# 458.308 Process Control & Design

## Lecture 9: Control System Design Based on Frequency Response Analysis

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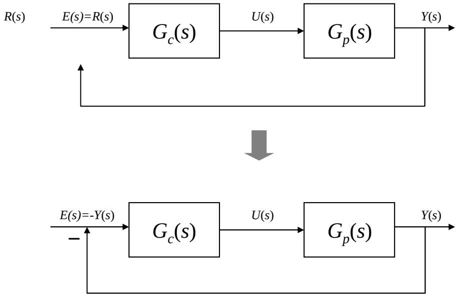
# Bode Stability Criterion

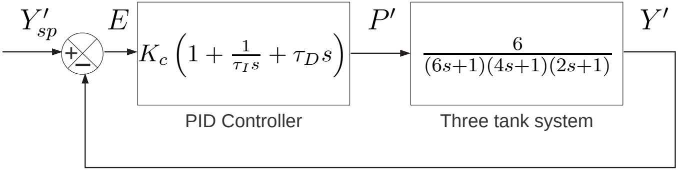
- Define the open-loop transfer function $G_{OL} \equiv GG_c, \ (G \triangleq G_v G_p G_m)$
- The phase angle plot of $G_{OL}$ starts from $0°$ (or $-90°$ if $G_c$ is a PI controller) at $\omega = 0$ (steady state) and keeps dropping as $\omega$ increases.
- The frequency at which the phase angle of $G_{OL}$ reaches $-180°$ is called the **critical frequency** ($\omega_c$).
- Stability Criterion: Closed-loop system is stable if A.R. of $G_{OL}(= |G_c(j\omega)G(j\omega)|) < 1$ at $\omega = \omega_c$
- If the criterion is not met, the system may diverge with growing oscillation of frequency $\omega_c$.

# Thought Experiment




- Assume that the set point is a sine wave.
- The controller has been tuned so that the output lags the set point by $180^{\circ}$
- Note that $-y(t)$ is exactly $180^{\circ}$ out of phase with $r(t)$, $\quad -y(t) = r(t)$.
- Consider the case where the set point signal, $r(t)$, is suddenly stopped and simultaneously the loop is closed. This means that the error signal will simply be $-y(t)$, identical to $r(t)$. Since it is identical to $r(t)$, then every signal on the control loop remains the same. The output continues to oscillate with the same frequency and magnitude as before the loop was closed (``nominally stable").
  If the AR is less than 1, the output decreases each time ``around the loop". In contrast, if $AR > 1$, the system gets unstable.

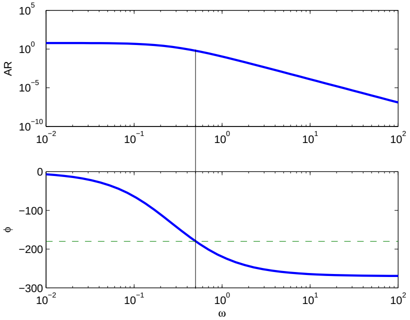
# Example




$$G_{OL} = \frac{6}{(6s+1)(4s+1)(2s+1)} \frac{K_c(1+\tau_I s+\tau_I \tau_D s^2)}{\tau_I s}$$

# P-Only Control

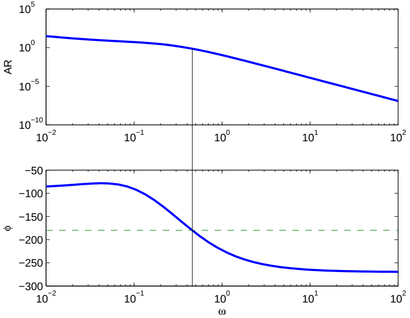
Bode Diagram of $G_{OL}$ with $K_c = 1$




- $\omega_c = 0.5,\ AR(G_{OL})$ at $\omega_c = 0.6$
- Closed loop is stable with $K_c = 1$
- Closed loop is stable for $K_c < 1/0.6 (= 1.67)$

# PI Control

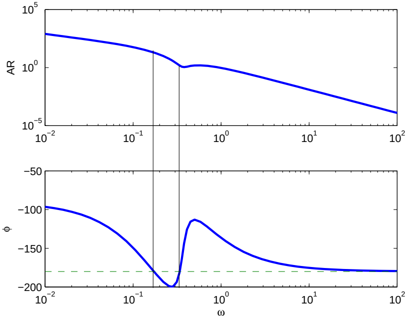
Bode Diagram of $G_{OL}$ with $K_c = 1,\ 1/\tau_I = 0.05,\ \tau_D = 0$




- $\omega_c = 0.457,\ AR(G_{OL})$ at $\omega_c = 0.732$
- Closed loop is stable with $K_c = 1$
- Closed loop is stable for $K_c < 1/0.732 (= 1.37)$

# PID Control

## Bode Diagram of $G_{OL}$ with $K_c = 1,\ 1/\tau_I = 1.33,\ \tau_D = 10$




- $\omega_c = 0.172,\ AR(G_{OL})$ at $\omega_c = 19.8$
- $\omega_c = 0.338,\ AR(G_{OL})$ at $\omega_c = 1.48$
- Multiple critical frequencies
- The closed loop is in fact stable with $K_c = 1$
- Bode stability criterion does not work.
- Nyquist criterion (14.3 of your textbook).

# Intuitive Interpretation of Bode Stability Criterion

- One chooses $K_c$ so that the steady-state loop gain is positive ($G(0),\ K_c > 0$): Right direction of control at steady state.
- Phase angle starts from $0°$ (or $-90°$) and reaches $-180°$ at some frequency ($\omega_c$).
- At this frequency, negative feedback becomes positive feedback and feedback results in control actions of wrong direction. Feedback does harm (adding to error) rather than good at this frequency.
- Loop gain at this frequency (AR of $G_{OL}$ at $\omega = \omega_c$) needs to be sufficiently small ($< 1$) in order to prevent the closed loop from going unstable.

# Gain Margin and Phase Margin

- Idea: Shouldn't be very close to the limit of stability as the process dynamics can change. Give some margin.
- Gain Margin:
  - G.M. = 1/(A.R. of $G_{OL}$ at $\omega = \omega_c$)
  - The gain of the process (or the controller) can increase by a factor of G. M. before the closed-loop becomes unstable.
- Phase Margin:
  - P.M. = $180^\circ$ + ($\phi$ of $G_{OL}$ at $\omega$ where A.R. of $G_L$ reaches 1)
  - The phase angle of the process (or the controller) can decrease by a factor of P.M. before the closed-loop becomes unstable.

# Relationship with Continuous Cycling and Z-N Tuning

- With P only control and $K_c = 1$, $G_{OL} = G$. Plot Bode diagram of $G$ only.
- Let $\omega_c$ be the frequency where phase angle of $G$ becomes $-180°$.
- Ultimate Gain: $K_{cu} = 1/$(A.R. of $G$ at $\omega = \omega_c$).
    - P controller with such a $K_c$ makes the A.R. of $G_L(= K_c G)$ to be 1 at $\omega = \omega_c$.
    - Gain margin with $K_c = K_{cu}$ is 1 (no margin).
- Ultimate period: $P_u = 2\pi/\omega_c$
- Z-N tuning rule uses the information on $K_{cu}$ and $P_u$ ($\omega_c$ and A.R. of $G$ at $\omega = \omega_c$) to define P/PI/PID parameters with sufficient gain and phase margins.