458.308 Process Control & Design

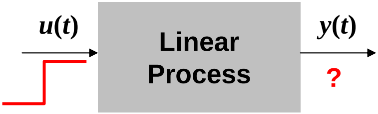
Lecture 4a: Models for Control -- Simple Dynamics

Jong Min Lee

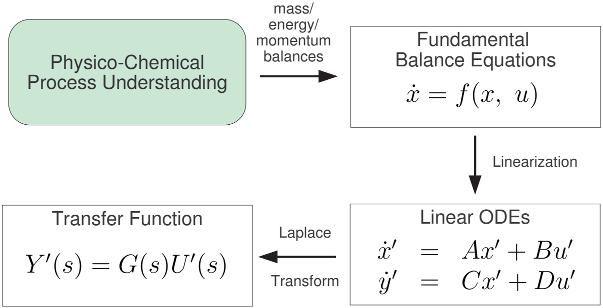
Chemical & Biomolecular Engineering
Seoul National University

Model Requirement for Control

For effective design of control system, we must understand the input-output (MV or disturbance to CV) dynamics.

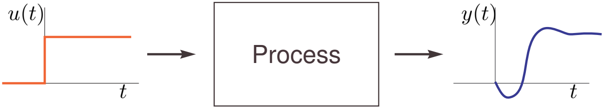


Obtaining the Model: Method 1



- There may not be sufficient process understanding.
- One may have too many equations, thus complicating the model development process and the final model form.
- Must add sensor/control valve calibration and dynamics.

Obtaining the Model: Method 2



1. Structure Selection
(1st order, 2nd order...)

2. Parameter Fitting

↓

Linear ODE	or	Transfer Function
$\dot{x}' = Ax' + Bu'$ $\dot{y}' = Cx' + Du'$		$Y'(s) = G(s)U'(s)$

- The model is restricted to low-order linear differential equations.
- Must understand how the order and parameter values of differential equation affect the response to various forcing functions (e.g., step, pulse) -- ``Make Friends w/ Transfer Functions."

Standard Process Inputs

Learning the Vocabulary

1. Step function

$$U_S(t) = \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases}$$

$$U_S(s) = \int_0^\infty M e^{-st} dt = -\frac{M}{s} e^{-st}\Big|_0^\infty = \frac{M}{s}$$

2. Ramp function

$$U_R(t) = \begin{cases} 0 & t < 0 \\ at & t \geq 0 \end{cases}$$

$$U_S(s) = \int_0^\infty a t e^{-st} dt = \frac{at}{-s} e^{-st}\Big|_0^\infty - \int_0^\infty \frac{a e^{-st}}{-s} dt = 0 - 0 + \frac{1}{s}\int_0^\infty a e^{-st} dt = \frac{1}{s}\left(\frac{a}{s}\right) = \frac{a}{s^2}$$

3. Rectangular pulse function

$$U_{RP}(t) = \begin{cases} 0 & t < 0 \\ h & 0 \le t < t_w \\ 0 & t \ge t_w \end{cases}$$

$$U_{RP}(s) = \int_0^{t_w} h \cdot e^{-st} dt = \frac{h}{s}\left(1 - e^{-t_w s}\right)$$

4. Trigonometric functions

$$U_{\sin}(t) = \begin{cases} 0 & t < 0 \\ A \sin wt & t \ge 0 \end{cases}$$

$$\begin{aligned} e^{jwt} &\triangleq \cos wt + j \sin wt \\ e^{-jwt} &\triangleq \cos wt - j \sin wt \end{aligned}$$

$$\begin{aligned} U_{\sin}(s) &= \mathcal{L}\left(A \frac{e^{jwt}}{2}\right) - \mathcal{L}\left(A \frac{e^{-jwt}}{2}\right) \\ &= \frac{A}{2}\left(\frac{1}{s-jw}\right) - \frac{A}{2}\left(\frac{1}{s+jw}\right) \\ &= \frac{Aw}{s^2+w^2} \end{aligned}$$

5. Impulse function: Idealization of pulse of short duration

$$U(s) = 1$$

1^{st} Order System

$$\tau \frac{dy}{dt} + y = Ku \quad \longleftrightarrow \quad Y(s) = \frac{K}{\tau s + 1} U(s)$$

- Monotonic exponential increase to a step change in input u
- Two adjustable parameters
  - K: Gain -- output change / input change at steady state
  - τ: Time constant -- speed of change (the system settles after $\sim 3\tau$.)

1^{st} Order System

- Responses to pulse / impulse changes?
- Examples: Continuous Stirred Tank

Pure Capacity (Integrating) System

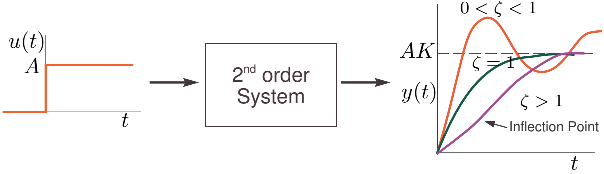
$$\frac{dy}{dt} = Ku \longleftrightarrow Y(s) = \frac{K}{s}U(s)$$

- Output is a time-integral of the input

$$y(t) = K\int_0^t u(\xi)\,d\xi$$

- To a step change, the output will grow at a constant rate (until something happens -- overflow)
- Example: Storage tank with an outlet pump

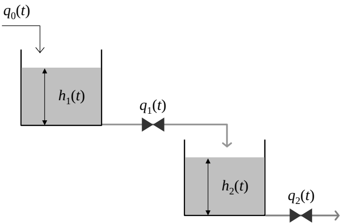
2^{nd} Order System

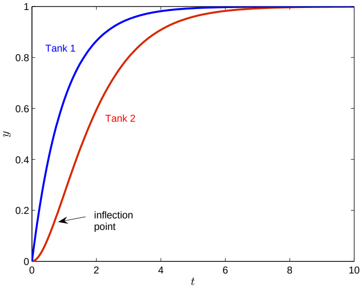


$$\tau^2 \frac{d^2 y}{dt^2} + 2\tau\zeta \frac{dy}{dt} + y = KU \longleftrightarrow Y(s) = \frac{K}{\tau^2 s^2 + 2\tau\zeta s + 1} U(s)$$

- A wider variety of possible responses to a step change
- Three adjustable parameters
  - K (gain): output change/input change at steady state
  - τ (time constant): speed of change
  - ζ (damping coefficient): degree of damping

Example of Overdamped ($\zeta > 1$) 2^{nd} Order System





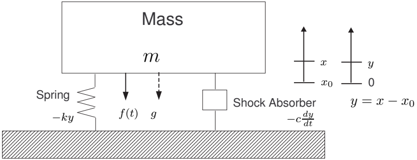
- Tank 1: $A_1 \frac{dh_1}{dt} = q_0 - \frac{h_1}{R_1}$
- Tank 2: $A_2 \frac{dh_2}{dt} = \frac{h_1}{R_1} - \frac{h_2}{R_2}$

$$Y_1(s) = \frac{K_1}{\tau_1 s + 1} U(s)$$

$$Y_2(s) = \frac{K_2}{\tau_2 s + 1} Y_1(s)$$

$$Y_2(s) = \frac{K_1 K_2}{(\tau_1 s + 1)(\tau_2 s + 1)} U(s)$$

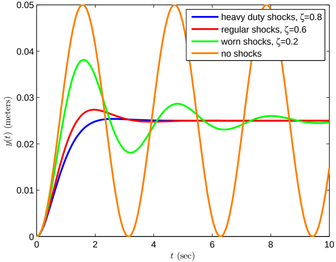
Example of Underdamped ($\zeta < 1$) 2^{nd} Order System



$$m\frac{d^2y}{dt^2} = -c\frac{dy}{dt} - ky + f(t)$$

$$\tau = \sqrt{\frac{m}{k}},\ K = \frac{1}{k},\ \zeta = \sqrt{\frac{c^2}{4mk}}$$

- Step Response (step change of $50N$ in $f(t)$)



- $k = 2000N/m$
- $m = 500kg$
- $c = 1600N/(m/s) \longrightarrow \zeta = 0.8$
- $c = 1200N/(m/s) \longrightarrow \zeta = 0.6$
- $c = 400N/(m/s) \longrightarrow \zeta = 0.2$

Why Study Underdamped Systems?

- It is true that very few chemical processes in their natural form exhibit underdamped dynamics.
- There are many mechanical structures (e.g., bridge towers, flexible beams) that display underdamped dynamics.
- Feedback controller may cause the closed-loop process to exhibit underdamped dynamics.

Characterization of 2^{nd} Order Underdamped System's Response

Rise time	t_r	Settling time	t_s
Time to first peak	t_p	Overshoot	$a/b = \exp\left(-\frac{\pi\tau}{\sqrt{1-\zeta^2}}\right)$
Decay ratio	$c/a = \exp\left(-\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right)$	Period of oscillation	$P = \frac{2\pi\tau}{\sqrt{1-\zeta^2}}$

- Measuring decay ratio gives ζ.