$C = (N_{\perp}^2 - P)(SN_{\perp}^2 - RL)$$

RF waves in plasmas

□ Polarization

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S}$$

$$\frac{E_z}{E_x} = \frac{N_{\parallel} N_{\perp}}{N_{\perp}^2 - P}$$

$$\frac{E_+}{E_-} = \frac{R - N^2}{L - N^2}, \quad E_+ = \frac{E_x + iE_y}{\sqrt{2}}, \quad E_- = \frac{E_x - iE_y}{\sqrt{2}}$$

□ Group velocity

$$v_g = -\frac{\partial H / \partial \vec{k}}{\partial H / \partial \omega}$$

$$\frac{\tan \theta_g}{\tan \theta} = \frac{v_{g\perp}}{v_{g\parallel}} \left(\frac{v_{p\perp}}{v_{p\parallel}} \right)^{-1} = \frac{SN_{\perp}^2 + P(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{PN_{\parallel}^2 + (N_{\parallel}^2 - P)(SN_{\parallel}^2 - RL)}$$

RF waves in plasmas

□ Cut-off /Resonance

cutoff: $N = 0, \lambda = \infty$; *wave is evanescent.*

resonance: $N = \infty, \lambda = 0$; *wave is locally piled up.*

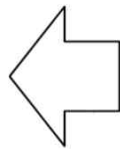
□ Cut-off

$$N, C = 0$$

$P = 0$: *O wave cutoff*

$R = 0$: *R wave cutoff*

$L = 0$: *L wave cutoff*



$$AN^4 + BN^2 + C = 0$$

$$A = S \sin^2 \theta + P \cos^2 \theta, \theta = \angle(\vec{B}, \vec{N})$$

$$B = -RL \sin^2 \theta - SP(1 + \cos^2 \theta), R = S + D, L = S - D$$

$$C = PRL$$

□ Resonance

$$N = \infty,$$

$$\theta = (0, \frac{\pi}{2}) \Rightarrow A = S \sin^2 \theta + P \cos^2 \theta = 0; \text{resonance cone wave}$$

$$\theta = 0 (\text{parallel}) \Rightarrow R, L = \infty; \text{cyclotron resonance}$$

$$\theta = \frac{\pi}{2} (\text{perpendicular}) \Rightarrow S = 0; \text{UHR, LHR}$$

RF waves in plasmas

□ Perpendicular / Parallel propagation

$$AN_{\perp}^4 + BN_{\perp}^2 + C = 0$$

$$A = S$$

$$B = N_{\parallel}^2(S + P) - (SP + RL)$$

$$C = P(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)$$

$$N_{\parallel} = 0,$$

$$N_{\perp}^2 = \frac{RL}{S} \quad (X \text{ wave})$$

$$N_{\perp}^2 = P \quad (O \text{ wave})$$

$$AN_{\parallel}^4 + BN_{\parallel}^2 + C = 0$$

$$A = P$$

$$B = N_{\perp}^2(S + P) - 2SP$$

$$C = (N_{\perp}^2 - P)(SN_{\perp}^2 - RL)$$

$$N_{\perp} = 0,$$

$$N_{\parallel}^2 = R \quad (R \text{ wave}),$$

$$N_{\parallel}^2 = L \quad (L \text{ wave})$$

□ Perpendicular / Parallel Cut-off

$$P = 0 : O \text{ wave cutoff}$$

$$R = 0 : X \text{ wave cutoff}$$

$$L = 0 : X \text{ wave cutoff}$$

$$R = 0 : R \text{ wave cutoff}$$

$$L = 0 : L \text{ wave cutoff}$$

□ Perpendicular / Parallel Resonance

$$S = 0 : X \text{ wave resonance (UHR, LHR)}$$

$$R = \infty : R \text{ wave resonance}$$

$$L = \infty : L \text{ wave resonance}$$

RF waves in plasmas

□ Cut-off

$$P = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} = 0 \Rightarrow X \cong 1 \quad : O \text{ wave cutoff} \quad X = \frac{\omega_{pe}^2}{\omega^2}, Y = \frac{\omega_{ce}}{\omega}, \delta = \frac{m_e}{m_i}$$

$$R = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \frac{\omega}{\omega + \Omega_{cs}} = 0 \Rightarrow Y \cong -X + 1 : R(X) \text{ wave cutoff}$$

$$L = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \frac{\omega}{\omega - \Omega_{cs}} = 0 \Rightarrow -\delta Y^2 + Y \cong X - 1 : L(X) \text{ wave cutoff}$$

□ Resonance

$$S = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_{cs}^2} = 0 \Rightarrow Y = (-X + 1)^{1/2} : \text{Upper Hybrid resonance}$$

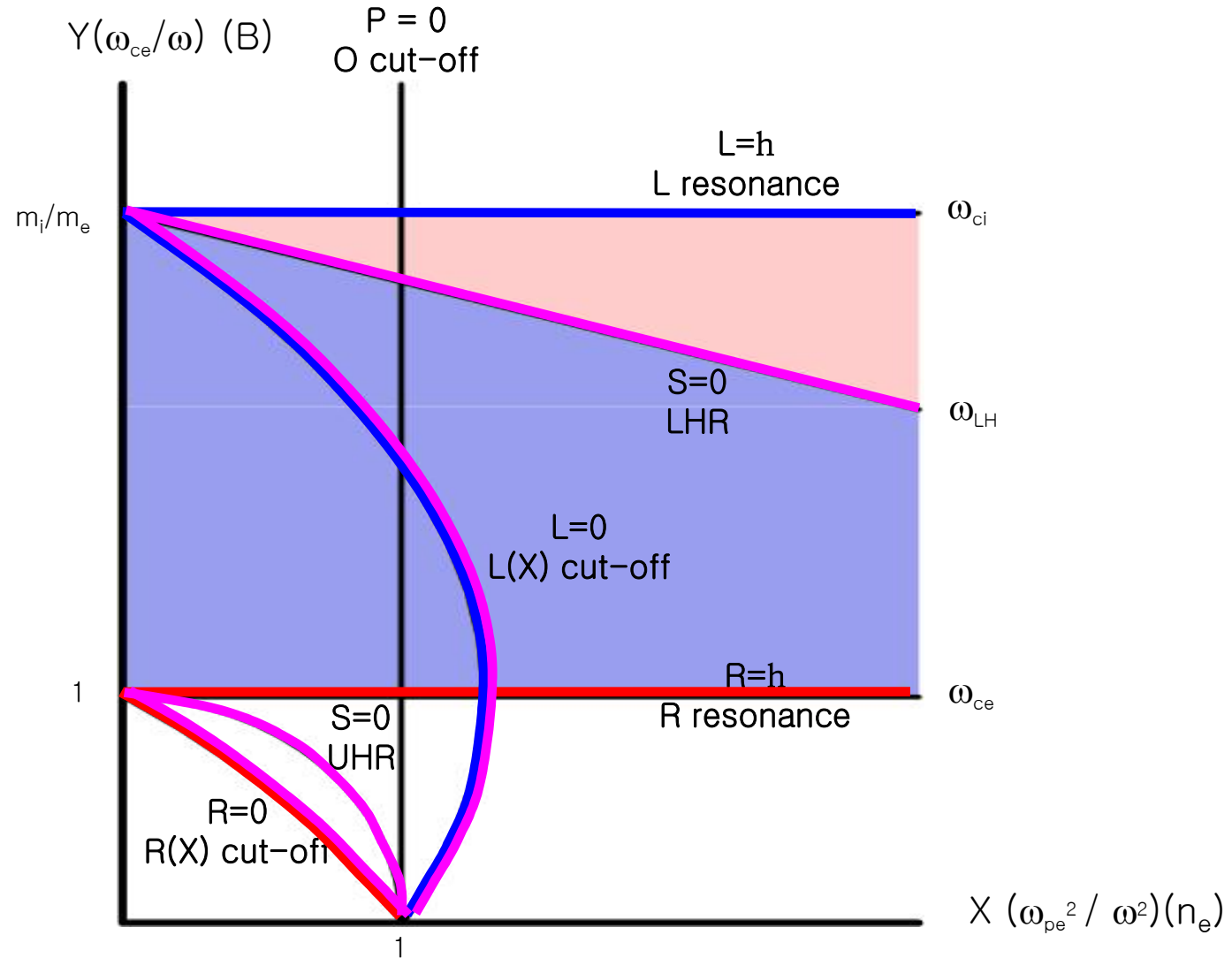
$$\Rightarrow Y^3 - \delta XY^2 + X = 0 : \text{Lower Hybrid resonance}$$

$$R = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \frac{\omega}{\omega + \Omega_{cs}} = \infty \Rightarrow \omega = \omega_{ce} \Rightarrow Y = 1 : \text{electron cyclotron resonance}$$

$$L = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \frac{\omega}{\omega - \Omega_{cs}} = \infty \Rightarrow \omega = \omega_{ci} \Rightarrow Y = \delta^{-1} ; \text{ion cyclotron resonance}$$

RF waves in plasmas

□ CMA diagram



RF waves in plasmas

□ Polarization of 4 wave branches

$$N_{\perp}^2 = \frac{RL}{S} \quad (X \text{ wave})$$

$$N_{\perp}^2 = P \quad (O \text{ wave})$$

X wave

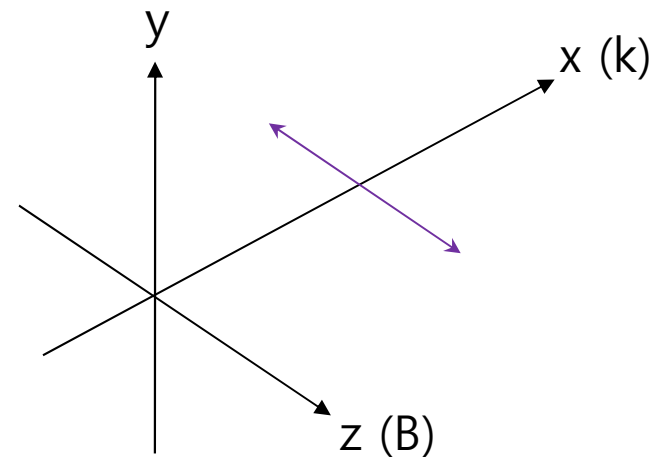
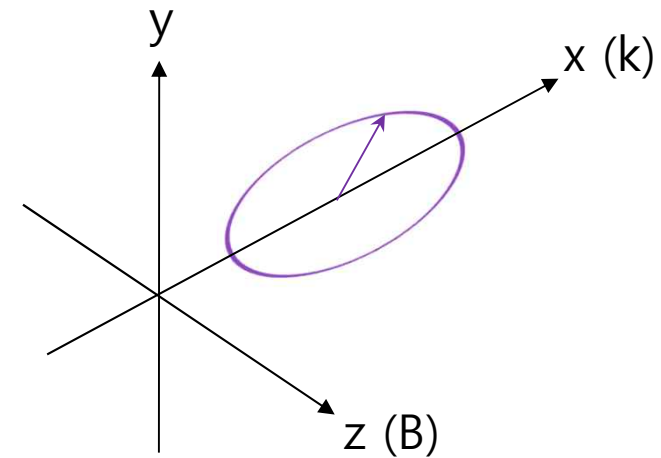
$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_{\perp}^2 - S} = \frac{iSD}{RL - S^2} = -i \frac{S}{D}$$

$$\frac{E_z}{E_x} = \frac{N_{\parallel} N_{\perp}}{N_{\perp}^2 - P} = 0 \quad \frac{\tan \theta_g}{\tan \theta} = \frac{1+P}{P} \sim 1$$

O wave

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_{\perp}^2 - S} = \frac{iD}{P - S}$$

$$\frac{E_z}{E_x} = \frac{N_{\parallel} N_{\perp}}{N_{\perp}^2 - P} = \infty \quad \frac{\tan \theta_g}{\tan \theta} = \frac{S + RL}{RL}$$



RF waves in plasmas

□ Polarization of 4 wave branches

$$N_{\parallel}^2 = R \quad (R \text{ wave}),$$

$$N_{\parallel}^2 = L \quad (L \text{ wave})$$

R wave

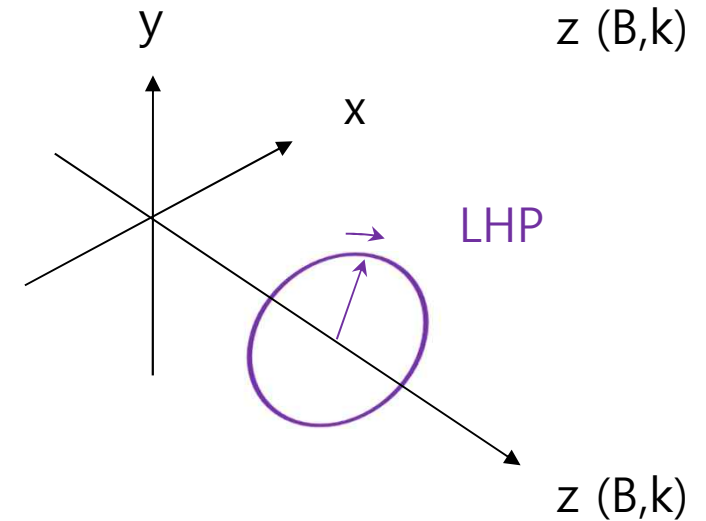
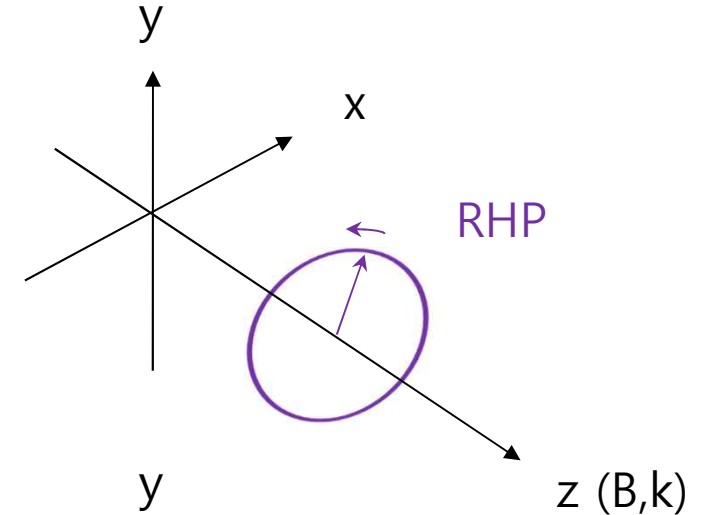
$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_{\parallel}^2 - S} = i \quad \frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = \frac{R - N_{\parallel}^2}{L - N_{\parallel}^2} = 0$$

$$\frac{E_z}{E_x} = \frac{N_{\parallel}N_{\perp}}{N_{\perp}^2 - P} = 0 \quad \frac{\tan \theta_g}{\tan \theta} = \frac{P(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{PR + (R - P)(SR - RL)} = 0$$

L wave

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_{\parallel}^2 - S} = -i \quad \frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = \frac{R - N_{\parallel}^2}{L - N_{\parallel}^2} = \infty$$

$$\frac{E_z}{E_x} = \frac{N_{\parallel}N_{\perp}}{N_{\perp}^2 - P} = 0 \quad \frac{\tan \theta_g}{\tan \theta} = \frac{P(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{PR + (R - P)(SR - RL)} = 0$$



RF waves in plasmas

□ Electrostatic waves

$$E = -\nabla\phi = -ik\phi \Rightarrow E \parallel k$$

$$\vec{N} \times \vec{N} \times E_0 = \vec{\epsilon}_c E_0$$

$$\vec{N} \cdot (\vec{N} \times \vec{N} \times E_0) = \vec{N} \cdot \vec{\epsilon}_c E_0 : \text{Wave equation parallel to propagation}$$

$$\vec{N} \cdot \vec{\epsilon}_c E_0 = 0$$

$$\vec{N} \cdot \vec{\epsilon}_c (E_{\parallel} + E_{\perp}) = 0, \angle(\vec{N}, E_{\parallel}) = 0, \angle(\vec{N}, E_{\perp}) = 90^\circ$$

$$\Rightarrow (\vec{N} \cdot \vec{\epsilon}_c \cdot \vec{N}) E_{\parallel} = 0$$

$$\Rightarrow SN_{\perp}^2 + PN_{\parallel}^2 = 0 : \text{Dispersion relation of cold electrostatic waves}$$

□ For X waves

if $S = 0$ at UHR, LHR in X wave

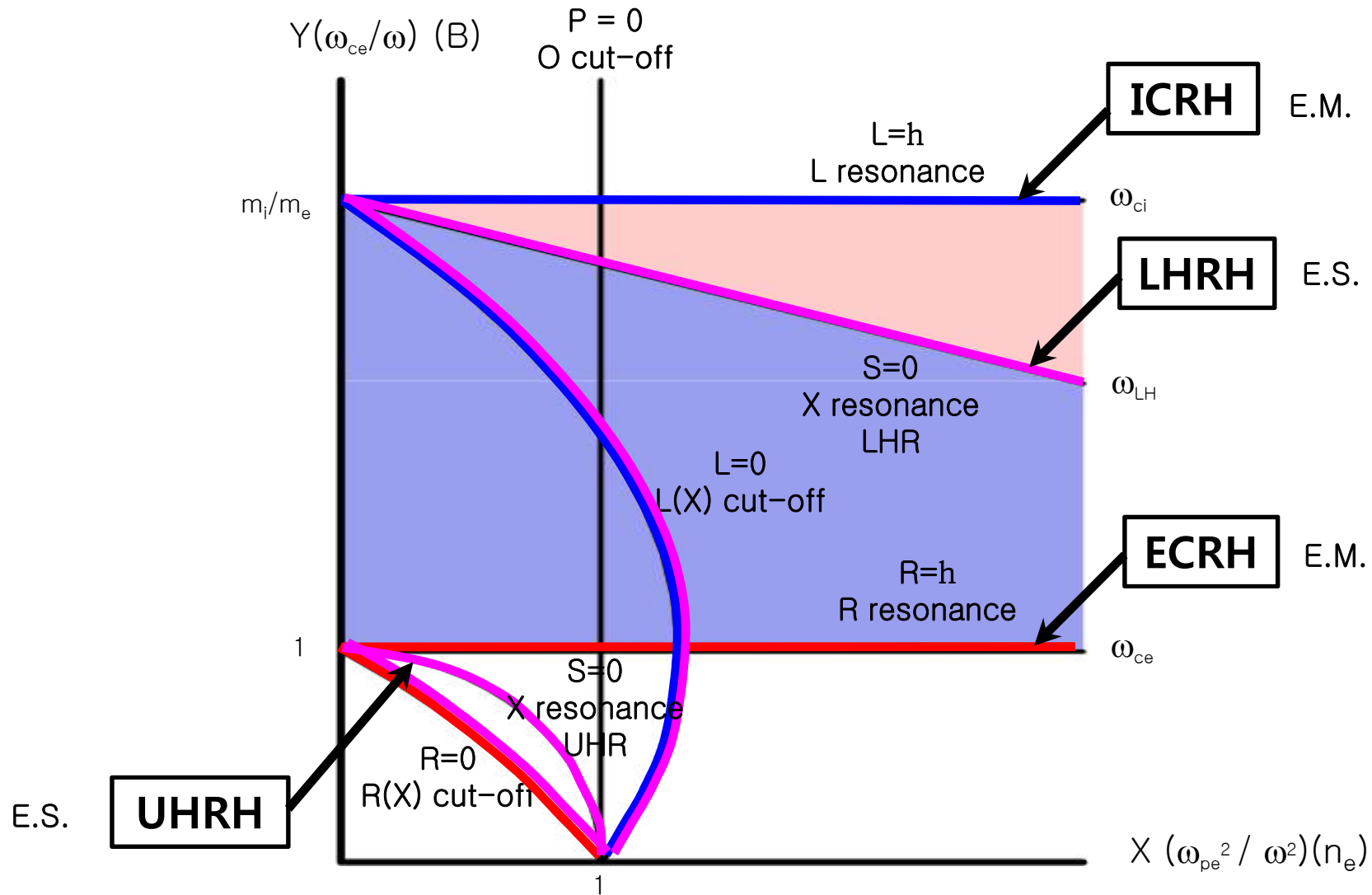
$$N_{\perp}^2 = \frac{RL}{S} \text{ (X wave)} \rightarrow \infty \text{ at UHR, LHR}$$

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_{\perp}^2 - S} = \frac{iSD}{RL - S^2} = -i \frac{S}{D} \rightarrow 0$$

\Rightarrow Purely x polarization $\Rightarrow k = k_x \parallel E_x \therefore$ X wave becomes e.s. at UHR, LHR

RF waves in plasmas

- What kinds of waves can be used? We should use **resonances**.



RF waves in plasmas

□ Oblique injection

RF wave is launched obliquely but almost perpendicular to magnetic field in tokamak with fixed parallel refractive index which just changes in the major radius direction.

$$AN_{\perp}^4 + BN_{\perp}^2 + C = 0$$

$$A = S$$

$$B = N_{\parallel}^2(S + P) - (SP + RL)$$

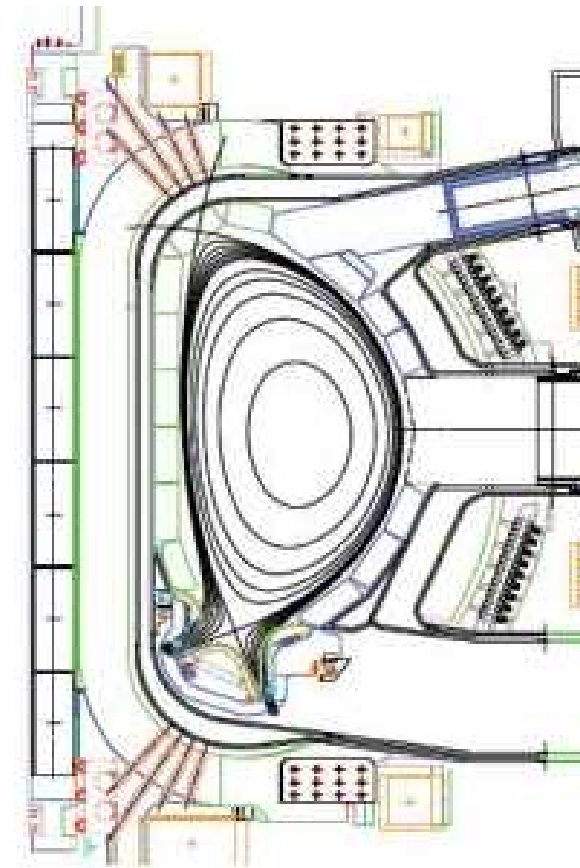
$$C = P(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)$$

$$N_{\perp}^2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\cong -\frac{P(N_{\parallel}^2 - S)}{S} + \frac{PD^2 N_{\parallel}^2}{S[(N_{\parallel}^2 - S)(S - P) + D^2]}$$

$$-\frac{S(N_{\parallel}^2 - S) + D^2}{S} - \frac{PD^2 N_{\parallel}^2}{S[(N_{\parallel}^2 - S)(S - P) + D^2]}$$

$$\cong -\frac{P(N_{\parallel}^2 - S)}{S}, -\frac{(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{(N_{\parallel}^2 - S)} : \text{Low frequency range}$$



RF waves in plasmas

□ Polarization of oblique injection

$$\begin{aligned}
 N_{\perp}^2 &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \\
 &\cong -\frac{P(N_{\parallel}^2 - S)}{S} + \frac{PD^2 N_{\parallel}^2}{S[(N_{\parallel}^2 - S)(S - P) + D^2]}, -\frac{S(N_{\parallel}^2 - S) + D^2}{S} - \frac{PD^2 N_{\parallel}^2}{S[(N_{\parallel}^2 - S)(S - P) + D^2]} \\
 &\cong -\frac{P(N_{\parallel}^2 - S)}{S}, -\frac{(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{(N_{\parallel}^2 - S)} : \text{Low frequency range}
 \end{aligned}$$

□ Slow wave and Fast wave (Low frequency range)

Slow wave

$$\begin{aligned}
 \frac{E_y}{E_x} &= \frac{iD}{N^2 - S} = \frac{iSD}{(N_{\parallel}^2 - S)(S - P)} \\
 \frac{E_z}{E_x} &= \frac{N_{\parallel} N_{\perp}}{N_{\perp}^2 - P} = -\frac{SN_{\perp}}{PN_{\parallel}}
 \end{aligned}$$

Fast wave

$$\begin{aligned}
 \frac{E_y}{E_x} &= \frac{iD}{N^2 - S} = \frac{i(N_{\parallel}^2 - S)}{D} \\
 \frac{E_z}{E_x} &= \frac{N_{\parallel} N_{\perp}}{N_{\perp}^2 - P} \quad \frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = -\frac{R - N_{\parallel}^2}{L - N_{\parallel}^2}
 \end{aligned}$$

RF waves in plasmas

- Polarization near ion cyclotron resonance of oblique injection
- Slow wave and Fast wave

Slow wave $\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iSD}{(N_{\parallel}^2 - S)(S - P)} \rightarrow -\frac{iD}{S} \rightarrow i, E_+ \rightarrow 0 : RHP, \text{ near } ICR$

$$\frac{E_z}{E_x} = \frac{N_{\parallel}N_{\perp}}{N_{\perp}^2 - P} = -\frac{SN_{\perp}}{PN_{\parallel}} \rightarrow \infty$$

Fast wave $\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{i(N_{\parallel}^2 - S)}{D} \rightarrow -\frac{iS}{D} \rightarrow i, E_+ \rightarrow 0 : RHP, \text{ near } ICR$

$$\frac{E_z}{E_x} = \frac{N_{\parallel}N_{\perp}}{N_{\perp}^2 - P} \rightarrow 0$$

- *There are no cyclotron absorption near fundamental ion cyclotron resonances for obliquely injected cold slow or fast waves (This result is similar for electron cyclotron resonance).*

RF waves in plasmas

- ICRF fast wave has not favorable LHP near fundamental ion cyclotron resonance.

$$N_{\perp}^2 \cong -\frac{(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{(N_{\parallel}^2 - S)}$$

$$\frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = -\frac{R - N_{\parallel}^2}{L - N_{\parallel}^2} = -\frac{D + S - N_{\parallel}^2}{-D + S - N_{\parallel}^2}$$

- If $(N_{\parallel}^2 - S) = 0$, then

$$N_{\perp}^2 \rightarrow \infty,$$

$$\frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = -\frac{R - N_{\parallel}^2}{L - N_{\parallel}^2} = -\frac{D + S - N_{\parallel}^2}{-D + S - N_{\parallel}^2} = 1$$

- More rigorous absorption can be obtained from the hot dielectric tensor.
- $(N_{\parallel}^2 - S) = 0$ can be achieved with multi-species (major and minority ion species).
- There are two regimes with respect to the minority fraction, $\eta = n_m/n_M$.

$\eta < \eta_c$: Minority heating regime

$\eta > \eta_c$: Ion - Ion hybrid resonance regime

RF waves in plasmas (Summary I)

- ❑ One obtain 4 wave branches from cold dielectric tensors.
- ❑ And there are four wave resonances.
- ❑ We should use the resonance for plasma heating.
- ❑ However, there is very weak collision in fusion plasmas.
- ❑ Therefore, there is only weak power absorption even in resonances.
- ❑ In addition, there is no cyclotron resonance heating for obliquely injected waves.

- ❑ As a result, we should analyze the power absorption with a hot dielectric tensor.
- ❑ It means that wave power absorption in fusion plasmas is possible via kinetic effect.

RF waves in plasmas (Power absorption)

- Power absorption can be represented as follows.

$$\begin{aligned}
 P_{abs} &= \frac{1}{2} \text{Re}[J \cdot E^*] \\
 &= \frac{1}{2} \text{Re}[E^* \cdot \vec{\sigma} \cdot E] \\
 &= \frac{1}{2} \text{Re}[(-i\varepsilon_0\omega)E^* \cdot (\vec{\varepsilon}_r - \vec{I}) \cdot E] \\
 &= \frac{1}{2} \varepsilon_0\omega[E^* \cdot \vec{\varepsilon}_A \cdot E]
 \end{aligned}$$

$$\begin{aligned}
 \vec{\varepsilon}_r &= \vec{\varepsilon}_H + i\vec{\varepsilon}_A \\
 \vec{\varepsilon}_H &= \frac{1}{2}[\vec{\varepsilon}_r + \vec{\varepsilon}_r^{T*}] \\
 \vec{\varepsilon}_A &= \frac{1}{2i}[\vec{\varepsilon}_r - \vec{\varepsilon}_r^{T*}]
 \end{aligned}$$

- For Maxwellian plasmas

$$P_{abs} = \frac{1}{2} \varepsilon_0 \omega \sum_{i,j} E_i^* \cdot \vec{\varepsilon}_{Aij} \cdot E_j$$

$$\lim_{k_{\parallel} \rightarrow 0} \frac{\omega}{k_{\parallel} v_{ths}} e^{-\frac{(\omega - n\Omega_{cs})^2}{k_{\parallel}^2 v_{ths}^2}} = \sqrt{\pi} \delta\left(\frac{\omega - n\Omega_{cs}}{\omega}\right)$$

$$P_{LD} \cong \frac{1}{2} \varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} 2\sqrt{\pi} \frac{\omega^3}{k_{\parallel}^3 v_{ths}^3} e^{-\frac{\omega^2}{k_{\parallel}^2 v_{ths}^2}} |E_z|^2 \quad (n=0)$$

$$P_{MP} \cong \frac{1}{2} \varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \frac{k_{\perp}^2 v_{ths}^2}{\Omega_{cs}^2} \sqrt{\pi} \frac{\omega}{k_{\parallel} v_{ths}} e^{-\frac{\omega^2}{k_{\parallel}^2 v_{ths}^2}} |E_y|^2 \quad (n=0)$$

$$P_{\Omega} \cong \frac{1}{2} \varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \frac{k_{\parallel}^2 v_{ths}^2}{\Omega_{cs}^2} g \sqrt{\pi} \frac{\omega}{k_{\parallel} v_{ths}} e^{-\frac{(\omega - \Omega_{cs})^2}{k_{\parallel}^2 v_{ths}^2}} |E|^2 \quad (n=1)$$

$$P_{n\Omega} \cong \frac{1}{2} \varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \left(\frac{k_{\perp}^2 v_{ths}^2}{2\Omega_{cs}^2}\right)^{n-1} \sqrt{\pi} \frac{\omega}{k_{\parallel} v_{ths}} e^{-\frac{(\omega - n\Omega_{cs})^2}{k_{\parallel}^2 v_{ths}^2}} |E_{\pm}|^2 \quad (n \geq 2)$$

RF waves in plasmas (Landau damping)

□ Landau damping

$$P_{LD} \cong \frac{1}{2} \epsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} 2\sqrt{\pi} \frac{\omega^3}{k_{\parallel}^3 v_{ths}^3} e^{-\frac{\omega^2}{k_{\parallel}^2 v_{ths}^2}} |E_z|^2$$

- Optimum phase velocity : $\frac{\omega}{k_{\parallel}} \sim v_{ths}$
- Electric field parallel to magnetic field is required.
- Low frequency is better for given E_z field.
- Slow wave has large E_z electric field.

□ General form(non-Maxwellian plasmas) & Picture

$$P_{LD} \cong \frac{1}{2} \epsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \pi \frac{\omega}{|k_{\parallel}|} \int_0^{\infty} \left(-v_{\parallel} \frac{\partial F_s}{\partial v_{\parallel}} \right)_{v_{\parallel}=\omega/k_{\parallel}} v_{\perp} dv_{\perp} |E_z|^2$$

- It requires negative particle distribution near particle phase velocity.

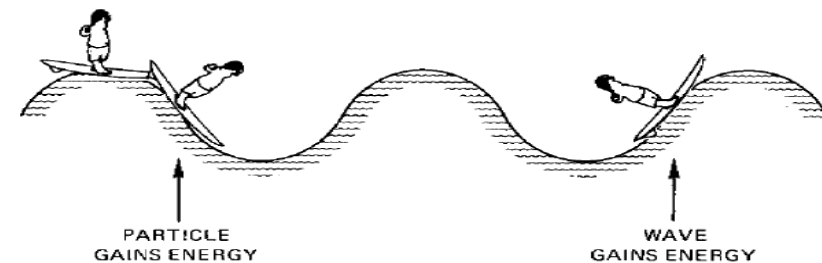
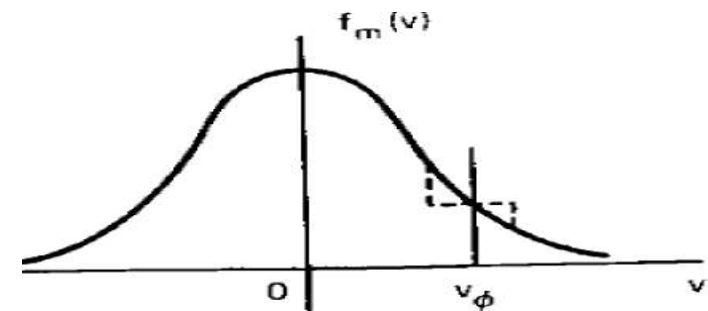


FIGURE 7-17 Customary physical picture of Landau damping.

RF waves in plasmas (TTMP)

□ TTMP : Transit Time Magnetic Pumping

$$P_{MP} \cong \frac{1}{2} \varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \frac{k_{\perp}^2 v_{ths}^2}{\Omega_{cs}^2} \sqrt{\pi} \frac{\omega}{k_{\parallel} v_{ths}} e^{-\frac{\omega^2}{k_{\parallel}^2 v_{ths}^2}} |E_y|^2$$

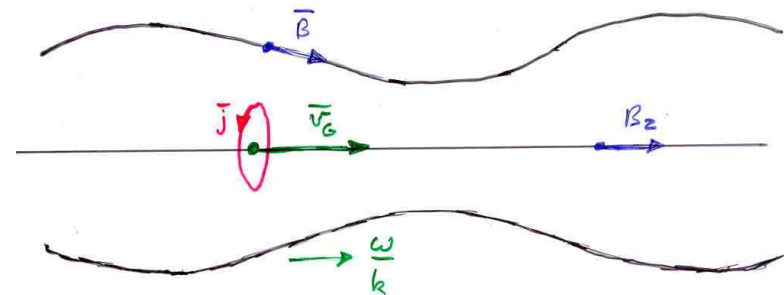
- Optimum phase velocity : $\frac{\omega}{k_{\parallel}} \sim v_{ths}$
- E_y perpendicular to magnetic field is required.
- Low frequency is better for given E_y .
- Fast wave has large E_y electric field (B_z).

□ Picture

- Driving force comes from the gradient of wave magnetic field which gyrating particles by external magnetic field feel during parallel motion in phase of phase velocity.

$$F_{MP} \cong -\mu \nabla B_z$$

- It is similar to Landau damping in view that it gain energy from wave during motion in phase of wave phase velocity except that it just gain energy from wave magnetic field instead of electric field



R. Koch, "Summer school in KAIST" 2009

RF waves in plasmas (Cyclotron damping)

□ Fundamental cyclotron damping

$$P_{\Omega} \cong \frac{1}{2} \varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \frac{k_{\parallel}^2 v_{ths}^2}{\Omega_{cs}^2} g \sqrt{\pi} \frac{\omega}{k_{\parallel} v_{ths}} e^{-\frac{(\omega - \Omega_{cs})^2}{k_{\parallel}^2 v_{ths}^2}} |E| \leftarrow \frac{1}{2} \varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \sqrt{\pi} \frac{\omega}{k_{\parallel} v_{ths}} e^{-\frac{(\omega - \Omega_{cs})^2}{k_{\parallel}^2 v_{ths}^2}} |E_{\pm}|^2$$

- There is no power absorption without parallel wave number.
- It is because the field polarization is RHP.

□ Harmonic cyclotron damping

$$P_{n\Omega} \cong \frac{1}{2} \varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \left(\frac{k_{\perp}^2 v_{ths}^2}{2\Omega_{cs}^2} \right)^{n-1} \sqrt{\pi} \frac{\omega}{k_{\parallel} v_{ths}} e^{-\frac{(\omega - n\Omega_{cs})^2}{k_{\parallel}^2 v_{ths}^2}} |E_{\pm}|^2$$

- Harmonic cyclotron damping is possible due to FLR(Finite Larmor Radius) effect.
- If Larmor radius is comparable to wavelength, the gyrating particles feel the non-uniform electric field during one gyration period.
- As a result, it is accelerated in average by the LH or RH circulating wave electric field with harmonic frequency.
- Power absorption decreases as the harmonic number increases if $k_{\perp} r_L < 1$. Therefore, Landau damping or TTMP becomes important for high harmonic heating in HHFW heating on ST.

RF waves in plasmas (Current Drive)

- ❑ One can calculate RF heating from a hot dielectric tensor of Maxwellian plasmas. However, one cannot obtain current drive by the power absorption since the Maxwell distribution Function is symmetric in velocity space. In addition, the power absorption can be different for non-Maxwellian plasmas.
- ❑ Therefore, we should know the changed asymmetric particle distribution by the heating.
- ❑ It can be obtained from Vlasov equation with collision(Fokker-Planck equation) in longer time scale than the wave period.

$$\frac{df_s}{dt} = \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \mathbf{a} \cdot \nabla_{\mathbf{v}} f_s = C(f_s), \quad \mathbf{a} = \frac{eZ_s}{m_s} \left[\mathbf{E} + \mathbf{v} \times (\mathbf{B} + \mathbf{B}_0) \right]$$

$$f_s = F_s(t, r, \mathbf{v}) + \tilde{f}_s(t, r, \mathbf{v})$$

$$\begin{aligned} \frac{dF_s}{dt} &= \frac{\partial F_s}{\partial t} + \mathbf{v} \cdot \nabla F_s + \frac{eZ_s}{m_s} [\mathbf{v} \times \mathbf{B}_0] \cdot \nabla_{\mathbf{v}} F_s = -\frac{eZ_s}{m_s} [\mathbf{E} + \mathbf{v} \times \mathbf{B}] \cdot \nabla_{\mathbf{v}} \tilde{f}_s + C(F_s) \\ &= Q(F_s) + C(F_s) \end{aligned}$$

Quasi-linear term by waves

RF waves in plasmas (Current Drive)

- Quasi-linear operator can be represented as follows.

$$Q(F_s) = \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} \left[v_\perp \left(D_{v_\perp v_\perp} \frac{\partial F_s}{\partial v_\perp} + D_{v_\perp v_\parallel} \frac{\partial F_s}{\partial v_\parallel} \right) \right] + \frac{\partial}{\partial v_\parallel} \left(D_{v_\parallel v_\perp} \frac{\partial F_s}{\partial v_\perp} + D_{v_\parallel v_\parallel} \frac{\partial F_s}{\partial v_\parallel} \right)$$

$$D_{v_\perp v_\perp} = \frac{\pi}{2\omega} \left(\frac{Ze}{m_s} \right)^2 \sum_n \delta \left(\frac{\omega - n\Omega_{cs} - k_\parallel v_\parallel}{\omega} \right) |d_\perp^{(n)} E|^2$$

$$D_{v_\perp v_\parallel} = D_{v_\parallel v_\perp} = \frac{\pi}{2\omega} \left(\frac{Ze}{m_s} \right)^2 \sum_n \delta \left(\frac{\omega - n\Omega_{cs} - k_\parallel v_\parallel}{\omega} \right) \text{Re} \left[d_\perp^{(n)*} E \cdot d_\parallel^{(n)} E \right]$$

$$D_{v_\parallel v_\parallel} = \frac{\pi}{2\omega} \left(\frac{Ze}{m_s} \right)^2 \sum_n \delta \left(\frac{\omega - n\Omega_{cs} - k_\parallel v_\parallel}{\omega} \right) |d_\parallel^{(n)} E|^2$$

$$d_\perp^{(n)} E = \frac{1}{\sqrt{2}} \left(1 - \frac{k_\parallel v_\parallel}{\omega} \right) \left[J_{n-1} \left(\frac{k_\perp v_\perp}{\Omega_{cs}} \right) E_+ e^{-i\psi} + J_{n-1} \left(\frac{k_\perp v_\perp}{\Omega_{cs}} \right) E_- e^{i\psi} \right] + \frac{v_\parallel}{v_\perp} \frac{n\Omega_{cs}}{\omega} J_n \left(\frac{k_\perp v_\perp}{\Omega_{cs}} \right) E_z$$

$$d_\parallel^{(n)} E = \frac{1}{\sqrt{2}} \frac{k_\parallel v_\perp}{\omega} \left[J_{n-1} \left(\frac{k_\perp v_\perp}{\Omega_{cs}} \right) E_+ e^{-i\psi} + J_{n-1} \left(\frac{k_\perp v_\perp}{\Omega_{cs}} \right) E_- e^{i\psi} \right] + \left(1 - \frac{n\Omega_{cs}}{\omega} \right) J_n \left(\frac{k_\perp v_\perp}{\Omega_{cs}} \right) E_z$$

RF waves in plasmas (Current Drive)

- Quasi-linear operator can be represented as follows.

$$Q(F_s) = \frac{1}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} \left[v_{\perp} \left(D_{v_{\perp}v_{\perp}} \frac{\partial F_s}{\partial v_{\perp}} + D_{v_{\perp}v_{\parallel}} \frac{\partial F_s}{\partial v_{\parallel}} \right) \right] + \frac{\partial}{\partial v_{\parallel}} \left(D_{v_{\parallel}v_{\perp}} \frac{\partial F_s}{\partial v_{\perp}} + D_{v_{\parallel}v_{\parallel}} \frac{\partial F_s}{\partial v_{\parallel}} \right)$$

$$D_{v_{\perp}v_{\perp}} = \frac{\pi}{2\omega} \left(\frac{Ze}{m_s} \right)^2 \sum_n \delta \left(\frac{\omega - n\Omega_{cs} - k_{\parallel}v_{\parallel}}{\omega} \right) |d_{\perp}^{(n)} E|^2$$

$$D_{v_{\perp}v_{\parallel}} = D_{v_{\parallel}v_{\perp}} = \frac{\pi}{2\omega} \left(\frac{Ze}{m_s} \right)^2 \sum_n \delta \left(\frac{\omega - n\Omega_{cs} - k_{\parallel}v_{\parallel}}{\omega} \right) \text{Re} \left[d_{\perp}^{(n)*} E \cdot d_{\parallel}^{(n)} E \right]$$

$$D_{v_{\parallel}v_{\parallel}} = \frac{\pi}{2\omega} \left(\frac{Ze}{m_s} \right)^2 \sum_n \delta \left(\frac{\omega - n\Omega_{cs} - k_{\parallel}v_{\parallel}}{\omega} \right) |d_{\parallel}^{(n)} E|^2$$

$$d_{\perp}^{(n)} E = \frac{1}{\sqrt{2}} \left(1 - \frac{k_{\parallel}v_{\parallel}}{\omega} \right) \left[J_{n-1} \left(\frac{k_{\perp}v_{\perp}}{\Omega_{cs}} \right) E_+ e^{-i\psi} + J_{n-1} \left(\frac{k_{\perp}v_{\perp}}{\Omega_{cs}} \right) E_- e^{i\psi} \right] + \frac{v_{\parallel}}{v_{\perp}} \frac{n\Omega_{cs}}{\omega} J_n \left(\frac{k_{\perp}v_{\perp}}{\Omega_{cs}} \right) E_z$$

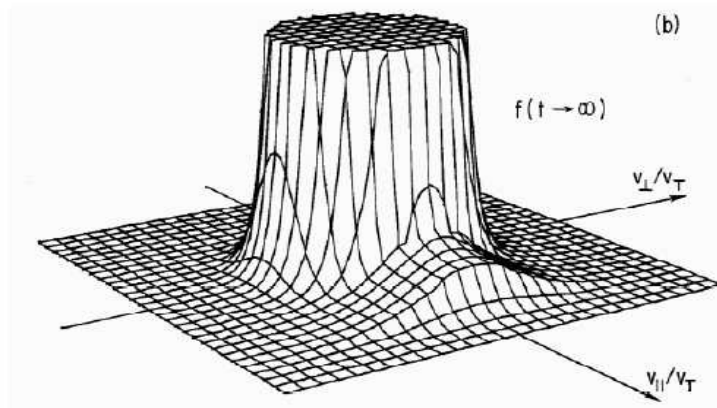
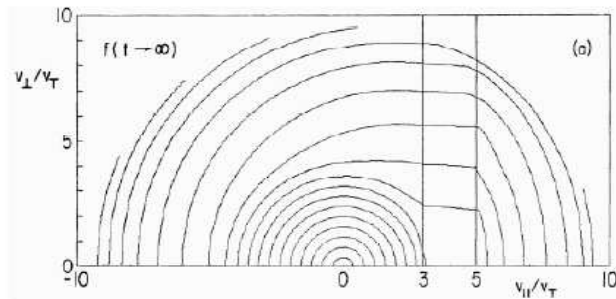
$$d_{\parallel}^{(n)} E = \frac{1}{\sqrt{2}} \frac{k_{\parallel}v_{\perp}}{\omega} \left[J_{n-1} \left(\frac{k_{\perp}v_{\perp}}{\Omega_{cs}} \right) E_+ e^{-i\psi} + J_{n-1} \left(\frac{k_{\perp}v_{\perp}}{\Omega_{cs}} \right) E_- e^{i\psi} \right] + \left(1 - \frac{n\Omega_{cs}}{\omega} \right) J_n \left(\frac{k_{\perp}v_{\perp}}{\Omega_{cs}} \right) E_z$$

RF waves in plasmas (Current Drive)

- Fokker Plank Equation for current drive by Landau damping can be represented as follows.

$$-n = 0, \frac{\omega}{k_{\parallel}} \sim v_{\parallel}$$

$$\frac{\partial F_e}{\partial t} - \frac{e}{m_s} E_0 \frac{\partial F_e}{\partial v_{\parallel}} = \frac{\partial}{\partial v_{\parallel}} \left(D_{v_{\parallel}v_{\parallel}} \frac{\partial F_e}{\partial v_{\parallel}} \right) + C(F_e)$$

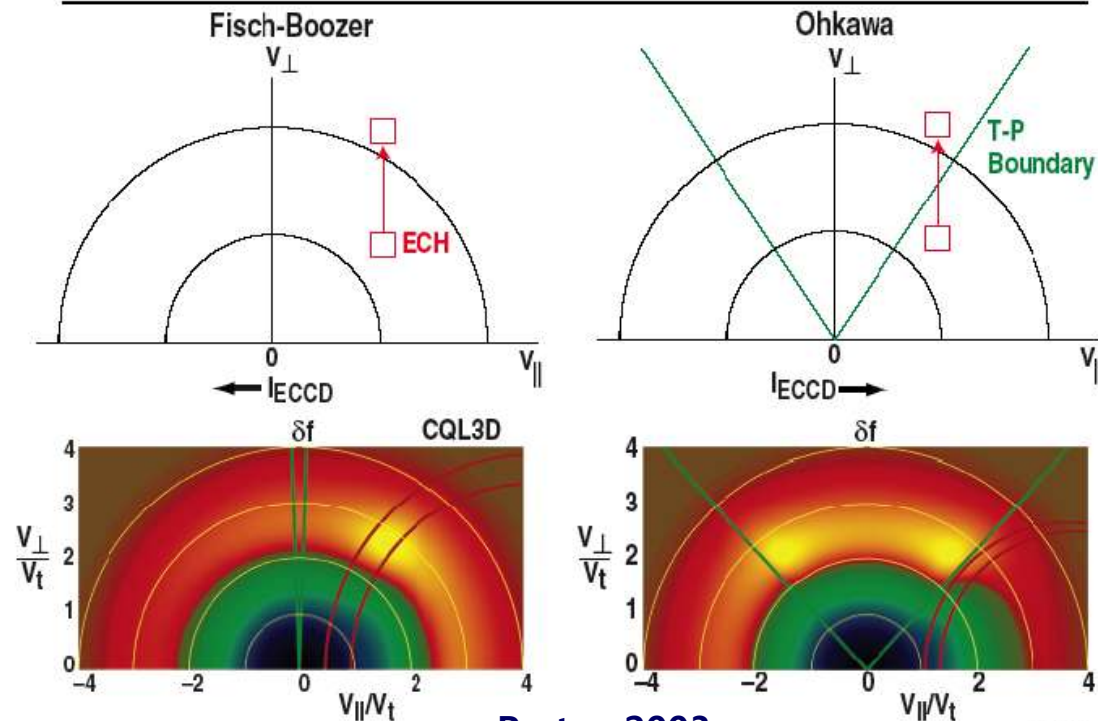


Karney & Fisch, 1979

RF waves in plasmas (Current Drive)

- Generally, current drive is possible if the distribution function is asymmetry in phase space.
 - Minority heating current drive / NB current drive
 - Ohkawa/Fisch-Boozer current drive (ECRF range)

ELECTRON CYCLOTRON CURRENT DRIVE IN TOROIDAL SYSTEMS IS DRIVEN BY TWO COMPETING EFFECTS



Prater, 2003

RF waves in plasmas (Current Drive)

- Current drive efficiency (rough estimation)

$$\left. \begin{aligned} \Delta E &= n_e m_e v_{\parallel} \Delta v_{\parallel} \\ j &= n_e e \Delta v_{\parallel} \\ p_d &= \Delta E v \end{aligned} \right\} \therefore \frac{j}{p_d} = \frac{e}{m_e v v_{\parallel}}$$

$$\frac{j}{p_d} \sim \begin{cases} 1/v_{\parallel} & : v \sim \text{const.} \text{ for low phase velocity : ICRF range} \\ v_{\parallel}^{-2} & : v \sim v_{\parallel}^{-3} \text{ for high phase velocity : LHRF range} \end{cases}$$

- Current drive efficiency (rigorous estimation)

$$\begin{aligned} \frac{j}{p_d} &= \frac{e}{m_e v_0 v_{Te}^3} \frac{2}{(5 + Z_{eff})} \frac{\hat{s} \cdot (\partial / \partial \vec{v})(v_{\parallel} v^3)}{\hat{s} \cdot (\partial / \partial \vec{v})v^2} \\ &= \frac{e}{m_e v_0 v_{Te}^3} \frac{2}{(5 + Z_{eff})} \frac{v^3 + 3v v_{\parallel}^2}{2v_{\parallel}} \text{ for parallel acceleration} \end{aligned}$$

- Current drive efficiency in practical units and Figure of merit

$$\frac{I}{P} = \frac{Aj}{2\pi R A p_d} = 0.061 \frac{T_e}{R n_e^{20} \ln \Lambda} \left(\frac{J}{P_d} \right) [A/W], \quad \frac{J}{P_d} = \frac{\hat{s} \cdot (\partial / \partial \vec{u})(u_{\parallel} u^3)}{\hat{s} \cdot (\partial / \partial \vec{u})u^2} \quad u = v/v_{th}$$

$$\eta = \frac{I}{P} R n_e^{20} [A/W/m^2]$$

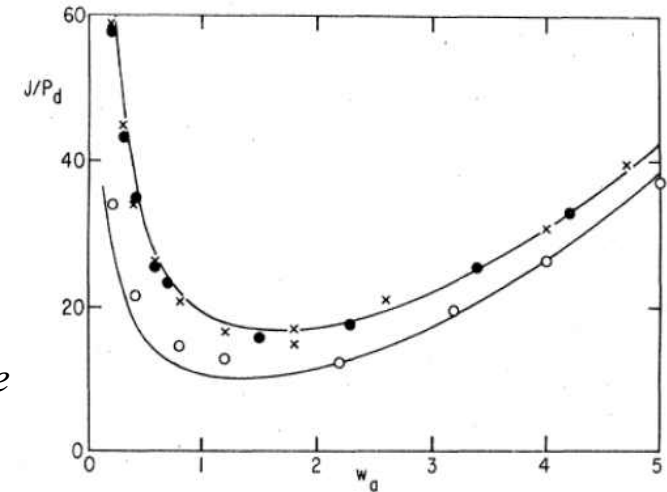
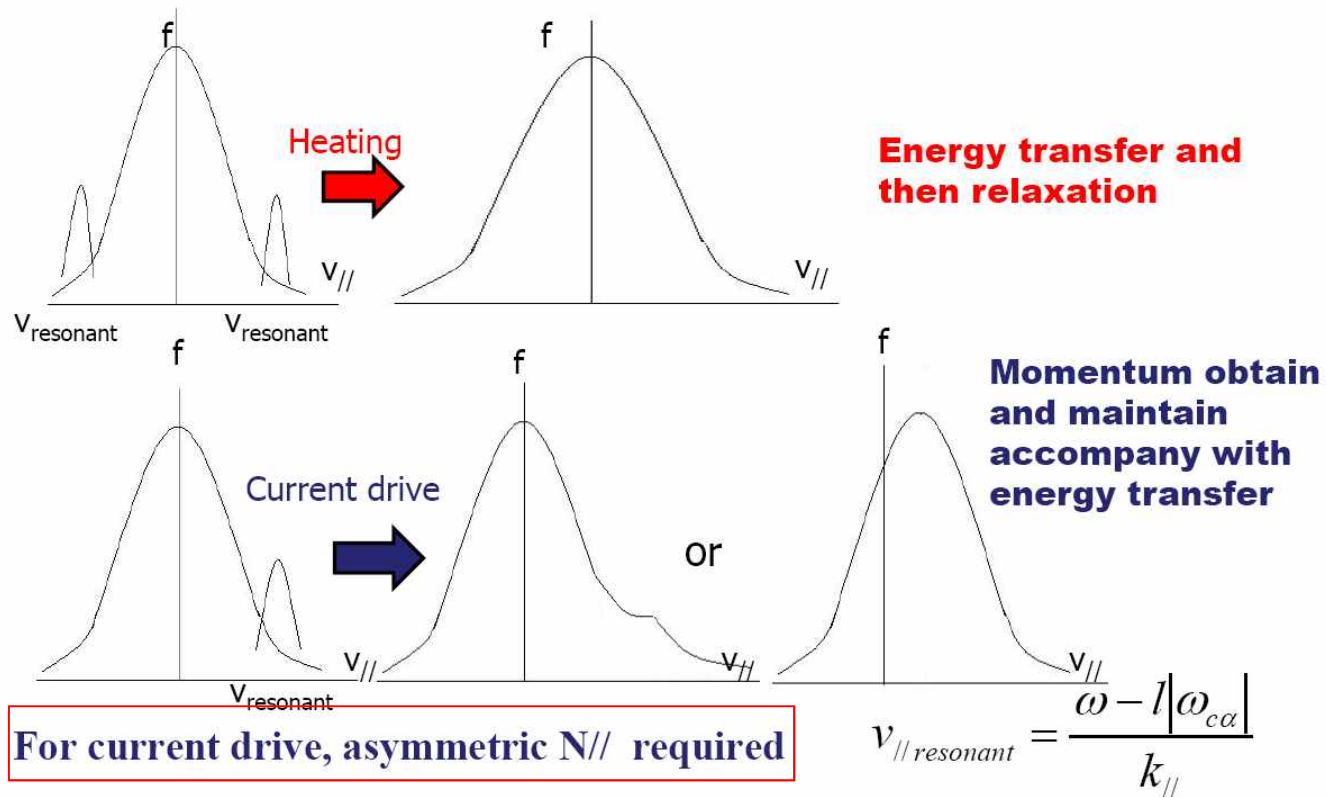


FIG. 21. Normalized J/P_d vs average normalized parallel-phase velocity w_d : \circ , Landau damping; \times , magnetic pumping; \bullet , Alfvén waves in the limit $D_{QL} \rightarrow 0$. The solid curves are rough semianalytic fits to the data (Fisch and Karney, 1981).

RF waves in plasmas (Heating)

- What is the difference between Current drive and Heating?

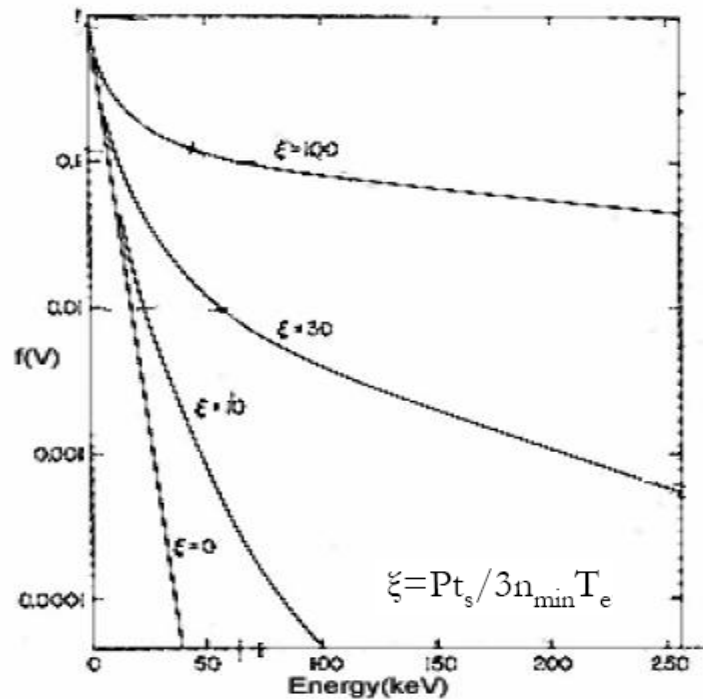
Heating and current drive



Z. Gao, "Summer school in KAIST" 2009

RF waves in plasmas (Heating)

- ICRF Harmonic or minority cyclotron heating



$$\ln f(v) = -\frac{E}{T_e(1+\xi)} \left[1 + \frac{R_f(T_e - T_f + \xi T_e)}{T_f(1+R_f+\xi)} K(E/E_f) \right]$$

Stix, "Waves in Plasmas" 1992

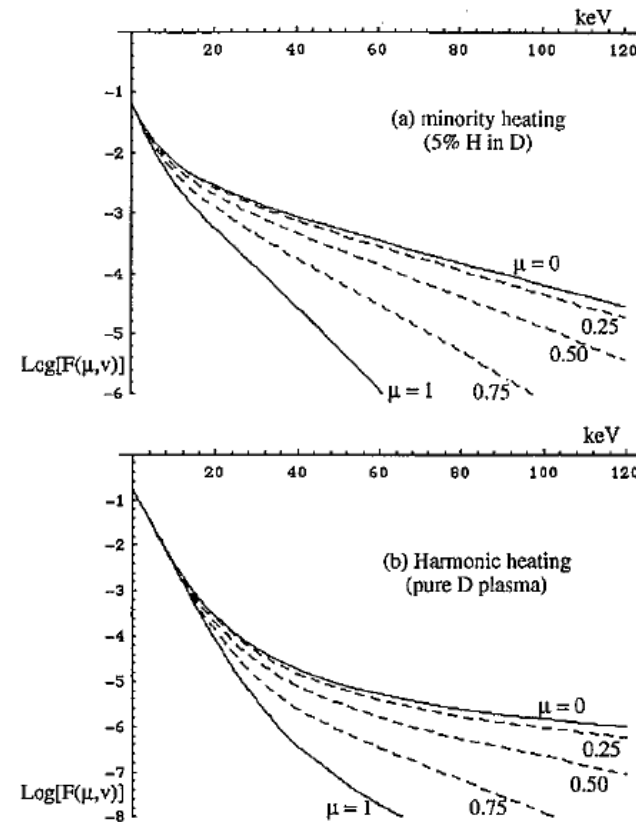


FIG. 43.11 Ion distribution function during ion cyclotron heating, $n_e = 8 \times 10^{13} \text{ cm}^{-3}$, $B_0 = 5 \text{ T}$, 'background' temperature 5 keV, 'linear' power density 0.5 W/cm^3 .

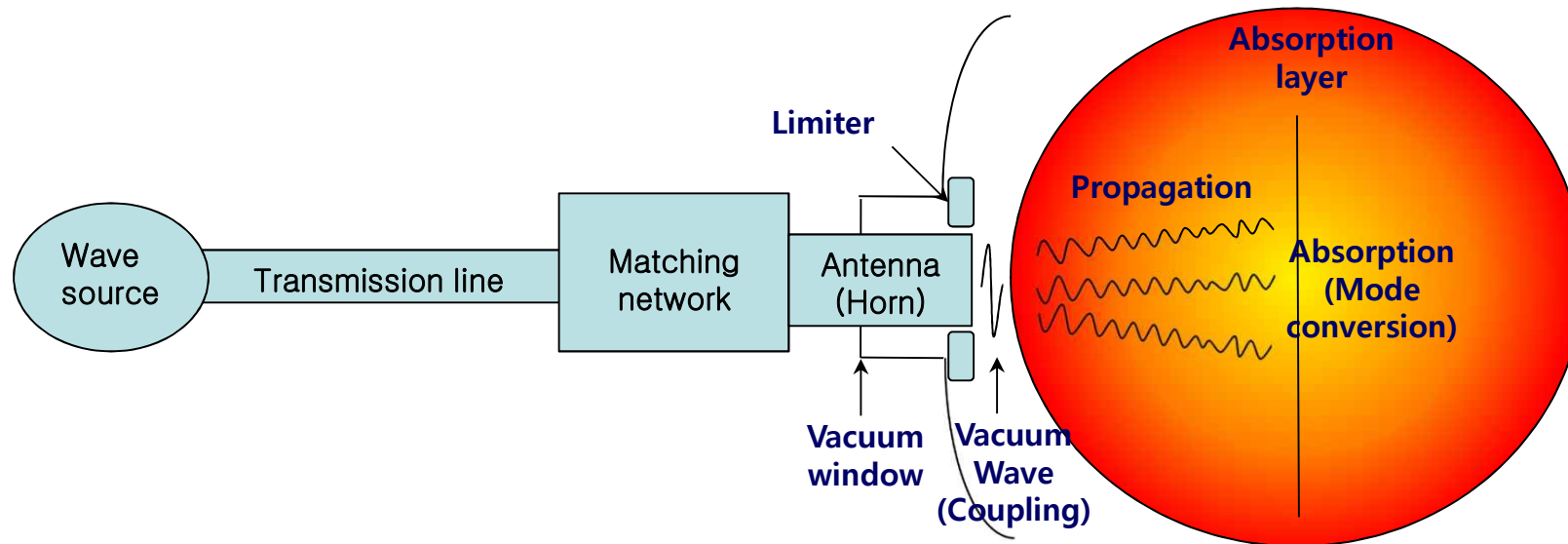
Brambila, "Kinetic theory of plasma waves" 1995

RF waves in plasmas (Summary II)

- General RF heating and current drive can be obtained through quasilinear Fokker-Planck equation.
- Heating and current drive is the result of the increase of high energy population in phase space.

Wave launching, propagation, absorption in fusion plasmas

- ❑ RF waves in fusion plasmas is usually launched from LFS(Low Field Side) with different launching structure for each frequency range.
- ❑ And it propagates through non-uniform plasmas.
- ❑ Finally, the wave power is absorbed near cyclotron resonance layer (harmonic cyclotron damping) and bulk plasmas (Landau damping or TTMP).
- ❑ Sometimes, the wave is mode converted into hot electrostatic wave branches(Ion or electron Bernstein waves) and finally absorbed through cyclotron resonance or Landau damping.



Wave launching, propagation, absorption in fusion plasmas (ICRF/LHRF/ECRF)

- RF waves in fusion plasmas is usually launched from LFS(Low Field Side) with different launching structure for each frequency range.

	Sources	Transmission	Coupling	Objectives
ICRF	Tube 25-100MHz 2 MW	Coaxial Line	Antenna (Current Strap)	Localised ion heating. Central CD Sawtooth control
LH	Klystron 1~5GHz 1MW	Waveguide	Waveguide grill	Off-axis CD for SS regimes. AT scenarios. Assisted ramp-up.
ECRF	Gyrotron 50~200GHz 1MW	Waveguide	Horn	Heating. Central CD. MHD control (NTM). Plasma start-up

Wave launching, propagation, absorption in fusion plasmas (ICRF/LHRF/ECRF)

- Full wave and WKB approach

Full Wave Approach

$$\begin{aligned}\nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times B &= \mu_0 \left(\epsilon_0 \frac{\partial E}{\partial t} + J_{rf} \right)\end{aligned}$$

1D analytic Approach
(Mode Conversion Study)

2D/3D Numerical Simulation
(TORIC/AORSA/...)

WKB Approach
(Spatially slowly varying medium)

$$\begin{aligned}E &= E_0 e^{i\Psi}, \\ B &= B_0 e^{i\Psi}, \\ \Psi &= k(\vec{r}, t) \cdot \vec{r} - \omega(\vec{r}, t)t \\ \left(\nabla \Psi = k(\vec{r}, t), \frac{\partial \Psi}{\partial t} = -\omega(\vec{r}, t) \right)\end{aligned}$$

$$\begin{aligned}\frac{d\vec{r}}{dt} &= -\frac{\partial H / \partial \vec{k}}{\partial H / \partial \omega} \\ \frac{d\vec{k}}{dt} &= -\frac{\partial H / \partial \vec{r}}{\partial H / \partial \omega}\end{aligned}$$

Ray Tracing Equation
(TORAY/GENRAY/...)

Uniform Plasmas

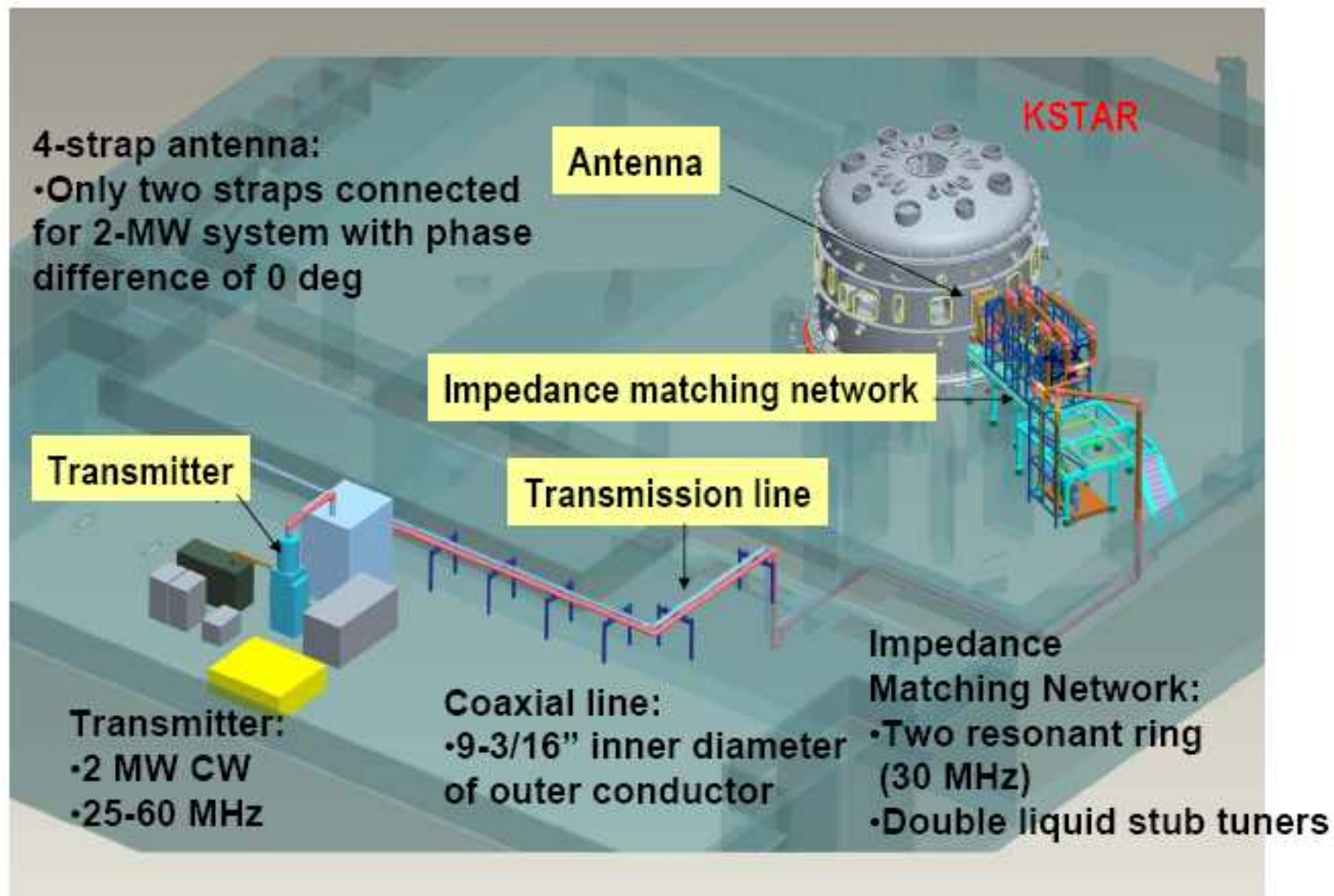
$$\begin{aligned}E &= E_0 e^{i\Psi}, \\ B &= B_0 e^{i\Psi}, \\ \Psi &= k \cdot \vec{r} - \omega t\end{aligned}$$

$$\begin{aligned}\vec{N} \times \vec{N} \times E_0 &= \vec{\epsilon}_c E_0 \\ (\vec{N}^2 - \vec{\epsilon}_c) E_0 &= 0 \\ H \equiv \det(\vec{N}^2 - \vec{\epsilon}_c) &= 0\end{aligned}$$

Dispersion Relation

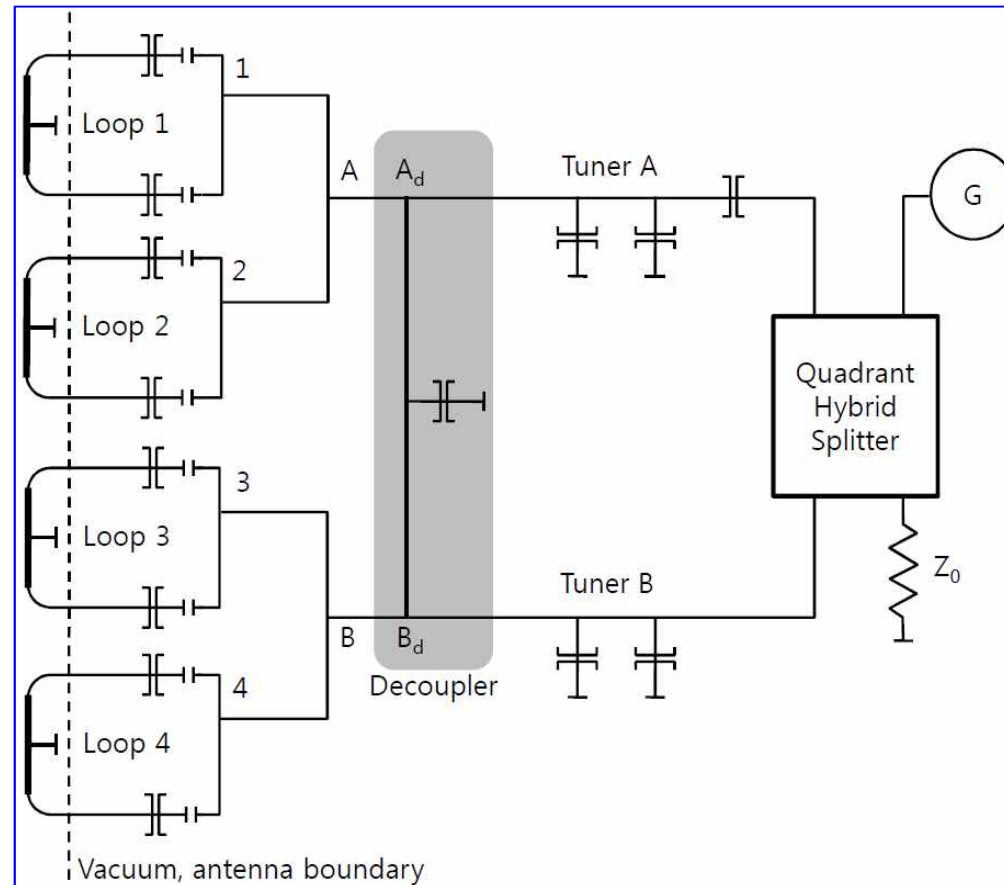
Wave launching, propagation, absorption in fusion plasmas (ICRF)

□ ICRF launching and Transmission Coupling System in KSTAR



Wave launching, propagation, absorption in fusion plasmas (ICRF)

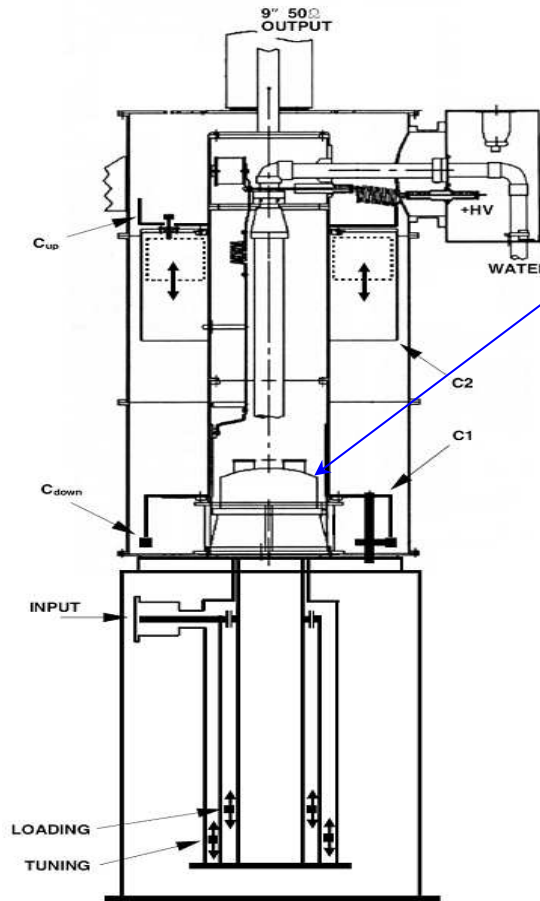
- ICRF launching and Transmission Coupling System in KSTAR



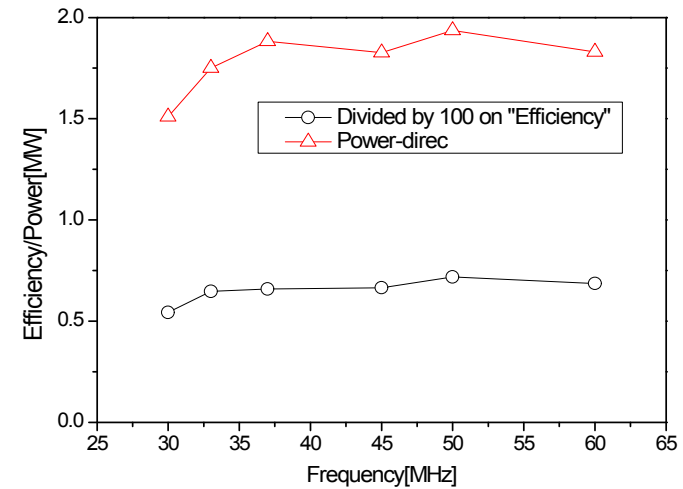
Schematic Resonant loop/matching system

Wave launching, propagation, absorption in fusion plasmas (ICRF)

- ICRF wave generator: Transmitter (Tetrode tube)



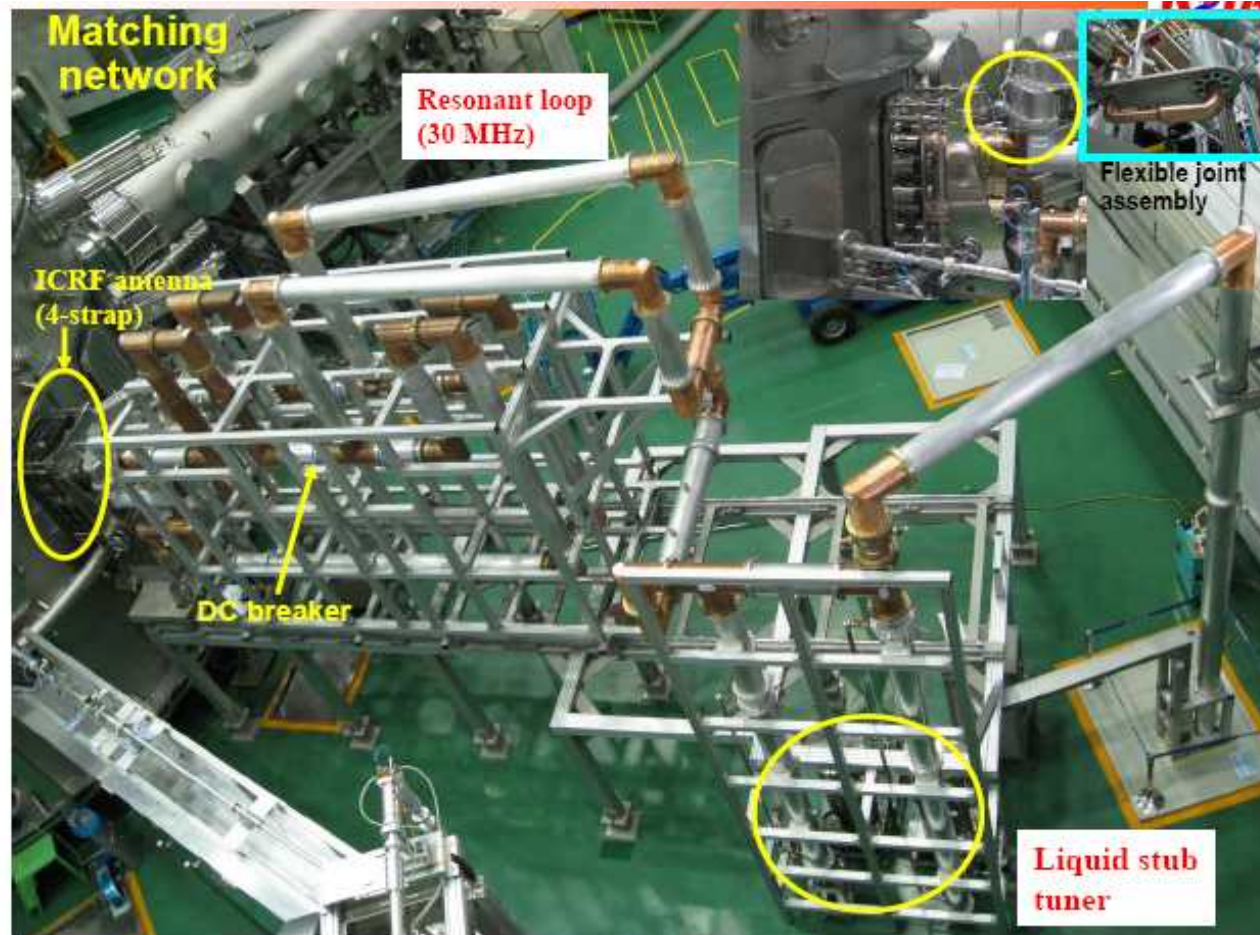
- Tetrode (4CM2500KG)
- 20~60 MHz
- 2 MW 300 sec



Transmitter FPA(Final Power Amplifier)

Wave launching, propagation, absorption in fusion plasmas (ICRF)

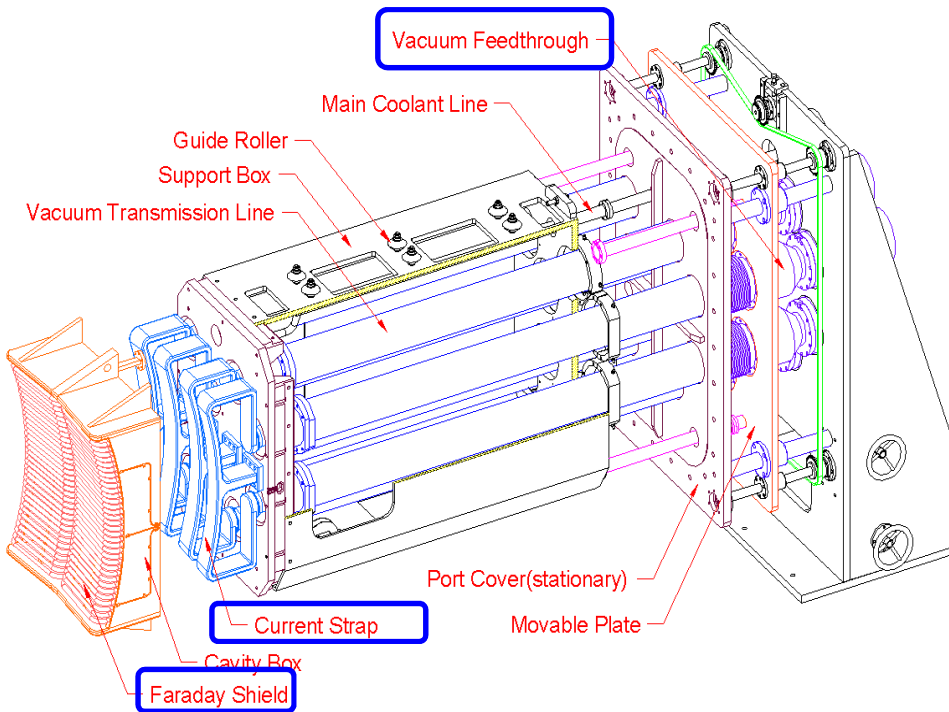
□ ICRF Resonant loop and Matching System



KSTAR Resonant loop and matching system

Wave launching, propagation, absorption in fusion plasmas (ICRF)

ICRF launcher: Antenna



- 4 strap
- 0-0- π - π : Heating
- 0-0- $\pi/2$ - $\pi/2$: Current



Radiation pattern for resonance heating antennas

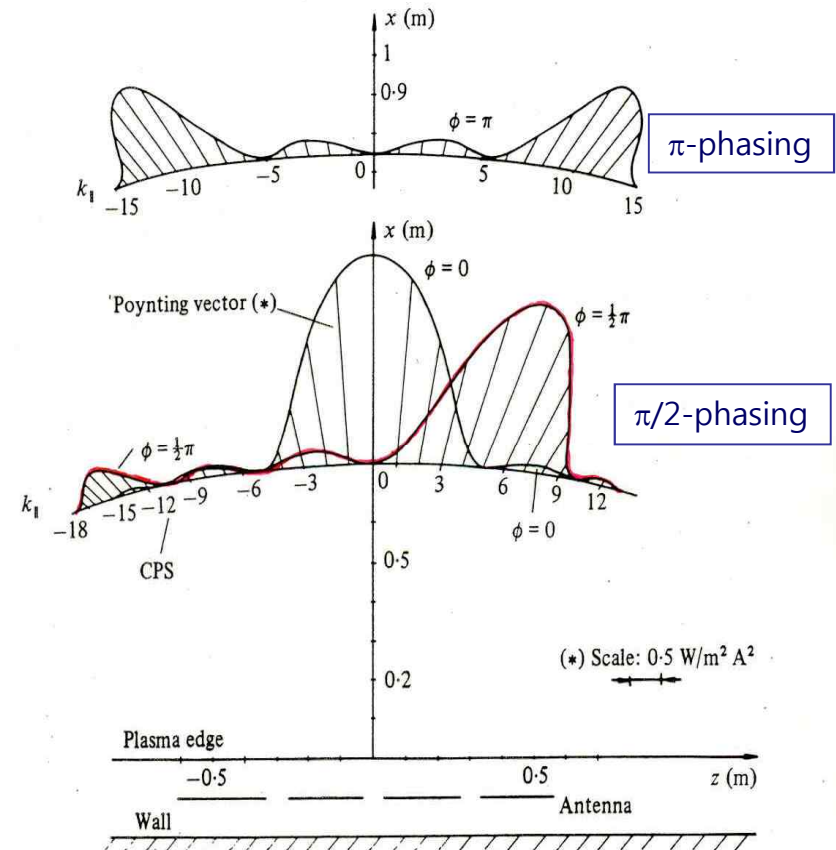


FIGURE 4. Distribution of the Poynting vector on a CPS in uniform plasma for an antenna array

Wave launching, propagation, absorption in fusion plasmas (ICRF)

□ ICRF Antenna

- Electric field is perpendicular to magnetic field in ICRF fast wave.
- Stray Ez field is screened by Faraday shield.

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{i(N_{\parallel}^2 - S)}{D} \rightarrow -\frac{iS}{D}$$

$$N_{\perp}^2 = -\frac{(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{(N_{\parallel}^2 - S)}$$

$$\frac{E_z}{E_x} = \frac{N_{\parallel}N_{\perp}}{N_{\perp}^2 - P} \rightarrow 0$$

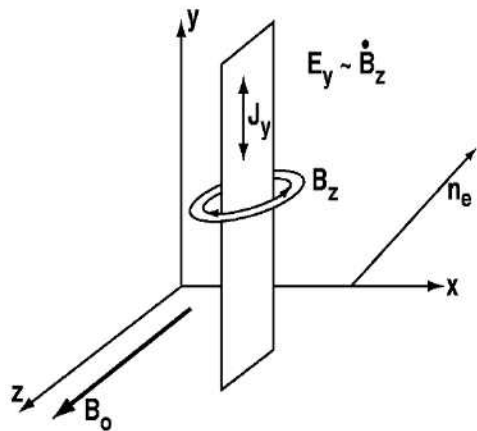
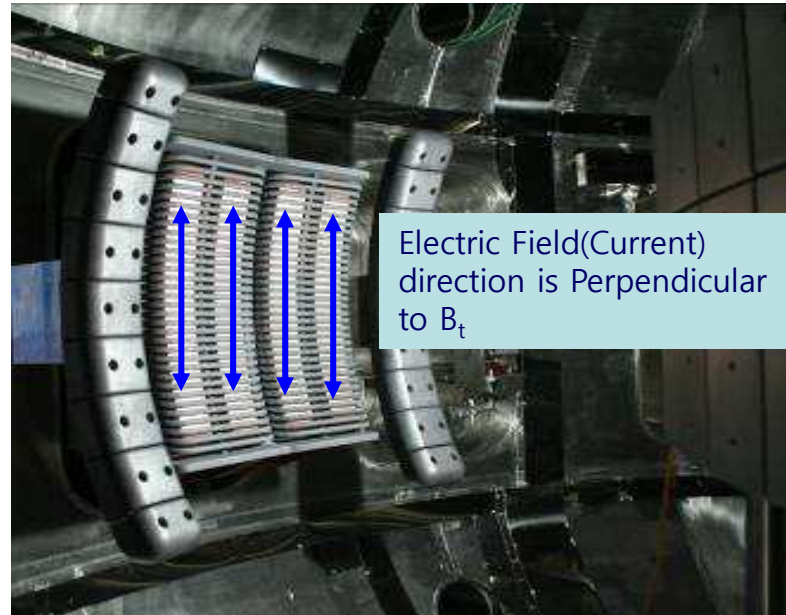


FIG. 3. Geometry of an inductive coupling element ("loop antenna") used for exciting the fast wave in the ICRF.

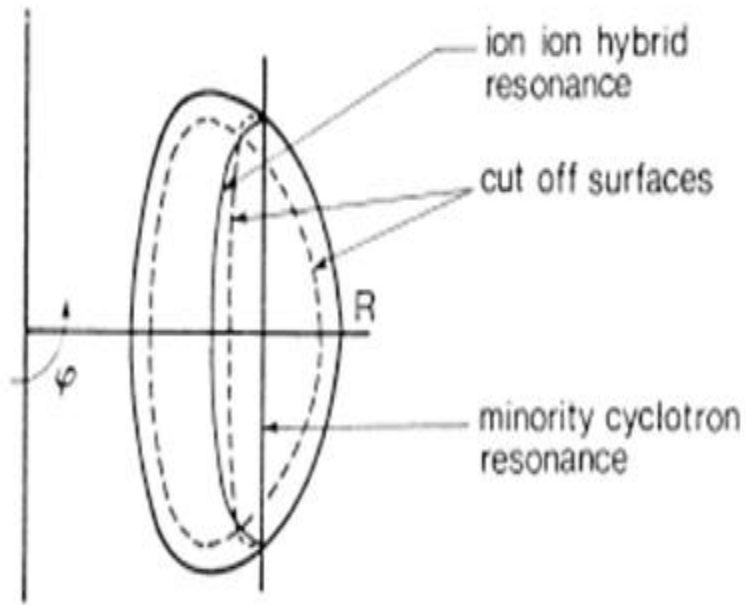


Electric Field(Current) direction is Perpendicular to B_t

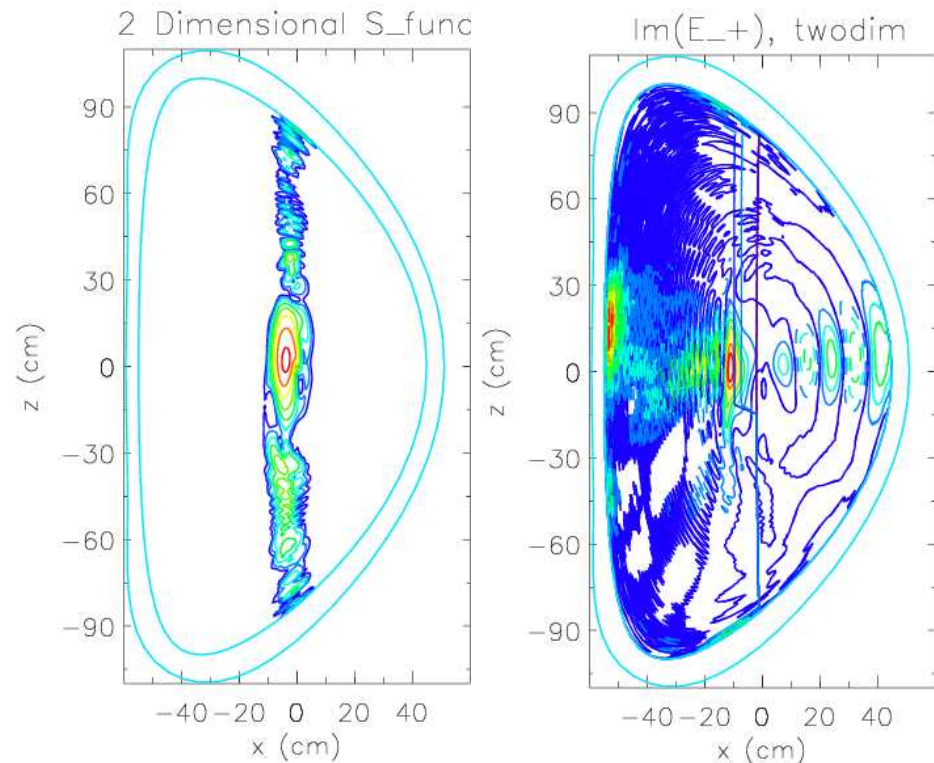
Wave launching, propagation, absorption in fusion plasmas (ICRF)

□ ICRF FW propagation and absorption (**Fundamental Minority Heating**)

- ICRF fast wave wavelength is comparable to system size. Therefore, full wave approach is required.



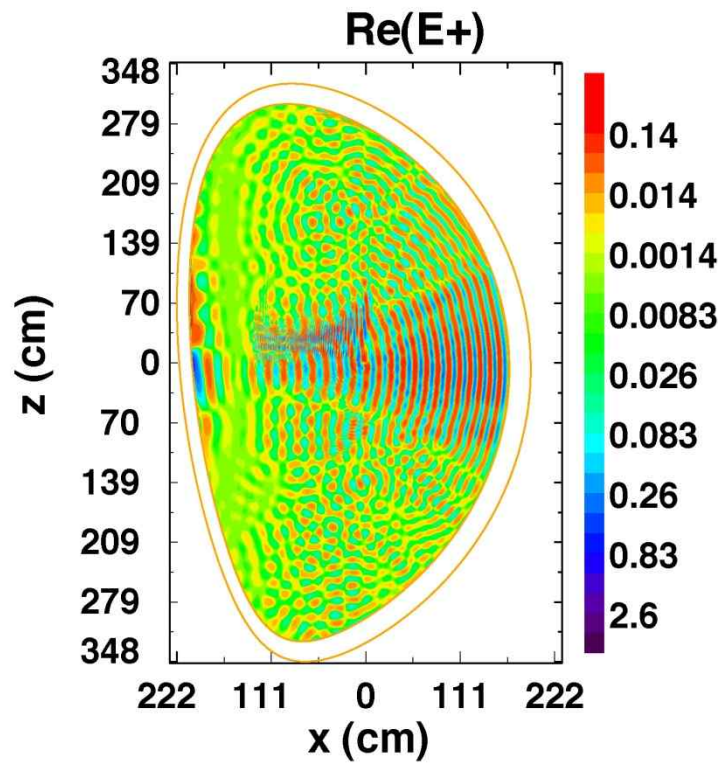
Cut-off/Resonances in minority heating scheme



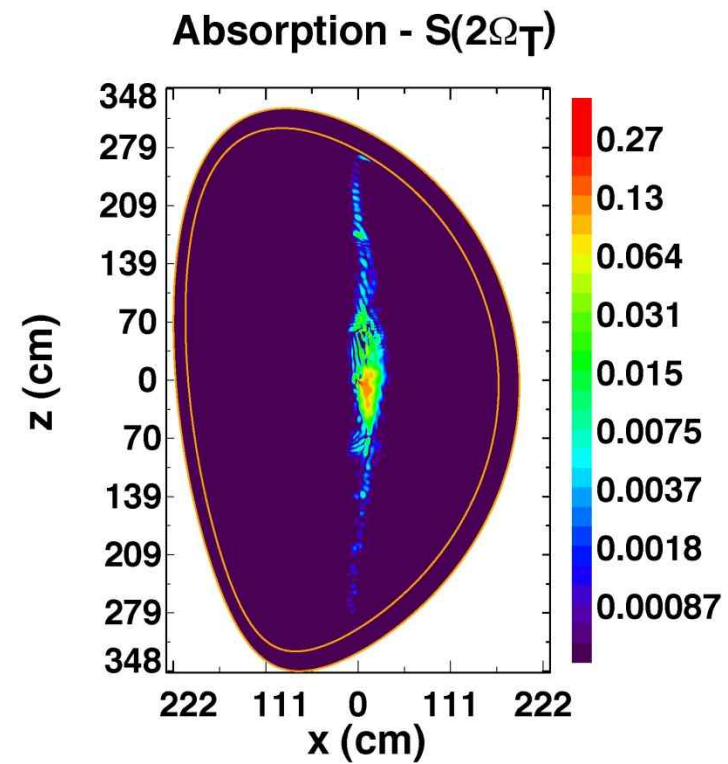
D(H) Minority Heating Scheme in KSTAR
Wang, 2009

Wave launching, propagation, absorption in fusion plasmas (ICRF)

- ICRF FW propagation and absorption (Second Harmonic Heating)



LHP wave field of T 2nd Harmonic Heating in ITER



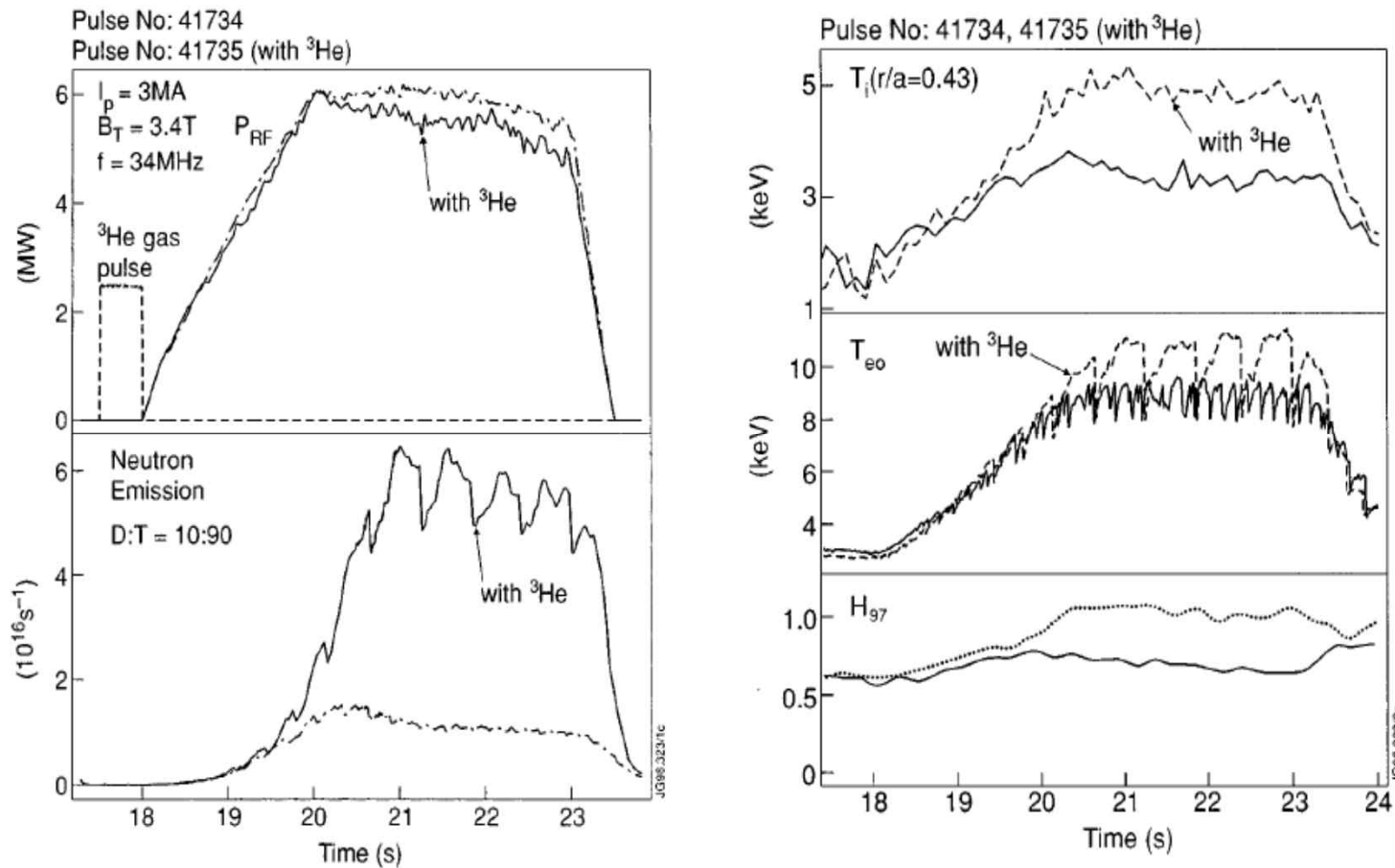
T power absorption profile

D. B. Batchelor, PAC, 2005

Wave launching, propagation, absorption in fusion plasmas (ICRF)

□ Experimental results

T 2nd Harmonic + He3 minority Heating

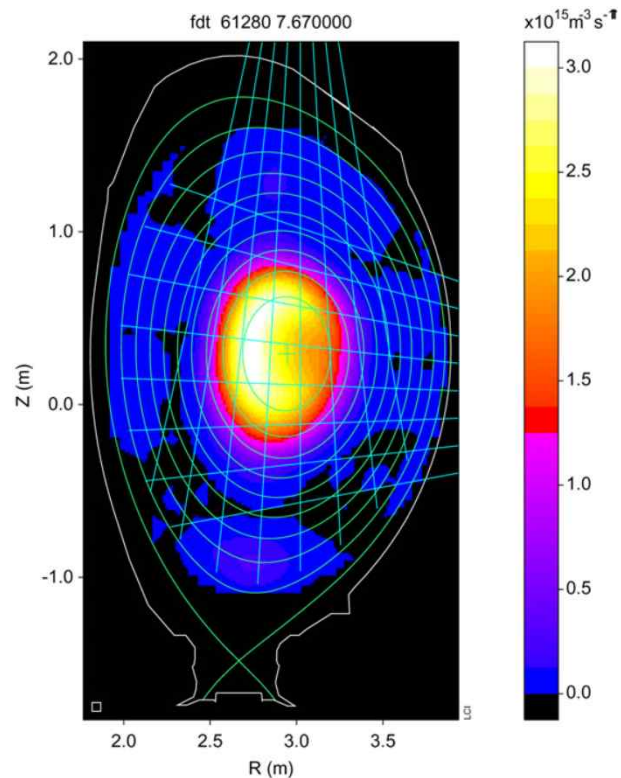


JET[Start et al. 1999]

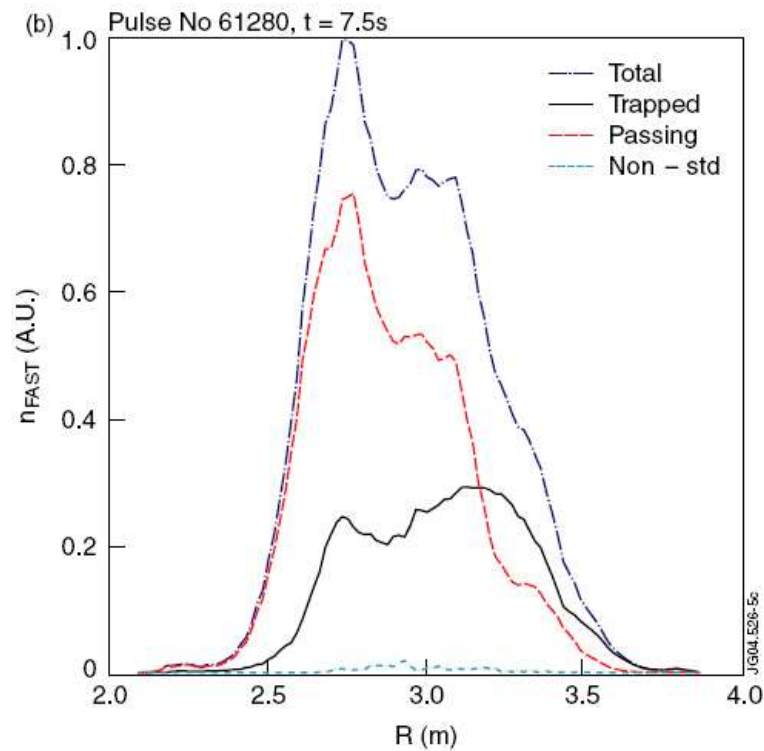
Wave launching, propagation, absorption in fusion plasmas (ICRF)

□ Experimental results

Neutron Thomography



SELFO simulation

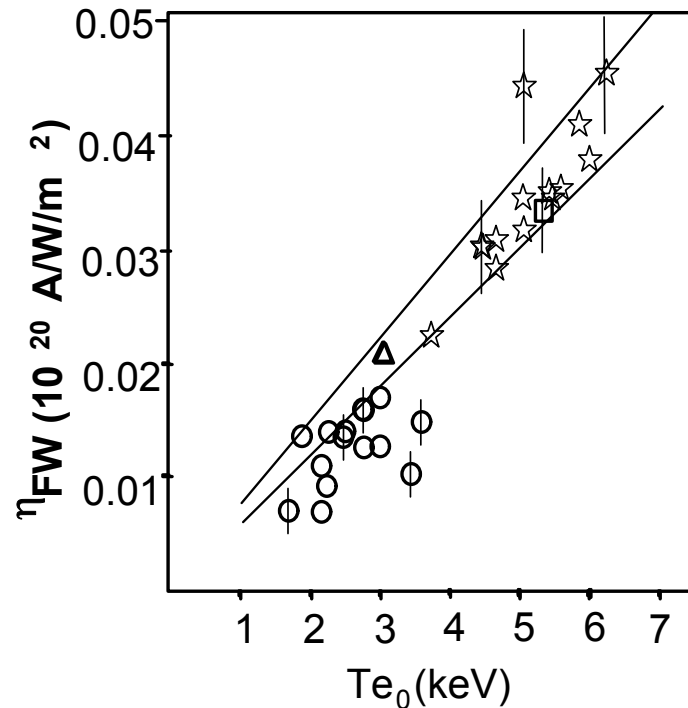


JET [Lamalle et al. 2006]

Wave launching, propagation, absorption in fusion plasmas (ICRF)

- Experimental results (Current drive)

Figure of merit of fast wave current drive versus central temperature



○ : L-mode in DIII-D.

△ : L-mode in Tore Supra.

□ : VH-mode in DIII-D.

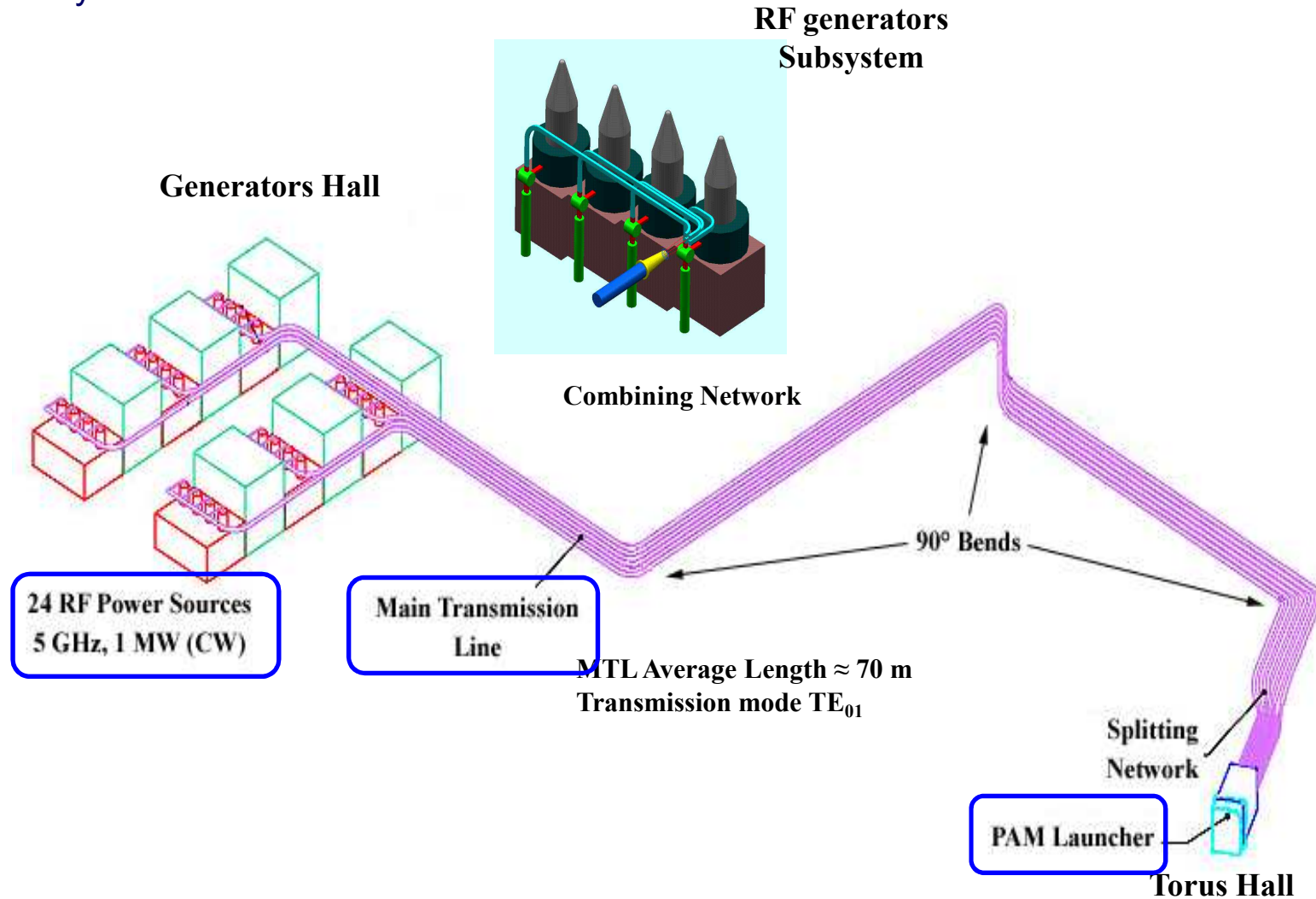
* : NCS L-mode in DIII-D.

- : lower and upper bounds of the simulations
(RT code CURRAY/ FW code ALCYON)

ITER Physics Basis 1999

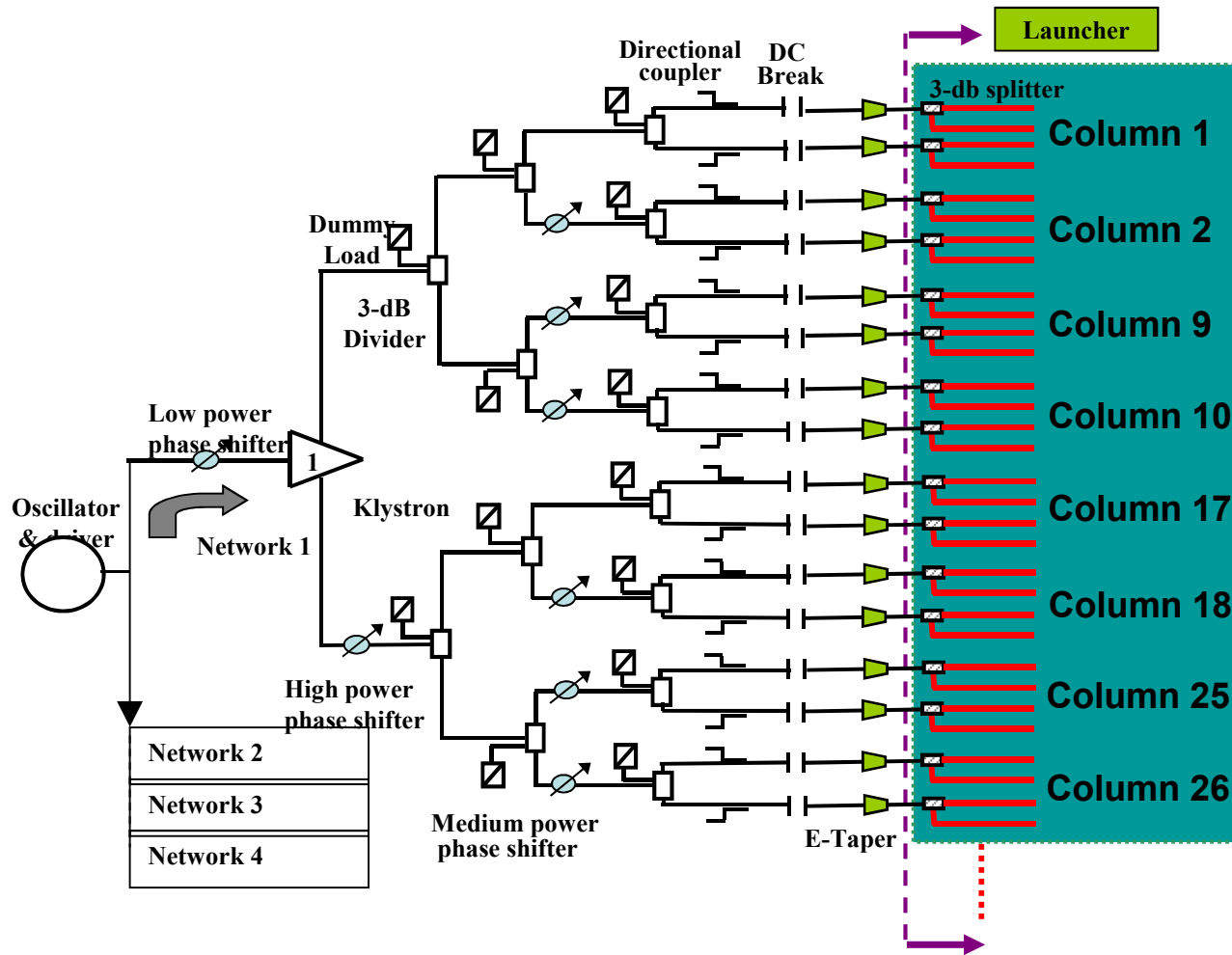
Wave launching, propagation, absorption in fusion plasmas (LHRF)

□ LHRF System for ITER



Wave launching, propagation, absorption in fusion plasmas (LHRF)

□ LHRF System Schematic

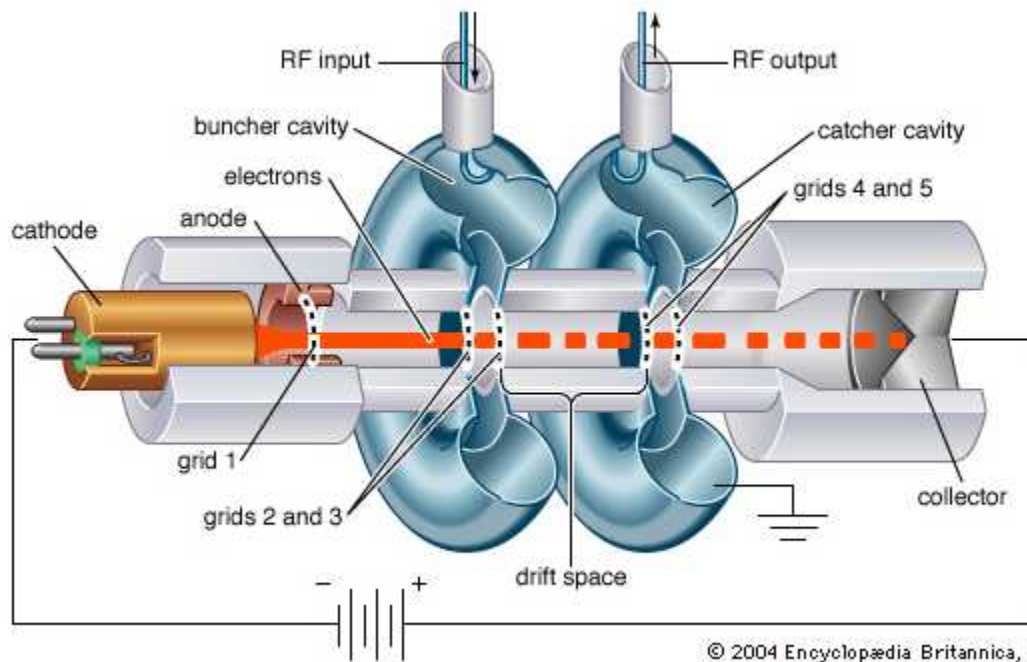


Wave launching, propagation, absorption in fusion plasmas (LHRF)

□ LHRF Sources (Klystron)



500 kW klystron for ITER

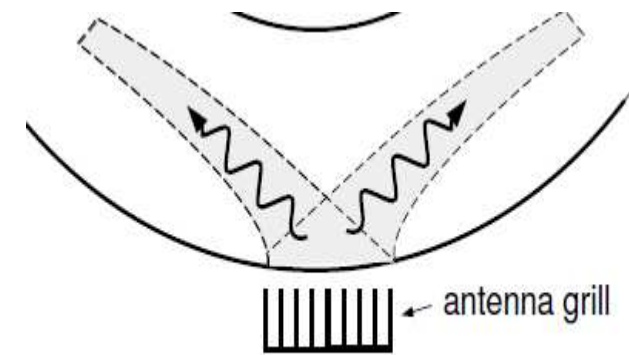
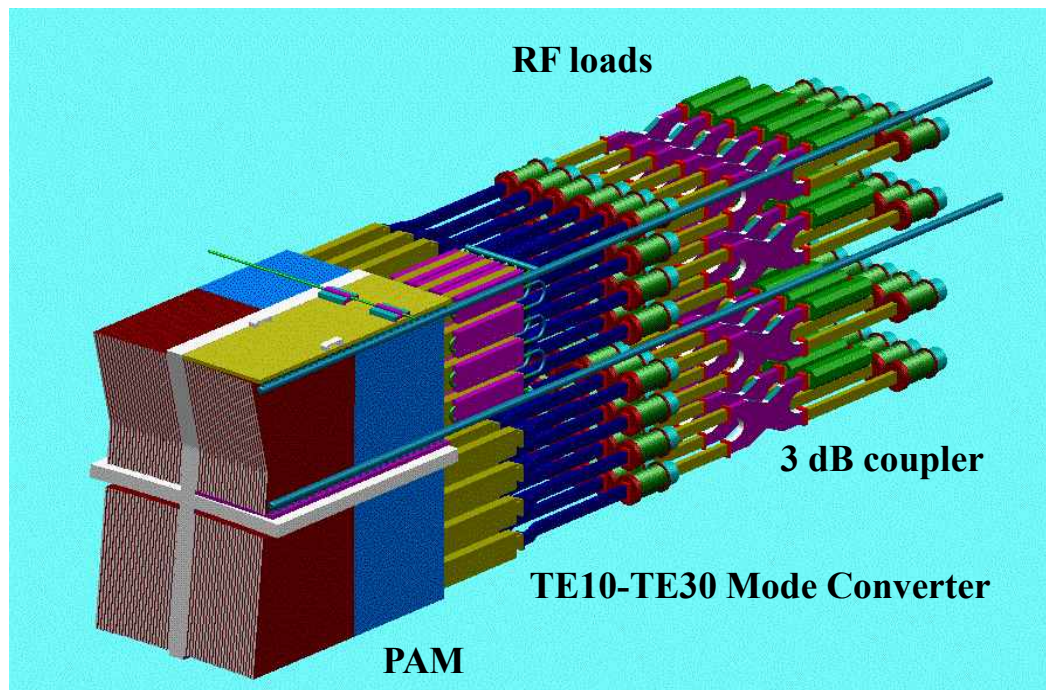


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Schematic of klystron structure

Wave launching, propagation, absorption in fusion plasmas (LHRF)

- LHRF Launcher: Waveguide grill



LHRF launcher for ITER

Wave launching, propagation, absorption in fusion plasmas (LHRF)

□ LHRF SW Launcher & Accessibility condition

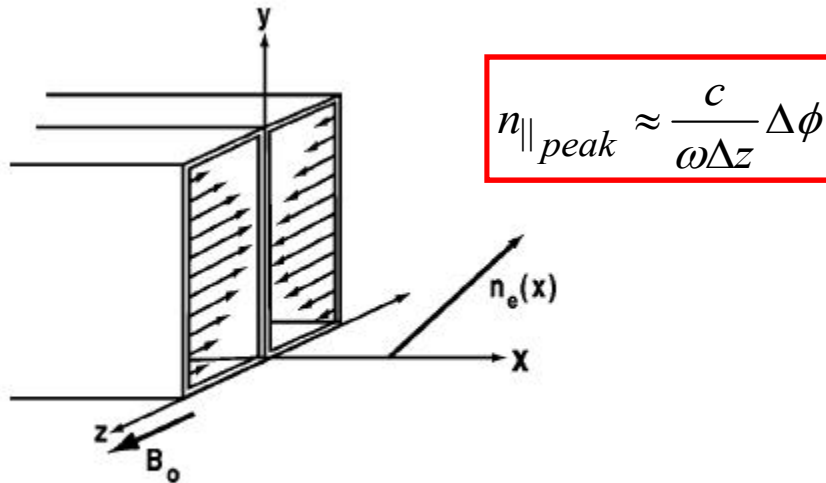
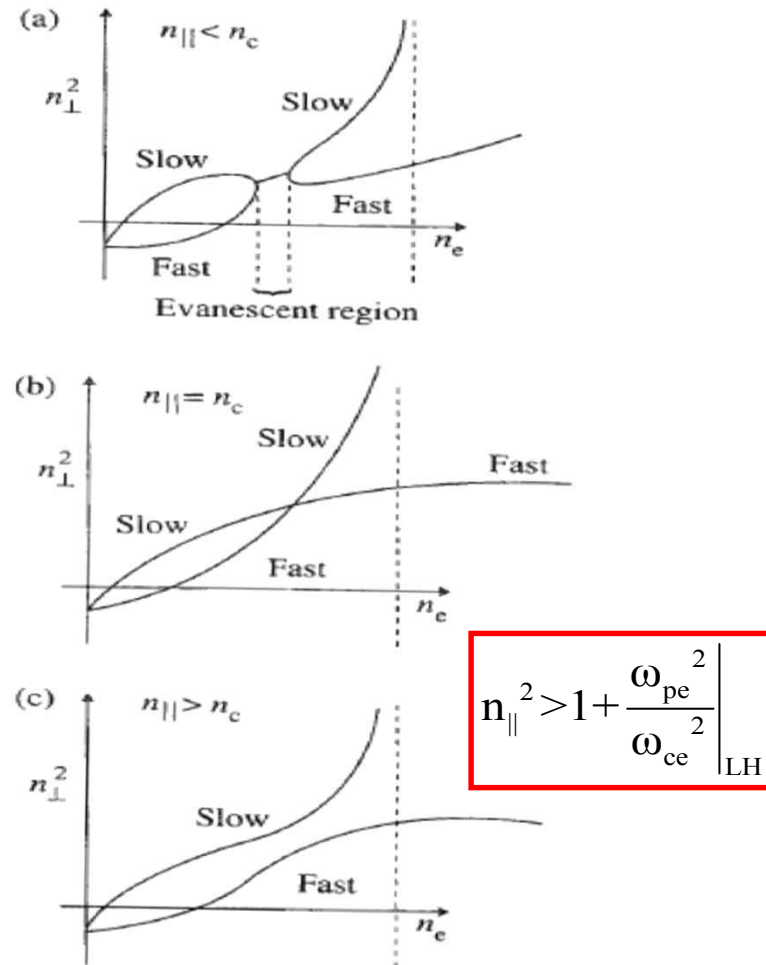


FIG. 6. Geometry of a phased array of open-ended waveguides used to excite lower hybrid waves in the LHRF.

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iSD}{(N_{||}^2 - S)(S - P)} \rightarrow -\frac{iD}{S}$$

$$\frac{E_z}{E_x} = \frac{N_{||} N_{\perp}}{N_{\perp}^2 - P} = -\frac{SN_{\perp}}{PN_{||}} \rightarrow \pm \infty$$

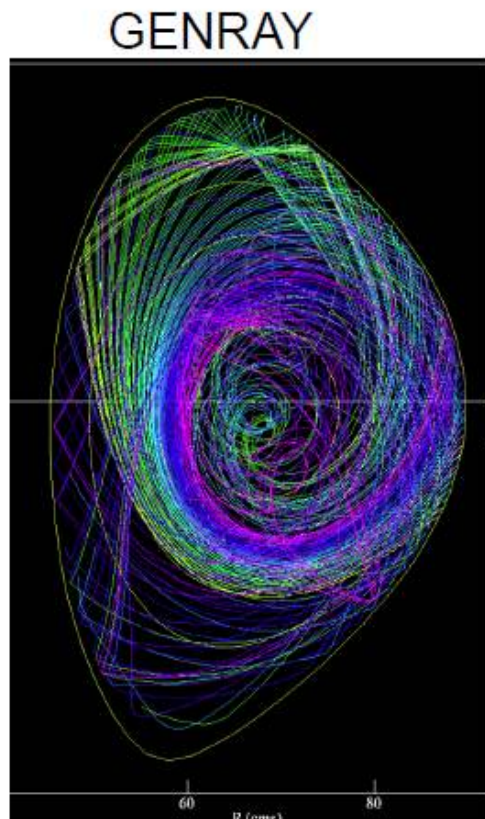
$$N_{\perp}^2 = -\frac{P(N_{||}^2 - S)}{S}$$



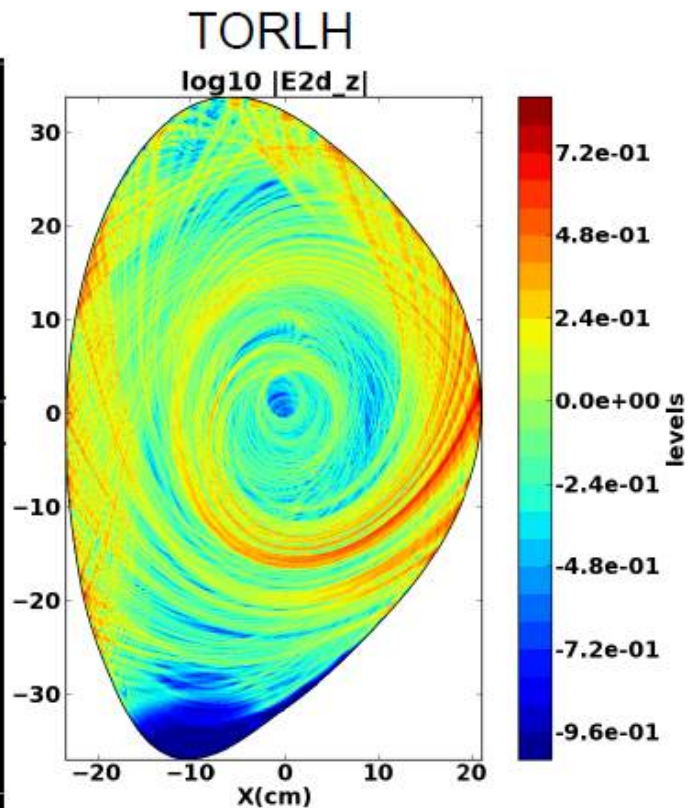
Wesson, Tokamaks, 2007

Wave launching, propagation, absorption in fusion plasmas (LHRF)

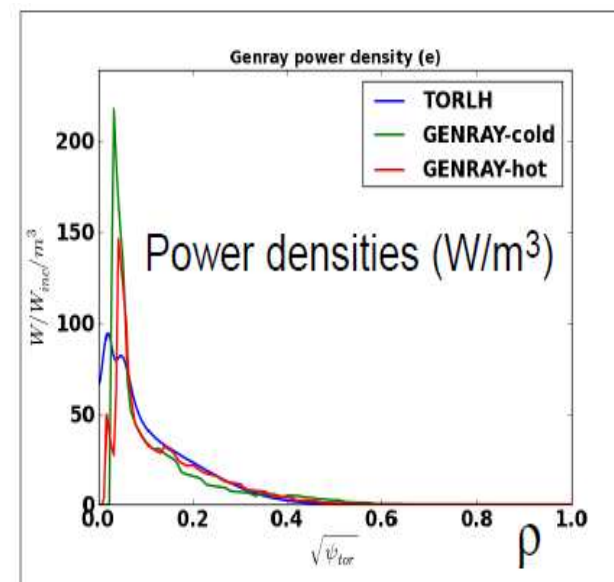
□ Propagation & Absorption



Petrov, CompX



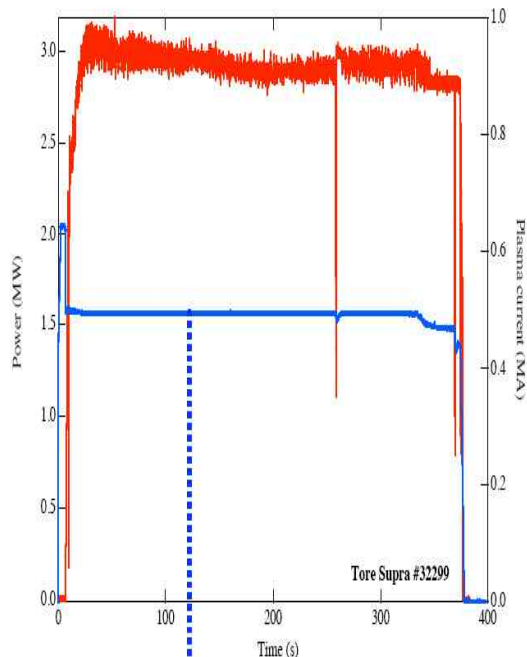
J. C. Wright, POP, 2009



Petrov, CompX

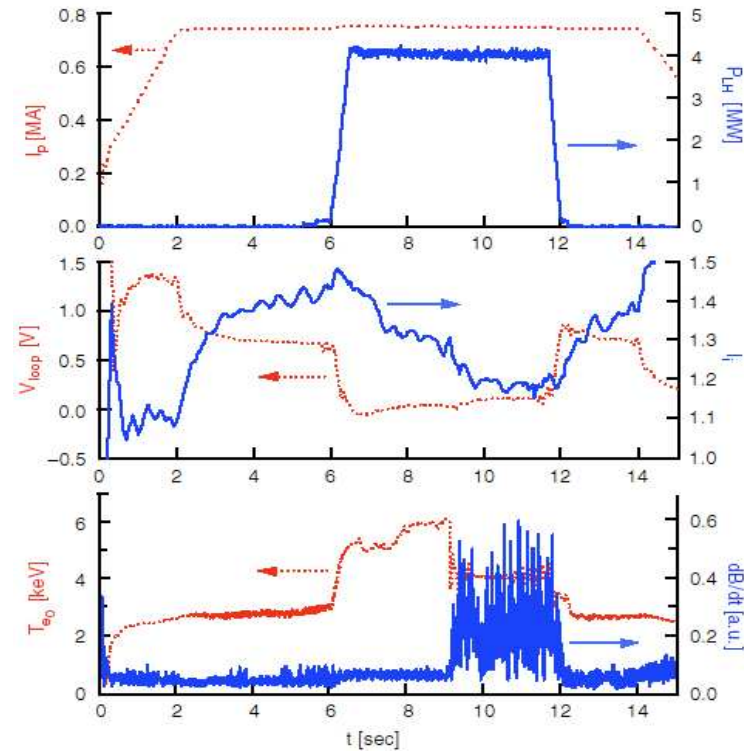
Wave launching, propagation, absorption in fusion plasmas (LHRF)

□ Experimental results (Full non-inductive current drive)



- Full LHCD (6 min.)
- 2 antennas
- $n_{H0} = 1.7 \pm 0.2$
- $P_{lh} = 3$ MW
- directivity: 0.6 & 0.7

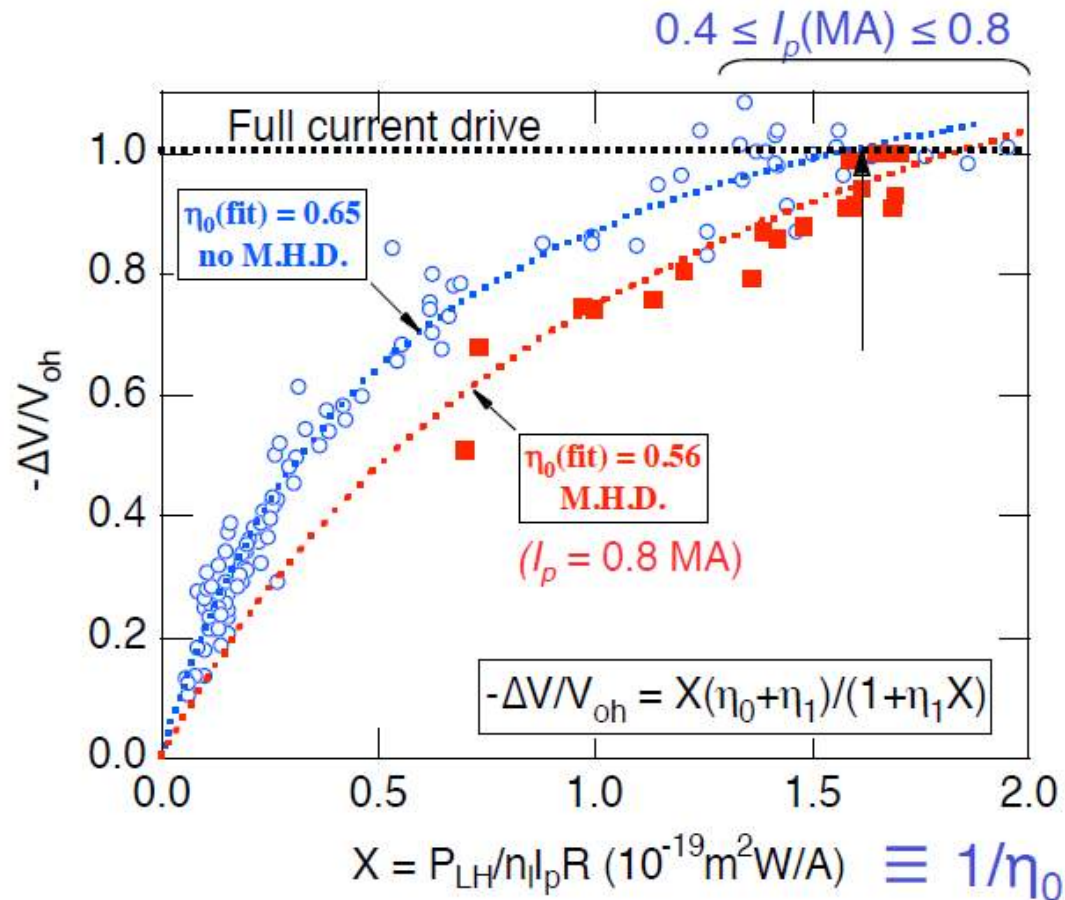
$I_p \approx 500$ kA



Y. Peysson, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (LHRF)

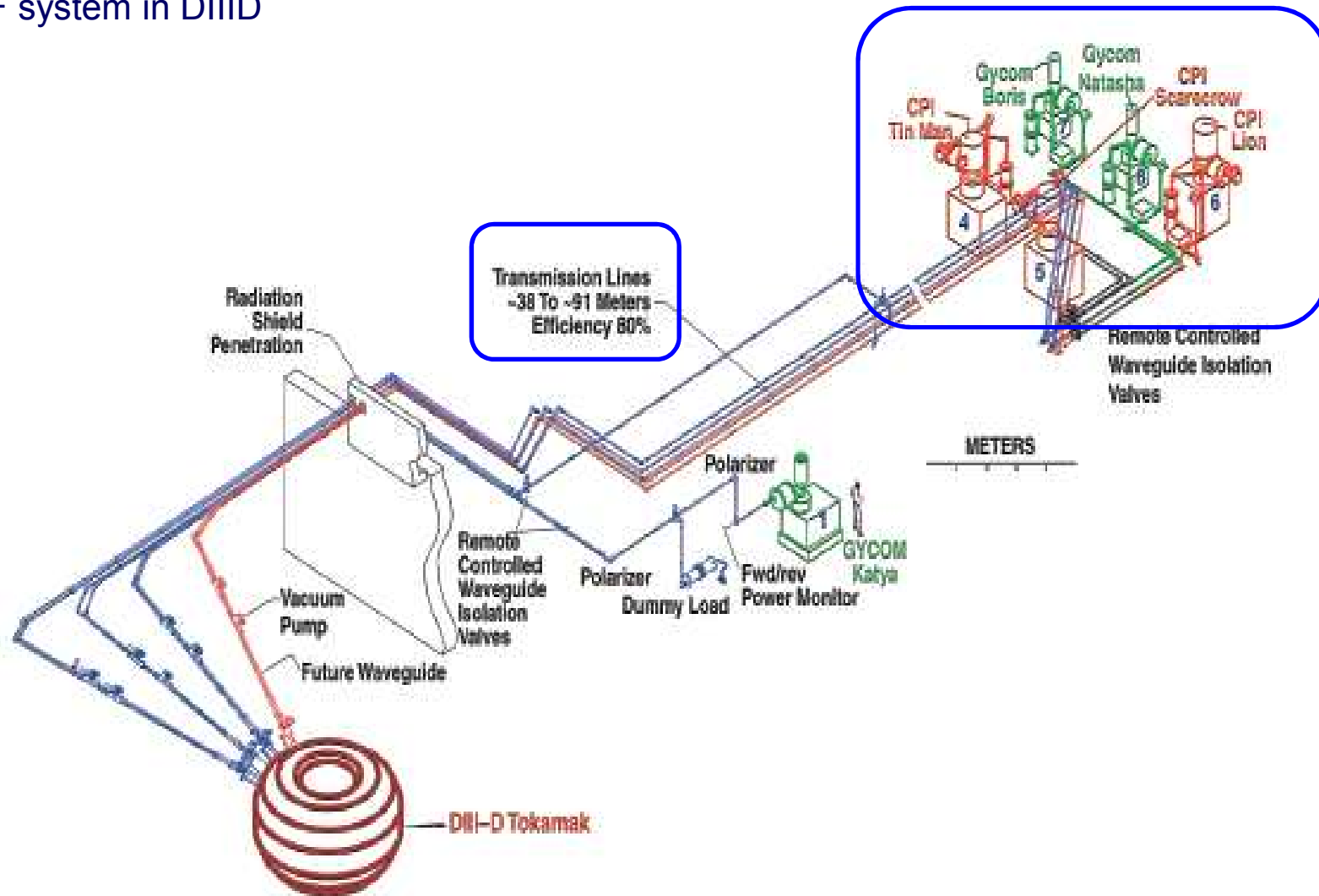
- Experimental results (Current drive efficiency)



Y. Peysson, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)



- ECRF system in DIII-D



Wave launching, propagation, absorption in fusion plasmas (ECRF)

- ECRF source: Gyrotron

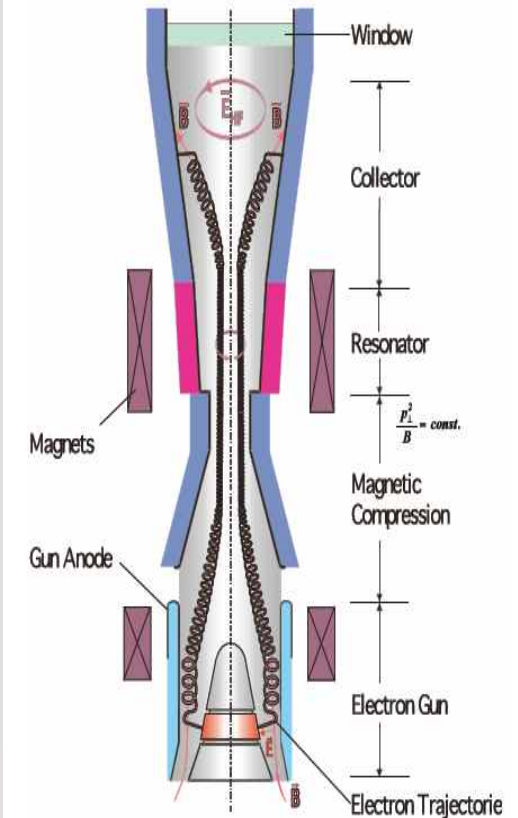
High-Power Gyrotrons for Fusion Plasma Applications

<p>ITER: TOSHIBA/JAEA (JA) 170 GHz, 1 (0.8) MW 800 (3600) s, 55 (57) %</p>	<p>ITER: GYCOM/IAP (RF) 170 GHz, 1.05 (0.83) MW 116 (203) s, 52 (48) %</p>	<p>W7-X: CPI (USA) 140 GHz, 0.9 MW 1800 s, 35 %</p>	<p>W7-X: TED/FZK/CRPP (EU) 140 GHz, 0.92 MW 1800 s, 45 %</p>
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9 M. Thumm, IPP Institutskolloquium (HGW), June 19, 2009
KIT – The cooperation of Forschungszentrum Karlsruhe GmbH and Universität Karlsruhe (TH)

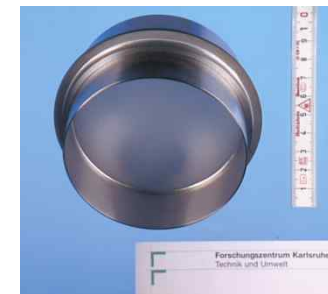
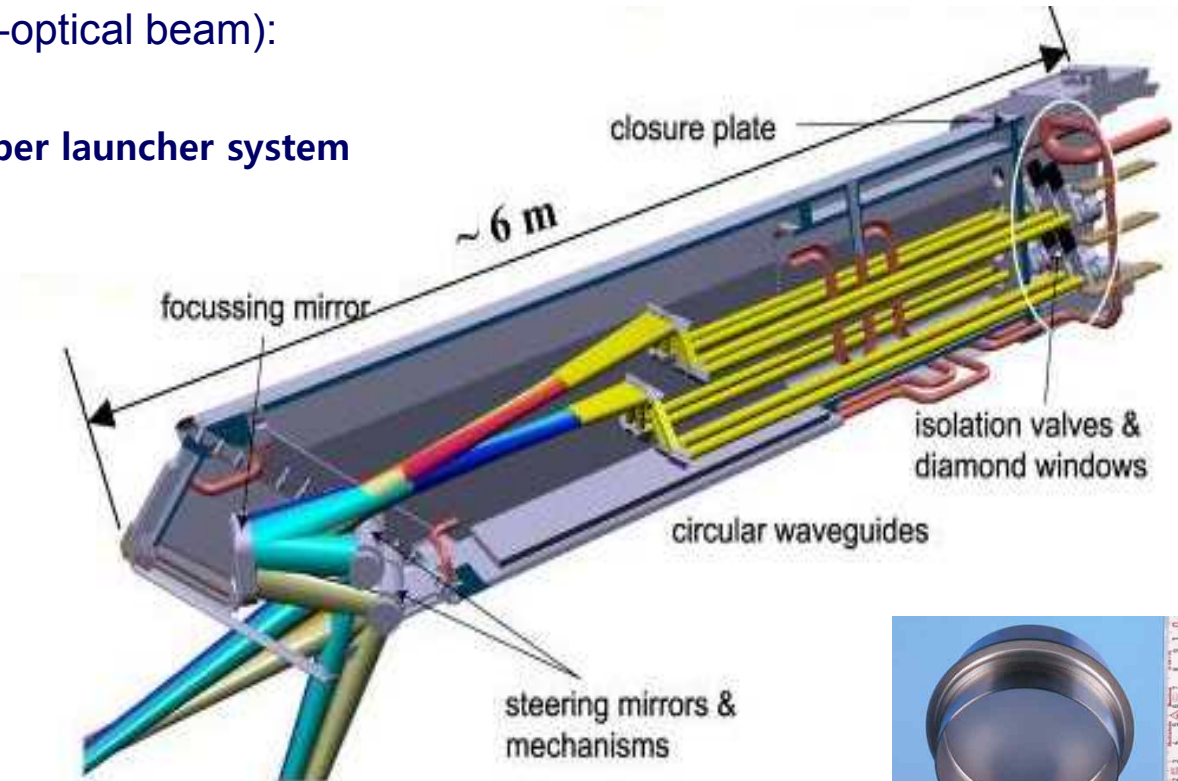
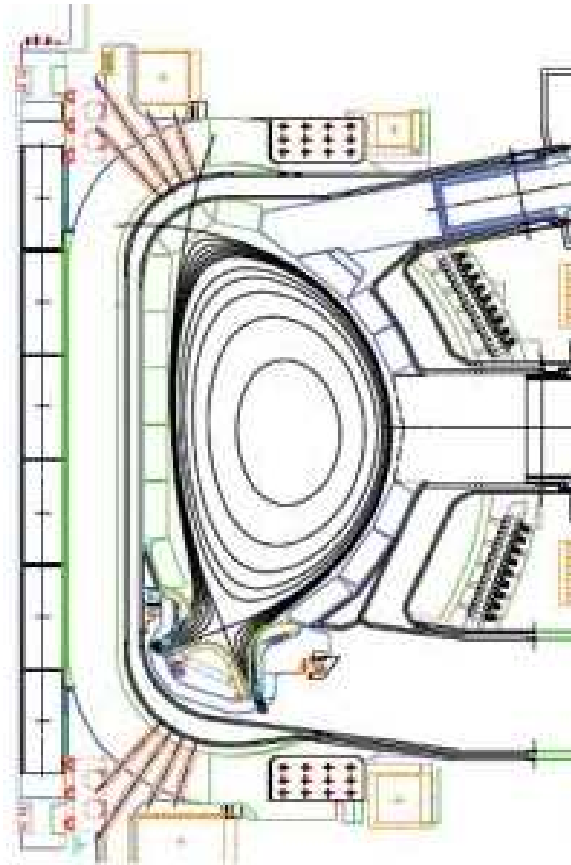
F Forschungszentrum Karlsruhe in der Helmholtz-Gemeinschaft
Universität Karlsruhe (TH) Research University • founded 1825



Wave launching, propagation, absorption in fusion plasmas (ECRF)

- ECRF launcher (Mirror: quasi-optical beam):

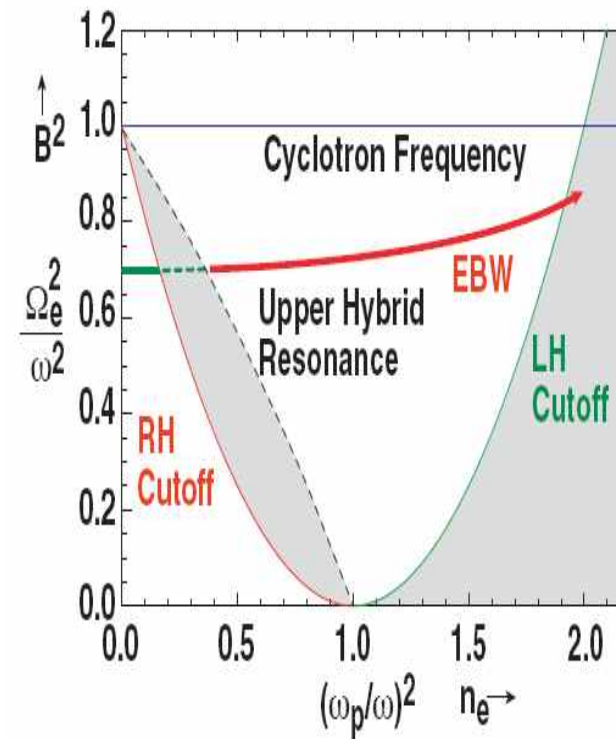
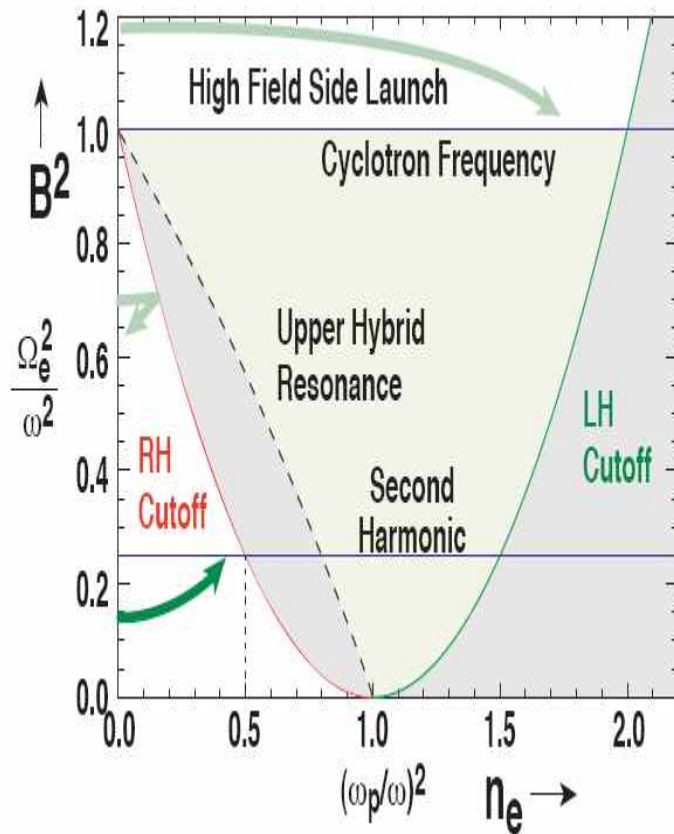
ITER ECRF upper launcher system



R. Prater, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)

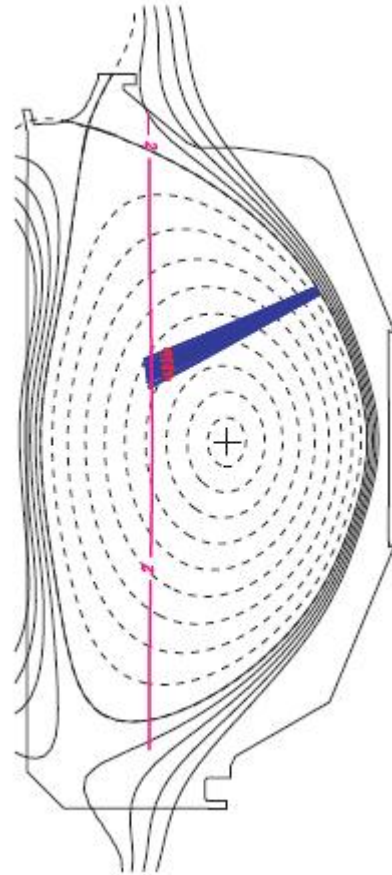
- ❑ O1, X2, X3 cyclotron heating and CD in tokamak
- ❑ XB, OXB EBW heating and CD in high beta ST



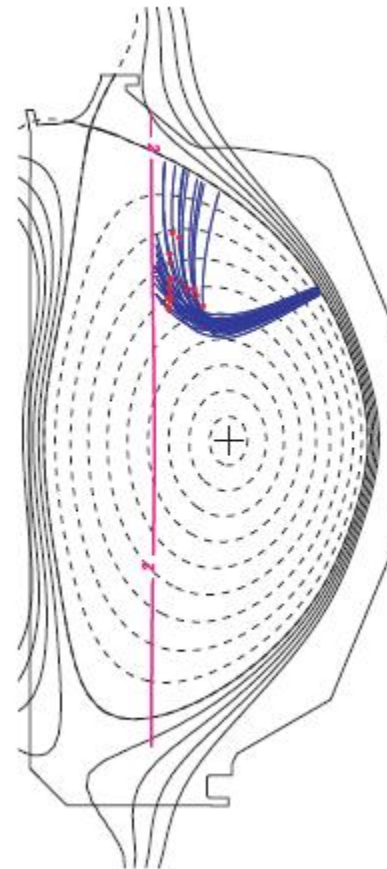
R. Prater, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)

□ Wave propagation



Low density under $R(X)$ cut-off

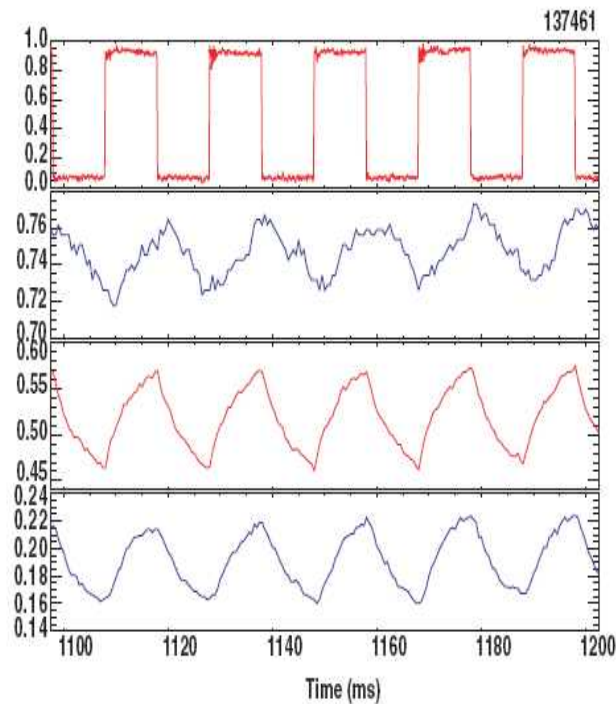
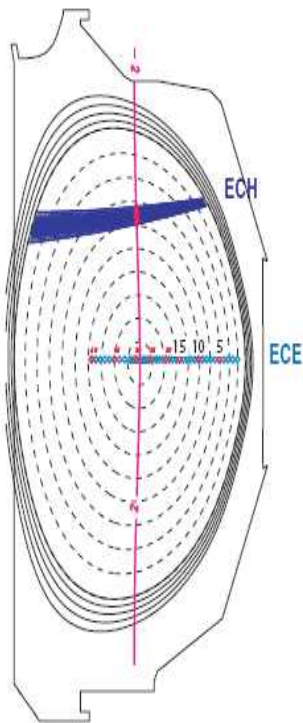


high density above $R(X)$ cut-off

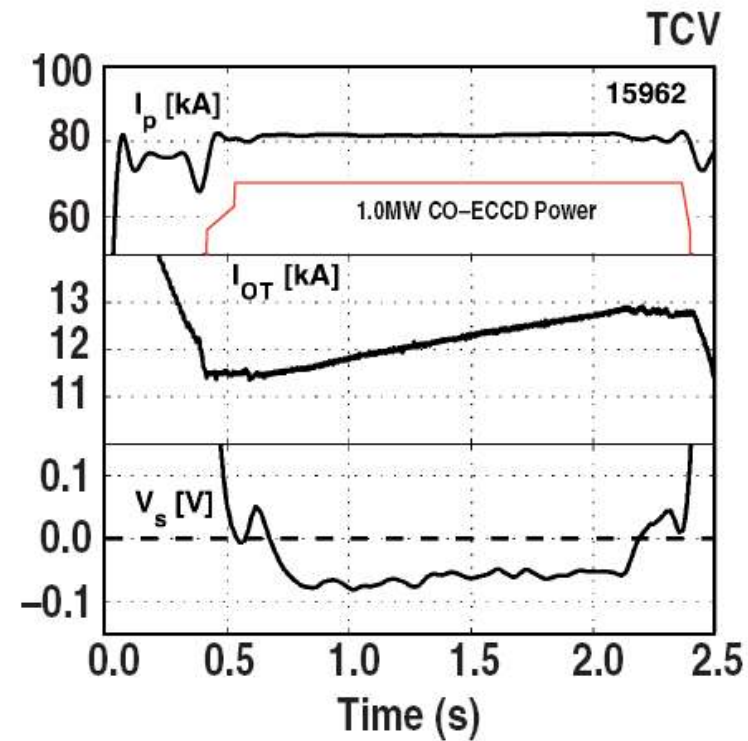
R. Prater, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)

- Experiments (heating and current drive)



X2 heating in DIID

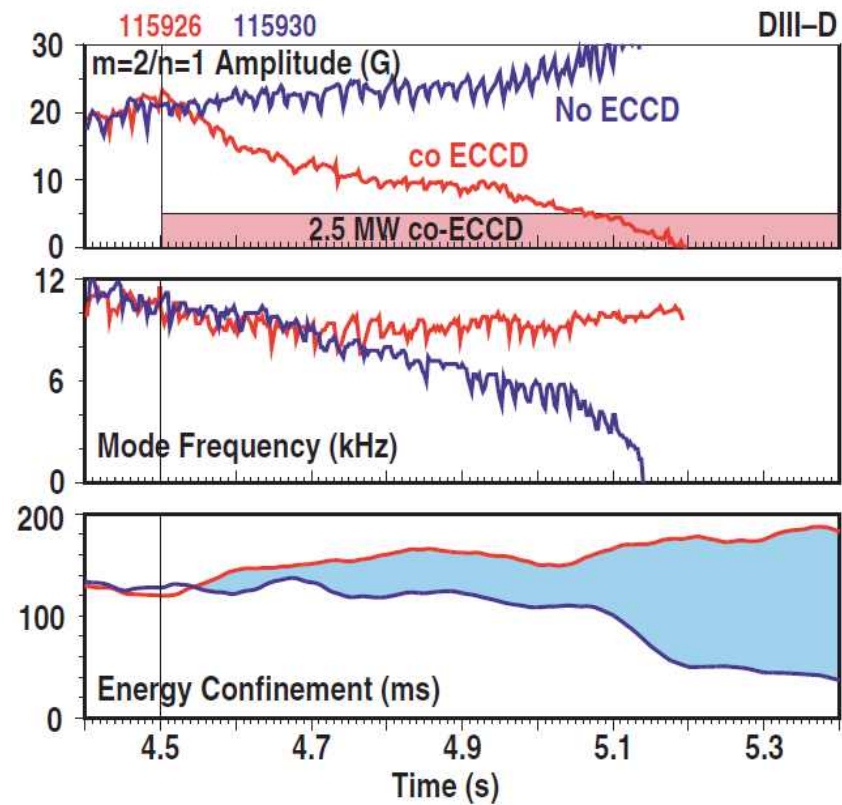
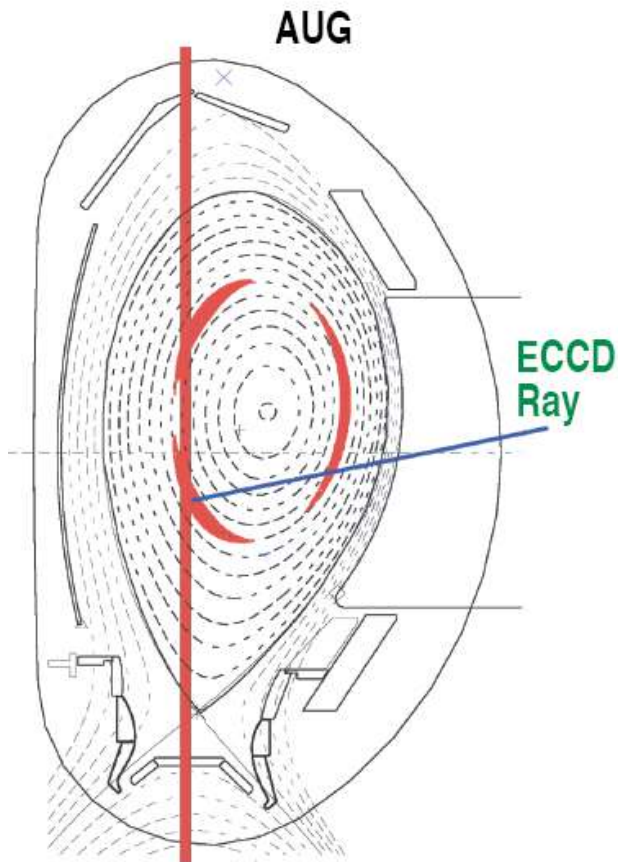


Full non-inductive CD in TCV

R. Prater, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)

- NTM stabilization

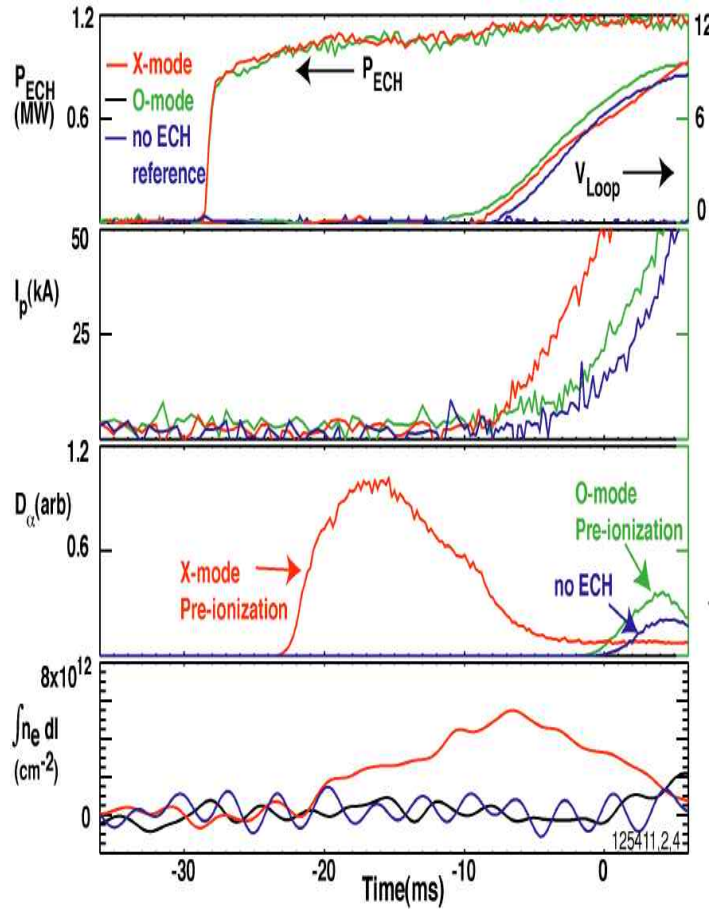


Suppression of 2/1 NTM by ECCD

R. Prater, Fusion summer school in KAIST, 2009

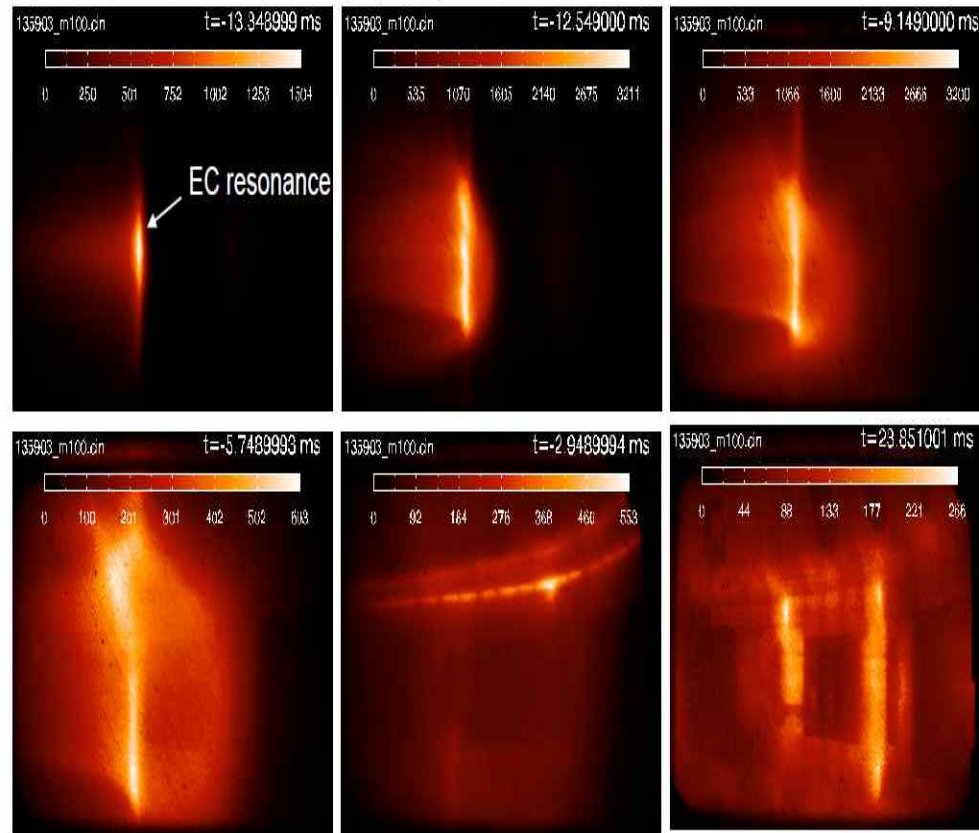
Wave launching, propagation, absorption in fusion plasmas (ECRF)

□ Start up



135903, D alpha filter, 5000 fps, 195 μs exposure

J. Yu, UCSD



R. Prater, Fusion summer school in KAIST, 2009

Summary

- ❑ RF waves have been successfully proven in tokamak experiments.
 - ICRF: Ion heating (Minority / 2nd Harmonic heating)
 - LHRF: Current drive (Landau damping)
 - ECRF: Pre-ionization and startup, NTM stabilization (Cyclotron damping of O1, X2, X3)

- ❑ There are still critical issues in RF systems to be solved (ICRF/LHRF).
 - Stable power transmission (arcing)
 - Power coupling

Reference

- ❑ T. Stix, “Waves in plasmas”, 1992
- ❑ M. Brambilla, “Kinetic theory of plasma waves”, 1998
- ❑ D. Swanson, “Plasma waves”, 2003
- ❑ Presentations on RF waves “Fusion Summer School in KAIST”, 2009