Microbial kinetics in reactors: further discussion II

Today's lecture

- Nutrient & e^- acceptor consumption
- Hydroysis
- Alternate rate expressions

Nutrient consumption

- For the consumption of nutrients for biomass production:

$$r_n = \gamma_n Y_{obs} r_{ut} = \gamma_n Y r_{ut} \frac{1+(1-f_d)b\theta_x}{1+b\theta_x}$$

r_n = rate of nutrient consumption $[M_n L^{-3} T^{-1}]$
γ_n = the stoichiometric ratio of nutrient mass to VSS for biomass $[M_n M_x^{-1}]$

Using $C_5H_7O_2N$ as cell formula:
γ_N = 14 g N/113 g VSS = 0.124 g N/g VSS
γ_P = 0.2 x 0.124 = 0.025 g P/g VSS (generally P = 0.2N)

Nutrient consumption in a CSTR

- Steady-state mass balance:

$$0 = QC_n^{\ 0} - QC_n + r_n V$$

$$\boxed{C_n = C_n^{\ 0} + r_n \theta}$$

if $C_n < 0$, nutrient-limiting

e$^-$-acceptor consumption

(e$^-$ acceptor used in a reactor)
= [(total O.D. in the effluent) - (total O.D. in the influent)]
x (conversion factor)

In terms of the use rate for a reactor ($\Delta S_a/\Delta t$):

$$\frac{\Delta S_a}{\Delta t} = \gamma_a \left[Q\left(S^0 + 1.42 X_v{}^0\right) \quad Q\left(S + SMP + 1.42 X_v\right)\right]$$

$$= \gamma_a Q\left[S^0 - S - SMP + 1.42\left(X_v{}^0 - X_v\right)\right]$$

γ_a = the stoichiometric ratio of acceptor mass to oxygen demand

for oxygen: 1 g O_2/g COD

for nitrate: 0.35 g NO_3^-–N/g COD

e^--acceptor consumption

To estimate the required mass rate of acceptor supply (ex: aeration [O_2] requirement),

the calculated e^- acceptor use rate, $\Delta S_a/\Delta t$ can be written as:

$$\frac{\Delta S_a}{\Delta t} = Q\left[S_a{}^0 - S_a\right] + R_a$$

(mass flow rate in) – (mass flow rate out)

Requirement of e^- acceptor addition

Nutrients and electron acceptors

Q: From the last lecture example, we calculated the following:

$$S^0 = 500\ mg\ BOD_L / L \qquad X_v = 221\ mg\ VSS / L$$

$$S = 1.7\ mg\ BOD_L / L \qquad SMP = 31.8\ mg\ BOD_L / L$$

$$X_i^0 = 50\ mg\ VSS / L \qquad r_{ut} = 249\ mg\ BOD_L / L$$

If influent N and P concentrations are 50 mg NH_4^+-N/L and 10 mg PO_4^{3-}-P/L, respectively, what are the effluent N and P concentrations?

If influent and effluent DO are 6 and 2 mg/L, respectively, how much O_2 should be supplied to the reactor?

Hydrolysis of particulates & polymers

- Particulates and polymeric substances account for a significant portion of BOD in wastewater
- >50% of BOD in typical sewage is particulates (SS)
- Particulates and large-MW compounds cannot penetrate the cell membrane

  → needs to be hydrolyzed to smaller molecules
- Catalyzed by extracellular enzymes
- The mechanism and kinetics of hydrolysis it not fully understood

Hydrolysis of particulates & polymers

One simple way of describing hydrolysis is to assume first-order kinetics for particulates (or polymers):

$$r_{hyd} = -k_{hyd} S_p$$

r_{hyd} = rate of accumulation of particulates (=dS_p/dt) [$M_s L^{-3} T^{-1}$]

k_{hdy} = first-order hydrolysis rate coefficient [T^{-1}]

S_p = concentration of particulates [$M_s L^{-3}$]

In a steady-state CSTR,

$$0 = Q\left({S_p}^0 - S_p\right) - k_{hyd} S_p V$$

$$\Rightarrow \quad S_p = \frac{{S_p}^0}{1 + k_{hyd}\theta}$$

Hydrolysis of particulates & polymers

- Effect of hydrolysis on dissolved substrates

  mass balance in a steady-state CSTR

$$0 = Q(S^0 - S) - \frac{\hat{q}S}{K + S} X_a V + k_{hyd} S_p V$$

$$0 = (S^0 - S) - \frac{\hat{q}S}{K + S} X_a \theta + k_{hyd} S_p \theta$$

→ Increase in S^0 by $k_{hyd} S_p \theta$

→ Increased biomass, but no change in dissolved substrates in the reactor

Hydrolysis of particulates & polymers

Q: Let's add hydrolysis on the previous example. Recall that without hydrolysis, the values calculated were as follows:

$$\theta = 2\ d \qquad S = 1.7\ mg\ BOD_L / L$$

$$X_a = 161\ mg\ VSS / L \qquad SMP = 31.8\ mg\ BOD_L / L$$

$$X_i = 60\ mg\ VSS / L \qquad Effluent\ COD = 347\ mg\ COD / L$$

$$X_v = 221\ mg\ VSS / L \qquad Effluent\ BOD_L = 216\ mg\ BOD_L / L$$

Assuming that the influent contains biodegradable particulate organic matter with a concentration of 100 mg COD/L and the hydrolysis rate coefficient is k_{hyd} = 0.2/d, recalculate the effluent VSS, COD, and BOD_L.

Hydrolysis of particulates & polymers

Summary of given values:

$$\theta = 2\ d$$

$$S^0 = 500\ mg\ BOD_L / L$$

$$\hat{q} = 20\ g\ BOD_L / g\ VSS - d$$

$$Y = 0.42\ g\ VSS_a / g\ BOD_L$$

$$K = 20\ mg\ BOD_L / L$$

$$b = 0.15 / d$$

$$f_d = 0.8$$

$$k_1 = 0.12\ g\ COD / g\ BOD_L$$

$$X_i^0 = 50\ mg\ VSS / L$$

$$k_2 = 0.09\ g\ COD / g\ VSS - d$$

$$\hat{q}_{UAP} = 1.8\ g\ COD / g\ VSS - d$$

$$K_{UAP} = 100\ mg\ COD / L$$

$$\hat{q}_{BAP} = 0.1\ g\ COD / g\ VSS - d$$

$$K_{BAP} = 85\ mg\ COD / L$$

$$S_p^0 = 100\ mg\ COD / L$$

$$k_{hyd} = 0.2 / d$$

Alternate rate expressions

- Contois equation

$$r_{ut} = -\frac{\hat{q}S}{BX_a + S}X_a$$

B = constant [M_s/M_x]

When $X_a \to \infty$, $r_{ut} = -\frac{\hat{q}}{B}S$

(at high biomass concentrations substrate utilization depends on *S*, not X_a)

Alternate rate expressions

- Moser equation

$$r_{ut} = -\frac{\hat{q}S}{K + S^{-\gamma}}X_a$$

γ = constant [unitless]

- Tessier equation

$$r_{ut} = -\hat{q}\left(1 - e^{S/K}\right)X_a$$

*Just **REMEMBER** that Monod Eq. is **NOT** the only option!!!*

Dual Monod equation

$$r_{ut} = -\hat{q}\frac{S}{K+S}\frac{A}{K_A + A}X_a$$

A = e^- acceptor concentration $[M_A/L^3]$

K_A = half-saturation coeff. for e^- acceptor $[M_A/L^3]$

- e^- acceptor can also be limiting!
- Can be reduced to single Monod Eq. if $A >> K_A$
- Terms for other limiting substances can be added as well