Reactor analysis

3. Continuous-stirred tank reactor

(1) 1^{st} order reaction of a contaminant

1) define control volume: the reactor

2) set mass balance (for the contaminant)

(mass rate of accumulation) = (rate of mass in) – (rate of mass out) + (rate of gain/loss)

$$\frac{dM}{dt} = V\frac{dC}{dt} = QC_0 - QC + (-kCV)$$

At steady state,

$$\frac{dC}{dt} = 0$$

$$0 = Q(C_0 - C) - kCV$$

3) solve the equation

$$C = \frac{C_0}{1 + k \cdot V / Q}$$

$V / Q = \theta$, hydraulic retention time – this is an average value for the fluid particles that enter the CSTR! (cf. PFR: all fluid particles have the same HRT)

$$C = \frac{C_0}{1 + k\theta}$$

(2) Bacterial growth following Monod kinetics

1) define control volume: the reactor (just the same)

2) set mass balance

i) For substrate

$$V\frac{dS}{dt}=QS^0-QS+r_{ut}\cdot V$$

at steady state,

$$0=QS^0-QS+r_{ut}\cdot V$$

$$0=\left(S^0-S\right)+\frac{\hat{q}S}{K+S}X_a\cdot\theta$$

ii) For active biomass,

$$V\frac{dX_a}{dt}=-QX_a+r_{net}\cdot V$$

at steady state,

$$0=-QX_a+r_{net}\cdot V$$

$$0=-X_a+\left(Y\frac{\hat{q}S}{K+S}X_a-bX_a\right)\cdot\theta$$

3) solve the equation

With some math:

$$S=K\frac{1+b\theta}{Y\hat{q}\theta-(1+b\theta)}$$

$$X_a=Y\frac{S^0-S}{1+b\theta}$$

<u>Special cases</u>

1. When $\Theta=\Theta_x$ is very small, $S=S_0$ and $X_a=0$: washout; not substrate removal and accumulation of active biomass

The active biomass is washed out before substantial growth occurs to show observable substrate utilization

Denote the Θ_x at washout as Θ_x^{min}, then, using the equation for S:

$$S = K\frac{1+b\theta_x}{Y\hat{q}\theta_x-(1+b\theta_x)}$$

$$S^0 = K\frac{1+b\theta_x^{min}}{Y\hat{q}\theta_x^{min}-(1+b\theta_x^{min})}$$

This gives

$$\theta_x^{min} = \frac{K+S^0}{S^0(Y\hat{q}-b)-bK}$$

2. For $\Theta > \Theta_x$, S declines with the increase in Θ_x. X_a initially increases, but reaches a maximum and then decreases as decay becomes dominant.

3. When $\Theta = \Theta_x$ is very large, the decay predominates and substrate concentration is not further reduced (S reaches S_{min})

$$S = K\frac{1+b\theta_x}{Y\hat{q}\theta_x-(1+b\theta_x)}$$

$$\theta_x^{min} \to \infty$$

$$S_{min} = K\frac{b}{Y\hat{q}-b}$$

(3) Bacterial growth following Monod kinetics including inert biomass

Mass balance for inert biomass (assume steady state):

$$0 = QX_i^0 - QX_i + (1-f_d)bX_aV$$

$$X_i = X_i^0 + X_a(1-f_d)b\theta$$

(This shows that operating at large Θ results in large accumulation of inert biomass)

The total volatile suspended solids (VSS) concentration, X_v is calculated as (used Θ instead of Θ_x)

$$X_v = X_i + X_a = X_i^0 + X_a(1 - f_d)b\theta_x + X_a = X_i^0 + Y(S^0 - S)\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$