Part IV Magnetic Properties of Materials

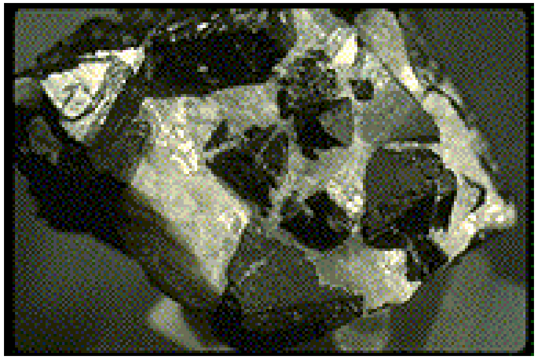
Chap. 14 Foundations of Magnetism

Chap. 15 Magnetic Phenomena and Their Interpretation- Classical Approach

Chap. 16 Quantum Mechanical Considerations

Chap. 17 Applications

14.1 Introduction



The magnetic rocks is lodestone which is the naturally occurring mineral magnetite, Fe_3O_4

How do lodestones become magnetic?

Lodestones are rich in magnetite, an iron oxide mineral. Lightning with a typical current of 1,000,000 Amps creates a magnetic field large enough to saturate the magnetization of nearby lodestone outcrops. This is a rare event- on average lightning strikes that close to any point on the Earths surface happens once in 1-10 million years.

14.2 Basic Concepts in Magnetism

14.2.1 Magnetic Poles

$$F = \frac{p_1 p_2}{d^2}$$

F = Force between two poles

p_1,p_2 = pole strength

d = distance between two poles

Propotionality constant = 1

$$F = pH$$

H = Field strength or field intensity

Propotionality constant = 1

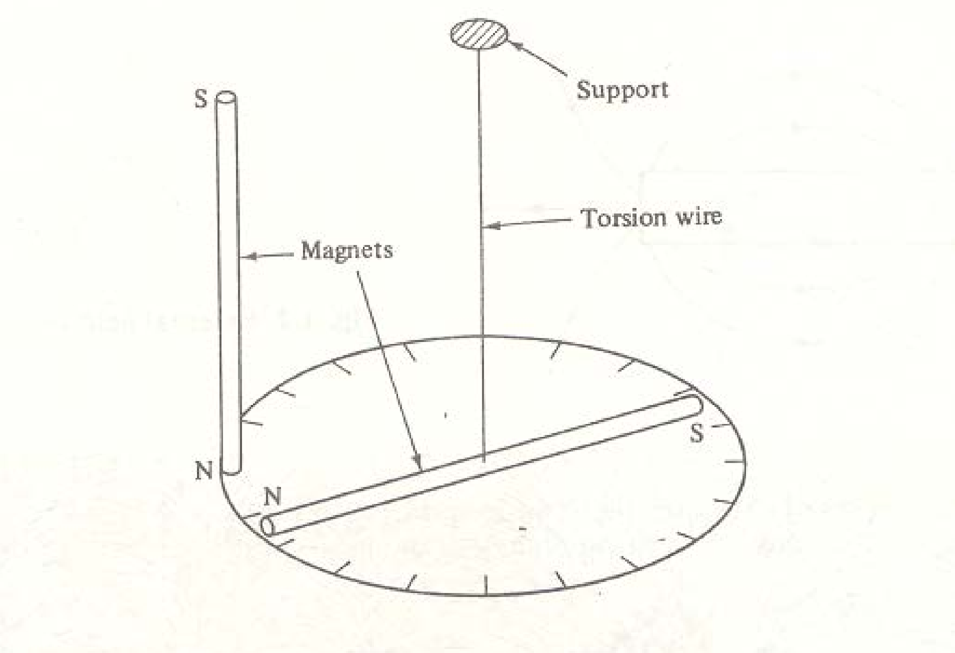


Fig. 1.1 Torsion balance for measuring the forces between poles.

Field of unit strength is one which exerts a force of one dyne on a unit pole.

$$H = \frac{p}{d^2}$$

1 Oe = 1 Line of force/cm² = 1 maxwell/cm²

14.2 Basic Concepts in Magnetism

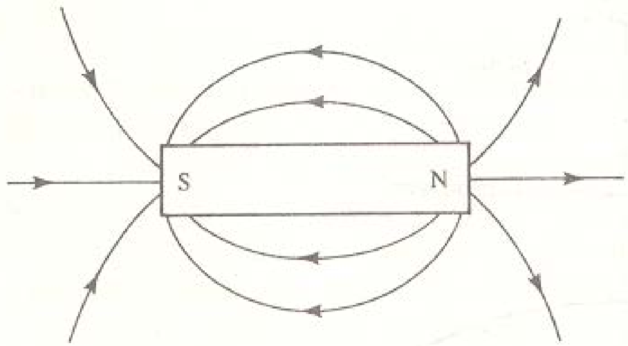


Fig. 1.2 External field of a bar magnet.

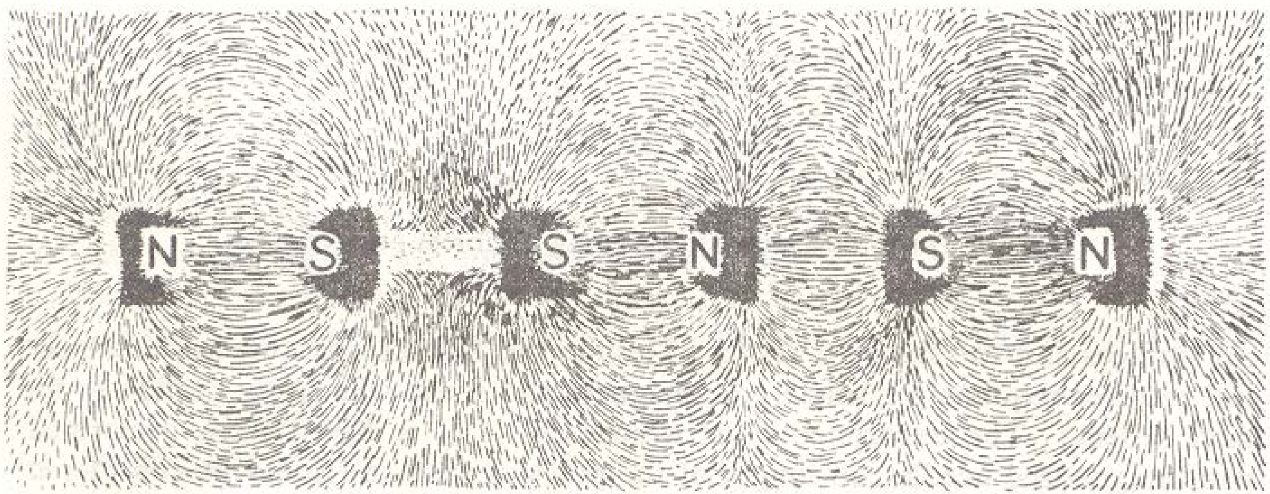
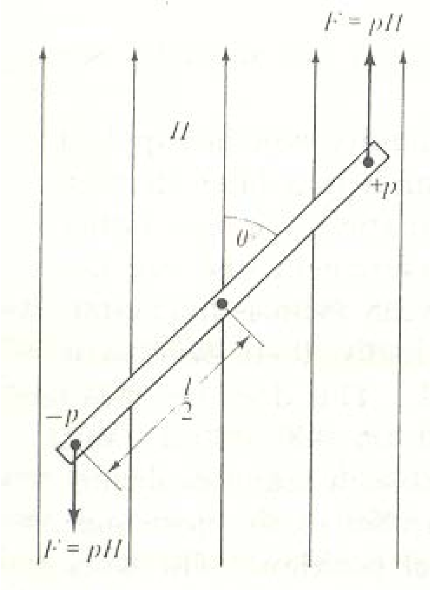


Fig. 1.3 Fields of bar magnets revealed by iron filings. (Courtesy Encyclopedia Brittanica).

14.2.2 Magnetic Moment



Moment of this couple :

$$\frac{l}{2}(pH\sin\theta)+\frac{l}{2}(pH\sin\theta)=pHl\sin\theta$$

When H = 1 Oe and θ = 90°, the moment is given by

$$m = pl$$

m = magnetic moment of the magnet

$$dE_p = 2(\frac{l}{2})(pH\sin\theta)\,d\theta = mH\sin\theta\,d\theta$$ **(E_p = potential energy)**

$$E_p = \int_{90^\circ}^{\theta} mH\sin\theta\,d\theta = -mH\cos\theta$$

$$E_p = -\boldsymbol{m}\cdot\boldsymbol{H}$$ **(Vector Form)**

14.2.3 Intensity of magnetization

$$M = \frac{m}{v} = \frac{pl}{v} = \frac{p}{v/l} = \frac{p}{a}$$

M = intensity of magnetization or magnetization

v = volume of the magnet

a = cross-sectional area of the magnet

p = pole strength

l = interpolar distance

$$\sigma = \frac{m}{w} = \frac{m}{v\rho} = \frac{M}{\rho} \ \text{emu/g}$$

σ = specific magnetization

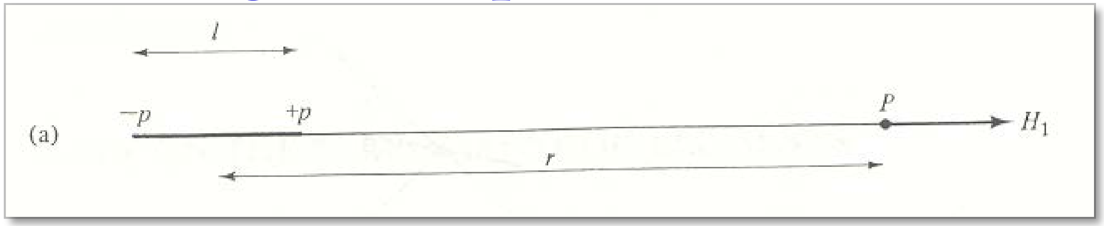
w = mass of magnet

ρ = density of magnet

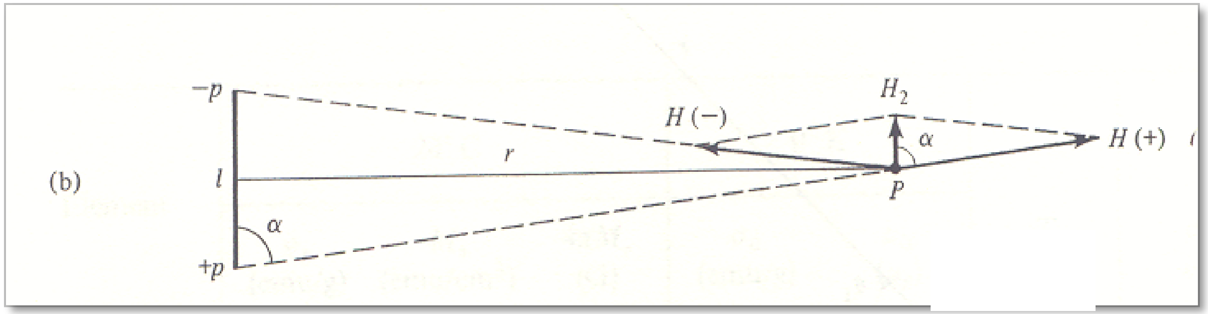
(A few writers use the abbreviation "emu" to mean regs/oersted cm^3 . Then M is in emu and σ in emu- cm^3/g)

14.2 Basic Concepts in Magnetism

14.2.4 Magnetic dipoles



$$H_1 = \frac{p}{[r-(l/2)]^2} - \frac{p}{[r+(l/2)]^2} = \frac{2prl}{[r^2-(l^2/4)]^2}$$

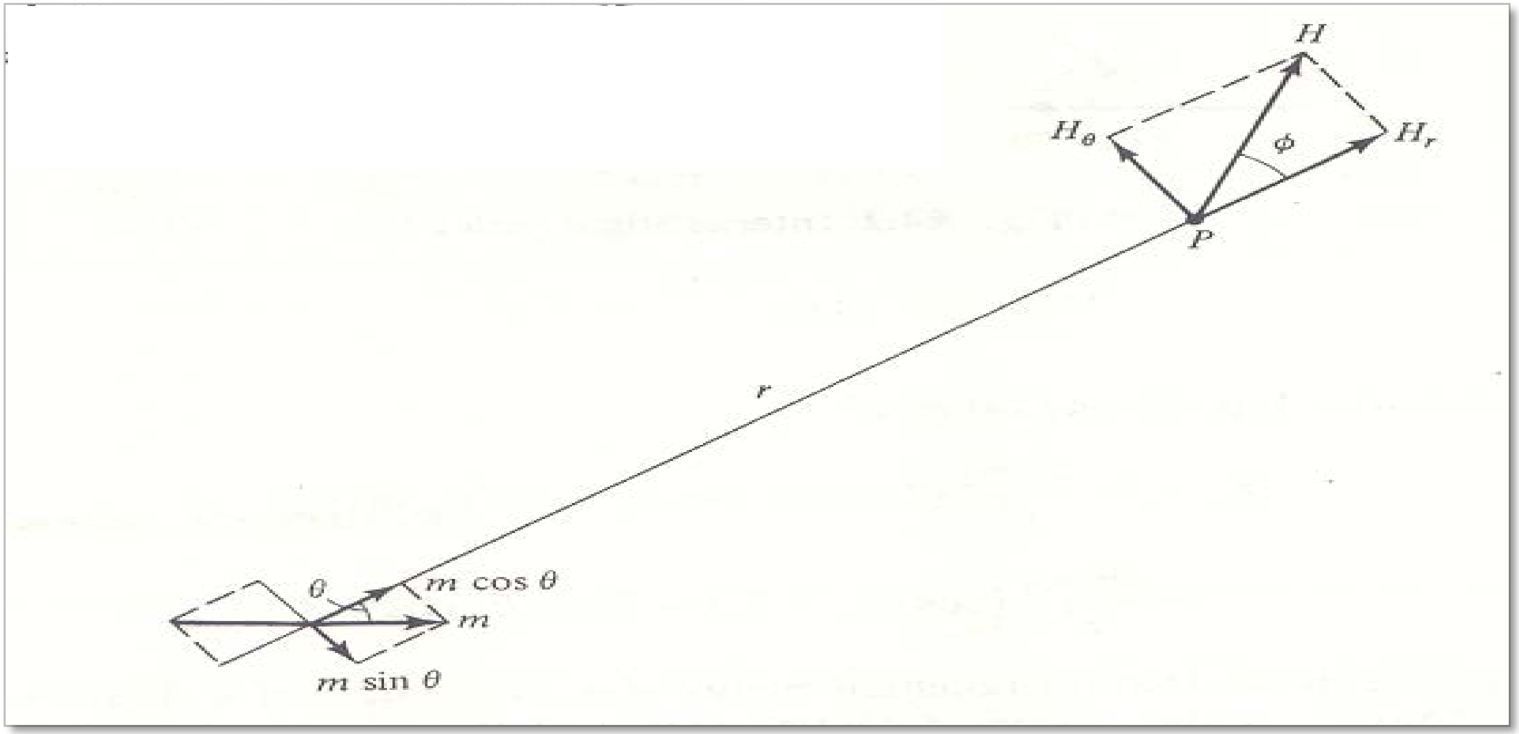


$$H_2 = 2H(+)\cos\alpha = 2\left[\frac{p}{r^2+(l^2/4)}\right]\left[\frac{l/2}{\{r^2+(l^2/4)\}^{1/2}}\right] = \frac{pl}{[r^2+(l^2/4)]^{3/2}}$$

If r is large compared to l, this expression becomes

$$H_2 = \frac{pl}{r^3} = \frac{m}{r^3}$$

14.2 Basic Concepts in Magnetism



$$H_r = \frac{2(m\cos\theta)}{r^3} \qquad H_\theta = \frac{m\sin\theta}{r^3}$$

$$H = (H_r^2 + H_\theta^2)^{1/2} = \frac{m}{r^3}(3\cos^2\theta + 1)^{1/2} \qquad \tan\phi = \frac{H_\theta}{H_r} = \frac{\tan\theta}{2}$$

14.2 Basic Concepts in Magnetism

$$H_p = H_r \cos\theta_2 - H_\theta \cos(90^\circ - \theta_2)$$

$$E_p = -m_2(H_r \cos\theta_2 - H_\theta \sin\theta_2)$$

$$H_r = \frac{2(m\cos\theta)}{r^3}$$

$$H_\theta = \frac{m\sin\theta}{r^3}$$

$$E_p = -\frac{m_1 m_2}{r^3}(2\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$$

$$= \frac{m_1 m_2}{r^3}[\cos(\theta_1 - \theta_2) - 3\cos\theta_1\cos\theta_2]$$

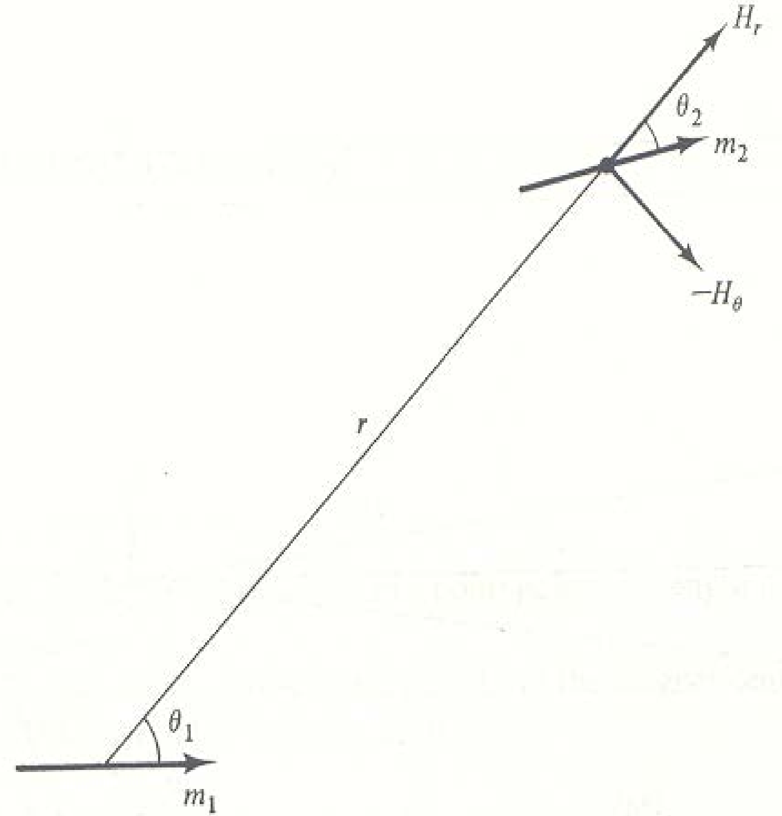
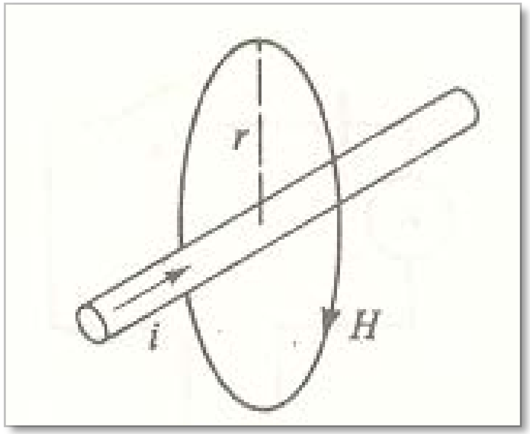


Fig. A4.2 Interacting dipoles.

14.2.5 Magnetic effects of currents



Outside the wire :

$$H = \frac{2i}{10r}\ \text{Oe}$$

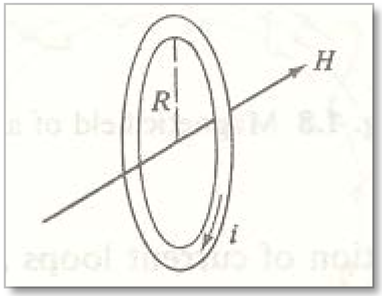
i = current in ampreres

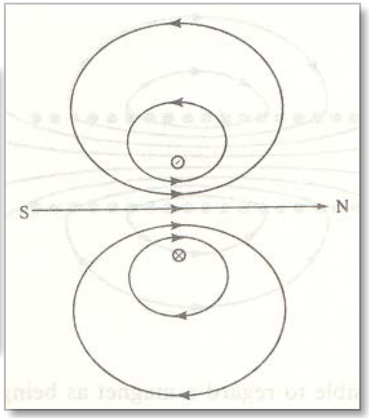
Inside the wire :

r = distance

$$H = \frac{2ir}{10r_0^2}\ \text{Oe}$$

r_0 = wire radius





Field at the center along the axis :

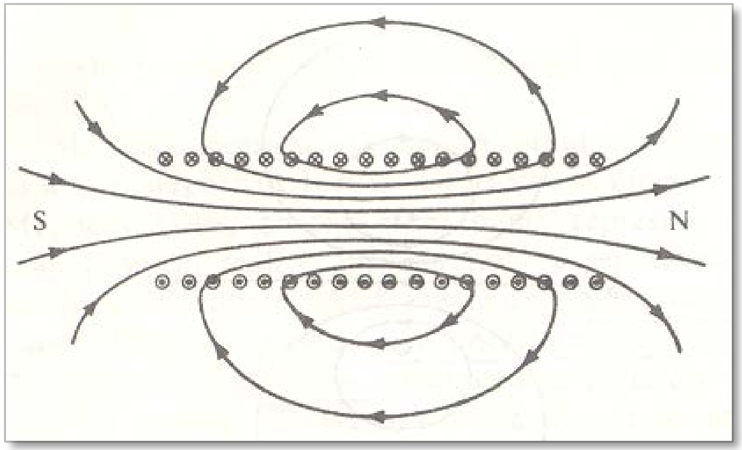
$$H = \frac{2\pi i}{10R}\ \text{Oe}$$

Magnetic moment :

$$m\ (\text{loop}) = \frac{\pi R^2 i}{10} = \frac{Ai}{10}\ \text{erg/Oe}$$

A = area of the loop in cm^2

14.2 Basic Concepts in Magnetism



Field at the center along the axis :

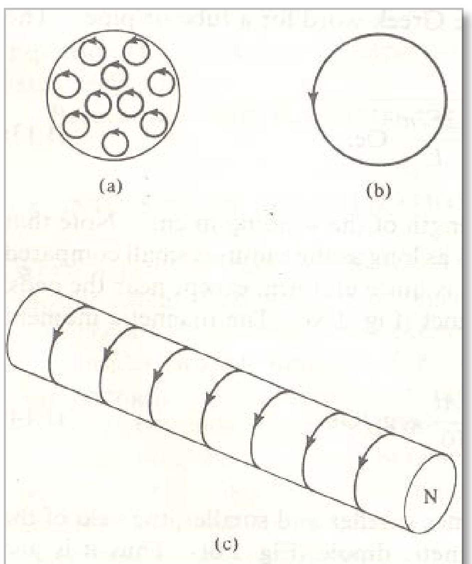
$$H = \frac{4\pi ni}{10L} = \frac{1.257ni}{L}\ \text{Oe}$$

Magnetic moment :

$$m\,(\text{solenoid}) = \frac{nAi}{10}\ \text{erg/Oe}$$

n = number of turns

L = length of the winding in cm



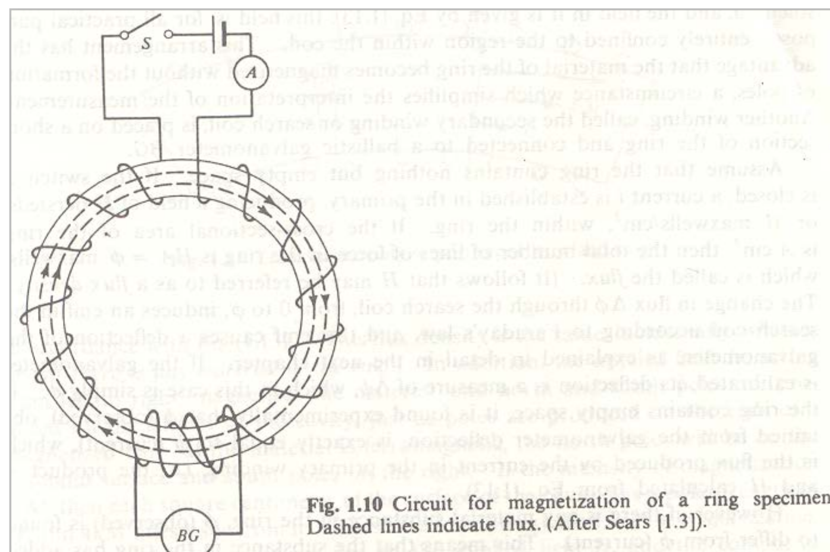


Fig. 1.10 Circuit for magnetization of a ring specimen. Dashed lines indicate flux. (After Sears [1.3]).

14.2.6 Varieties of magnetism

$$\phi = HA$$

ϕ= flux

H = field (maxwell/cm^2)

A = cross-sectional area of the ring (cm^2)

ϕ (observed) < ϕ (current), diamagnetic (for example, Cu, He)

ϕ (observed) > ϕ (current), paramagnetic (for example, Na Al)

or antiferromagnetic (for example, MnO, FeO)

ϕ (observed) $\gg$ ϕ (current), ferromagnetic (for example, Fe, Co, Ni)

or ferrimagnetic (for example, Fe_3O_4)

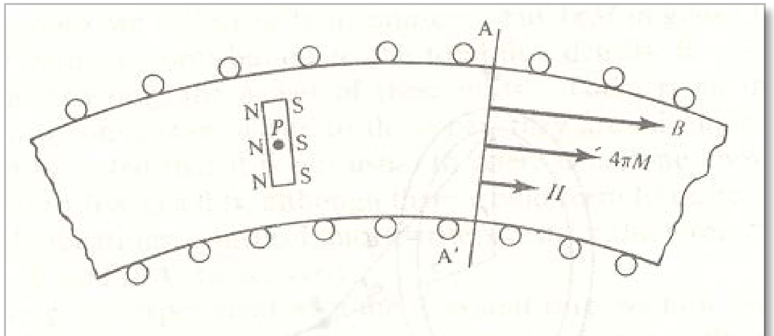


Fig. 1.11 Transverse cavity in portion of Rowland ring.

$$\boxed{B = H + 4\pi M}$$

14.2 Basic Concepts in Magnetism

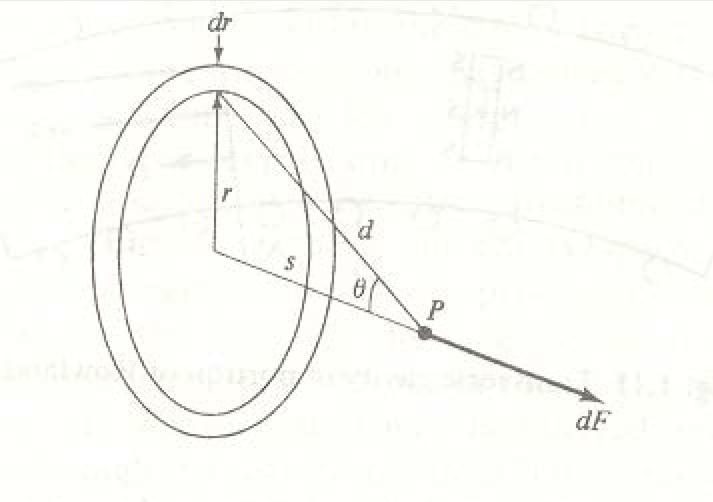


Fig. 1.12 Axial force on unit pole at P.

Total pole strength on this ring :

$$(M)(2\pi r)dr$$

$$dF = \frac{(2\pi Mr\,dr)(1)\cos\theta}{d^2}$$

$$r = d\sin\theta = (r^2 + s^2)^{1/2}\sin\theta$$

$$dF = 2\pi M\sin\theta\,d\theta$$

Since P is very near the face of the gap, the total force is obtained by integrating from θ = 0 to = $\pi/2$

$$F = \int_0^{\pi/2} 2\pi M\sin\theta d\theta = 2\pi M$$

The other face of the gap, covered with south poles, exerts an equal force in the same direction. Therefore, The force due to the poles on the gap surfaces is

$$F = 2\int_0^{\pi/2} 2\pi M\sin\theta d\theta = 4\pi M$$

$\therefore$ Total force on a unit pole at P : $B = H + 4\pi M$

14.2 Basic Concepts in Magnetism

The magnetic properties of a material are characterized not only by the magnitude and sign of M but also by the way in which M varies with H. The ratio of these two quantities is called the

susceptibility : $\kappa = \dfrac{M}{H}$ emu/cm^3 Oe

$$\chi = \kappa / \rho = \text{mass susceptibility (emu/g Oe), where } \rho = \text{density}$$

$$\chi_A = \chi A = \text{stomic susceptibility (emu/g atom Oe), where A = atomic weight}$$

$$\chi_M = \chi M' = \text{molecular susceptibility (emu/g mol Oe), where M}' = \text{molecular weight}$$

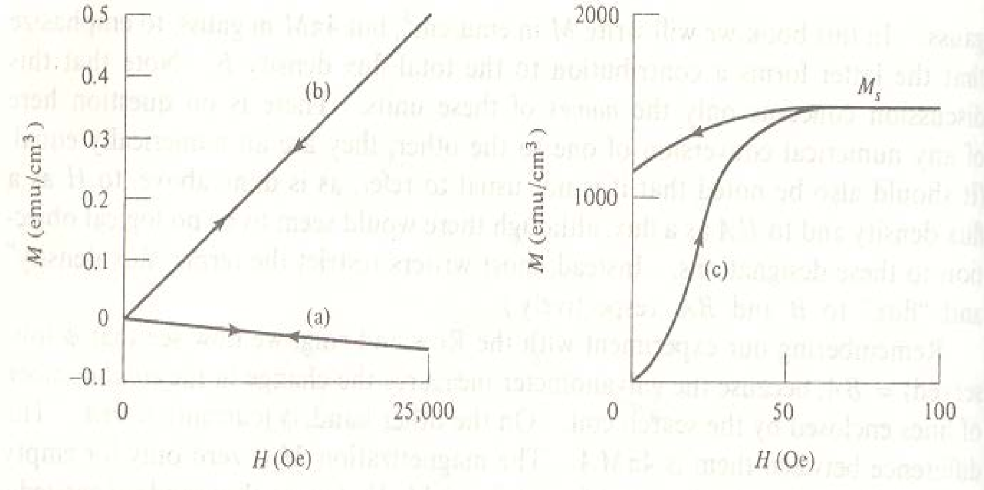


Fig. 1.13 Typical magnetization curves of (a) a diamagnetic, (b) a para- or antiferromagnetic, and (c) a ferro- or ferrimagnetic. (After Schoenberg [1.4]).

Saturation :

At large enough values of H, the Magnetization M becomes constant at saturation value of M_s.

Hysteresis or irreversibility :

After saturation, a decrease in H to zero does not reduce M to zero. Ferro- and ferrimagnetic materials can thus be made into permanent magnets

14.2 Basic Concepts in Magnetism

The ratio of B and H is the **permeability** :

$$\mu = \frac{B}{H}$$

Since $B = H + 4\pi M$

$$\frac{B}{H} = 1 + 4\pi \frac{M}{H}$$

$$\boxed{\therefore \mu = 1 + 4\pi\kappa}$$

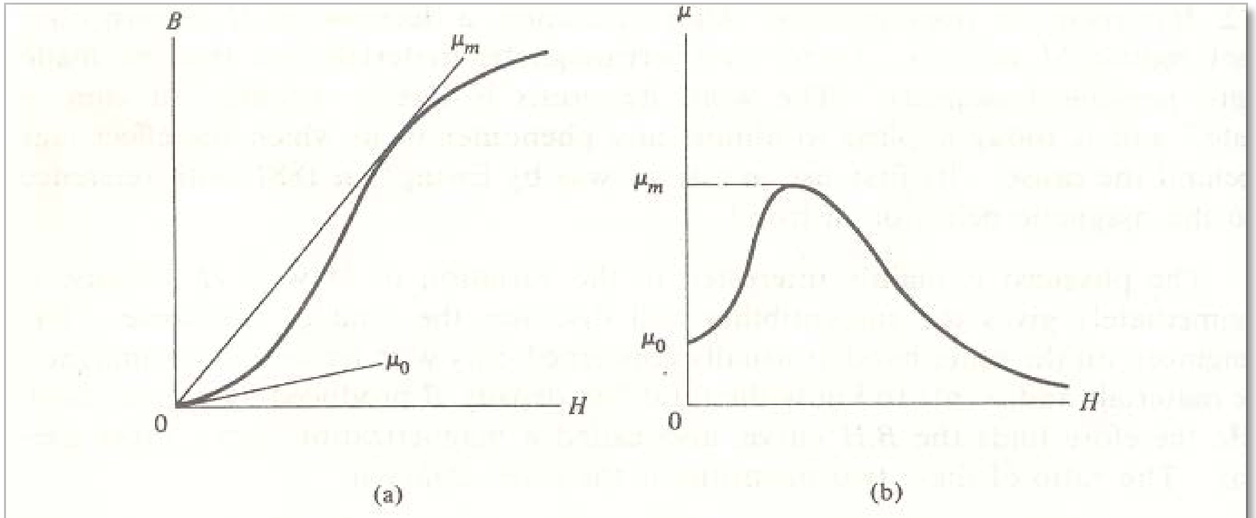


Fig. 1.14 (a) B versus H curve of a ferro- or ferrimagnetic, and (b) corresponding variation of μ with H.

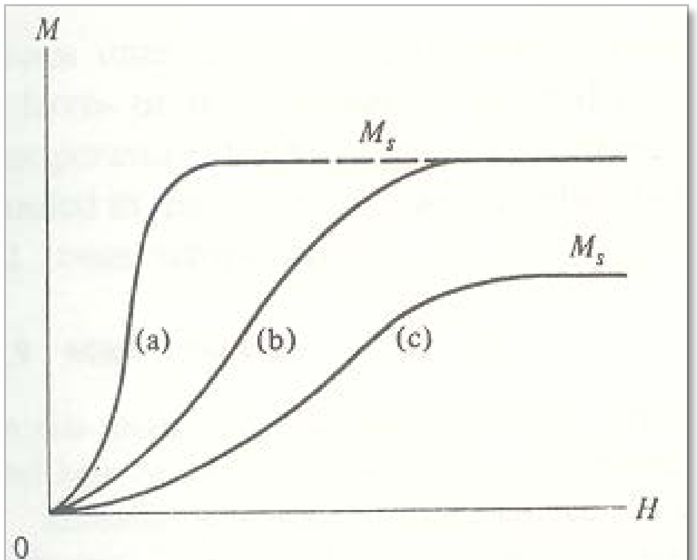
14.2 Basic Concepts in Magnetism

We can now characterize the magnetic behavior of various kinds of substances by their corresponding values of κ and μ :

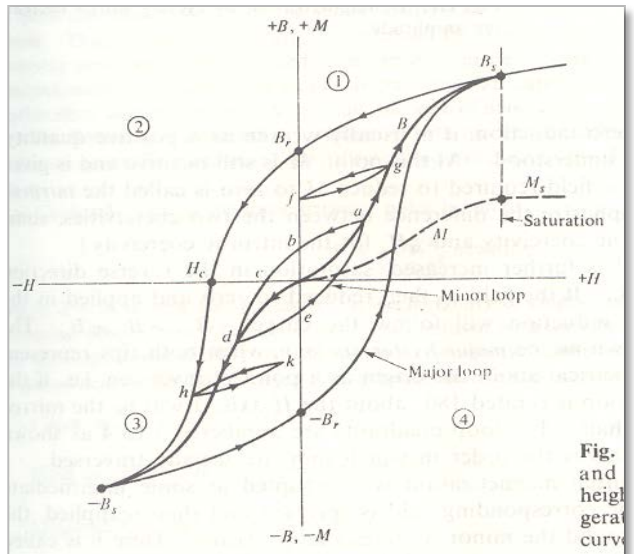
1. **Empty Space :** K = 0, since there is no matter to magnetize, and $\mu = 1$.
2. **Diamagnetic :** κ is small and negative, and μ slightly less than 1.
3. **Para- and antiferromagnetic :** κ s small and positive, and μ slightly greater than 1.
4. **Ferro- and ferrimagnetic :** κ and μ are large and positive, and both are functions of H.

14.2 Basic Concepts in Magnetism

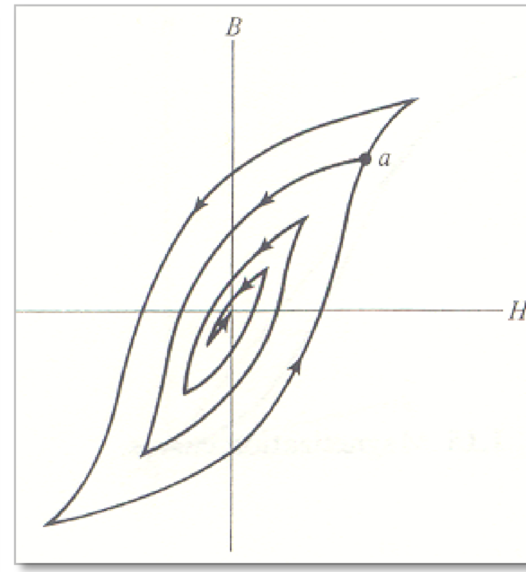
14.2.7 Magnetization curves and hysteresis loops



Magnetization curves



Magnetization curves and hysteresis loops. (The height of the M curve is exgerated relative to that of B curve.



Demagnetization by cycling with a decreasing amplitude.

14.3 Units

The scientific and technical literature on magnetism, particularly in the USA is still widely written in electromagnetic cgs(emu) units. In some European countries, and in many international scientific journals, the SI units are mandatory.

$$B = H + 4\pi M$$ (cgs units)

$$\frac{B}{H} = 1 + 4\pi \frac{M}{H} \qquad \Leftarrow \qquad \mu = \frac{B}{H}$$

$$\boxed{\therefore \mu = 1 + 4\pi\kappa}$$

14.3 Units

14.3.1 MKS units

Coulomb's law of the force between poles: $F = \dfrac{p_1 p_2}{4\pi\mu_0 d^2}$ newtons

Force on a pole: $F = pH$ newtons

Field of a pole: $H = \dfrac{p}{4\pi\mu_0 d^2}$ ampere-turns/meter

Magnetic moment: $m = pl$ weber-meter

Potential energy: $E_p = -mH\cos\theta$ joules

Magnetization: $M = \dfrac{m}{v} = \dfrac{\mathrm{p}}{\mathrm{a}}$ weber/meter^2

Field of straight wire: $H = \dfrac{i}{2\pi r}$ ampere/meter

Field of current loop: $H = \dfrac{i}{2R}$ ampere/meter

m (loop): $\mu_0 Ai$ weber-meter

Field of solenoid: $H = \dfrac{ni}{L}$ ampere/meter

m (solenoid): $\mu_0 nAi$ weber-meter

Volume susceptibility: $\kappa = \dfrac{M}{H}$ weber/ampere meter

Absolute permeability: $\mu = \dfrac{B}{H}$ weber/ampere meter

Relative permeability: $\mu_r = \dfrac{\mu}{\mu_0} = \dfrac{B}{\mu_0 H}$

H: 1 ampere-turn/m $= 4\pi \times 10^{-3}$ oersted

B: 1 weber/meter^2 $= 10^4$ gauss $=$ 1 tesla

M: 1 weber/meter^2 $= \dfrac{104}{4\pi}$ emu/cm^3

ϕ: 1 weber $= 10^8$ maxwells