Biofilm kinetics

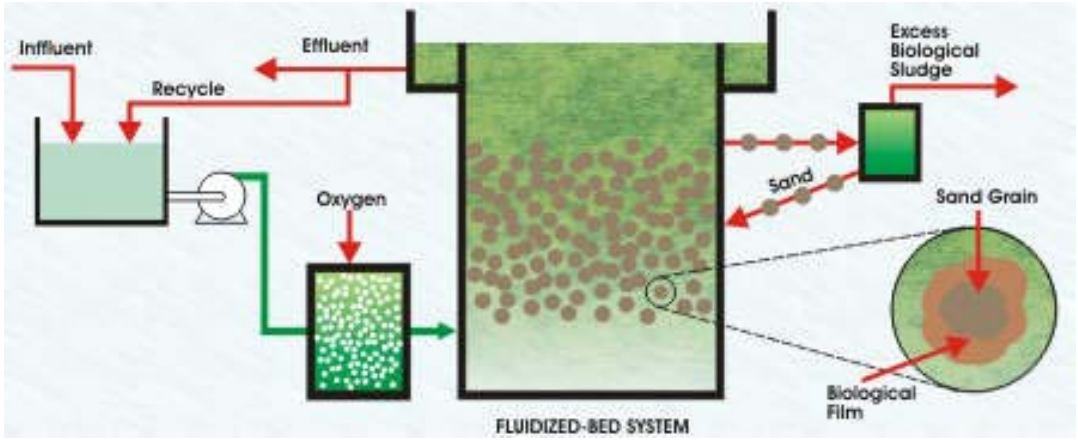
Today's lecture

- Biofilm processes
- Concept, assumptions, theory
- Substrate analysis

Biofilm processes

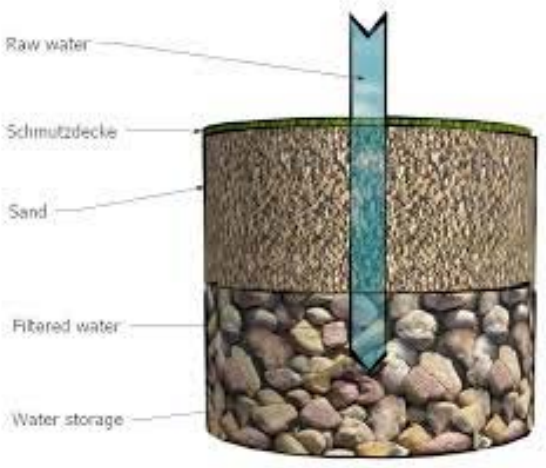
Trickling filter

Rotating biological contactor



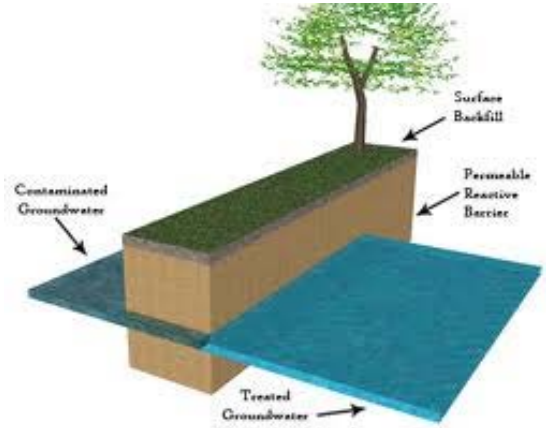
Fluidized bed bioreactor

Biofilm processes



Slow sand filtration
(Schmutzdecke)

Biological GAC treatment



In-situ bioremediation
of groundwater

Why microorganisms form biofilms?

- Possible answers:
  - Cells are continually exposed to a fresh substrate due to advection of substrates past the biofilm
  - For obligate consortia, the cells need to be located close to each other for substance exchanges
  - The cells in the biofilms create unique microenvironments that benefit themselves
  - The surface creates a unique microenvironment such as by adsorption of toxins or corrosive release of Fe^{2+}
  - The surface triggers a physiological change in the bacteria
  - The tight packing of cells in the aggregate alters the cells' physiology

Biofilm kinetics – key concept

- *Addition of a key mechanism: DIFFUSION*
  - Diffusion of substrates
  - Diffusion of e^- acceptors
  - Diffusion of nutrients

- Fick's Law of diffusion
  - Fick's 1st law: $J = -D\frac{\partial C}{\partial x}$
  - Fick's 2nd law: $\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$

J = flux of a substance [$ML^{-2}T^{-1}$]

D = diffusion coefficient [L^2T^{-1}]

C = concentration of a substance [ML^{-3}]

Assumptions for biofilm analysis

Idealizing a biofilm:

- The biofilm has a uniform biomass density X_f ($M_x L^{-3}$)
- It has a locally uniform thickness of L_f.
- Mass transport resistance can be important inside the biofilm and to the biofilm.
  - External mass transport (bulk liquid → surface of a biofilm): represented by an effective diffusion layer of thickness L (film theory)
  - Internal mass transport (within the biofilm): molecular diffusion (Fick's 2nd law)

Film Theory

- The entire resistance to mass transport resides in a stagnant film at the phase interphase.
- Equilibrium is obtained at the interface
- The bulk fluids are sufficiently well-mixed so that the concentration gradients in the bulk fluid are negligible
- The concentration gradient in the film is linear, following the steady state diffusion in a stagnant fluid.

Film Theory

The flux in the film is:

$$J = -\frac{D}{\delta}\left(C_{bulk} - C_{interphase}\right)$$

J = flux of a substance [$ML^{-2}T^{-1}$]

D = diffusion coefficient [$L^{2}T^{-1}$]

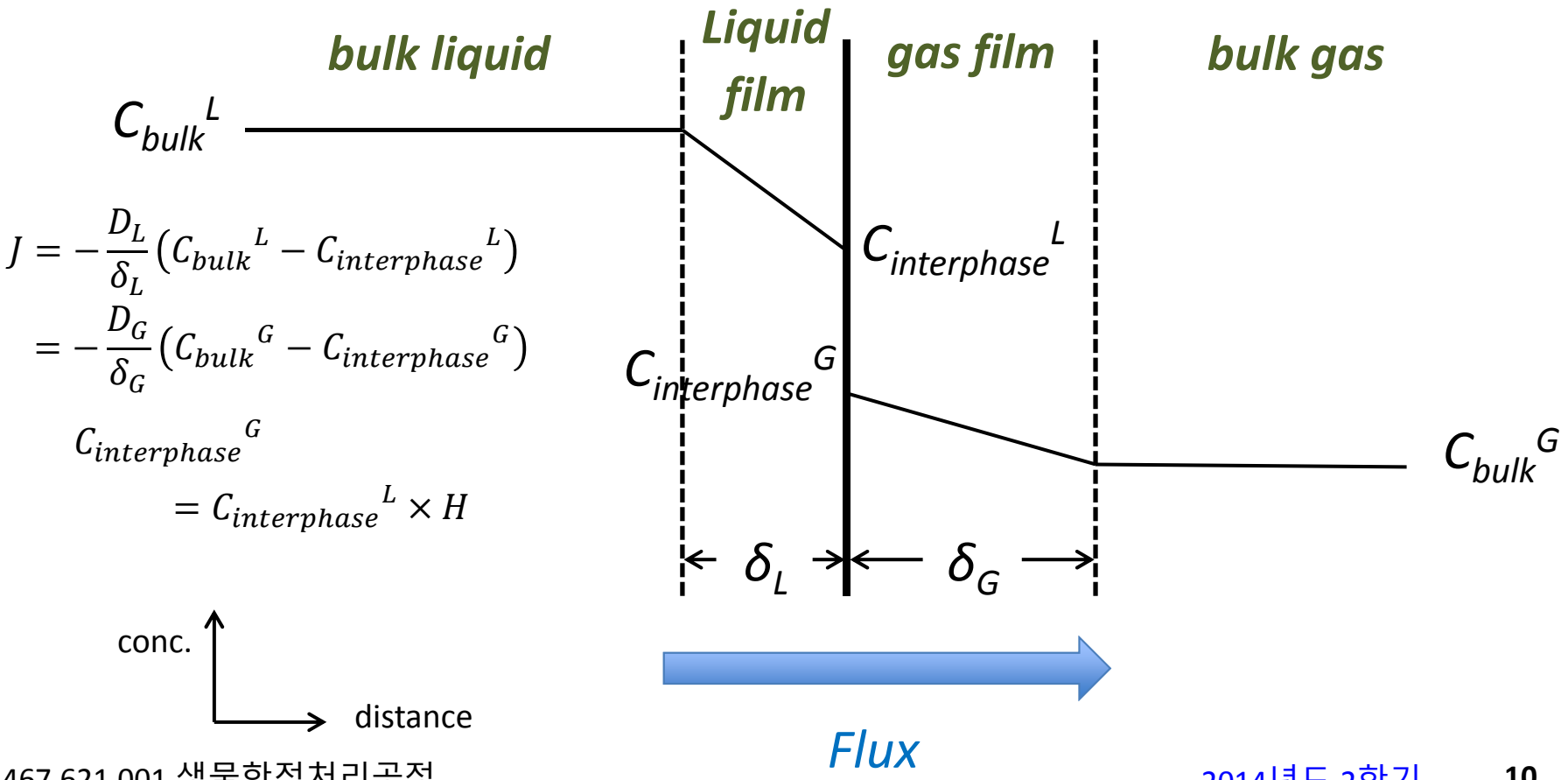
δ = film (effective diffusion layer) thickness [L]

C_{bulk} = concentration in the bulk fluid [ML^{-3}]

$C_{interphase}$ = concentration at the interphase [ML^{-3}]

Film Theory

ex: gas-liquid interface



$$J = -\frac{D_L}{\delta_L}\left(C_{bulk}{}^{L} - C_{interphase}{}^{L}\right)$$

$$= -\frac{D_G}{\delta_G}\left(C_{bulk}{}^{G} - C_{interphase}{}^{G}\right)$$

$$C_{interphase}{}^{G} = C_{interphase}{}^{L} \times H$$

Deep & shallow biofilm

- Deep biofilm: the substrate concentration approaches zero at some point in the film
  - Further increase in biofilm thickness does not increase the overall rate of substrate utilization
- Shallow biofilm: the substrate concentration remains above zero at all points in the film
  - Fully penetrated biofilm: a special case of shallow biofilm where the substrate concentration at the outer surface and the attachment surface are almost identical

Substrate analysis

The substrate utilization follows Monod kinetics:

$$r_{ut} = -\frac{\hat{q} X_f S_f}{K + S_f}$$

X_f = active biomass density within the biofilm [$M_x L^{-3}$]

S_f = substrate concentration at a point of the biofilm [$M_s L^{-3}$]

The molecular diffusion of the substrate (Fick's 2nd law):

$$r_{diff} = D_f \frac{d^2 S_f}{dz^2}$$

r_{diff} = rate of substrate accumulation due to diffusion [$M_s L^{-3} T^{-1}$]

D_f = molecular diffusion coeff. of the substrate in the biofilm [$M_s L^{-3}$]

z = depth dimension normal to the biofilm surface [L]

Substrate analysis

Combining the substrate utilization and diffusion, and assuming steady state,

$$0 = D_f \frac{d^2 S_f}{dz^2} - \frac{\hat{q} X_f S_f}{K + S_f}$$

Boundary condition I

no flux to the attachment surface $\left.\frac{dS_f}{dz}\right|_{z=L_f} = 0$

L_f = biofilm thickness [L]

Boundary condition II

Flux at the biofilm/water interface determined according to the film theory

$$J = \frac{D}{L}(S - S_s) = D_f \left.\frac{dS_f}{dz}\right|_{z=0} = D \left.\frac{dS}{dz}\right|_{z=0}$$

D = molecular diffusion coefficient in water

L = effective diffusion layer thickness [L]

S, S_s = substrate concentrations in the bulk liquid and at the biofilm/liquid interface, respectively [$M_s L^{-3}$]

Substrate analysis – analytical solution

$$0 = D_f \frac{d^2 S_f}{dz^2} - \frac{\hat{q} X_f S_f}{K + S_f} \quad \text{(+ two B.C.s)}$$

The first integration yields *J* and the second integration yields S_f, but closed-form solutions cannot be obtained for this non-linear form

Special Case I:

When $S_f << K$ for all parts of the biofilm, the substrate utilization is a first-order function of S_f

$$0 = D_f \frac{d^2 S_f}{dz^2} - k_1 X_f S_f$$

$k_1 = \hat{q}/K$, rate coefficient [$L^3 M_x^{-1} T^{-1}$]

Substrate analysis – analytical solution

Integration of the linear form of the equation gives:

$$J_1 = \frac{D_f S_s tanh\left(L_f/\tau_1\right)}{\tau_1} \qquad S_f = S_s \frac{cosh\left(\left(L_f - z\right)/\tau_1\right)}{cosh\left(L_f/\tau_1\right)}$$

J_1 = substrate flux into the biofilm [$M_s L^{-2} T^{-1}$]

$\tau_1 = \sqrt{D_f/k_1 X_f}$ = first-order, standard biofilm depth dimension

- Dimensionless parameter for the deepness of the biofilm:

  $L_f/\tau_1 > 1$: deep biofilm

  $L_f/\tau_1 << 1$: fully penetrated biofilm

Substrate analysis – analytical solution

$$0 = D_f \frac{d^2 S_f}{dz^2} - \frac{\hat{q} X_f S_f}{K + S_f} \quad \text{(+ two B.C.s)}$$

Special Case II:

For deep biofilm, S_w is known as 0

$$J_{deep} = \left[2\hat{q} X_f D_f \left(S_s + K \, ln\left(\frac{K}{K + S_s} \right) \right) \right]^{1/2}$$

$k_1 = \hat{q}/K$, rate coefficient [$L^3 M_x^{-1} T^{-1}$]