Chapter 3
Resistive Circuits

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Analysis of Resistive circuit

- **KCL(Kirchhoff Current Law), KVL(Kirchhoff Voltage Law)** 등의 이론을 도입하여 저항회로를 분석하자**.**

KCL과 KVL은?

-**Maxwell equations**을 부분적으로 재해석한 것이다**.**

From Maxwell's Equations

	미분형	적분형
Faraday의 전자유도법칙	$\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$	$\oint_C \vec{E} \cdot d\vec{l} = -\dfrac{\partial \int_s \vec{B} \cdot d\vec{S}}{\partial t}$
전하 보존의 법칙	$\vec{\nabla} \cdot \vec{J} + \dfrac{\partial \rho}{\partial t} = 0$	$\oint_S \vec{J} \cdot d\vec{S} + \dfrac{\partial \int_V \rho dV}{\partial t} = 0$

KVL, KCL에서의 가정

(1) 전파**(傳播)** 효과가 무시될 만큼 계가 작다.

즉**,** 계가 순간적으로**,** 동시적으로 변화한다 →

집중정수계 **(lumped constants system).**

(2) 계에 알짜 전하**(net charge)**는 없다.

(3) 계의 구성 부품 간에 자기적인 결합은 없다.

KCL은 '연속방정식(전하 보존의 법칙)'으로부터

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \qquad \oint \vec{J} \cdot d\vec{s} + \frac{\partial \int_v \rho dv}{\partial t} = 0$$

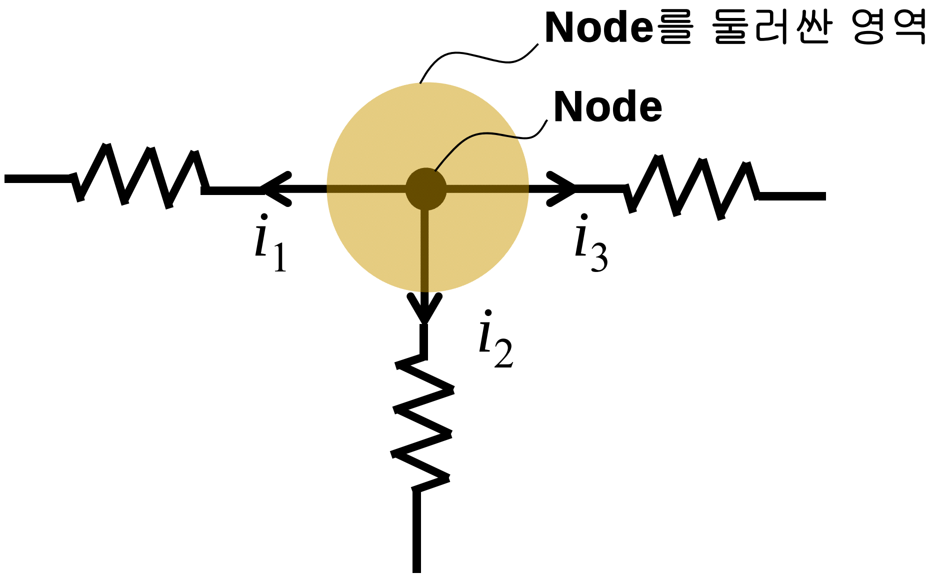
- 가정 **(2)** 는 회로에 알짜 전하가 없으므로 제 **2** 항의 전하 밀도가 영이다**.**

$$\rho = 0 \qquad \rightarrow \quad \frac{\partial \rho}{\partial t} = 0$$

따라서**,** $\vec{\nabla} \cdot \vec{J} = 0 \Rightarrow \quad \oint_S \vec{J} \cdot d\vec{S} = 0$

발산**(divergence):** 어떤 영역의 부피를 통과하며 나가는 알짜 양

KCL continued



- 회로에서는 전류가 존재하는 곳이 도선뿐이다.
- 따라서, 영역의 표면 면적분이 도선에 흐르는 전류의 합으로 표현된다.

$$\oint_S \vec{J} \cdot d\vec{S} = 0 \Rightarrow \sum_j i_j = 0$$

KVL : Faraday의 전자유도 법칙

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial \int_s \vec{B} \cdot d\vec{s}}{\partial t}$$

- 가정 **(3)** 은 구성 부품간의 자기적인 결합이 없으므로 회로가 만드는 면을 통과하는 자속의 시간적인 변화가 영이다.

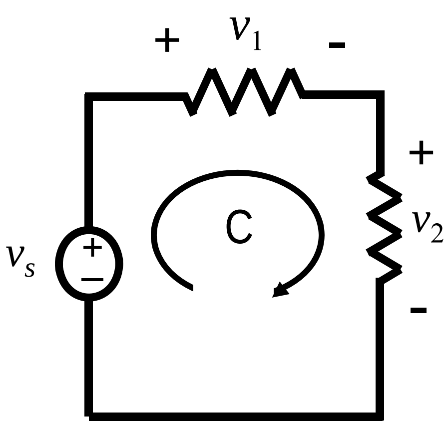
$$\frac{\partial \vec{B}}{\partial t} = 0.$$

- 따라서,

$$\vec{\nabla} \times \vec{E} = 0 \Rightarrow \oint_C \vec{E} \cdot d\vec{l} = 0$$

Stoke's theorem

KVL continued

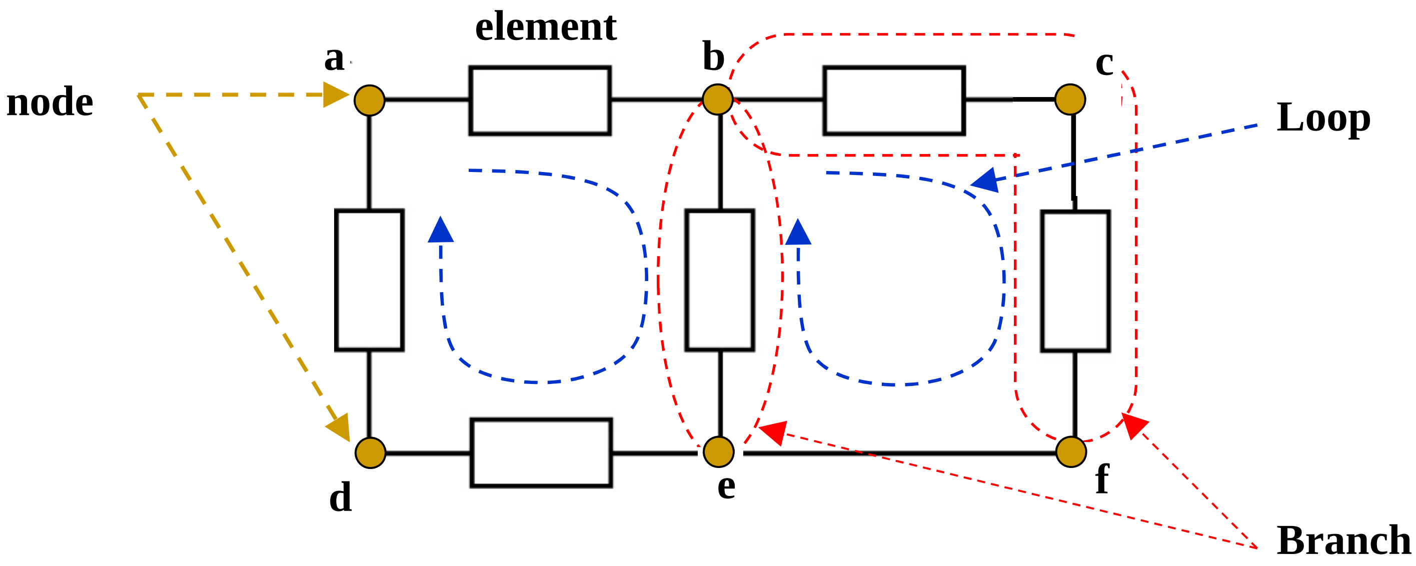


- 회로가 만드는 면을 통과하는 자계의 시간적인 변화가 영이면 회로를 따라서 전계를 적분하면 영이 된다.
- 그런데, 전계가 존재하는 곳은 소자에서만 이므로 회로의 선적분은 소자에서의 전압 강하의 합으로 표현된다.

$$\oint_C \vec{E} \cdot d\vec{l} = 0 \Rightarrow \sum_j v_j = 0$$

Definition of Branch/ Node/ Loop

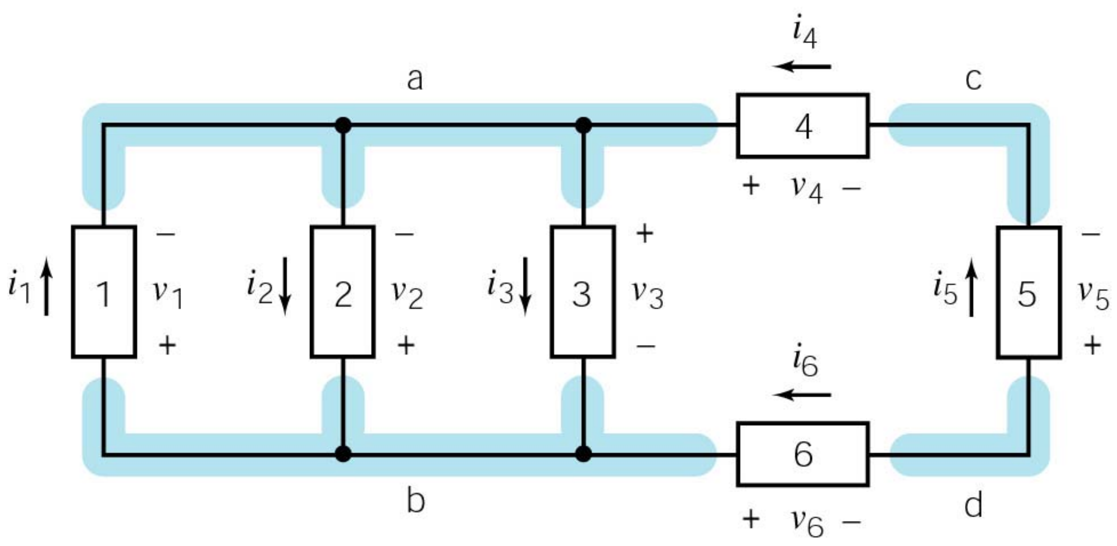
- **Branch**
  - 세 개 이상의 소자가 만나는 점 사이의 회로 (b-a-d-e, b-e, b-c-f-e)
- **Node**
  - 소자(element)와 소자가 만나는 점 (a,b,c,d,e,f)
- **Loop**
  - 닫혀있는 폐회로 (a-b-e-d, b-c-f-e)



Kirchhoff's Current Law

- *Kirchhoff's current law* **(KCL):**

  The algebraic sum of the currents into a node at any instant is zero.



(c)

<Node a>

$$-i_1 + i_2 + i_3 - i_4 = 0$$

$$i_2 + i_3 = i_1 + i_4$$

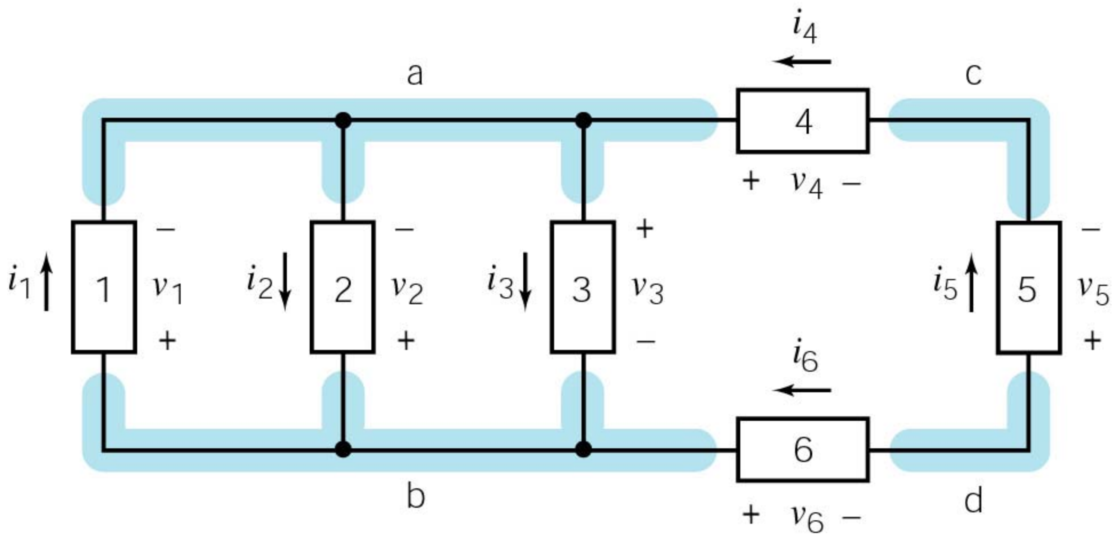
<Node b>

$$i_1 = i_2 + i_3 + i_6$$

Kirchhoff's Voltage Law

- ***Kirchhoff's voltage law*** **(KVL):**

  The algebraic sum of the voltages around any loop in a circuit is identically zero for all time.



(c)

<Loop 3,4,5,6>

$$v_4 - v_5 - v_6 - v_3 = 0$$

<Loop 1,4,5,6>

$$v_4 - v_5 - v_6 + v_1 = 0$$

<Loop 1,2>

$$-v_2 + v_1 = 0$$

Example 3.2-2 *Kirchhoff's Laws*

- Consider the circuit shown in Figure 3.2-4a. Determine the power supplied by element C and the power received by element D.

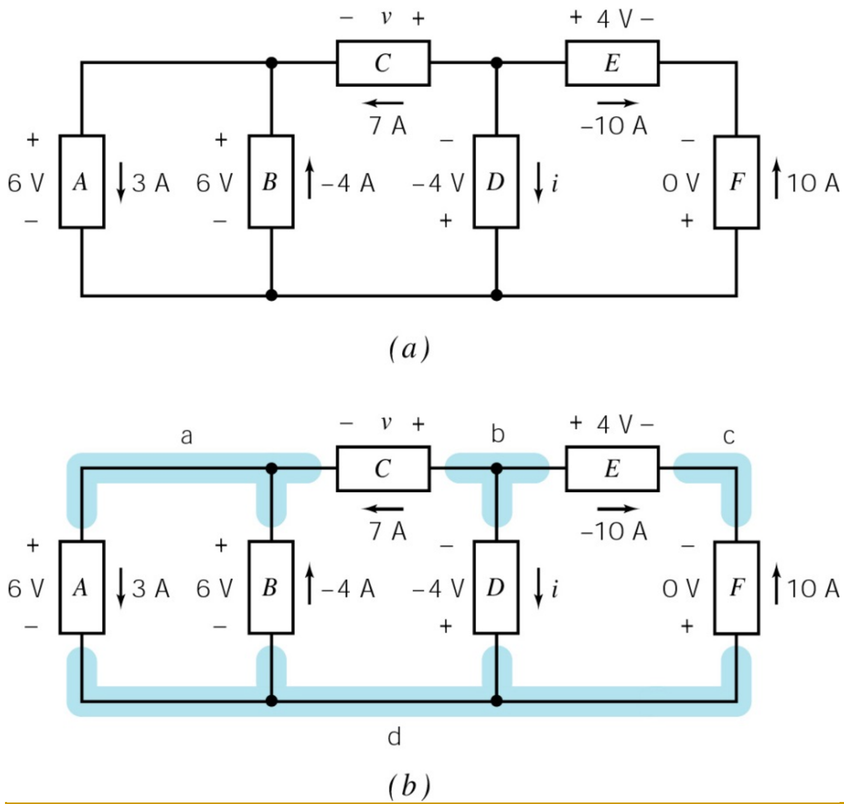
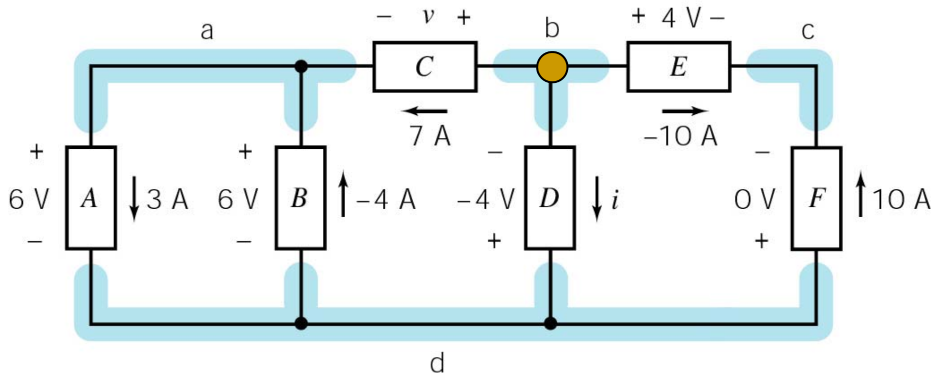


Figure 3.2-4 (p. 58)
(*a*) The circuit considered in Example 3.3.2 and (*b*) the circuit redrawn to emphasize the nodes.

Solution



(b)

1. Apply KVL to the loop consisting of elements C,D and B

$$-v-(-4)-6=0 \quad \Rightarrow \quad v=-2\text{V}$$

$$p_c = v(7) = (-2)(7) = -14\text{W}$$ C supplies 14W.

2. Apply KCL at node b

$$7+(-10)+i=0 \quad \Rightarrow \quad i=3\text{A}$$

$$p_D = (-4)i = (-4)(3) = -12\text{W}$$ D receives 12W.

Example 3.2-3 *Ohm's and Kirchhoff's Laws*

- Consider the circuit shown in Figure 3.2-5. Notice that the passive convention was used to assign reference directions to the resistor voltages and currents. This anticipates using Ohm's law. Find each current and each voltage when R_1=8Ω, v_2=-10V, i_3=2A, and R_3=1Ω. Also, determine the resistance R_2.

2 Unknowns
i1,i2

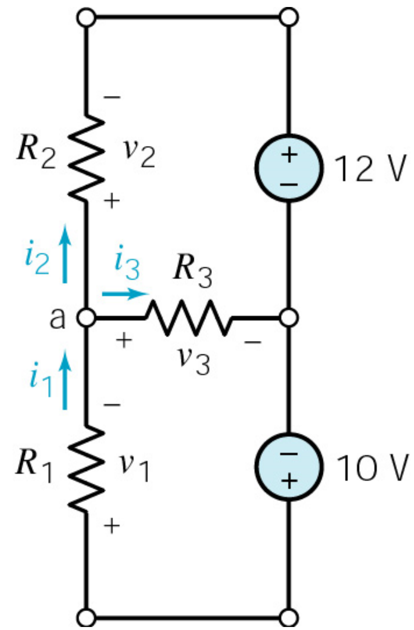


Figure 3.2-5 (p. 59)
Circuit with two constant-voltage sources.

Department of Electrical and Computer Engineering, SNU
Prof. SungJune Kim

Solution

1. At Node a

$$i_1 - i_2 - i_3 = 0$$

2. Using Ohm's law for R_3

$$v_3 = R_3 i_3 = 1(2) = 2\text{V}$$

3. KVL for the bottom loop incorporating v_1,v_3, and the 10V source

$$-10 + v_1 + v_3 = 0$$

$$v_1 = 10 - v_3 = 8\text{V}$$

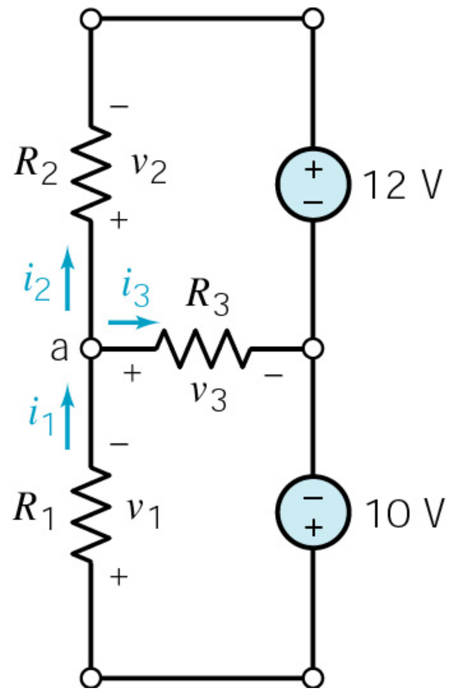
4. Ohm's law for the resistor R_1

$$i_1 = v_1 / R_1 = 8/8 = 1\text{A}$$

$$i_2 = i_1 - i_3 = 1 - 2 = -1\text{A}$$ ← $\boldsymbol{i_1}$=1A, $\boldsymbol{i_3}$=2A

5. Resistance R_2

$$R_2 = v_2 / i_2 = -10/-1 = 10\Omega$$



Example 3.2-4 *Ohm's and Kirchhoff's Laws*

- Determine the value of the current, in amps, measured by the ammeter in Figure 3.2-6a.

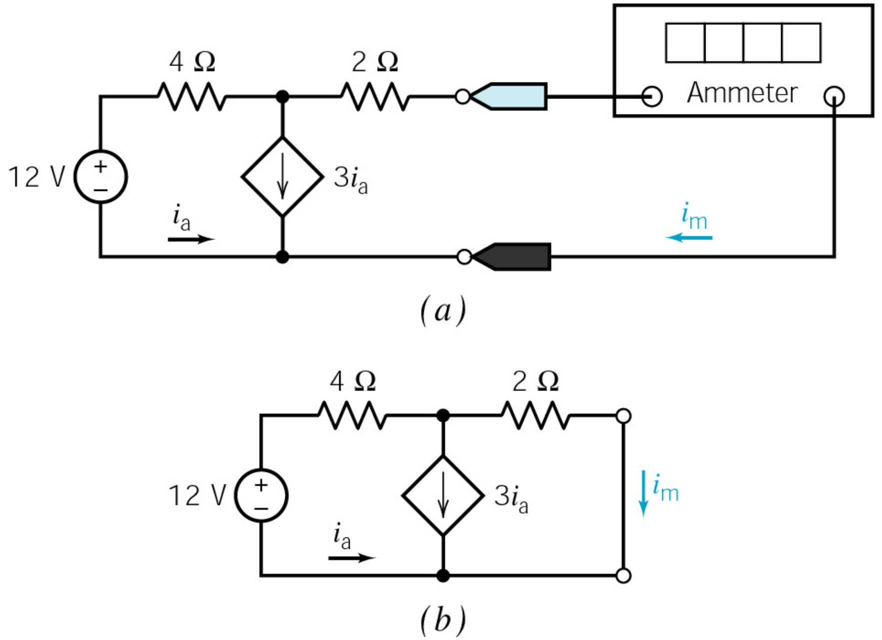
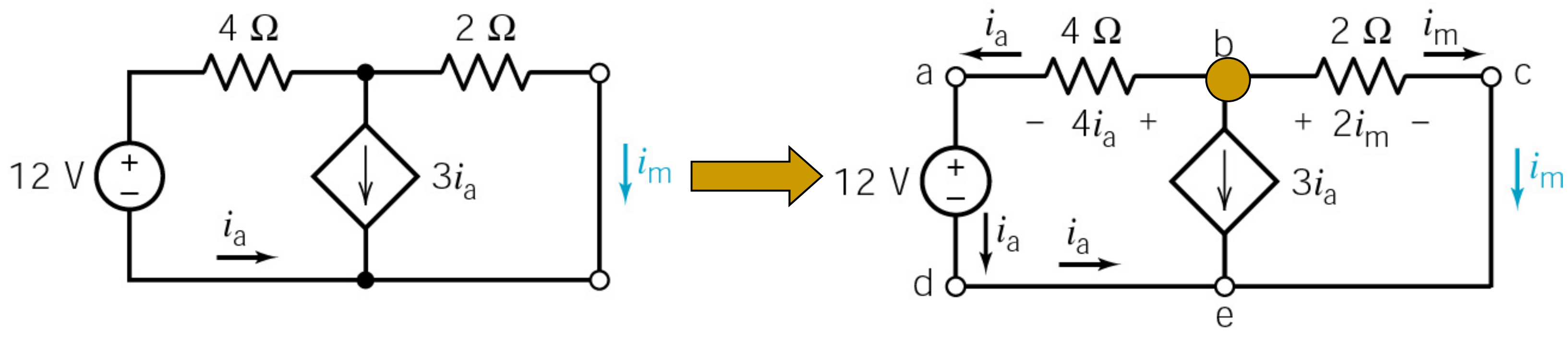


Figure 3.2-6 (p. 59)
(*a*) A circuit with dependent source and an ammeter. (*b*) The equivalent circuit after replacing the ammeter by a short circuit.

Solution



1. Apply KCL at node b

$$-i_a - 3i_a - i_m = 0$$

2. Apply KVL to closed path a-b-c-e-d-a

$$0 = -4i_a + 2i_m - 12 = -4\left(-\frac{1}{4}i_m\right) + 2i_m - 12 = 3i_m - 12$$

Solving this equation $\quad i_m = 4\mathrm{A}$

Example 3.2-5 *Ohm's and Kirchhoff's Laws*

- Determine the value of the voltage, in volts, measured by the voltmeter in Figure 3.2-8a.

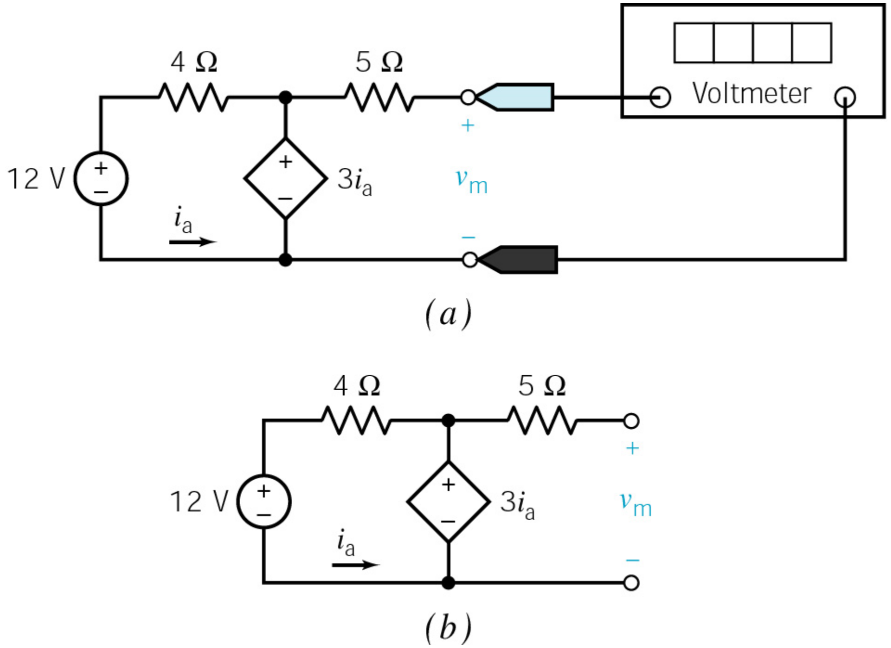
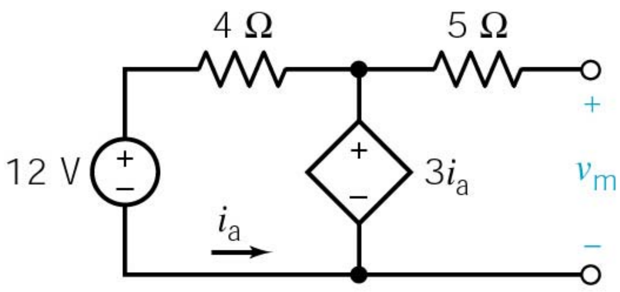
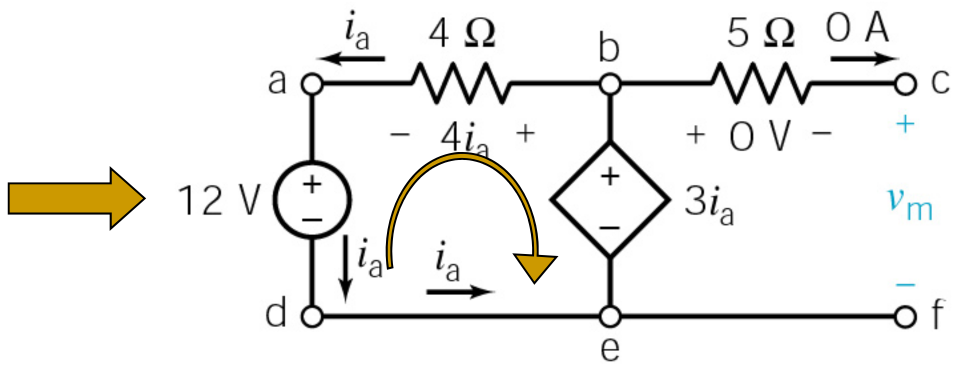


Figure 3.2-8 (p. 60)
(*a*) A circuit with dependent source and a voltmeter. (*b*) The equivalent circuit after replacing the voltmeter by an open circuit.

Solution





1. Apply KVL to the closed path a-b-e-d-a

$$-4i_a + 3i_a - 12 = 0$$

$$i_a = -12\text{A}$$

Finally $$v_m = 3i_a = 3(-12) = -36\text{V}$$

Series Resistors and Voltage Division

- 그림 3.3-1의 단일 루프 회로에서 **KCL**과 **KVL**과 옴의 법칙을 적용해 보면,

KCL at each node **(a, b, c, d)**

$$a : i_s = i_1$$
$$b : i_1 = i_2$$
$$c : i_2 = i_3$$
$$d : i_3 = i_s$$
$$i_s = i_1 = i_2 = i_3$$

KVL around the loop

$$v_1 + v_2 + v_3 - v_s = 0$$

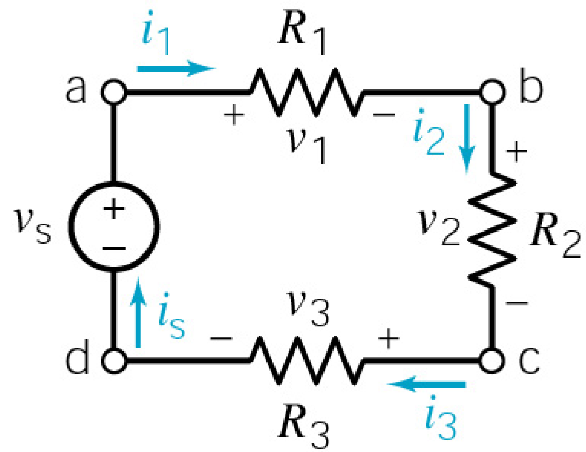


Figure 3.3-1 (p. 63)
Single-loop circuit with a voltage source v_s.

Ohm's Law

$$R_1 i_1 + R_2 i_2 + R_2 i_3 - v_s = 0 \quad \Rightarrow \quad R_1 i_1 + R_2 i_1 + R_2 i_1 = v_s$$

Series Resistors and Voltage Division

- 그림 3.3-2a의 단일 루프 회로에서 i_1을 계산

$$i_1 = \frac{v_s}{R_1 + R_2 + R_3}$$

- n번째 저항 R_n에 걸리는 전압 v_n은

$$v_n = i_1 R_n = \frac{v_s R_n}{R_1 + R_2 + R_3}$$

- 따라서 전압은 직렬로 연결된 각각의 저항에 나누어 진다.
  이러한 회로를 ***voltage divider***라고 한다.

$$\boxed{v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v_s}$$

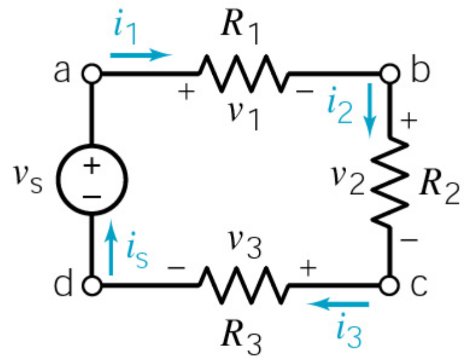


Figure 3.3-2a (p. 64)

Series Resistors and Voltage Division

- 직렬로 연결된 N개의 저항이 각각 $R_1, R_2 \ldots R_N$의 저항값을 가질때, 이를 하나의 저항값을 갖는 저항 성분 R_S으로 나타내면,

$$R_s = R_1 + R_2 + \cdots + R_N$$

- 파워 계산 (in Fig.3.3-2a)

$$p = i_s{}^2 R_1 + i_s{}^2 R_2 + i_s{}^2 R_3$$

$$p = i_s{}^2 (R_1 + R_2 + R_3) = i_s{}^2 R_s$$

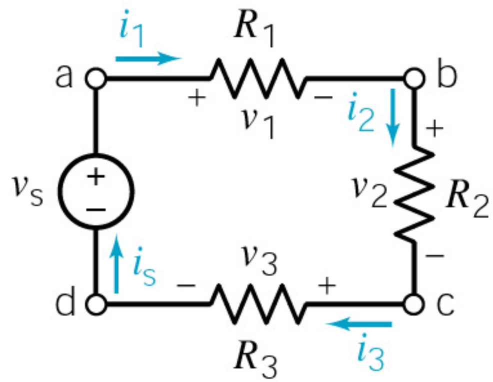


Figure 3.3-2a (p. 64)

Example 3.3-1 *Voltage Division*

- Consider the two similar voltage divider circuits shown in Figure 3.3-3. Use voltage division to determine the values of the voltage v_2 in Figure 3.3-3a and the voltage v_b in Figure 3.3-3b.

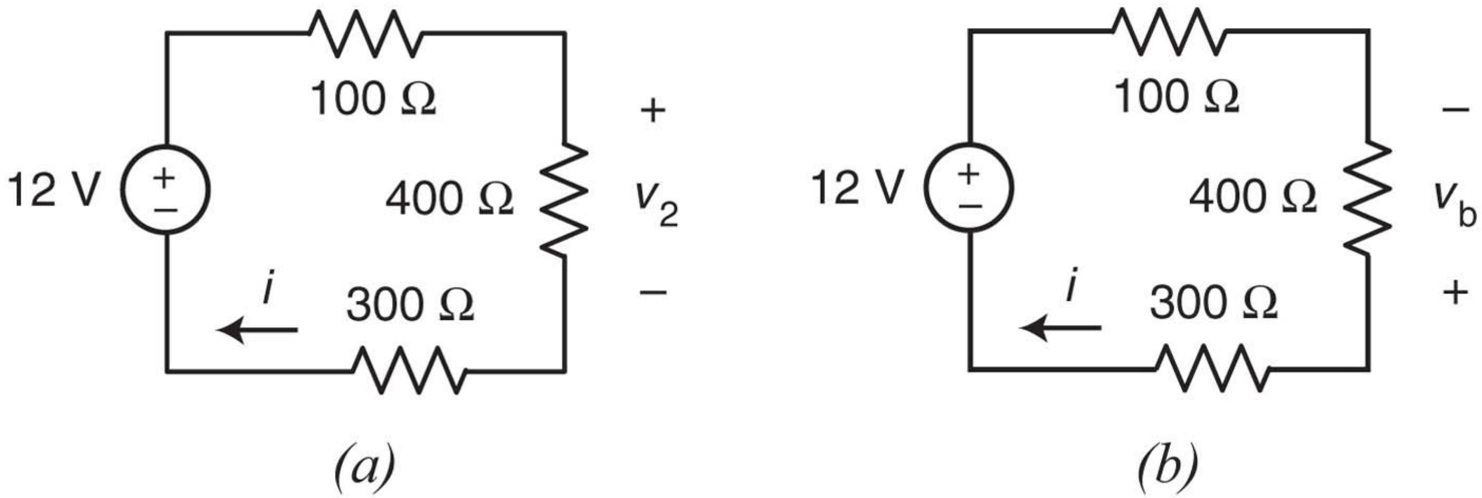


Figure 3.3-3 (p. 65)
Two similar voltage divider circuits.

Solution

1. In (a), the current in the loop is given by

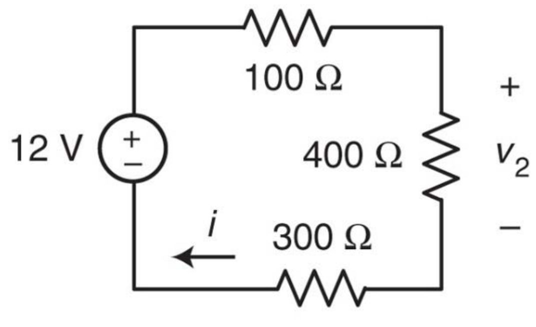
$$i = \frac{12}{100+400+300} = 0.015A = 15mA$$

2. Using voltage division,

$$v_2 = \frac{400}{100+400+300}(12) = 6V$$

3. As a check,

$$6 = v_2 = 400(i) = 400(0.015)$$



(a)

4. In (b), the current in the loop is given by, again,

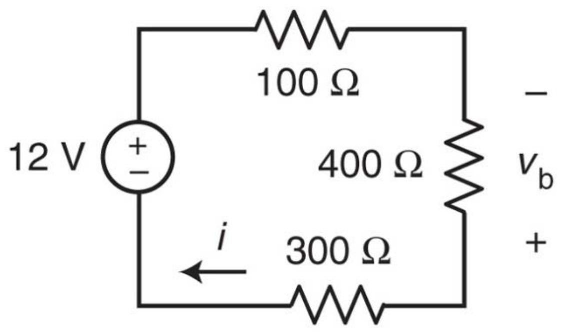
$$i = \frac{12}{100+400+300} = 0.015A = 15mA$$

5. The voltage v_b is same as v_2 **except for polarity.**

$$v_2 = -v_b, \qquad v_2 = \frac{400}{100+400+300}(12) = 6V$$

6. As a check,

$$-6 = v_b = -400(i) = -400(0.015)$$



(b)

Example 3.3-2 ***Series Resistors***

- For the circuit of Figure 3.3-4a, find the current measured by the ammeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

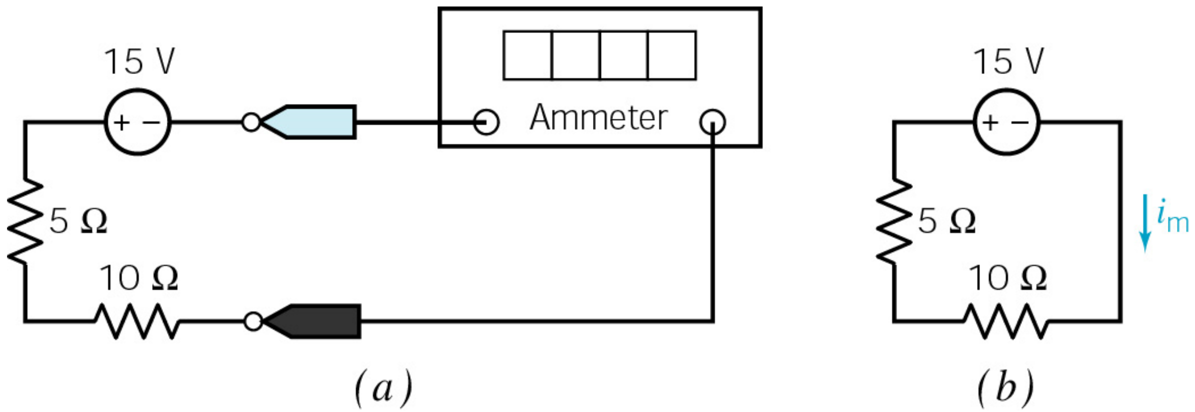
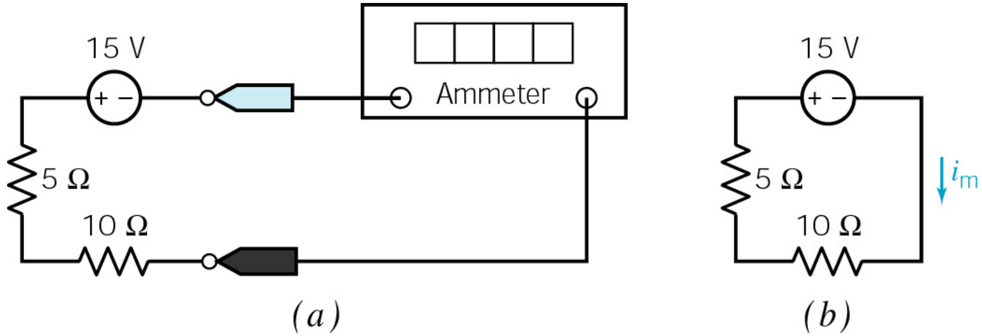


Figure 3.3-4 (p. 66)
(*a*) A circuit containing series resistors. (*b*) The circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter i_m.

Solution



- Applying KVL gives

$$15 + 5i_m + 10i_m = 0$$

The current measured by the ammeter is

$$i_m = -\frac{15}{5+10} = -1\text{A}$$

The total power absorbed by the two resistors is $p_R = 5i_m^2 + 10i_m^2 = 15(1^2) = 15(\text{W})$

The power supplied by the source is $p_s = -v_s i_m = -15(-1) = 15(\text{W})$

Thus, the power supplied by the source is equal to that absorbed by the series connection of resistors.

Parallel Resistors and Current Division

- **The Resisters in Figure 3.4-2 are connected in parallel.**
  Apply KCL at node a

$$i_s - i_1 - i_2 = 0$$

or $$i_s = i_1 + i_2$$

From Ohm's law

$$i_1 = \frac{v}{R_1} \quad and \quad i_2 = \frac{v}{R_2}$$

Then $$i_s = \frac{v}{R_1} + \frac{v}{R_2}$$

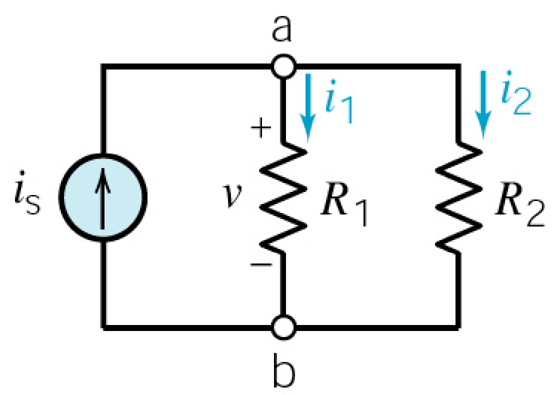


Figure 3.4-2 (p. 68)
Parallel circuit with a current source.

Recall that we defined conductance G as the inverse of resistance R. We may rewrite,

$$i_s = G_1 v + G_2 v = (G_1 + G_2) v$$

Parallel Resistors and Current Division

- The equivalent circuit for this parallel circuit is a conductance G_p, as shown in Figure 3.4-3,where

$$G_p = G_1 + G_2$$

The equivalent resistance for the two-resistor circuit is found from

$$G_p = \frac{1}{R_1} + \frac{1}{R_2}$$

Since $G_p = 1/R_p$, we have

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$\boxed{R_p = \frac{R_1 R_2}{R_1 + R_2}}$$

Figure 3.4-3 (p. 69)
Equivalent circuit for a parallel circuit.

Note that the total conductance, G_p, increases as additional parallel elements are added and that the total resistance, R_p, declines as each resistor is added.

Parallel Resistors and Current Division

- **The circuit shown in Figure 3.4-2 is called a current divider circuit since it divides the source current. Note that**

$$i_1 = G_1 v$$

Also, since $i_s=(G_1+G_2)v$, we solve for v, obtaining

$$v = \frac{i_s}{G_1 + G_2}$$

Therefore,

$$i_1 = \frac{G_1 i_s}{G_1 + G_2}$$

Similarly,

$$i_2 = \frac{G_2 i_s}{G_1 + G_2}$$

It terms of two resistances, R_1 and R_2,

$$\boxed{i_2 = \frac{R_1 i_s}{R_1 + R_2}}$$

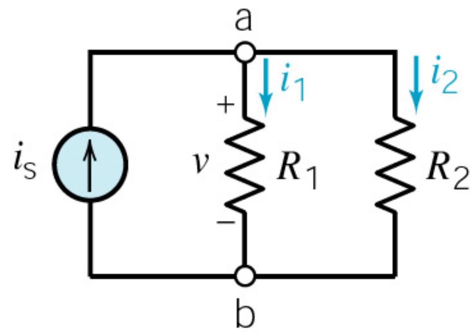


Figure 3.4-2 (p. 66)
Parallel circuit with a current source.

Figure 3.4-3 (p. 67)
Equivalent circuit for a parallel circuit.

Parallel Resistors and Current Division

- **Let us consider the more general case of current division with a set of *N* parallel conductors as shown in Figure 3.4-4. The KCL gives**

$$i_s = i_1 + i_2 + i_3 + \cdots + i_N$$

for which

$$i_n = G_n v$$

for n=1,…,N. We may write

$$i_s = (G_1 + G_2 + G_3 + \cdots + G_N)v$$

Therefore

$$i_s = v \sum_{n=1}^{N} G_n$$

Since i_n=$G_v n_v$, we may obtain v from Eq. 3.4-10 and substitute it in Eq. 3.4-8, obtaining

$$i_n = \frac{G_n i_s}{\sum_{n=1}^{N} G_n}$$

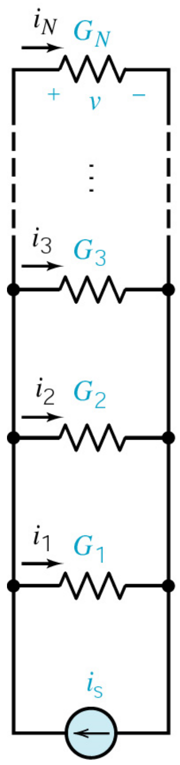


Figure 3.4-4 (p. 70)
Set of *N* parallel conductance with a current source i_s.

Parallel Resistors and Current Division

- **Recall that the equivalent circuit, Figure 3.4-3, has an equivalent conductance Gp such that**

$$G_p = \sum_{n=1}^{N} G_n \qquad \text{Eq. 3.4-12}$$

Therefore

$$i_n = \frac{G_n i_s}{G_p}$$

which is the basic equation for the current divider with N conductances. Of course, Eq. 3.4-12 can be rewritten as

$$\boxed{\frac{1}{R_P} = \sum_{n=1}^{N} \frac{1}{R_n}}$$

Example 3.4-1 *Parallel Resistors*

- For the circuit of Figure 3.4-5 find (a) the current in each branch, (b) the equivalent circuit, and (c) the voltage *v*, The resistors are

$$R_1 = \frac{1}{2}\Omega, \quad R_2 = \frac{1}{4}\Omega, \quad R_3 = \frac{1}{8}\Omega$$

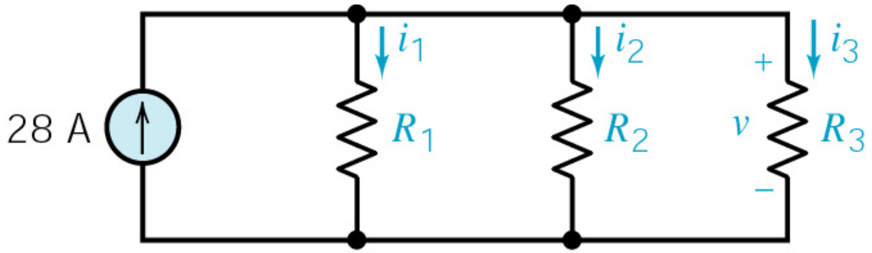
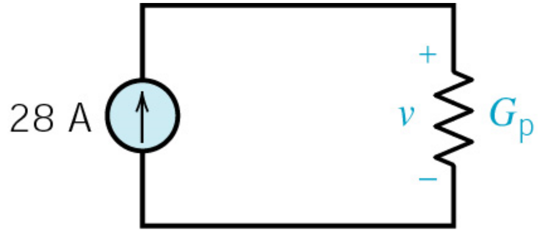


Figure 3.4-5 (p. 70)
Parallel circuit for Example 3.4-1.

Solution

<Equivalent circuit>



- The current divider follows the equation $$i_n = \frac{G_n i_s}{G_p}$$

  equivalent conductance G_p $$G_p = \sum_{n=1}^{N} G_n = G_1 + G_2 + G_3 = 2+4+8 = 14S$$

  Recall that the units for conductance are siemens (S). Then

  $$i_1 = \frac{G_1 i_s}{G_p} = \frac{2}{14}(28) = 4(\mathrm{A})$$

  Similarly, $$i_2 = \frac{G_2 i_s}{G_p} = \frac{4}{14}(28) = 8(\mathrm{A})$$

  and $$i_1 = \frac{G_3 i_s}{G_p} = 16(\mathrm{A})$$

  Since $i_n = G_n v$, we have $$v = \frac{i_1}{G_1} = \frac{4}{2} = 2(\mathrm{V})$$

Example 3.4-2 ***Parallel Resistors***

- For the circuit of Figure 3.4-7a, find the voltage measured by the voltmeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

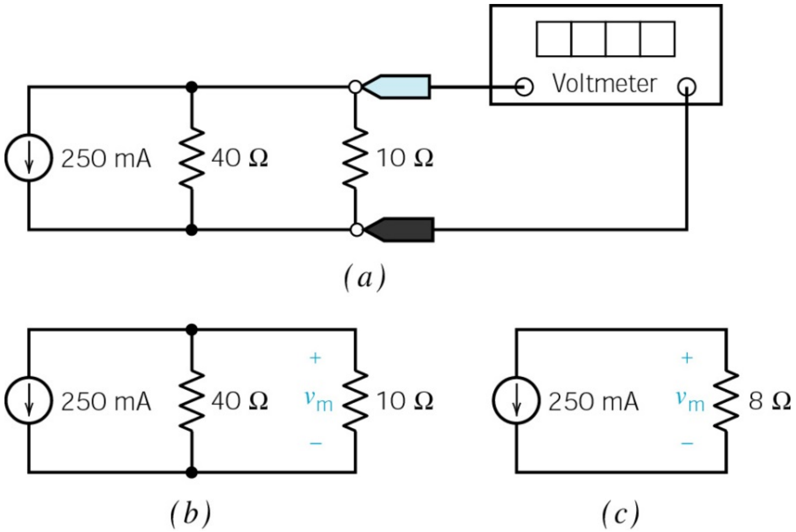
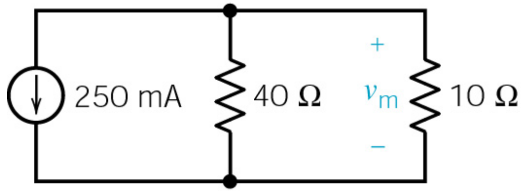


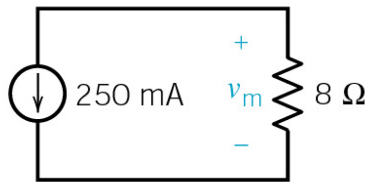
Figure 3.4-7 (p. 71)
(*a*) A circuit containing parallel resistors. (*b*) The circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter v_m. (*c*) The circuit after the parallel resistors have been replaced by an equivalent resistance.

Solution

<Equivalent circuit>



(b)



(c)

- The resistance of this equivalent resistor $$\frac{40 \cdot 10}{40+10} = 8(\Omega)$$

  voltage $\boldsymbol{v_m}$ $$v_m = 8(-0.25) = -2(\mathrm{V})$$

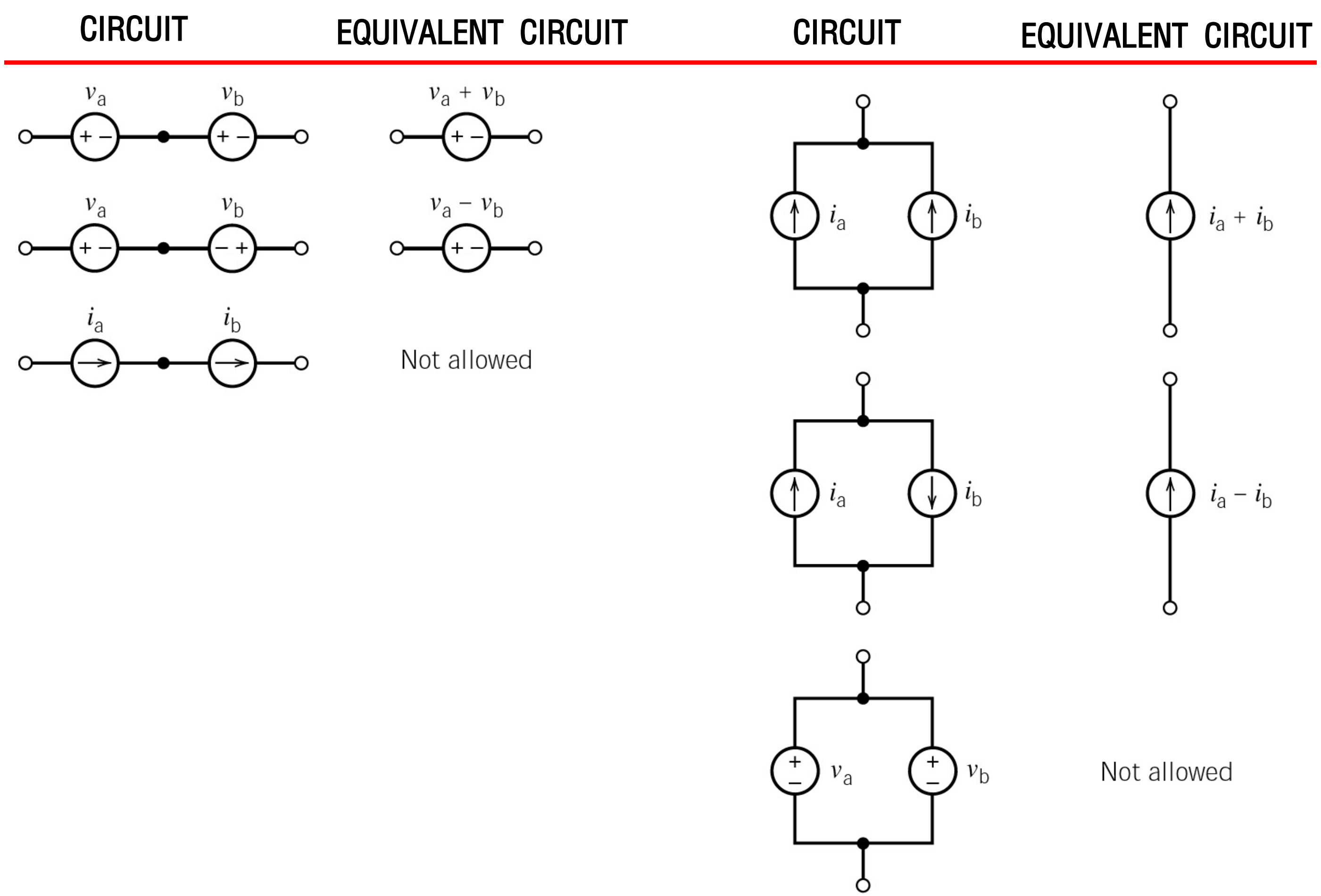
  The power absorbed by the resistor $$p_R = \frac{v_m^2}{40} + \frac{v_m^2}{10} = \frac{2^2}{40} + \frac{2^2}{10} = 0.1 + 0.4 = 0.5(\mathrm{W})$$

  The power supplied by the current source $$p_s = 2(0.25) = 0.5(\mathrm{W})$$

The power absorbed by the two resistors is equal to that supplied by the source.

Series Voltage Sources and Parallel Current Source

Table 3.5-1 Parallel and Series Voltage and Current Source



Circuit Analysis

- **Consider the circuit shown in Figure 3.6-1.**

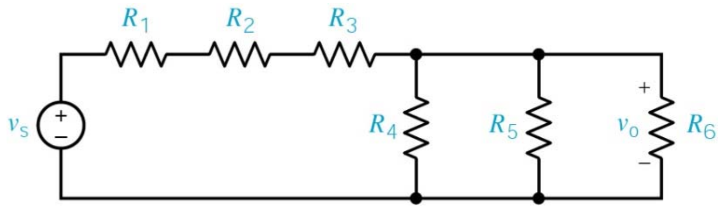


Figure 3.6-1 (p. 77)
Circuit with a set of series resistors and a set of parallel resistors.

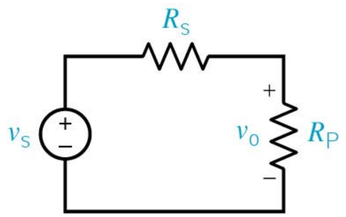


Figure 3.6-2 (p. 77)
Equivalent circuit for the circuit of Figure 3.6-1.

The equivalent series resistance is $R_s = R_1 + R_2 + R_3$

The equivalent parallel resistance is $R_p = \dfrac{1}{G_p}$

where $G_p = G_4 + G_5 + G_6$

Then, using the voltage divider principle, we have $v_o = \dfrac{R_p}{R_s + R_p} v_s$

Example 3.6-1 ***Series and Parallel Resistors***

- Determine the value of the current I for the circuit shown in Figure 3.6-4.

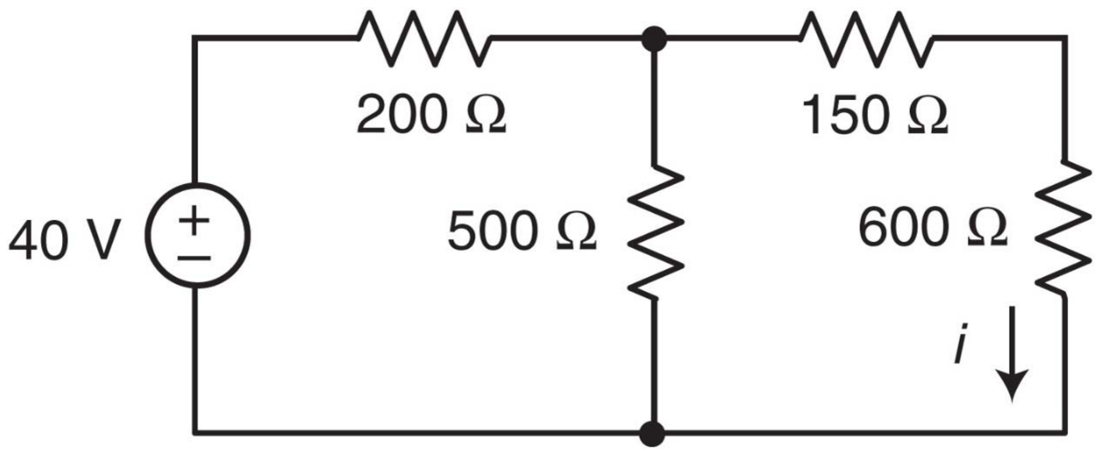


Figure 3.6-4 (p. 78)
The Circuit considered in Example 3.6-1

Solution

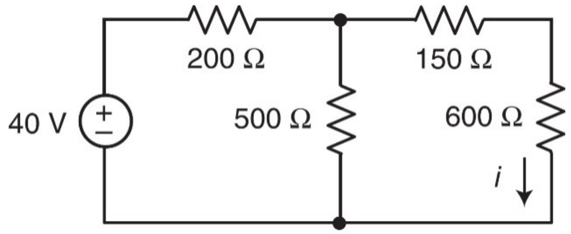


Figure 3.6-4

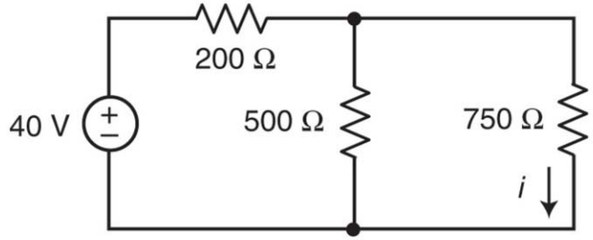


Figure 3.6-5(a)

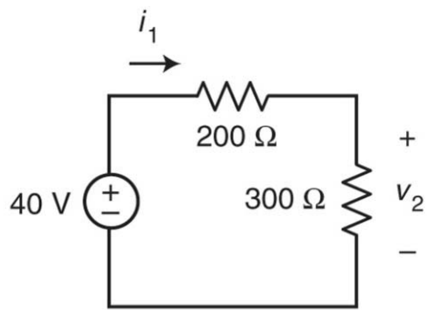


Figure 3.6-5(b)

1. The equivalent resistance of the 150- and 600 Ω resistors in Figure 3.6-4 $R_S = 150 + 600 = 750\Omega$
2. The equivalent resistance of the 500- and 750 Ω resistors in Figure 3.6-5(a) $R_P = \dfrac{500(750)}{500+750} = 300\Omega$
3. The voltage across the 300 Ω resistor in Figure 3.6-5(b) $v_2 = \dfrac{300}{200+300}(40) = 24V$
4. The current in the resistors in Figure 3.6-5(b) $i_1 = \dfrac{40}{200+300} = 0.08A = 80mA$
5. The current i is related to the current i_1 by current division $i = \dfrac{500}{500+750} i_1 = (0.4)(80) = 32mA$
6. As a check, we can also calculate the current i using Ohm's law $i = \dfrac{v_2}{750} = \dfrac{24}{750} = 32mA$

Example 3.6-2 ***Equivalent Resistance***

- The circuit in Figure 3.6-6a contains an ohmmeter. An ohmmeter is an instrument that measures resistance in ohms. The ohmmeter will measure the equivalent resistance of the resistor circuit connected to its terminals. Determine the resistance measured by the ohmmeter in Figure 3.6-6a

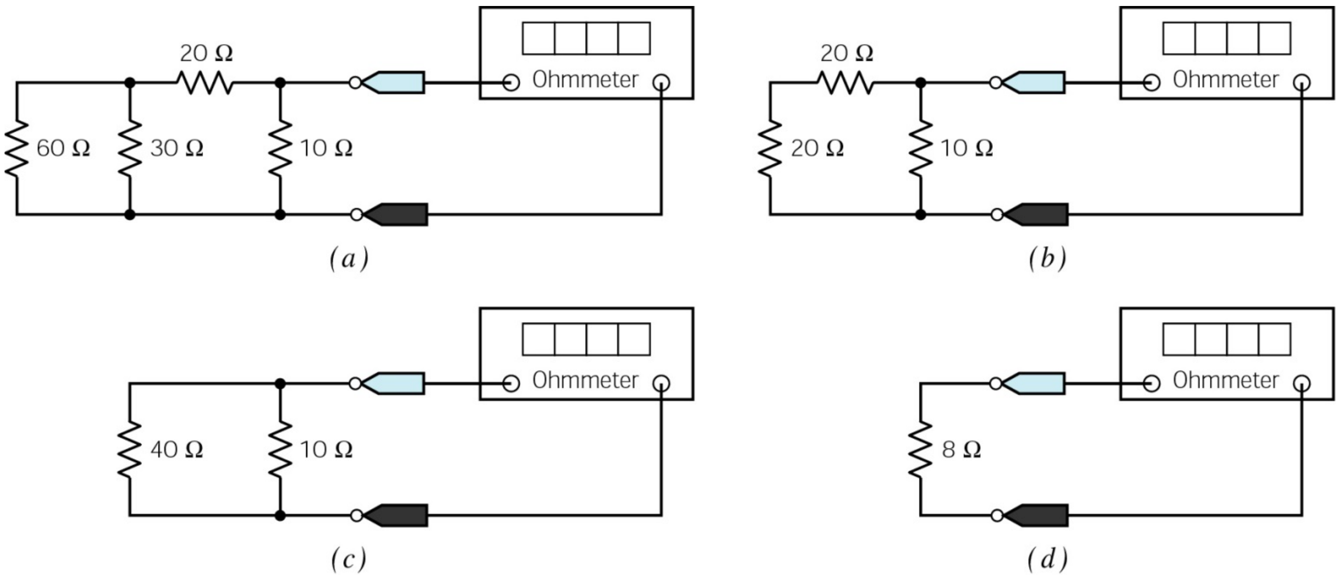
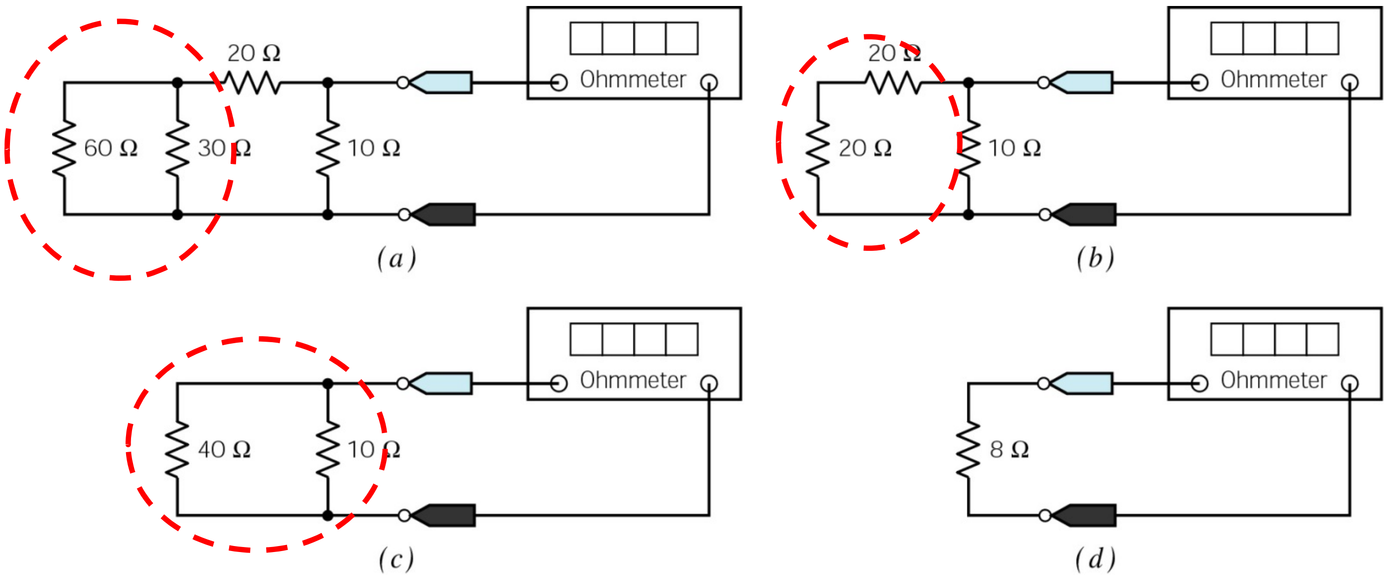


Figure 3.6-6 (p. 80)

Solution

<Equivalent circuit>



a. Equivalent resistance of 30Ω and 60Ω

$$\frac{60 \cdot 30}{60+30} = 20(\Omega)$$

b. Series equivalent resistor of 20Ω and 20Ω

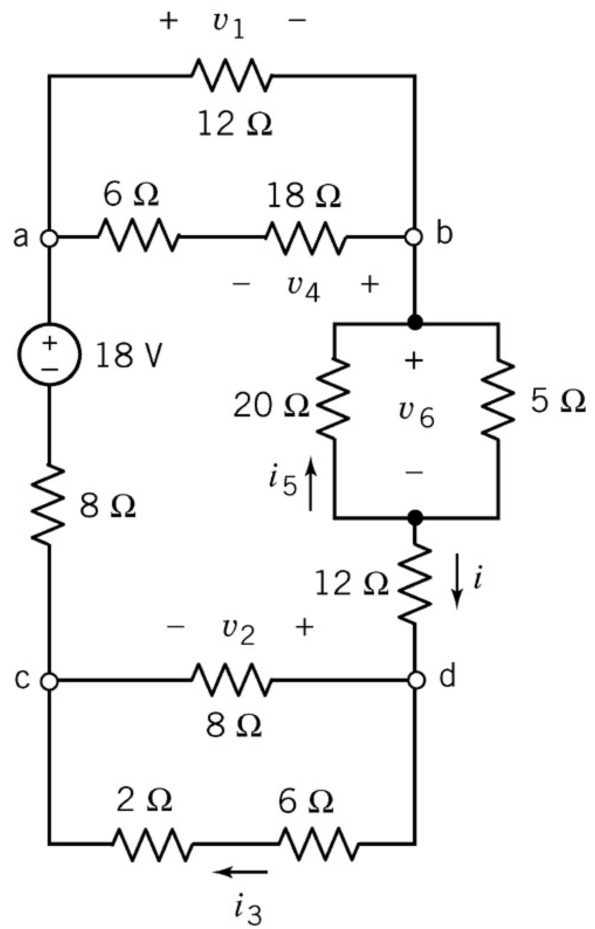
$$20 + 20 = 40(\Omega)$$

c. parallel equivalent resistor of 40Ω and 10Ω

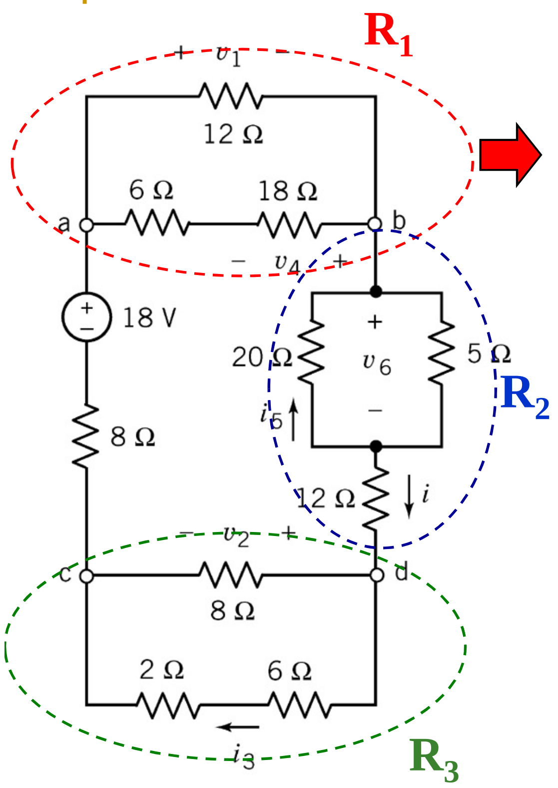
$$\frac{40 \cdot 10}{40+10} = 8(\Omega)$$

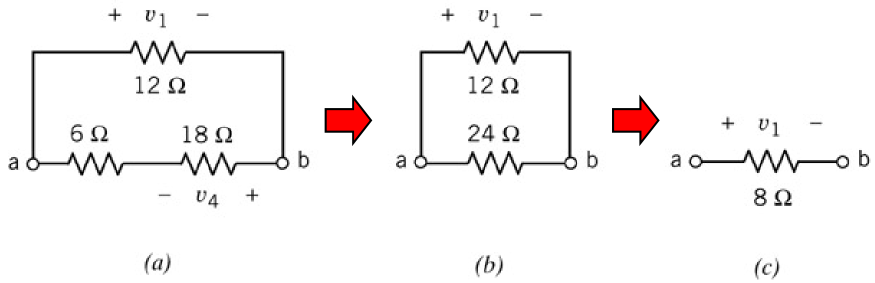
Example 3.6-3 ***Circuit Analysis Using Equivalent Resistances***

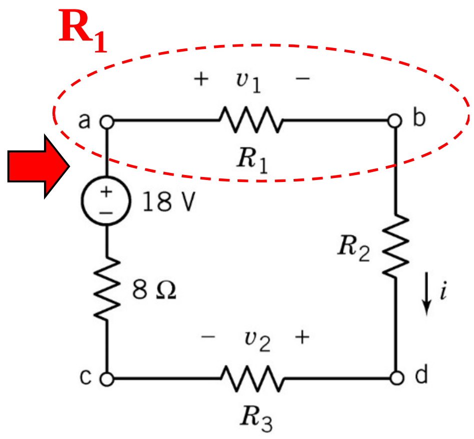
- Determine the values of $\boldsymbol{i_3}$, $\boldsymbol{v_4}$, $\boldsymbol{i_5}$, and $\boldsymbol{v_6}$ in circuit shown in Figure 3.6-7



Solution – step1







:3개의 저항을 하나의 등가저항 모델로 변환

- example of resistor R_1

(a) three resistors at the top of the circuit

(b) series resistor (6Ω ,18Ω) → 24Ω

(c) parallel resistor(12Ω, 24Ω) → 8Ω

Similarly,

$$R_1 = 12 \| (6+18) = 8\Omega$$

$$R_2 = 12 + (20 \| 5) = 16\Omega$$

$$R_3 = 8 \| (2+6) = 4\Omega$$

Solution – step2,3

Step 2 :

- Apply KVL

$$R_1 i + R_2 i + R_3 i + 8i - 18 = 0 \quad \Rightarrow \quad i = \frac{18}{R_1 + R_2 + R_3 + 8} = \frac{18}{8 + 16 + 4 + 8} = 0.5(\mathrm{A})$$

- Ohm's law

$$v_1 = R_1 i = 8(0.5) = 4(\mathrm{V}) \quad \textit{and} \quad v_2 = R_3 i = 4(0.5) = 2(\mathrm{V})$$

Step 3 :

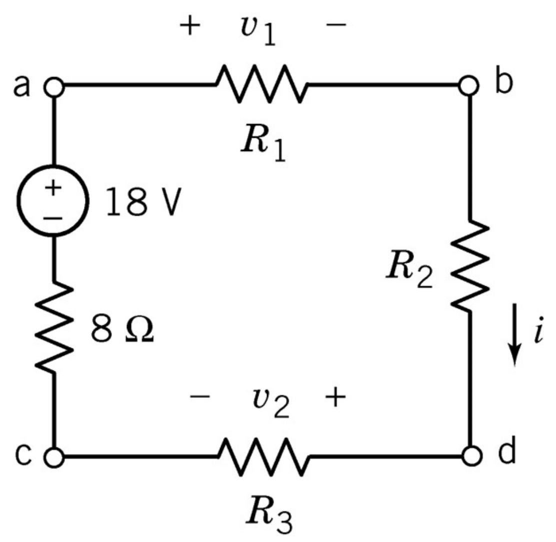
- using voltage division, current division, Ohm's law

$$i_3 = \frac{8}{8 + (2+6)} i = \frac{1}{2}(0.5) = 0.25(\mathrm{A})$$

$$v_4 = -\frac{18}{6+18} v_1 = -\frac{3}{4}(4) = -3(\mathrm{V})$$

$$i_5 = -\frac{5}{20+5} i = -\left(\frac{1}{5}\right)(0.5) = -0.1(\mathrm{A})$$

$$v_6 = (20 \| 5) i = 4(0.5) = 2(\mathrm{V})$$



Analyzing Resistive Circuit Using MATLAB

- **The computer program MATLAB is a tool for making mathematical calculations.**
- **Example**
  - **Consider the circuit shown in Figure 3.7-1**
    Plot the current I versus the input voltage Vs

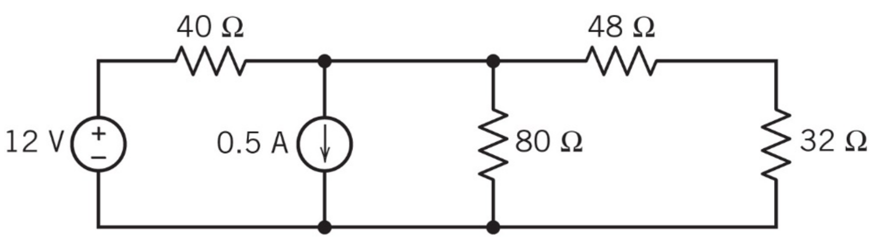


Figure 3.7-1 (p. 82)
The circuit considered in Example 3.7-1.

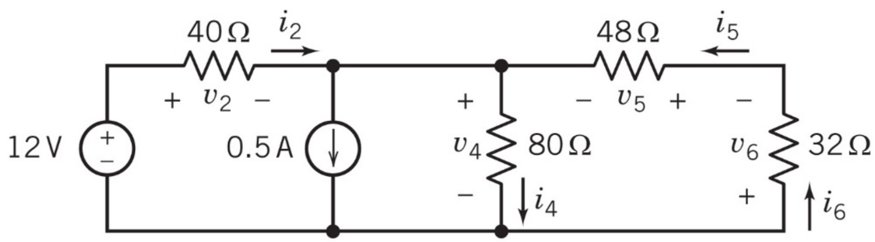
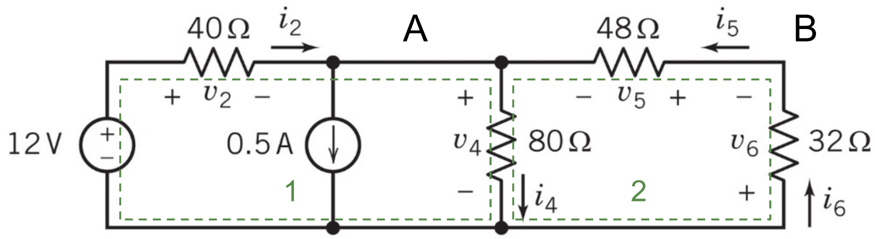


Figure 3.7-2 (p. 83)
After labeling the resistor voltages and currents.

Analyzing Resistive Circuit Using MATLAB

- **Solution**



KCL at the Node A: $i_2 + i_5 = 0.5 + i_4$

KCL at the Node B: $i_5 = i_6$

KVL at the Loop 1: $12 = v_2 + v_4$

KVL at the Loop 2: $v_4 + v_5 + v_6 = 0$

Ohm's Law at resistors: $v_2 = 40\, i_2,\ v_4 = 80\, i_4,\ v_5 = 48\, i_5,\ v_6 = 32\, i_6$

$$12 = 40\, i_2 + 80\, i_4$$

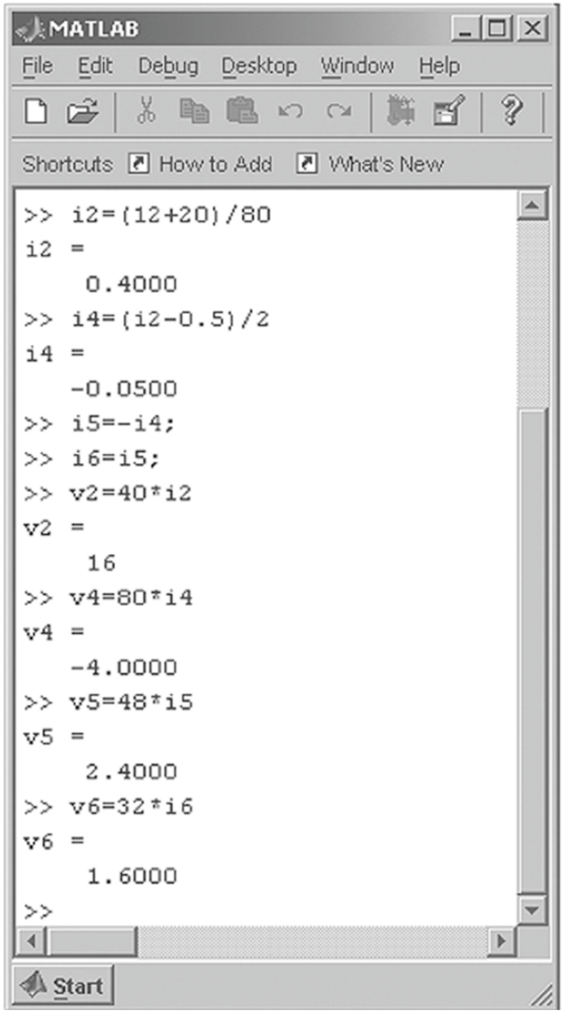
$$80\, i_4 + 48\, i_5 + 32\, i_6 = 0$$

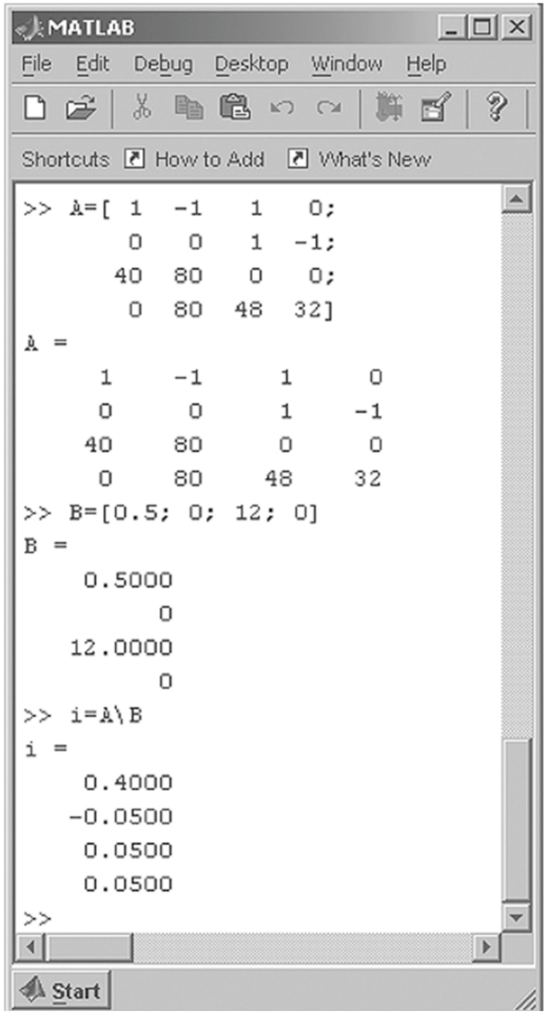
$$80\, i_4 + 48\, i_5 + 32\, i_5 = 0 \quad \Rightarrow \quad 80\, i_4 + 80\, i_5 = 0 \quad \Rightarrow \quad i_4 = -i_5$$

$$i_2 - i_4 = 0.5 + i_4 \quad \Rightarrow \quad i_2 = 0.5 + 2\, i_4$$

$$12 = 40\, i_2 + 80\left(\frac{i_2 - 0.5}{2}\right) = 80\, i_2 - 20 \quad \Rightarrow \quad i_2 = \frac{12 + 20}{80} = 0.4\ \text{A}$$

MATLAB input file

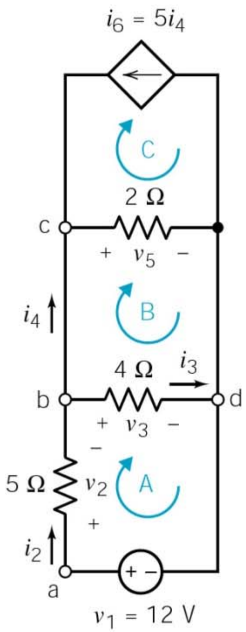




Example 3.8-1 *How can we check voltage and current values?*

- The circuit shown in Figure 3.8-1a was analyzed by writing and solving a set of simultaneous equation:

$$12 = v_2 + 4i_3, \quad i_4 = \frac{v_2}{5} + i_3, \quad v_5 = 4i_3, \quad \frac{v_5}{2} = i_4 + 5i_4$$



(a)

```
v2 := 0   i3 := 0   i4 := 0   v5 := 0
Given
    12 ≈ v2 + 4·i3            Apply KVL to loop A.
    i4 ≈ v2/5 + i3            Apply KCL at node b.
    v5 ≈ 4·i3                 Apply KVL to loop B.
    v5/2 ≈ i4 + 5·i4          Apply KCL at node c.

  Find (v2,i3,i4,v5) = [-60, 18, 6, 72]
```

$$\text{Find}(v2, i3, i4, v5) = \begin{bmatrix} -60 \\ 18 \\ 6 \\ 72 \end{bmatrix}$$

(b)

- The computer Mathcad was used to solve the equations as shown in Figure 3.8-1b. It was determined that

$$v_2 = -60\text{V}, \quad i_3 = 18\text{A}, \quad i_4 = 6\text{A}, \quad v_5 = 72\text{V}$$

How can we check that these currents and voltages are correct?

Solution

1. Ohm's law $$i_2 = \frac{v_2}{5} = \frac{-60}{5} = -12(\text{A})$$

2. Applying KCL at node b $$i_2 = i_3 + i_4 = 18 + 6 = 24(\text{A})$$

3. ***i_2** cannot be both **-12** and **24A**, so the values calculated for **v_2, i_3, i_4, v_5** cannot be correct.*

- We find a sign error in KCL equation corresponding to node b.

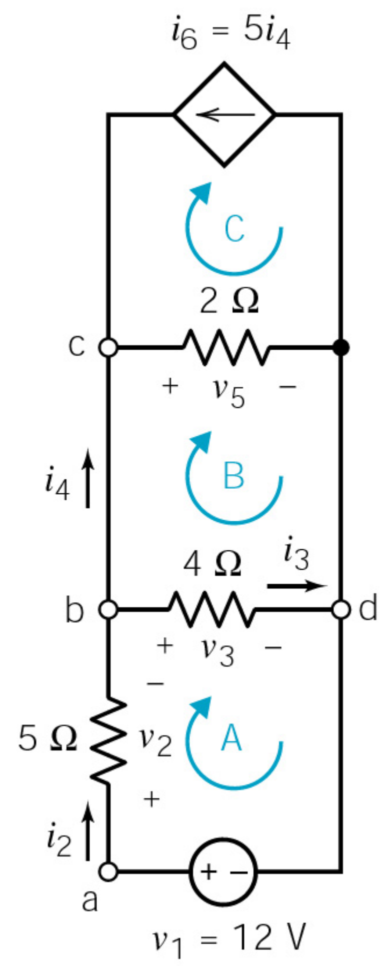
$$i_4 = \frac{v_2}{5} + i_3 \quad \Rightarrow \quad i_4 = \frac{v_2}{5} - i_3$$

after making this correction, ***v_2, i_3, i_4, v_5*** are calculated.

$$v_2 = 7.5(\text{V}), \quad i_3 = 1.125(\text{A}), \quad i_4 = 0.375(\text{A}), \quad v_5 = 4.5(\text{V})$$

$$i_2 = \frac{v_2}{5} = \frac{7.5}{5} = 1.5$$

$$i_2 = i_3 + i_4 = 1.125 + 0.375 = 1.5(\text{A})$$



Solution

- Additional check

1. Ohm's law

$$v_3 = 4i_3 = 4(1.125) = 4.5$$

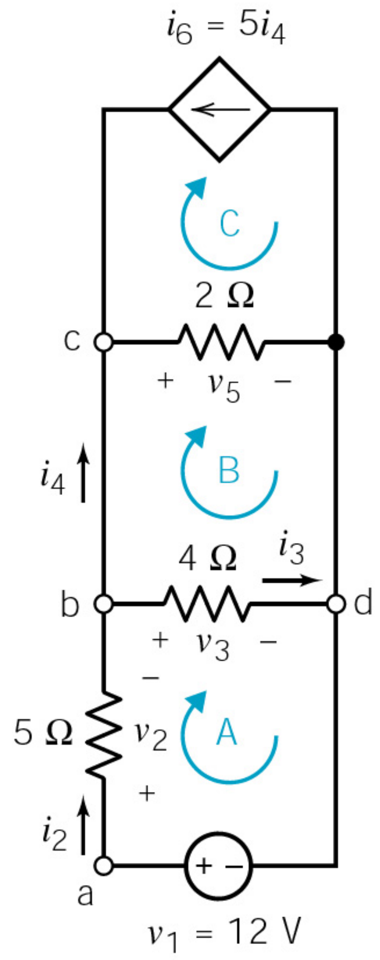
2. Applying KVL to the loop A

$$v_3 = 12 - v_2 = 12 - 7.5 = 4.5$$

3. Applying KVL to the loop B

$$v_3 = v_5 = 4.5$$

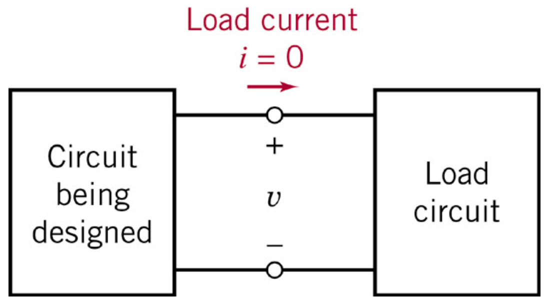
$$v_2 = 7.5(\mathrm{V}), \quad i_3 = 1.125(\mathrm{A}), \quad i_4 = 0.375(\mathrm{A}), \quad v_5 = 4.5(\mathrm{V})$$



Design Example – Adjustable voltage source

- **Describe the situation and the assumptions**

  The voltage ***v*** is the adjustable voltage. The circuit that uses the output of the circuit being designed is frequently called the "load". (load current ***i***=0)



- **State the goal**

  A circuit providing the adjustable voltage

$$-5\text{V} \leq v \leq +5\text{V}$$

  must be designed using the available components.

- **Generate a plan**

  - voltage range -5V to +5V
  - voltage source currents less than 100mA
  - possible to reduce the power absorbed by R_1, R_2, R_p

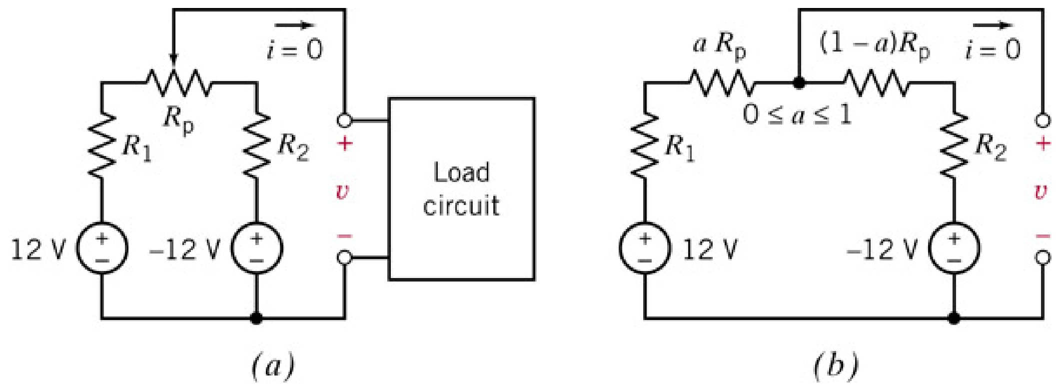
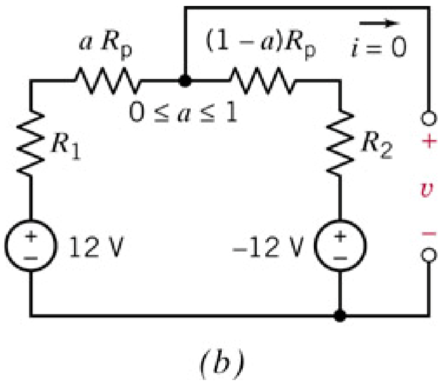


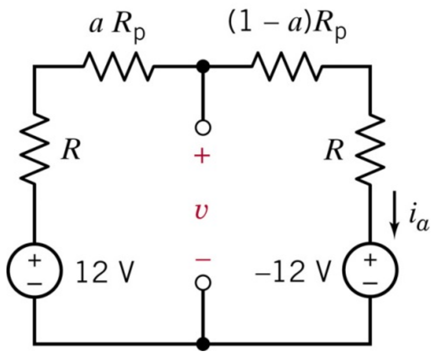
Figure 3.9-2
(a) A proposed circuit for producing the variable voltage, v
(b) the equivalent circuit after the potentiometer is modeled

Design Example – Adjustable voltage source

- **Act on the plan**



R_1=R_2=R



- Applying KVL to the outside loop yields

$$-12 + Ri_a + aR_p i_a + (1-a)R_p i_a + Ri_a - 12 = 0 \quad \Rightarrow \quad i_a = \frac{24}{2R + R_p}$$

- Applying KVL to the left loop

$$v = 12 - (R + aR_p)i_a$$

$$v = 12 - \frac{24(R + aR_p)}{2R + R_p}$$

$$5 = 12 - \frac{24R}{2R + R_p} \qquad \leftarrow a=0$$

$$R = 0.7R_p$$

- Suppose the potentiometer resistance is selected to be R_p=20kΩ

$$R = 14k\Omega$$

Design Example – Adjustable voltage source

- **Verify the proposed solution**
  - When a=1

$$v = 12 - \left(\frac{14k + 20k}{28k + 20k}\right)24 = -5$$ 는 다음을 만족→ $-5\text{V} \le v \le +5\text{V}$

  - Power absorbed by the three resistance

$$p = i_a^2(2R + R_p) = \frac{24^2}{2R + R_p}$$

  - 파워를 줄이기 위해 R_p를 가능한 큰 값으로 잡으면(R_p=50kΩ)

$$R = 0.7 \times R_p = 35k\Omega$$

$$-5\text{V} = 12 - \left(\frac{35k + 50k}{70k + 50k}\right)24 \le v \le 12 - \left(\frac{35k}{70k + 50k}\right)24 = 5\text{V}$$ 는 다음을 만족→ $-5\text{V} \le v \le +5\text{V}$

  - Power absorbed by the 3 resistance is

$$p = \frac{24^2}{50k + 70k} = 5mW$$

  - Power supply current is

$$i_a = \frac{24}{50k + 70k} = 0.2mA$$

which is below the 100mA that the voltage sources are able to supply. The design is complete!

Maxwell Equations

	미분형	적분형
Gauss 법칙	$\vec{\nabla}\cdot(\varepsilon\vec{E}) = \rho$	$\oint_S \varepsilon\vec{E}\cdot d\vec{S} = \int_v \rho dV$
자속 보존의 법칙	$\vec{\nabla}\cdot\vec{B} = 0$	$\oint_S \vec{B}\cdot d\vec{S} = 0$
Ampere의 둘레법칙	$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial(\varepsilon\vec{E})}{\partial t}$	$\oint_C \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{S} + \frac{\partial\int_s \varepsilon\vec{E}\cdot d\vec{S}}{\partial t}$
Faraday의 전자유도법칙	$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$	$\oint_C \vec{E}\cdot d\vec{l} = -\frac{\partial\int_s \vec{B}\cdot d\vec{S}}{\partial t}$
전하 보존의 법칙	$\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$	$\oint_S \vec{J}\cdot d\vec{S} + \frac{\partial\int_V \rho dV}{\partial t} = 0$

** 이 페이지는 김용권교수 강의자료2005에서 허락 받고 발췌한 것임.*