Microbial kinetics
Reactors I

Today's lecture

- Microbial growth kinetics
- Reactor design and analysis

Monod equation

$$\mu_{syn} = \left(\frac{1}{X_a} \frac{dX_a}{dt} \right)_{syn} = \hat{\mu} \frac{S}{K+S}$$

where μ_{syn} = specific growth rate due to synthesis (T^{-1})

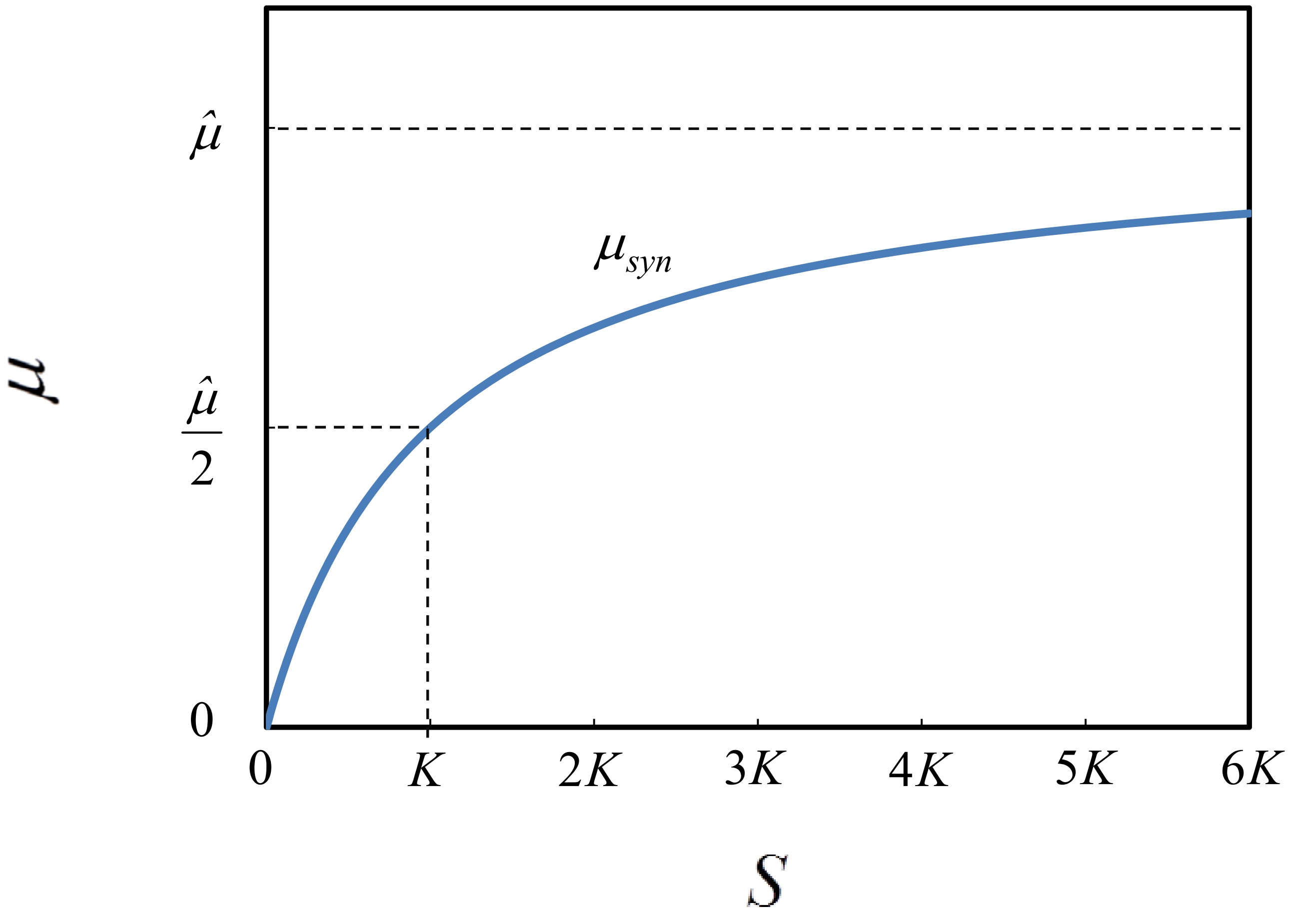
X_a = concentration of active biomass (M_xL^{-3})

S = concentration of the rate-limiting substrate (M_sL^{-3})

$\hat{\mu}$ = maximum specific growth rate (T^{-1})

K = half saturation coefficient (M_sL^{-3})

Monod equation



Typical values for K

Process	**K (mg substrate/L)**
Aerobic:	
organic mixtures	50-150 mg COD/L
single organics	1-10 mg COD/L
nitrification	0.4-2 mg NH_3-N/L
Anaerobic:	
denitrification	0.06-0.20 mg NO_3^--N/L
methane fermentation:	
acetate, propionate	600-900 mg COD/L
sewage sludge	2000-3000 mg COD/L

Growth kinetics with decay

- As discussed in the previous lecture, we assume decay is proportional to cell biomass

$$\left(\frac{dX_a}{dt}\right)_{decay} = -bX_a$$

in the form of specific growth rate,

$$\mu_{dec} = \left(\frac{1}{X_a}\frac{dX_a}{dt}\right)_{decay} = -b$$

where μ_{syn} = specific growth rate due to decay (T^{-1})

b = decay coefficient (T^{-1})

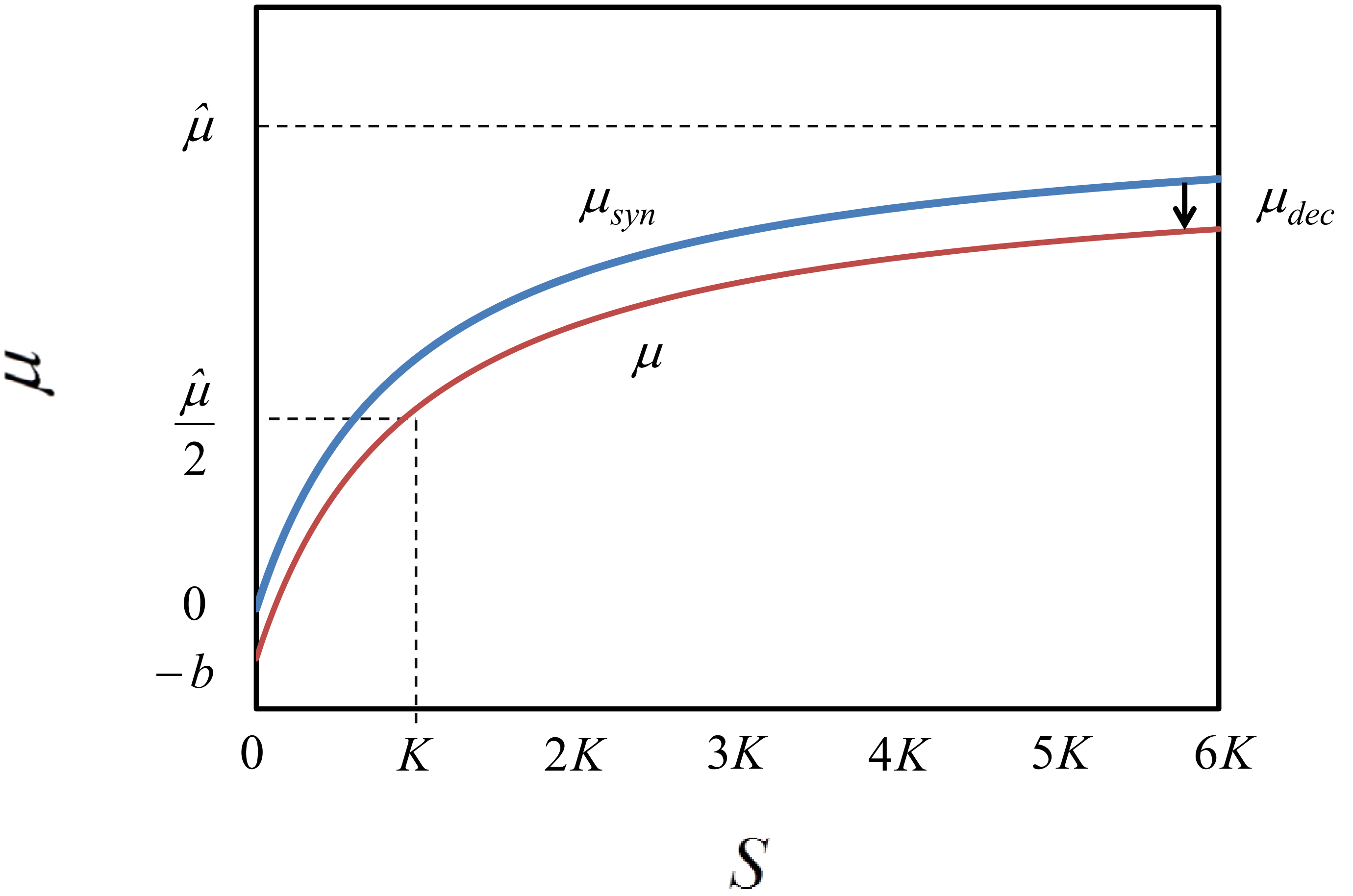
Overall bacterial growth kinetics

(Net growth) = (New growth) + (Decay)

$$\mu = \left(\frac{1}{X_a} \frac{dX_a}{dt} \right) = \mu_{syn} + \mu_{dec} = \hat{\mu} \frac{S}{K+S} - b$$

where μ = net specific growth rate (T^{-1})

Growth kinetics with decay



More on decay

$$\mu_{dec} = \left(\frac{1}{X_a} \frac{dX_a}{dt} \right)_{decay} = -b$$

- Most fraction (f_d≈0.8) is oxidized
- The other fraction (1-f_d ≈0.2) is accumulated as inert biomass

Rate of oxidation (respiration): $$\left(\frac{1}{X_a} \frac{dX_a}{dt} \right)_{resp} = -f_d b$$

Rate of conversion to inert biomass: $$\left(\frac{1}{X_a} \frac{dX_a}{dt} \right)_{inert} = -\frac{1}{X_a} \frac{dX_i}{dt} = -(1-f_d)b$$

X_i = inert biomass (M_xL^{-3})

Substrate utilization rate

Recall that,

$$Y = \frac{(g \text{ cells produced})}{(g \text{ substrate utilized})} = \frac{(dX_a / dt)_{\mathrm{syn}}}{-dS/dt}$$

and

$$\mu_{syn} = \left(\frac{1}{X_a}\frac{dX_a}{dt}\right)_{syn} = \hat{\mu}\frac{S}{K+S}$$

With the expression for Y, the Monod equation can be written as:

$$-\frac{1}{Y}\left(\frac{dX_a}{dt}\right)_{syn} = \frac{dS}{dt} = -\frac{\hat{\mu}}{Y}\frac{S}{K+S}X_a$$

Substrate utilization rate

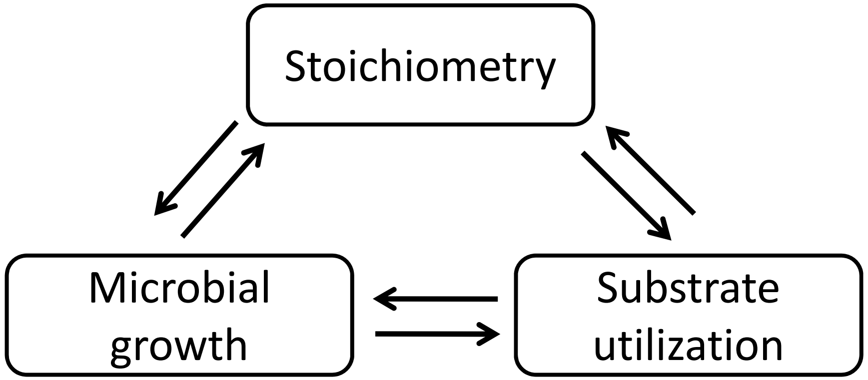
Substrate utilization rate, r_{ut} [$M_sL^{-3}T^{-1}$]

$$r_{ut} = \frac{dS}{dt} = -\frac{\hat{q}S}{K+S} X_a$$

$\hat{q} = \hat{\mu} / Y$; max. specific rate of substrate utilization ($M_s/M_x^{-1}T^{-1}$)

Recall that,

$$Y = f_s^{\,0} \frac{M_c}{n_e \cdot \left(8\,g\,COD / e^- - eq\right)}$$



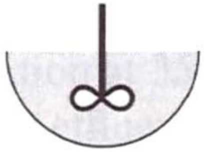
Stoichiometry

Microbial growth

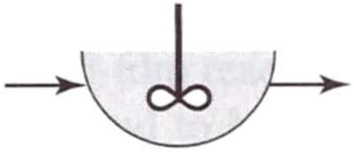
Substrate utilization

Reactors

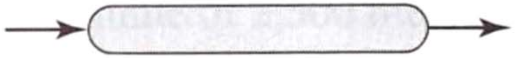
Suspended growth:



Batch reactor

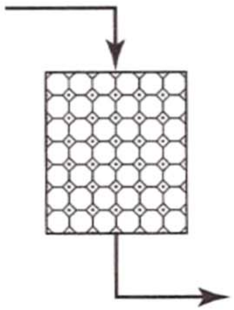


Continuous-stirred tank reactor

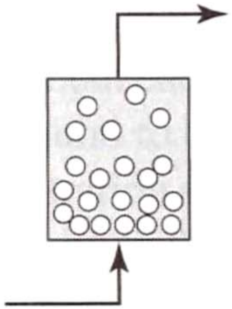


Plug-flow reactor

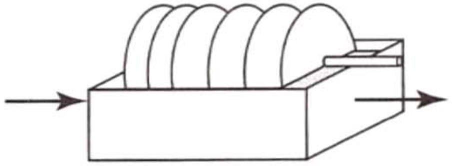
Attached growth:



Packed-bed reactor



Fluidized-bed reactor



Rotating biological contactor

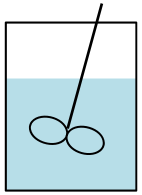
Suspended vs. attached growth

suspended growth

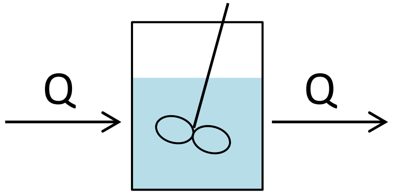
attached growth

Reactors for suspended growth

- **Batch reactor**



  - Bench-scale test systems
  - Some wastewater processes – “sequencing batch reactors”
- **Continuous-stirred tank reactor (CSTR)**



  - Activated sludge
  - Flocculator
- **Plug flow reactor (PFR)**

  - Disinfection
  - Long river/canal
  - Pipeline/aqueduct

Reactor analysis

1. Define control volume

2. Set mass balance (for a **single** substance!!!)

(mass rate of accumulation)

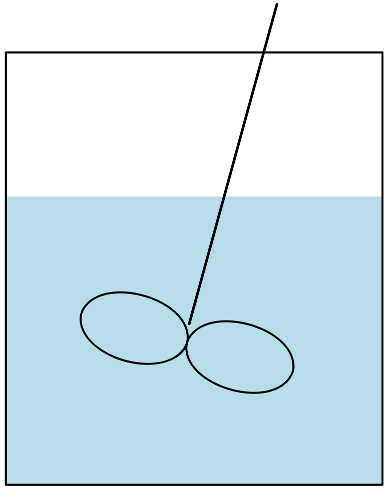
= (rate of mass in) – (rate of mass out)

+ (mass rate of gain/loss)

↖ Any processes related to gain/loss, but here we are interested in reactions!

3. Rearrange/solve the equation to a useful form

Reactor analysis: batch reactor



For 1^{st} order reaction of a contaminant,

(initial concentration = C_0)

$$\boxed{C / C_0 = e^{-kt}}$$

For bacterial growth following Monod kinetics,

(initial biomass & substrate conc. = X_a^0 and S^0)

$$\frac{dS}{dt} = -\frac{\hat{q}S}{K+S}\left[X_a{}^0 + Y\left(S^0 - S\right)\right]$$

(Eq. [5.10] in the Textbook)