Chapter 2

Energy Principle

Fundamental to the solution of many engineering problems is the correct application of Mechanics.

Mechanics **may be defined as the discipline concerned with the motion (including deformation) of a body or system of bodies subjected to body forces and surface forces.**

In general, we have used two methods to tackle these problems.

Newtonian Mechanics(Vector Mechanics) ~ Work-energy principle(Scalar)

Advantage ? Drawback ?

1. Newtonian mechanics

Utilizing the Newtonian Mechanics which is based on Newton's law of motion, we can solve these problems to a certain extent. For a body, shown in the figure, under a force F, the Newton's second law of motion states

Figure 1.1 : A body subjected to a force

$$\vec{F} = m\vec{a}_c = m\frac{d^2\vec{r}_c}{dt^2} \tag{1.1}$$

where m **is the mass of the body,**

$\vec{a}_c$ **is the acceleration at center of mass,**

$\vec{r}_c$ is the position vector of center of mass,

t is assumed to be same for all observers, i.e., no relativistic effects.

The primary goal of the Newtonian mechanists is either :

Type I: Given position $\vec{r}_c(t)$, obtain the force system $F(t)$

or

Type II: Given $F(t)$, determine the resultant motion $\vec{r}_c(t)$

~ The type I problems are in general tractable in that explicit rules for vector differentiation are known.: Kinematics

~ However the solution of Type II problems is quite different matter and only a few general solutions have been obtained.

This is in part due to the following 5 cases :

(a) All the forces on the body are required even such as <u>constraint forces</u> which are usually not known.

(b)No general formulae for <u>vector integration</u> are known

(c)The acceleration vector may be complex for particular coordinate systems.

(d)Eq. (1.1) is valid only in an inertial reference frame.

(e)Even for equilibrium problems ($a_c = 0$), the distribution of internal forces throughout the body is not a trivial problem. Indeed this is the aim of continuum mechanics !

2. Principle of Virtual Work(~ Energy principle!)

Mechanics problems can be approached in a manner strikingly different from

the Newtonian technique. (~ Free-body diagram: isolated, constraint!)

ex> Elevator :

The essence of this method is the work energy principle or more broadly the principle of virtual work.

Some mechanists attribute the founding of the work energy principle to a German scholar Leibniz(1646-1716) who was a fierce competitor of Newton.

Let W be the work done on a system and ΔK be the change of kinetic energy of the system.

Then this principle can be written as :

$$W = \Delta K \tag{1.2}$$

Introducing the concept of virtual displacements which are arbitrary displacements satisfying the geometrical constraints of a system, we can broaden the work energy principled and may state it as follows:

Assume that a mechanical system is in equilibrium under applied forces and prescribed geometrical constraints.

Then, the sum of all virtual work, denoted by δW **, done by the external and internal forces existing in the system in any arbitrary virtual displacements is zero, i.e.,**

$$\delta W = 0 \tag{1.3}$$

: This is called the principle of virtual work and was stated first by

John Bernouli(1667-1745) as "for all possible displacements, the sum of all the products of force and virtual velocity in direction of force must balance for equilibrium."

$$\delta W = 0 \sim \vec{F} \bullet \delta \vec{r} = 0$$

A number of mechanists and mathematicians contributed to development of the above branch of mechanics. Some of them are listed here.

Galileo (1564-1642) :

His work recognized that work is "product of a force and the displacement in the direction of the force.

D’Alembert(1717-1785) : introduced the D’Alembert’s principle, which states that a system can be considered to be in equilibrium if inertia force is taken into account, so that the principle of virtual work could be applied to a dynamic problem.

Lagrange(1736-1813) : developed the calculus of variations and introduced the concept of generalized coordinates. He founded analytical mechanics.

Hamilton(1805-1865) : gave first exact formulation of the principle of least action.

Hamiltion’s principle

The motion of a conservative system (all forces acting on the system are derived from a potential energy function V) occurs in such a way that the

definite integral

$$\int_{t1}^{t2} L dt = \int_{t1}^{t2} (T - V) dt$$

is stationary (an extremum) for the actual path of the system motion.

In this equation, $L = T - V$ **where T is the kinetic energy of the system.**

Hamilton's principle can be written mathematically as

$$\delta \int_{t1}^{t2} L dt = \delta \int_{t1}^{t2} (T - V) dt \qquad \textbf{(1.4)}$$

The branch of mechanics based on the principle of virtual work, Eq. (1.3), or Hamilton's principle, Eq. (1.4), is called variational formulation or Hamilton-Lagrangian mechanics.

The variational formulation has several advantages over Newtonian mechanics.

(a) The functional usually has a **definite meaning**.

In addition, since the **functional is a scalar quantity**, it is **invariant** under **coordinate transformations**.

(b) Variational principles sometimes lead to formulate for **upper or lower bounds** of the **exact solution of the problem under consideration**.

(c) **When** an elastic problem **cannot be solved exactly**, the **variational method** often provides an approximate formulation of the problem which yields a solution **compatible with the assumed degree of approximation**.

By the use of modern high speed computing machines, we can obtain a good approximate solution of the problem based on the variational

principles.

This is a very important fact which we are going to study the present course.

Aim of this course

In this class we are concerned with the study of deformable bodies based on the principle of virtual work, or various energy principles.

Object : we are interested in pursuing the following items in the solid mechanics:

1. **The strain field of the body.**
2. **The stress field of the body.**
3. **The deformation field of the body.**

The stress field and the strain field are connected by the constitute law which is determined by material property.

On the other hand, the kinematics of the solid body determines the relationship between the strain and the deformation fields.