Water quality II

Today's lecture

- Biochemical oxygen demand (BOD)
  - Concept
  - Measurement
  - Modeling
- DO dynamics in river
  - DO sag curve
  - Modeling DO in the river
  - Solution: Streeter-Phelps equation

Oxygen demand

- Biochemical oxygen demand (BOD)
  - The oxygen demand is measured by a bioassay
  - The water sample is inoculated with bacteria that degrade organic matter in water
  - The difference in DO in the water sample at the beginning and end of the test is used to calculate BOD

Oxygen demand

ThOD ≥ COD > BOD

- Some organic compounds may not be oxidized even with a strong oxidizing agent (ThOD ≥ COD)
- Some carbon is used for bacterial growth; some organic compounds are not biodegradable; some organic matter is converted to non-biodegradable materials (COD > BOD)

BOD Measurement

Step 1. Take the wastewater sample and dilute if needed. Fill the test bottle (usually 300 mL) with the (diluted) sample and a suspension of microorganisms (seed) if needed. Seal the bottle to prevent air intrusion/water evaporation.

$$Dilution\ factor = P = \frac{volume\ of\ wastwater\ sample}{volume\ of\ wastewater + dilution\ water}$$

The expected BOD of the diluted sample should be 2-6 mg/L.

* saturation DO concentration at 20°C: 9.17 mg/L

BOD Measurement

Step 2. Prepare blank samples (control) containing only the dilution water and the seed.

Step 3. Incubate the samples and blanks at 20°C in the dark. Usually the incubation time is 5 days.

Step 4. Measure the DO after incubation.

BOD Measurement

The BOD of the wastewater sample can be calculated as:

$$BOD_t = \frac{DO_{b,t} - DO_{s,t}}{P}$$

$DO_{b,t}$ = DO concentration in blank after t days of incubation

$DO_{s,t}$ = DO concentration in sample after t days of incubation

If the BOD of the seed is significant, the following equation should be used instead:

$$BOD_t = \frac{(DO_{s,i} - DO_{s,t}) - (DO_{b,i} - DO_{b,t})f}{P}$$

$DO_{s,i}$ = the initial DO of the sample

$DO_{b,i}$ = the initial DO of the blank

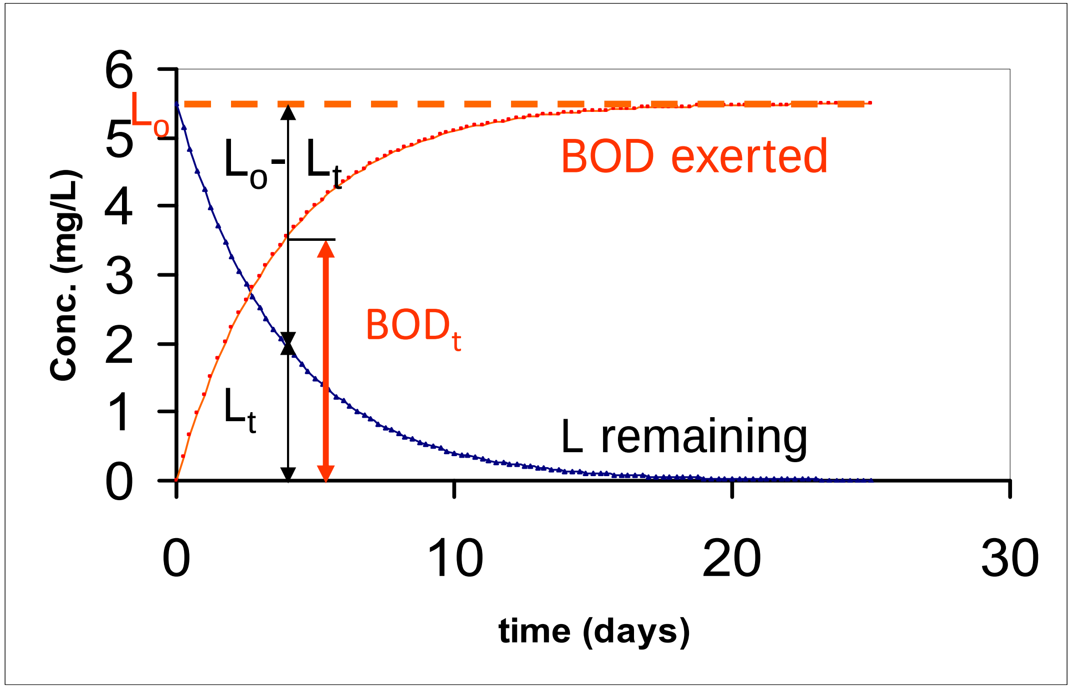
f = (volume of seed in sample) / (volume of seed in blank)

BOD Measurement

Q: The BOD of a wastewater sample was initially estimated to be 180 mg/L. What volume of the sample should be added to a 300-mL bottle?

Applying the calculated dilution factor, the DO values for the blank and diluted sample after 5 days of incubation were 8.7 and 4.2 mg/L, respectively. What is the BOD_5 of the sample? Assume that the sample and the blank were not seeded.

Modeling BOD



L = oxygen demand of remaining biodegradable organic chemicals (mg/L)

- L_t decreases with time and BOD_t increases with time
- $L_0 = L_t + BOD_t$
- L_0 (= BOD_∞): ultimate BOD

Modeling BOD

The degradation of organic compounds by microorganisms is modeled as a first-order reaction:

$$\frac{dL}{dt} = -kL$$

k = first-order reaction constant (day^{-1})

Integration of the equation gives:

$$L_t = L_0 e^{-kt}$$

As $BOD_t = L_0 - L_t$,

$$\boxed{BOD_t = L_0\left(1 - e^{-kt}\right)}$$

Modeling BOD

The magnitude of the BOD rate constant, *k* depends on:

1. Nature of waste: whether the waste is easily biodegradable or not
2. Ability of organisms to use waste: the microorganisms in the test bottle may not be ready to degrade the waste! (recall the “lag phase”)
3. Temperature

$$k_T = k_{20}\theta^{T-20}$$

k_T = BOD rate constant at temperature T (day^{-1})

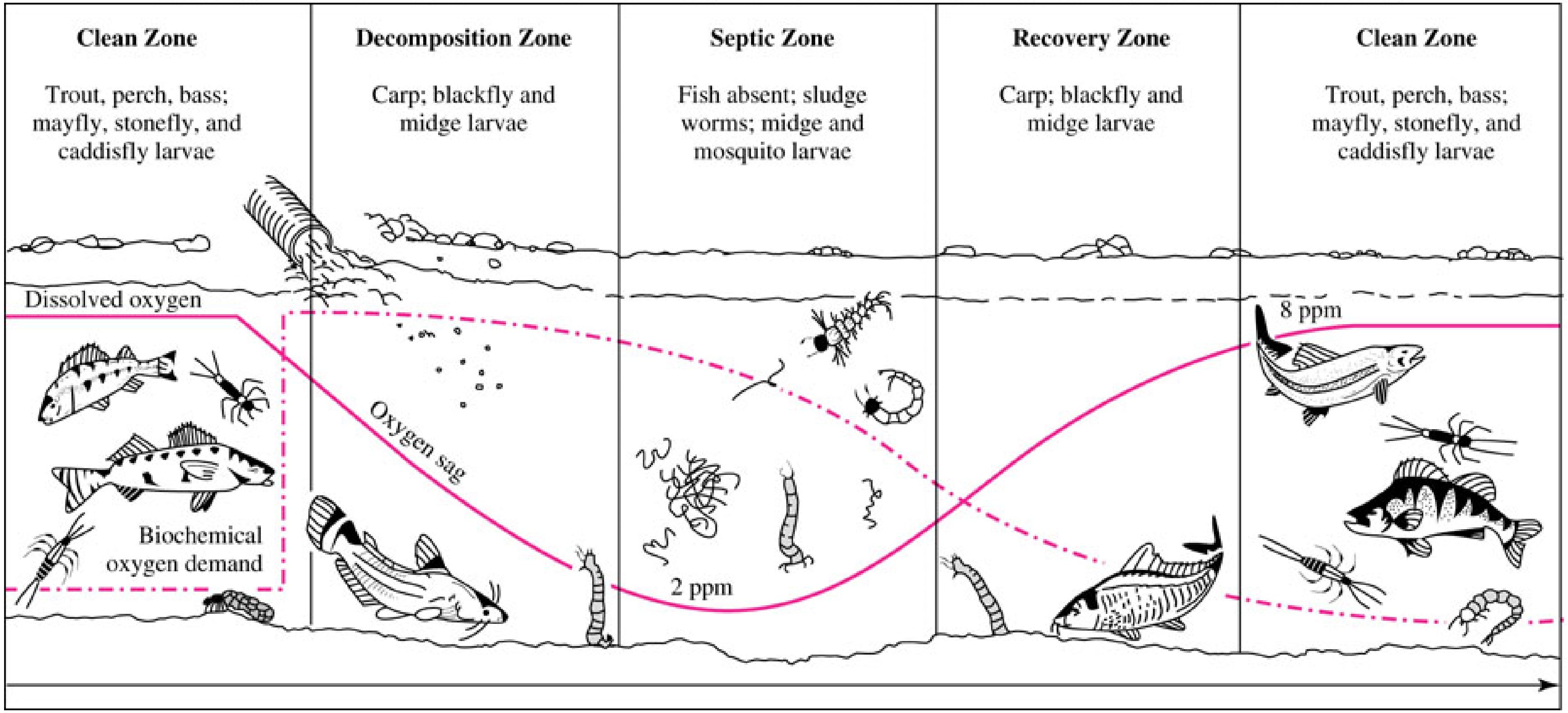
k_{20} = BOD rate constant at 20°C (day^{-1})

θ = temperature coefficient
(use 1.135 for 4-20°C and 1.056 for 20-30°C)

Modeling BOD

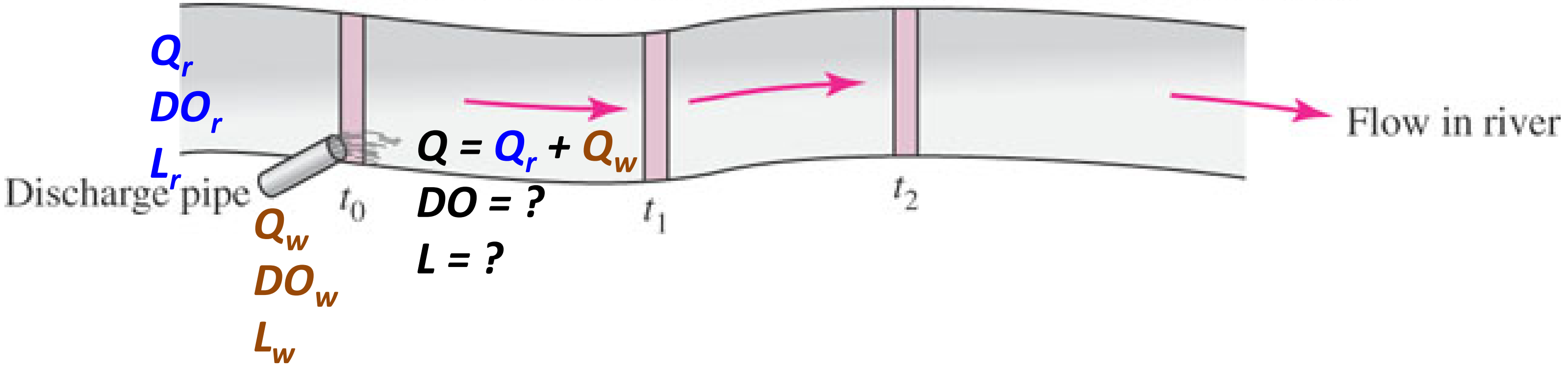
Q: The BOD_5 of a wastewater is 120 mg/L and the BOD rate constant is 0.115 day^{-1} at 20°C. What is the ultimate BOD? If the wastewater is incubated at 15°C with a supply of oxygen, how much oxygen will be used by microorganisms in three days?

DO sag curve



- Factors causing DO depletion: BOD in water (upstream + waste)
- Factors causing DO increase: reaeration from the atmosphere (+ photosynthesis – neglected)

Modeling the DO along a river



Q_r
DO_r
L_r

Q_w
DO_w
L_w

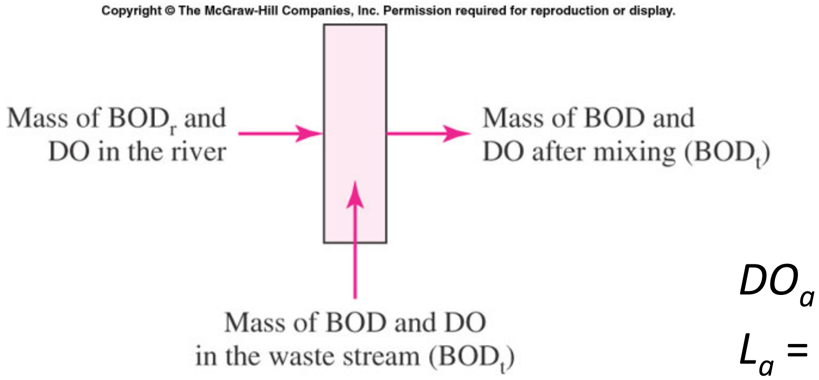
$Q = Q_r + Q_w$
$DO = ?$
$L = ?$

We will model the DO of a river receiving waste at time t_0. The river will be modeled as a PFR.

* The solution for this problem is known as “Streeter-Phelps equation”, a well-known equation derived by Streeter and Phelps in 1925.

Modeling the DO along a river

The DO and ultimate BOD at t_0 are calculated by a mass balance approach:



$$(Q_w + Q_r)DO_a = Q_w DO_w + Q_r DO_r$$

$$(Q_w + Q_r)L_a = Q_w L_w + Q_r L_r$$

DO_a = DO concentration right after mixing (mg/L)

L_a = ultimate BOD right after mixing (mg/L)

$$DO_a = \frac{Q_w DO_w + Q_r DO_r}{Q_w + Q_r} \qquad L_a = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r}$$

Modeling the DO along a river

The temperature after mixing is calculated in the same way:

$$T_a = \frac{Q_w T_w + Q_r T_r}{Q_w + Q_r}$$

T_a = temperature after mixing (°C or K)

Oxygen deficit

- Oxygen deficit (*D*): the difference between the saturation DO value and the actual DO concentration

$$D = DO_s - DO$$

Therefore, the oxygen deficit right after mixing is calculated as:

$$D_a = DO_s - \frac{Q_w DO_w + Q_r DO_r}{Q_w + Q_r}$$

D_a = oxygen deficit right after mixing (mg/L)

Modeling the DO along a river

- Rate of reaeration
  - Should depend on the stream velocity and depth
  - The reaeration coefficient, k_r [day^{-1}]

$$k_r = \frac{3.9u^{1/2}}{h^{2/3}}$$

u = average stream velocity (m/s)
h = average stream depth (m)

  - Rate of reaeration should also depend on oxygen deficit

$$Rate\ of\ reaeration = \left.\frac{d(DO)}{dt}\right|_{reaeration} = -\left.\frac{dD}{dt}\right|_{reaeration} = k_r D$$

  - Effect of temperature on k_r: faster mass transfer at higher temp.

$$k_{r,T} = k_{r,20}\theta^{T-20}$$

$k_{r,T}$ = reaeration coeff. at temperature T (day^{-1})
$k_{r,20}$ = reaeration coeff. at 20°C, obtained from
$k_{r,20} = 3.9u^{1/2}/h^{2/3}$ (day^{-1})
θ = temperature coefficient (use 1.024)

Modeling the DO along a river

- Rate of deoxygenation
  - Rate of oxygen consumption by microorganisms
  - Assume that the first-order deoxygenation rate constant is equal to the BOD rate constant, *k*
  - The assumption is valid for deep, slow-moving streams
  - The rate of deoxygenation

$$Rate\ of\ deoxygenation = -\left.\frac{d(DO)}{dt}\right|_{deoxygenation} = \left.\frac{dD}{dt}\right|_{deoxygenation}$$

$$= k_d L$$

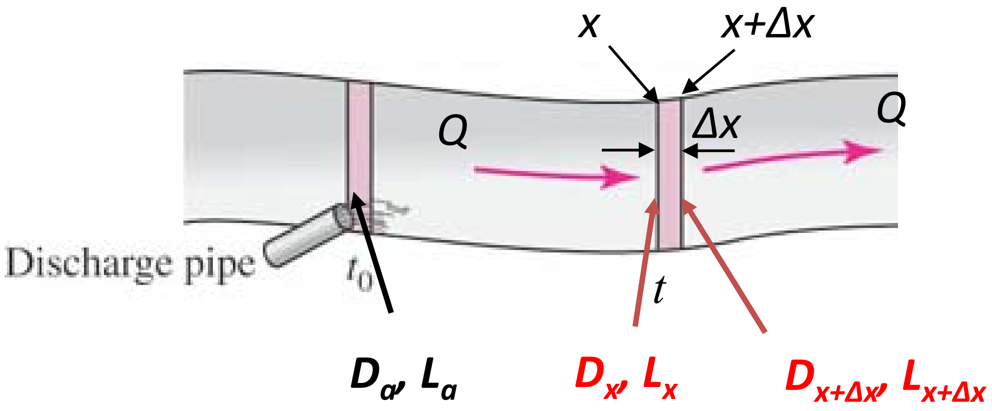
k_d = first-order deoxygenation rate constant [T^{-1}]

  - Effect of temperature on k_d: use the equation for *k*!

$$k_T = k_{20}\theta^{T-20}$$

θ = 1.135 for 4-20°C and
1.056 for 20-30°C

Modeling the DO along a river



Steady-state D (=DO_s-DO) balance for a thin plate at time t :

$$0 = QD_x - QD_{x+\Delta x} + k_d L_x \cdot \Delta V - k_r D_x \cdot \Delta V$$

ΔV = volume of the CV = $A \cdot \Delta x$

(A = cross-sectional area)

With rearrangements and Δx → 0, we obtain:

$$\frac{dD}{dt} = k_d L - k_r D$$

Modeling the DO along a river

Governing equation:

$$\frac{dD}{dt} = k_d L - k_r D$$

+ Boundary conditions:

at t=0, $D=D_a$ and $L=L_a$

Solution:

$$D_t = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_a\left(e^{-k_r t}\right)$$

D_t = oxygen deficit in a river after flowing downstream from the mixing point for time *t*

(Note $DO_t = DO_s - D_t$)

Critical point

- Critical point: the point where the DO is the lowest on the DO sag curve

$$t_c = \frac{1}{k_r - k_d} ln\left[\frac{k_r}{k_d}\left(1 - D_a \frac{k_r - k_d}{k_d L_a}\right)\right]$$

t_c = the time to the critical point [T]

- The critical deficit, D_c

$$D_c = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d t_c} - e^{-k_r t_c}\right) + D_a\left(e^{-k_r t_c}\right)$$

Modeling BOD

Q: A city disposes of 1.05 m^3/s of treated sewage having ultimate BOD of 28.0 mg/L and DO of 1.8 mg/L into a river. At the upstream from the outfall, the river flowrate is 7.08 m^3/s, the velocity is 0.37 m/s, and the ultimate BOD and DO of the river are 3.6 and 7.6 mg/L, respectively. At the river temperature, the saturation value of DO is 8.5 mg/L, the deoxygenation coefficient, k_d, 0.61 day^{-1}, and the reaeration coefficient, k_r 0.76 day^{-1}.

1) Calculate the ultimate BOD and DO just downstream from the outfall. Assume complete mixing.
2) Calculate the DO 16 km downstream from the outfall.
3) Calculate the critical time, distance, and the minimum DO.

Reading assignment

Textbook Ch 9 p. 392-418