Chapter 2
Circuit Elements

Seoul National University
Department of Electrical and Computer Engineering

Prof. SungJune Kim

Engineering and Linear Models

- **Engineers use Models to represent the elements of an electric circuit.**

- **A linear element satisfies both the properties of superposition and homogeneity.**

1. ***Superposition:***

$$i_1 \rightarrow v_1$$
$$i_2 \rightarrow v_2$$
$$i_1 + i_2 \rightarrow v_1 + v_2$$

1. ***Homogeneity:***

$$i \rightarrow v$$
$$ki \rightarrow kv$$

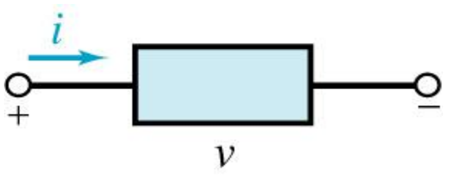


Figure 2.2-1 (p. 21)
An element with an excitation current *i* and a response *v*.

- A device that does not satisfy either the superposition or the homogeneity principle is said to be <u>nonlinear</u>.

Example 2.2-1 *A Linear Device*

- Consider the element represented by the relationship between current and voltage as

$$\boldsymbol{v = Ri}$$

Determine whether this device is linear.

Solution

- The response to a current i_1 is $v_1 = Ri_1$
- The response to a current i_2 is $v_2 = Ri_2$
- The sum of these response is $v_1 + v_2 = Ri_1 + Ri_2 = R(i_1 + i_2)$
- Since the sum of the response to i_1 and i_2 is equal to the response to i_1 and i_2, the principle of superposition is satisfied. Next consider the principle of homogeneity. Since

$$v_1 = Ri_1$$

- We have for an excitation $i_2 = ki_1$

$$v_2 = Ri_2 = Rki_1$$

- Therefore,

$$v_2 = kv_1$$

satisfies the principle of homogeneity. Because the element satisfies both the properties of superposition and homogeneity, it is linear.

Example 2.2-2 *A Nonlinear Device*

- Now let us consider an element represented by the relationship between current and voltage:

$$v = i^2$$

Determine whether this device is linear.

Solution

- The response to a current i_1 is $$v_1 = i_1^2$$
- The response to a current i_2 is $$v_2 = i_2^2$$
- The sum of these response is $$v_1 + v_2 = i_1^2 + i_2^2$$
- response to i_1 and i_2 is $$(i_1 + i_2)^2 = i_1^2 + 2i_1 i_2 + i_2^2$$
- Since, $$i_1^2 + i_2^2 \neq (i_1 + i_2)^2$$

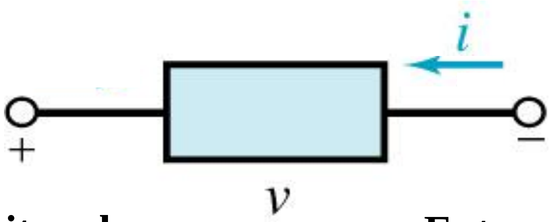
the principle of superposition is not satisfied. Therefore, the device is nonlinear.

Definition of Passive and Active elements

- We define an **active element** as an element that is capable of furnishing an average power greater than zero to some external device where the average is taken over an infinite time interval. Examples) Ideal voltage and current sources, amplifiers, transistors
- A **passive element** is defined as an element that cannot supply an average power that is greater than zero over an infinite time interval. Example) The resistor falls into this category. The energy it receives is usually transformed in to heat.
- Thus next definitions are possible: the average power for the active sign convention can be greater than zero for active elements, but can not be greater than zero for any t for passive elements.

Active and Passive Circuit Elements

An **active element** is capable of supplying energy.

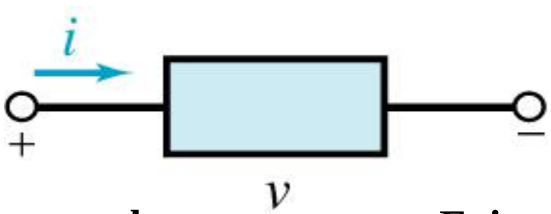


Exit node **Entry node**

Active element는 아래 수식을 만족하는 시간 t가 적어도 1개 있다.

$$w = \int_{-\infty}^{t} vi \, d\tau \ \geq \ 0$$

A **passive element** is not capable of supplying energy. It simply absorbs energy.



Entry node **Exit node**

Passive element는 모든 시간 t에 대하여 아래 수식을 만족한다.

$$w = \int_{-\infty}^{t} vi \, d\tau \ \geq \ 0$$

Example 2.3-1 *An Active Circuit Element*

- A circuit has an element represented by Figure 2.3-1*b* where the current is a constant 5A and the voltage is a constant 6V. Find the energy supplied over the time interval 0 to *T*.

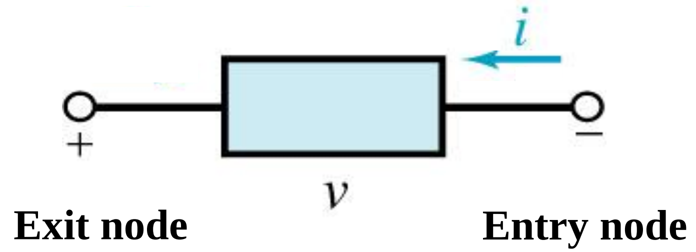


Figure 2.3-1 *b* (p. 24)

Solution

- Since the current enters the negative terminal, the energy ***supplied by*** the element is given by

$$w = \int_0^T (6)(5)d\tau = 30T\,(\mathrm{J})$$

- Thus, the device is a generator or an active element, in this case a dc battery.

Resistors

- **Resistivity, ρ**
  - The ability of a material to resist the flow of charge.
- **Resistance, R**
  - The physical property of an element or device that impedes the flow of current.
  - Unit of resistance R: Ohm (Ω)

$$R = \frac{\rho L}{A}$$

ρ: resistive
L: length
A: cross sectional area

- **Ohm's law**

$$v = Ri$$

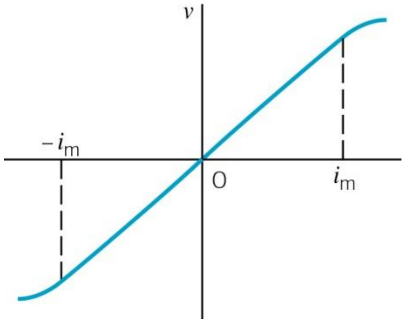


Figure 2.4-3 (p. 26)
A resistor operating within its specified current range $\pm i_m$, can be modeled by Ohm's law.

Resistors (cont’d)

- **Resistor (저항)**
  - An element that has a resistance *R*
  - in Figure 2.4-4

$$i_a = -i_b$$

$$v = Ri_a$$

Replacing ia by $-i_b$ gives

$$v = -Ri_b$$

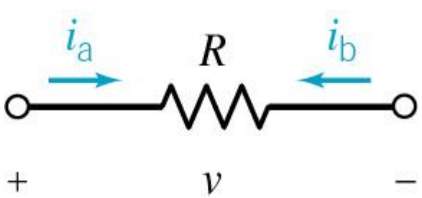
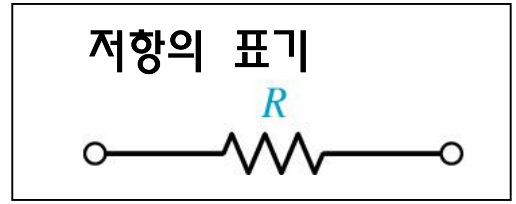


Figure 2.4-4 (p. 24)
A resistor with element current and element voltage.



- **Conductance (G)**
  - G=1/R
  - Unit of conductance: mhos(℧)

$$i = Gv$$

Resistors (cont'd)

- **Power delivered to a resistor**

$$p = vi = \frac{v^2}{R} = i^2 R$$

- **Energy delivered to resistor**

$$w = \int_{-\infty}^{t} p\, d\tau = \int_{-\infty}^{t} i^2 R\, d\tau$$

- **Resistance** is a measure of an element's ability to dissipate power irreversibly.

Example 2.4-1 *Power Dissipated by a Resistor*

- Let us devise a model for a car battery when the lights are left on and the engine is off. We have all experienced or seen a car parked with its lights on. If we leave the car for a period, the battery will "run down" or "go dead." An auto battery is a 12-V constant-voltage source, and the lightbulb can be modeled by a resistor of 6 ohms. The circuit is shown in Figure 2.4-8. Let us find the current i, the power p, and the energy supplied by the battery for a four-hour period.

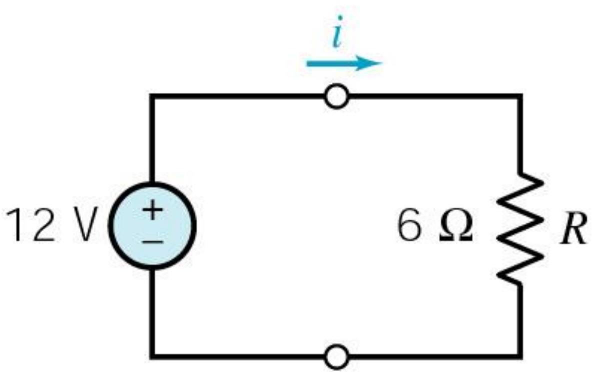
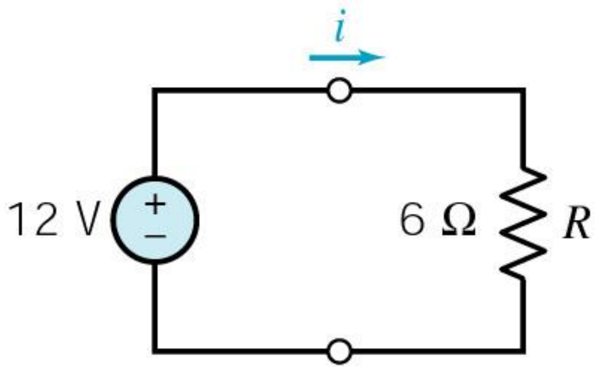


Figure 2.4-8 (p. 27)
Model of a car battery and the headlight lamp.

Solution



- According to Ohm's law, we have

$$v = Ri$$

- Since $\boldsymbol{v}$=12V and $\boldsymbol{R}$=6Ω, we have $\boldsymbol{i}$ = 2A.
  In order to find the power delivered by the battery, we use

$$p = vi = 12(2) = 24(\mathrm{W})$$

- Finally, the energy delivered in the four-hour period is

$$w = \int_0^t p\,d\tau$$

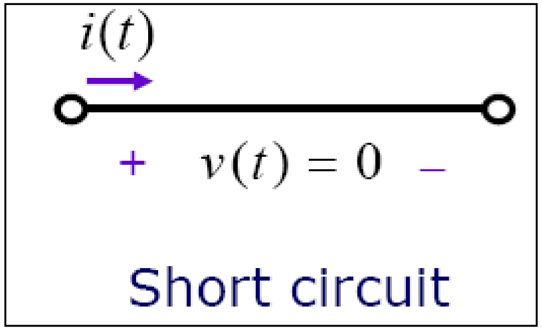
$$= 24t = 24(60 \times 60 \times 4) = 3.46 \times 10^5 (\mathrm{J})$$

Independent Sources

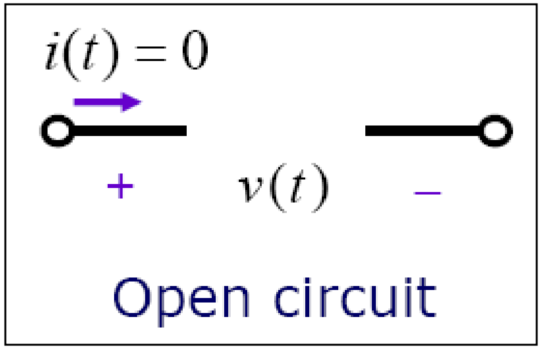
- **Source**
  - Voltage or current generator capable of supplying energy to a circuit
- **Independent source**
  - Voltage or current generator not dependent on other circuit variables
- **Ideal source**
  - Voltage or current generator independent of the current through the voltage source or the voltage across the current source
  - The voltage of an **ideal voltage source** is given to be a specified function, say $v(t)$. The current is determined by the rest of the circuit.
  - The current of an **ideal current source** is given to be a specified function, say $i(t)$. The voltage is determined by the rest of the circuit.

Independent Sources (cont'd)

- ***Short circuit***
  - Ideal voltage source ***v(t)*=0**
  - The current is determined by the rest of the circuit.
  - A resistor with resistance ***R*=0**

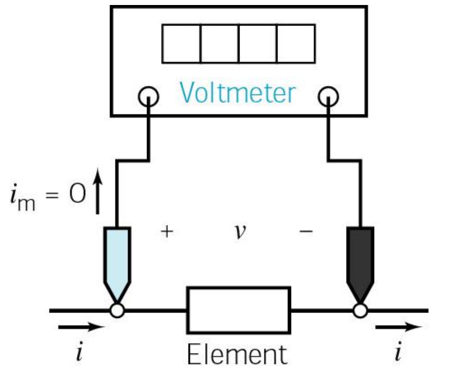


- ***Open circuit***
  - Ideal current source ***i(t)*=0**
  - The voltage is determined by the rest of the circuit.
  - A resistor with conductance ***G*=0 (*R*=**∞)

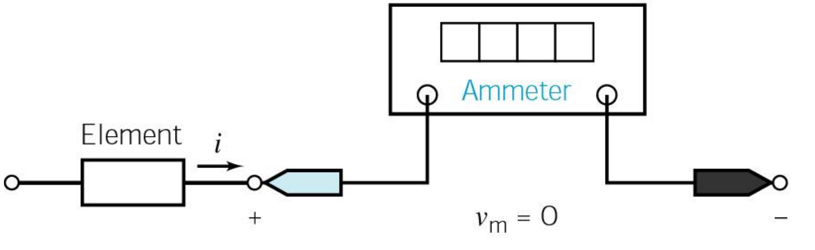


Voltmeters and Ammeters

- **Ideal voltmeters** **act like** **<u>open circuits</u>.**



- **Ideal ammeters** **act like** **<u>short circuits</u>.**



Voltmeters and Ammeters (cont’d)

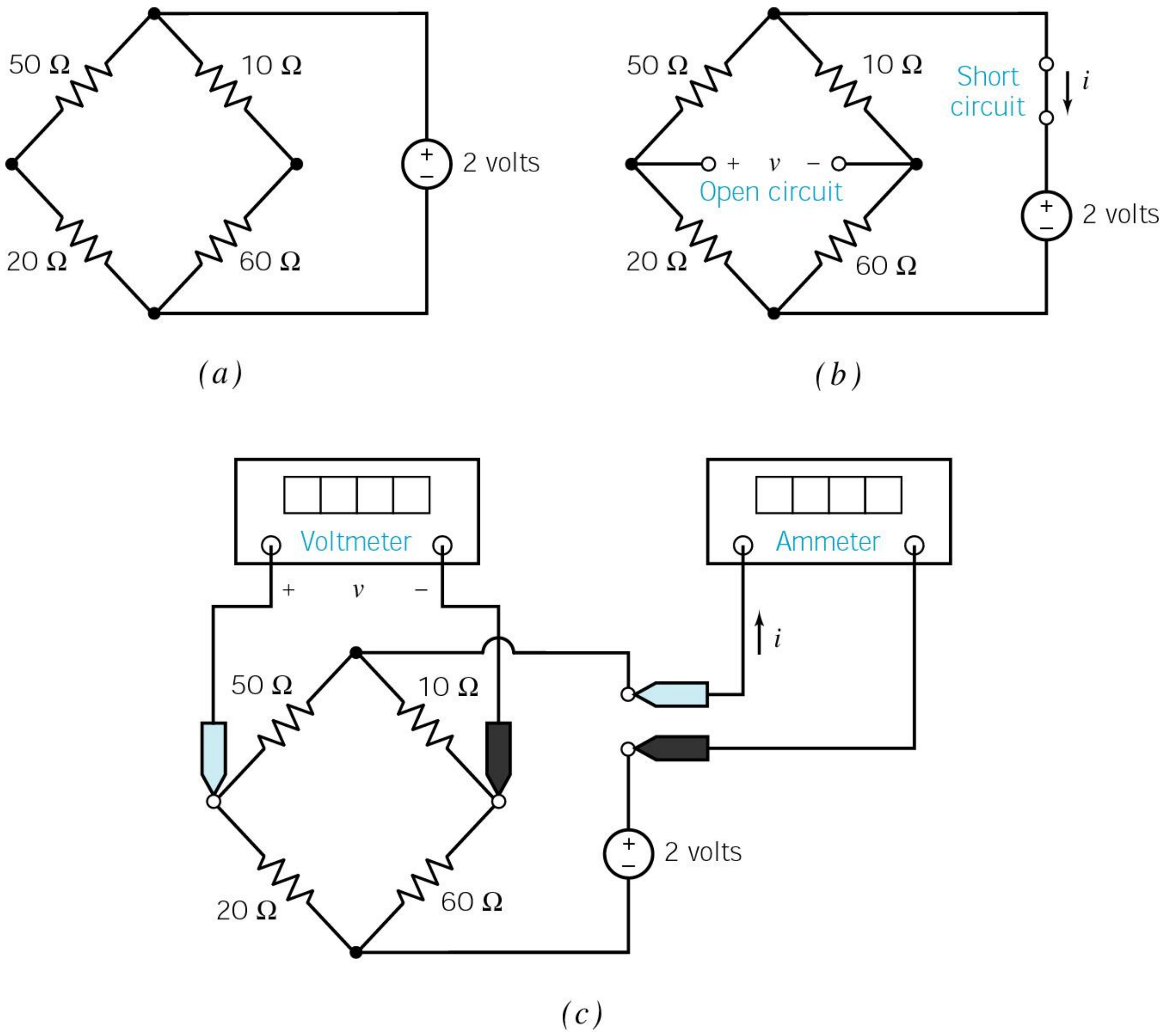
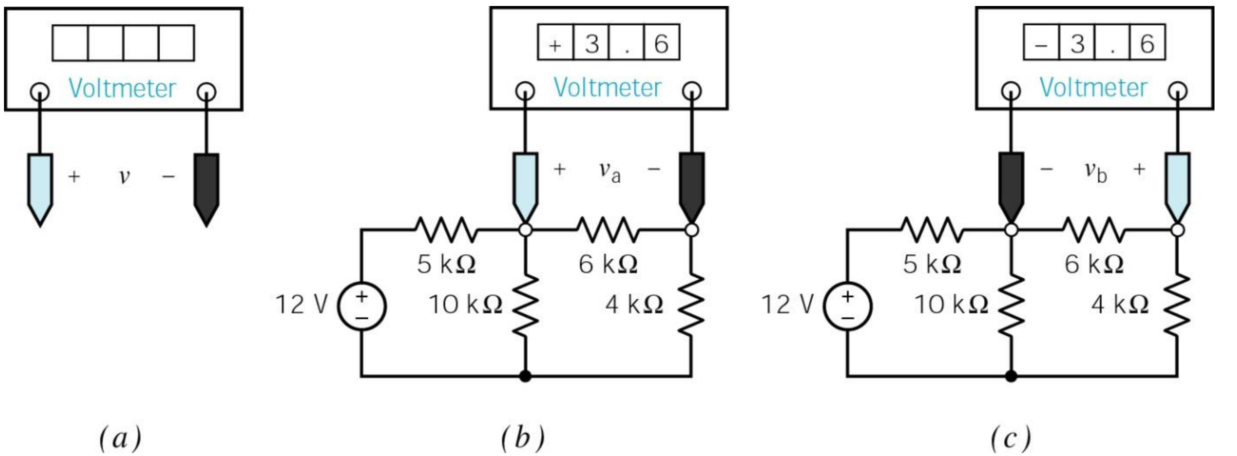


Figure 2.6-3 (p. 31)
(*a*) An example circuit,
(*b*) plus an open circuit and a short circuit.
(c) The open circuit is replaced by a voltmeter, and the short circuit is replaced by an ammeter. All resistances are in ohms.

Voltmeters and Ammeters (cont'd)

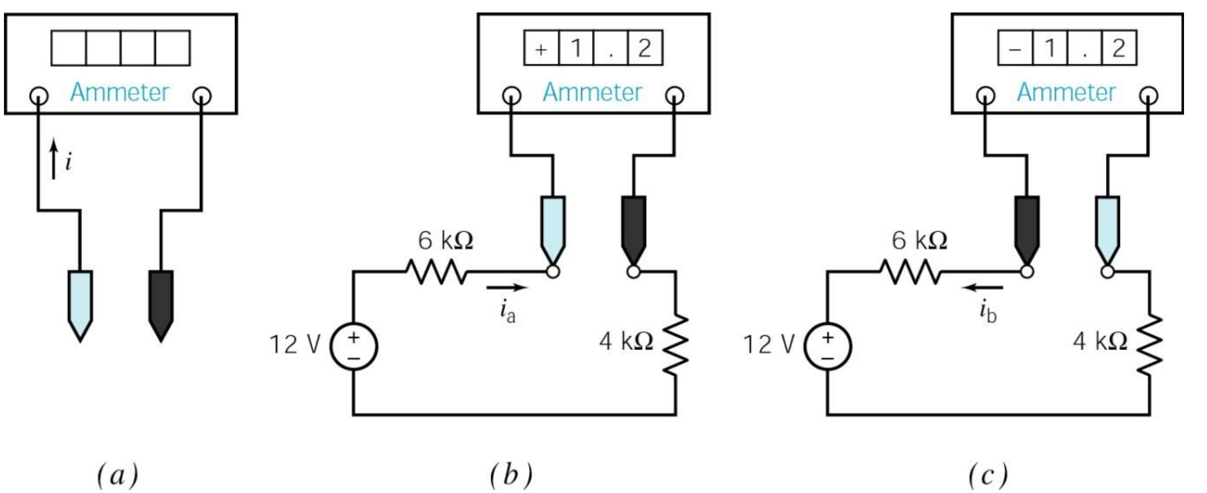
Voltmeter

(Figure 2.6-4)



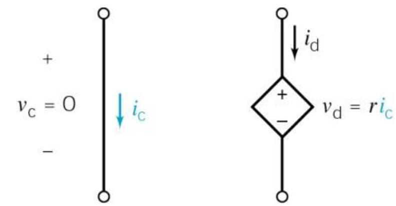
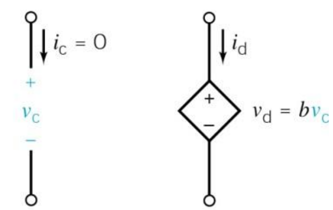
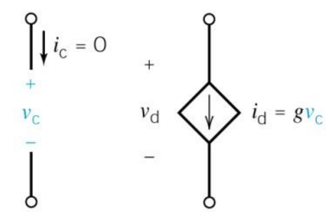
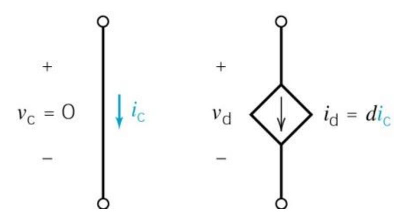
Ammeter

(Figure 2.6-5)



Dependent Sources

Table 2.7-1 Dependent Source

DESCRIPTION	SYMBOL
Current-Controlled Voltage Source (CCVS) *r* is the gain of the CCVS. *r* has units of volts/ampere.	
Voltage-Controlled Voltage Source (VCVS) *b* is the gain of the VCVS. *b* has units of volts/volt.	
Voltage-Controlled Current Source (VCCS) *g* is the gain of the VCCS. *g* has units of amperes/volt.	
Current-Controlled Current Source (CCCS) *d*i is the gain of the CCCS. *d* has units of amperes/ampere.	

Dependent Sources (cont'd)

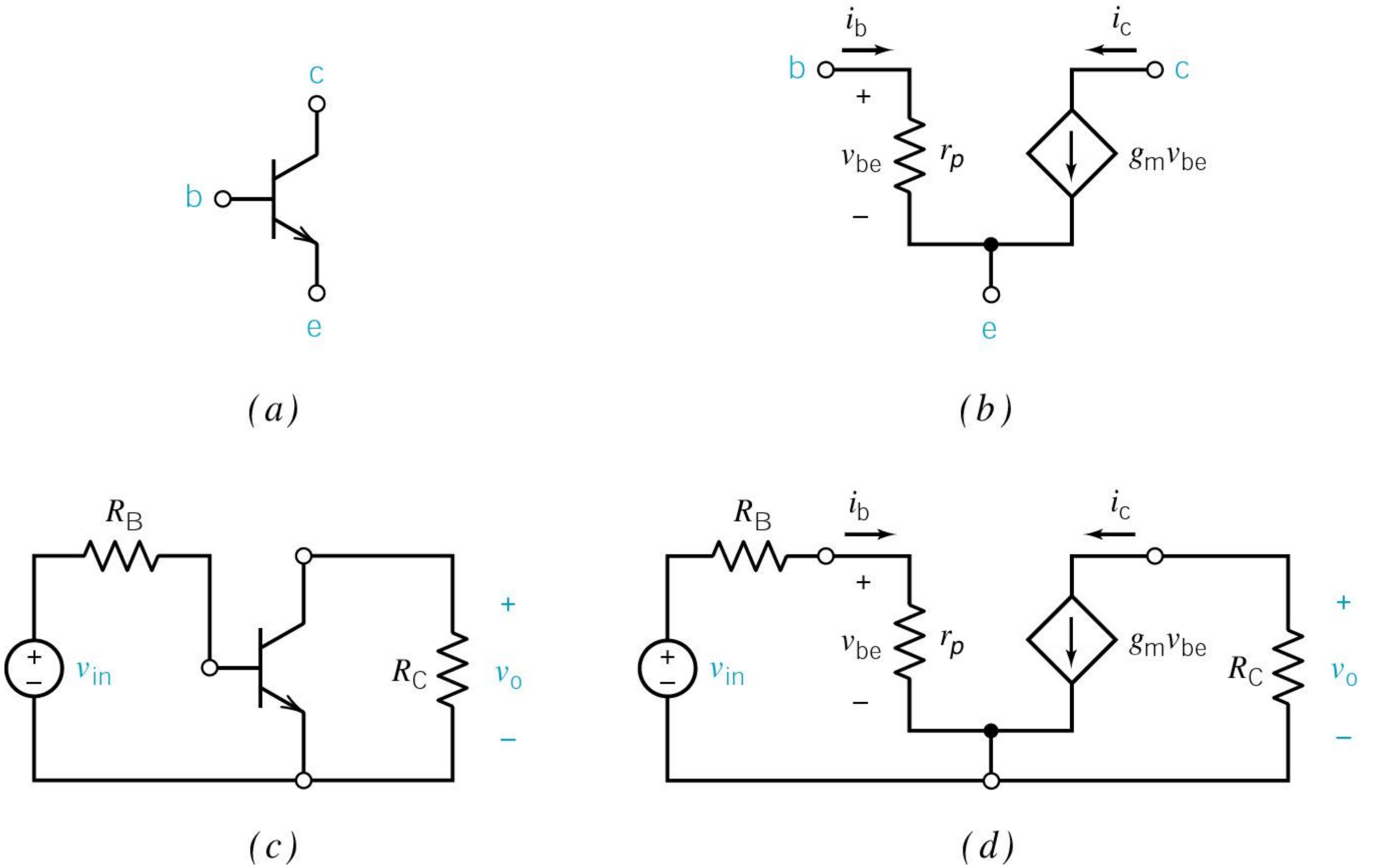


Figure 2.7-2 (p. 35)
(*a*) A symbol for a transistor. (*b*) A model of the transistor. (*c*) A transistor amplifier. (*d*) A model of the transistor amplifier.

Example 2.7-1 *Power and Dependent Sources*

- Determine the power absorbed by the VCVS in Figure 2.7-3

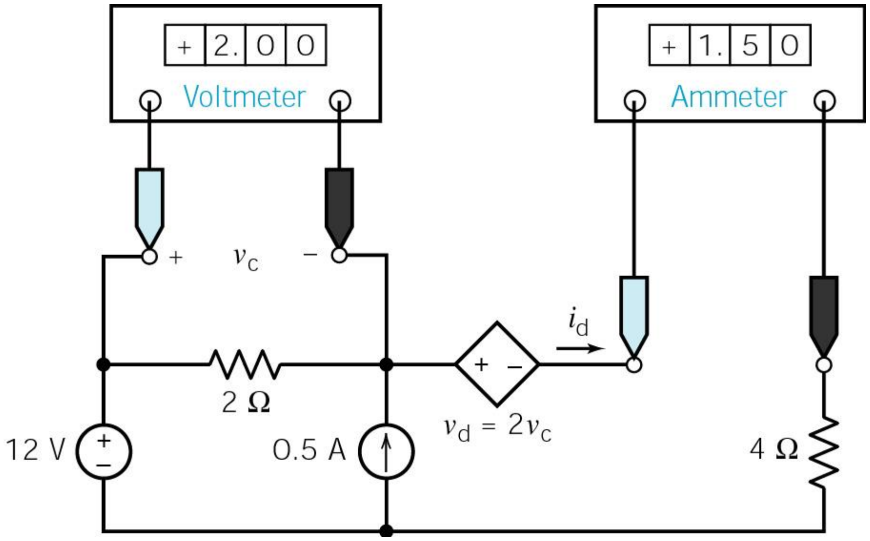


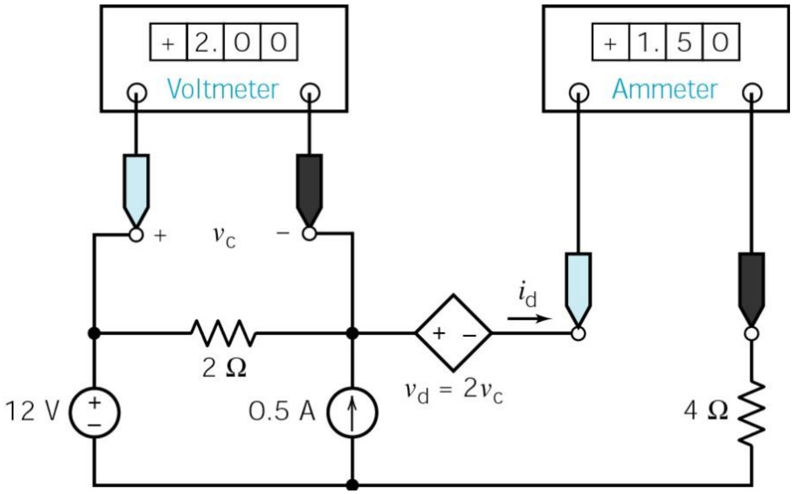
Figure 2.7-3 (p. 36)
A circuit containing a VCVS. The meters indicate that the voltage of the controlling element is $v_c = 2.0$ volts and that the current of the controlled element is $i_d = 1.5$ amperes.

Solution

- The VCVS consists of an open circuit and a controlled voltage source. There is no current in the open circuit, so no power is absorbed by the open circuit.

- The voltage, $\boldsymbol{v_c}$, across the open circuit is the controlling signal of the VCVS.
  The voltmeter measures $\boldsymbol{v}_c$ to be

$$v_c = 2(\mathrm{V})$$



- The voltage of the controlled voltage source is

$$v_d = 2v_c = 4(\mathrm{V})$$

- The ammeter measures the current in the controlled voltage source to be

$$i_d = 1.5(\mathrm{A})$$

- The element current, id, and voltage, vd, adhere to the passive convention. Therefore

$$p = i_d v_d = (1.5)(4) = 6(\mathrm{W})$$

  is the power absorbed by the VCVS

Transducers

- **Transducers**
  - Devices that convert physical quantities to electrical quantities.
  - Ex) potentiometers & temperature sensors

  - **Potentiometer**
    - Convert position to resistance

  - **Temperature sensors**
    - Convert temperature to current

Transducers (cont'd)

- **Potentiometer**

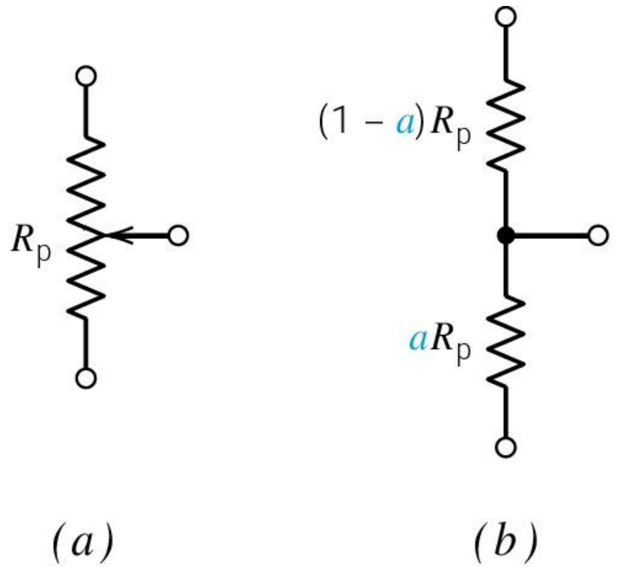


Figure 2.8-1 (p. 37)
(a) The symbol and (b) a model for the potentiometer.

- **Temperature sensors**

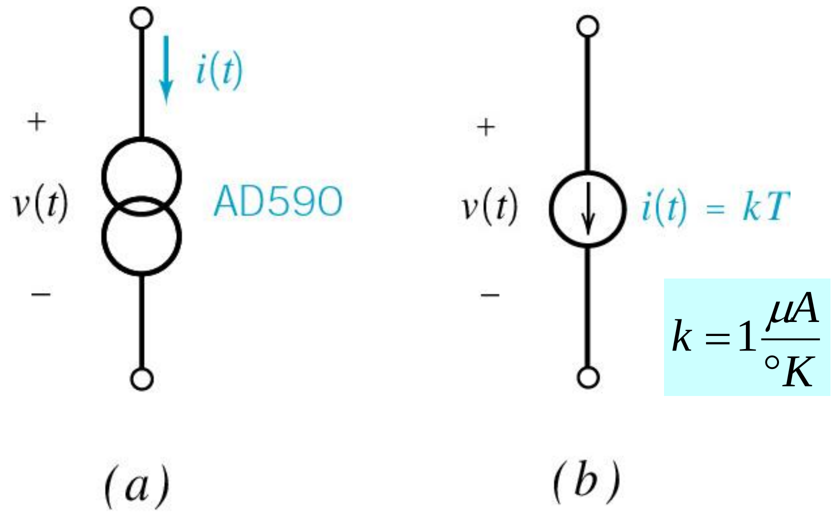
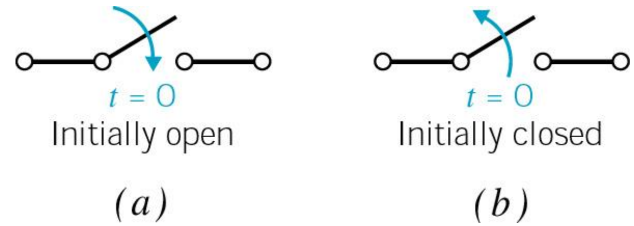


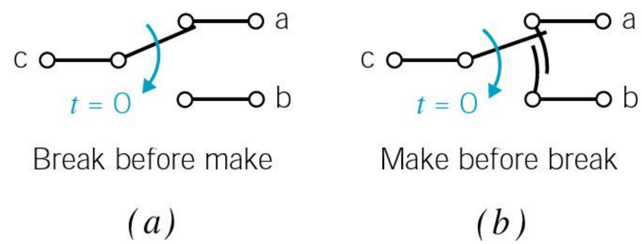
Figure 2.8-3 (p. 38)
(a) The symbol and (b) a model for the temperature sensor.

Switch

- **Switches have two distinct states: open and closed. Ideally, a switch acts as a short circuit when it is closed as an open circuit when it is open.**

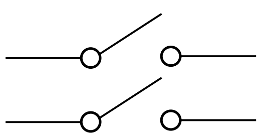


- **SPST switch**
  - Single-pole, single-throw



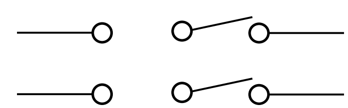
- **SPDT switch**
  - Single-pole, double-throw

- **DPST switch**
  - Double-pole, single-throw



DPDT switch

Double-pole, double-throw



Example 2.9-1 *Switches*

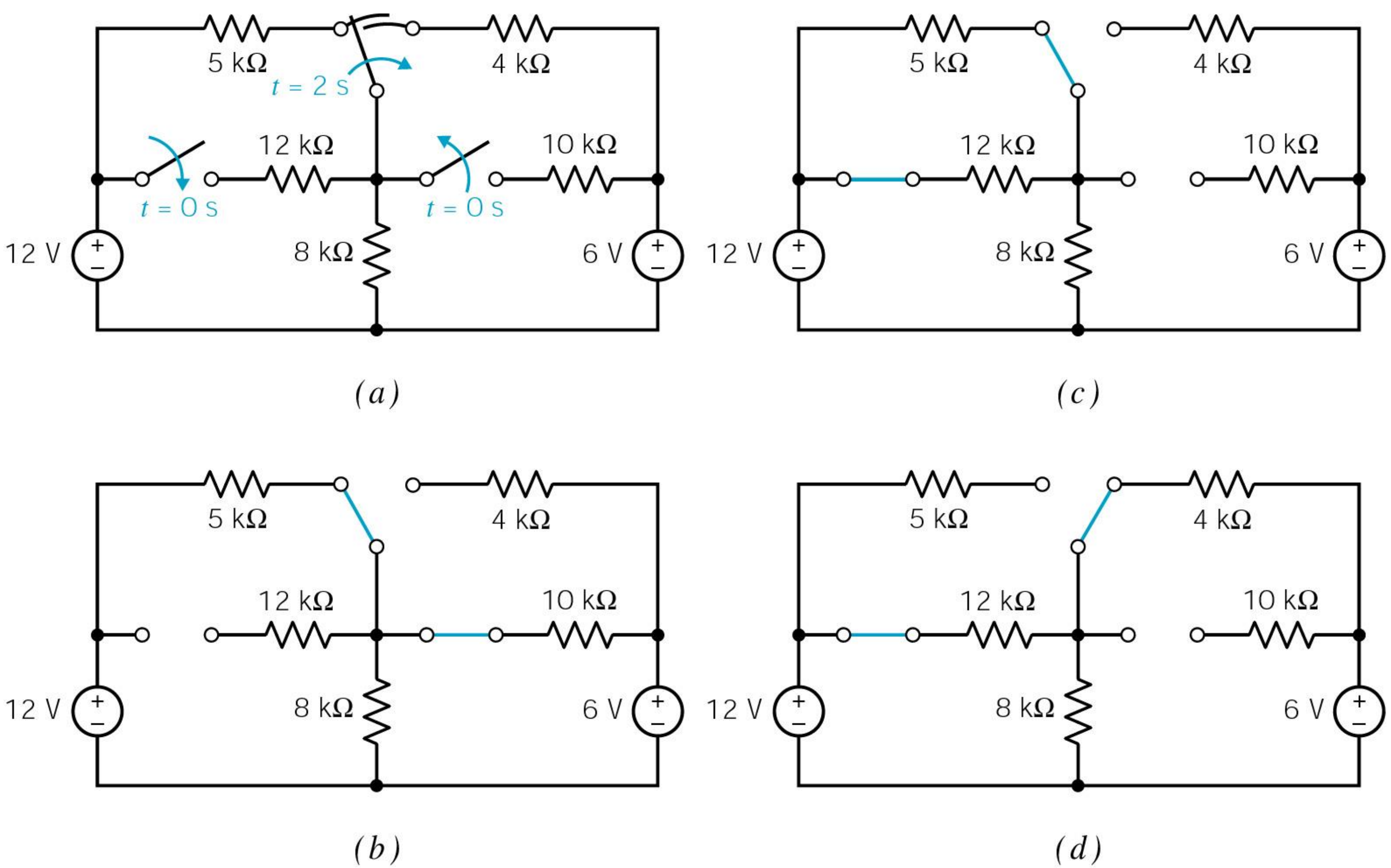


Figure 2. 9-3 (p. 40)
(*a*) A circuit containing several switches. (*b*) The equivalent circuit for $t \leq 0$ s. (*c*) The equivalent circuit for $0 < t < 2$ s. (*d*) The equivalent circuit for $t > 2$ s.

Example 2.10-1 *How can we check voltage and current values?*

- The meter in the circuit of Figure 2.10-1 indicate that v_1=-4V, v_2=8V and that i=1A. How can we check that the value of v_1, v_2, and i have been measured correctly? Let's check the values of v_1, v_2, and i in two ways:

(a) Verify that the given values satisfy Ohm's law for both resistors.

(b) Verify that the power supplied by the voltage source is equal to the power absorbed by the resistors.

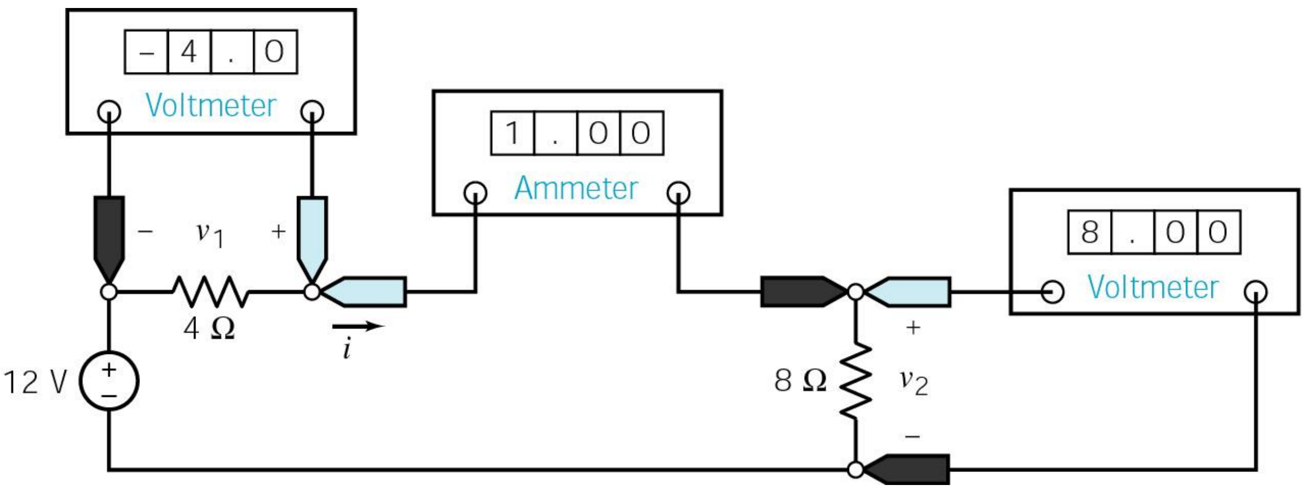
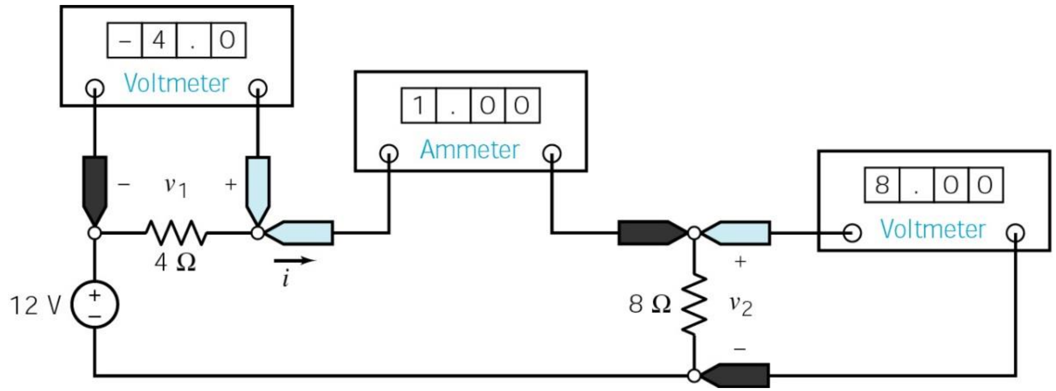


Figure 2.10-1 (p. 41)
circuit with meters.

Solution

(a) Verify that these values satisfy Ohm's law.



- **8Ω 저항에서**

  전류는 (+)에서 (-)방향으로 흐르므로

  $$\text{Ohm's law}: \quad v_2 = 8i$$

  전압계와 전류계로 측정한 결과
  v_2=8V, i=1A이므로 수식을 만족한다.

- **4Ω 저항에서**

  전류는 (-)에서 (+)방향으로 흐르므로

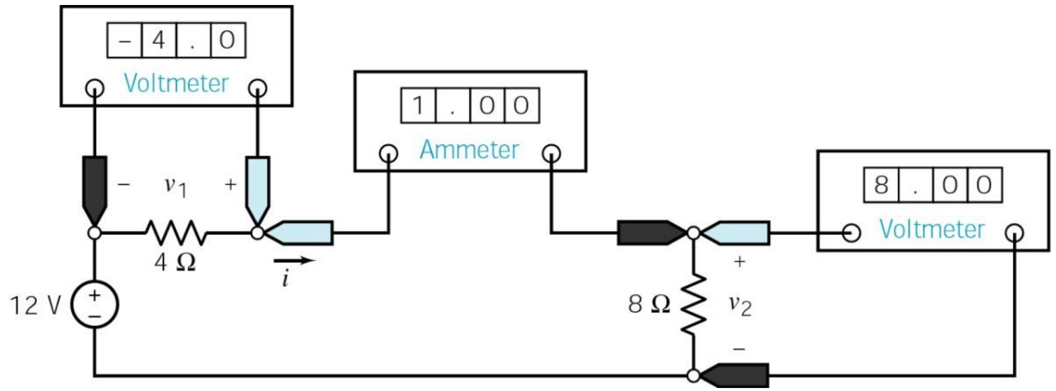
  $$\text{Ohm's law}: \quad v_1 = 4(-i)$$

  전압계와 전류계로 측정한 결과
  v_2=-4V, i=1A이므로 수식을 만족한다.

Ohm's law is satisfied!

Solution

(b) Verify that the power supplied by the voltage source is equal to the power absorbed by the resistors.



- **12V 전압원에서 (supplied power)**

$$p = vi = 12 \cdot 1 = 12W$$

- **4Ω 저항에서 (absorbed power)**

$$p = i^2 R = (1)^2 4 = 4W$$

- **8Ω 저항에서 (absorbed power)**

$$p = i^2 R = (1)^2 8 = 8W$$

The power supplied by the voltage source is equal to the power absorbed by the resistors!

Design Example – Temperature Sensor

- **Describe the situation and the assumptions**

  In order for the temperature transducer to operate properly, its element voltage must be between 4 volts and 30 volts. The power supplies and resistors will be used to establish this voltage. An ammeter will be used to measure the current in the temperature transducer.

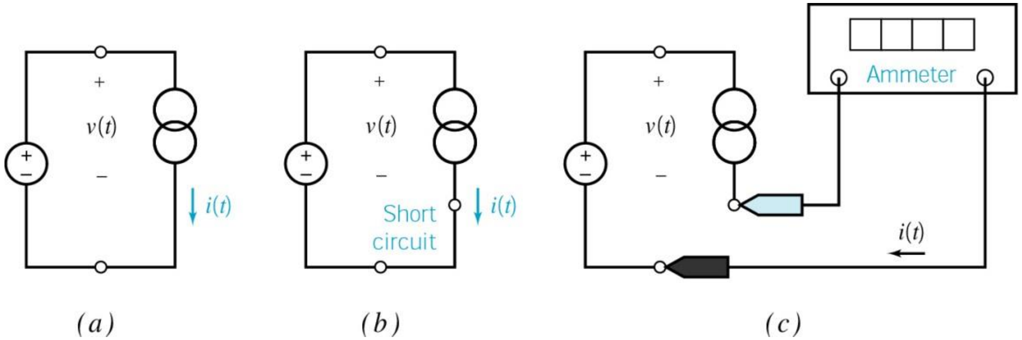
- **State the goal**

  Use the power supplies and resistors to cause the voltage, v, of the temperature transducer to be between 4 volts and 30 volts.

  Use an ammeter to measure the current, i, in the temperature transducer.

- **Generate a plan**

  Model the power supply as an ideal voltage source and the temperature transducer as an ideal current source



(a) Measuring temperature with a temperature sensor. (b) Adding a short circuit.
(c) Replacing the short circuit by an ammeter.

Design Example – Temperature Sensor

- **Act on the plan**

  The power absorbed by the transducer is ***p=vi***

  Choosing ***v*** as small as possible makes the power absorbed as small as possible (***v***=10V)

- **Verify the proposed solution**

  Suppose the temperature of the water is 80.6˚F (=27˚C =300˚K)

$$i = \left(1\frac{\mu\text{A}}{°\text{K}}\right)300°\text{K} = 300\,\mu\text{A}$$

  Suppose the ammeter in Figure 2.11-3a reads 300μA

$$T = \frac{300\,\mu\text{A}}{1\,\mu\text{A}/°\text{K}} = 300°\text{K} = 27°\text{C} = 80.6°\text{F}$$

  The graph in Figure 2.11-3b indicates that a sensor current of 300 temperature of 27˚C

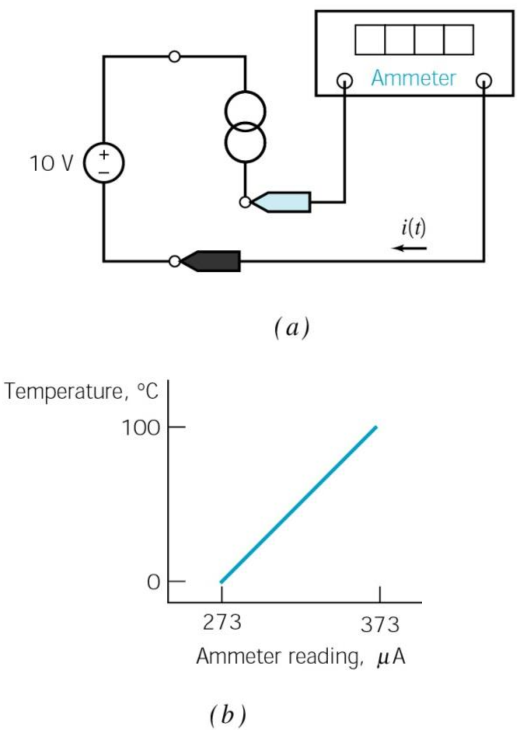


Figure 2.11-3
(*a*) Final design of a circuit that measures temperature with a temperature sensor. (*b*) Graph of temperature versus ammeter current.

정리 summary

- **Linearity의 정의**
- **Active and Passive devices의 정의**
- **Dependent Sources: voltage controlled current source등**
- **Voltmeters and Ammeters**
- **Potentiometers as a variable resistor**
- **AD590 as current source**
- **Switches의 종류**