Chapter 4
Methods of Analysis of Resistive Circuits

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Node Voltage Analysis of Circuits with Current Sources

- The ***nodes*** of a circuit are the places where the elements are connected together.
- ***n*** nodes will require ***n-1*** KCL equations.
- The voltage at any node of the circuit, relative to the reference node, is called a ***node voltage***.

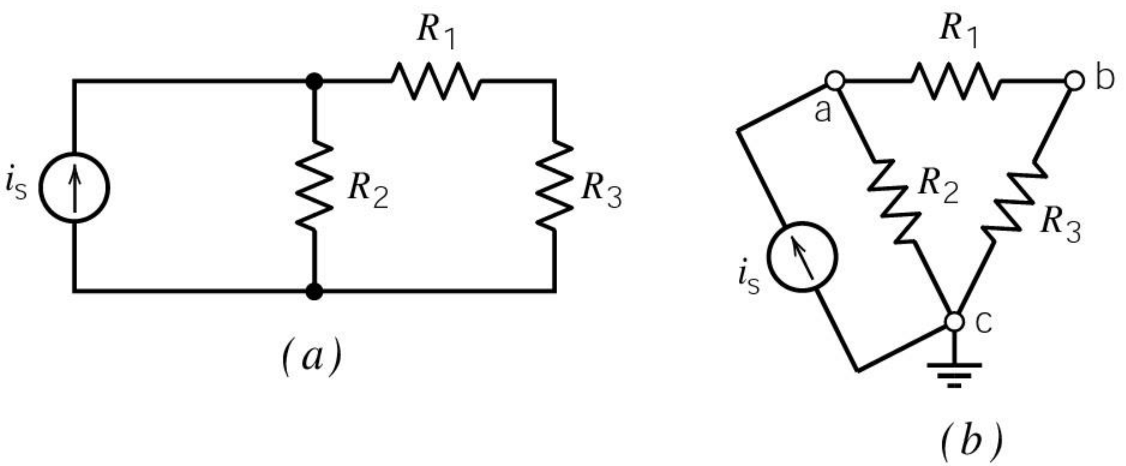


Figure 4.2-1 (b)

- 3 *node* (a, b, c)
- 2 *node voltage* (v_a, v_b)
- v_c: reference node

Figure 4.2-1 (p. 112)
(*a*) A circuit with three nodes. (*b*) The circuit after the nodes have been labeled and a reference node has been selected and marked.

Node Equations

- **To write a set of node equations, we do two things:**

1. Express element currents as functions of the node voltage.
2. Apply Kirchhoff's current law (KCL) at each of the nodes of the circuit, except for the reference node.

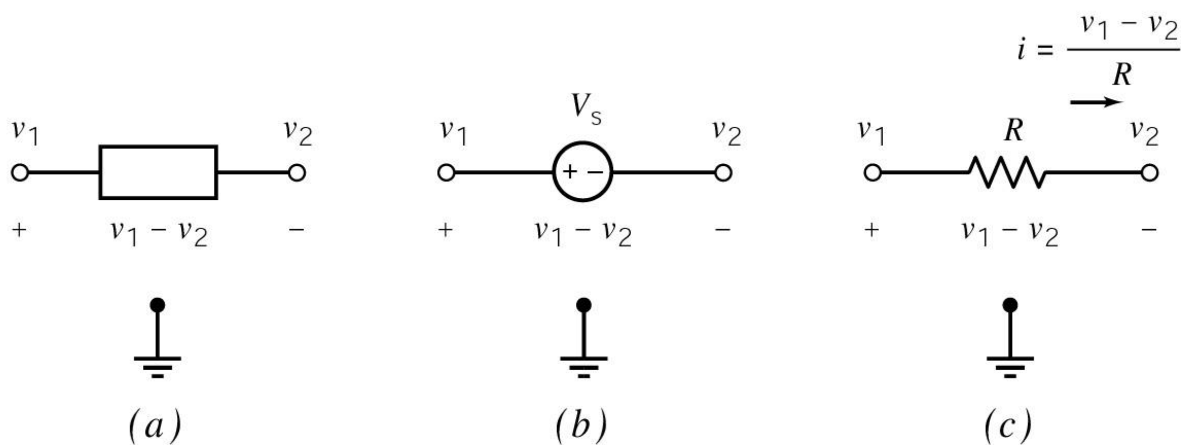


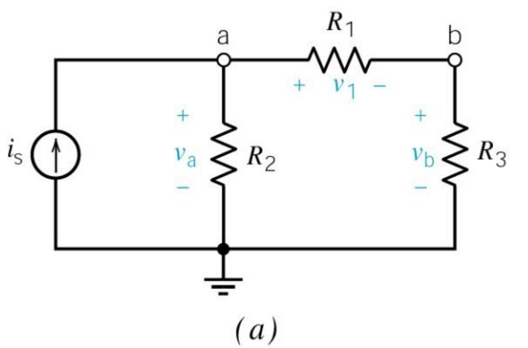
Figure 4.2-3 (p. 114)
Node voltages, v_1 and v_2, and element voltage, v_1-v_2, of a (a) generic circuit element, (b) voltage source, and (c) resistor.

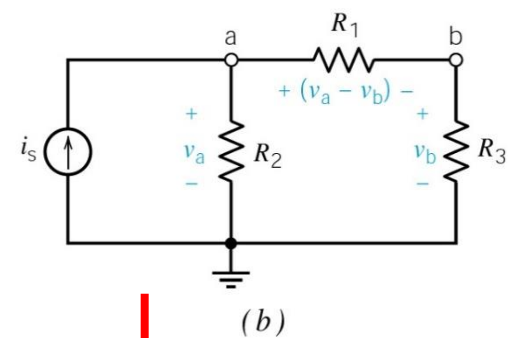
Rules

- *Rule #1:Express currents in terms of node voltages: Count the number of unknown node voltages and the required number of KCL equations to solve.*

Example of node equations

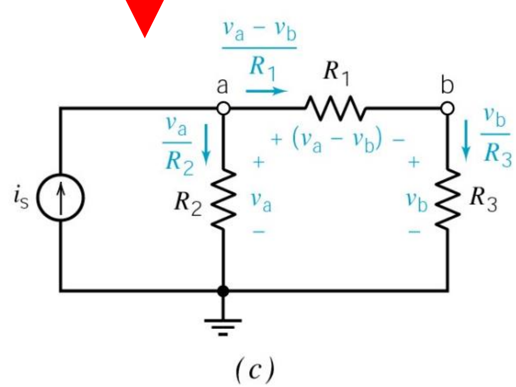
- Example of node equation
  1. Express element currents as functions of the node voltage





(*b*) The resistor voltages expressed as functions of the node voltages.

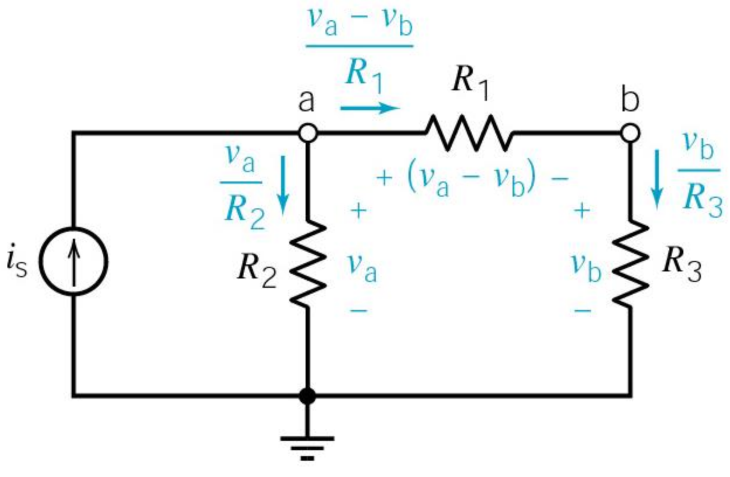
Figure 4.2-4 (p. 114)
(*a*) A circuit with three resistors.



(*c*) The resistor currents expressed as functions of the node voltages.

Example of node equations

- Example of node equation
  2. Apply Kirchhoff's current law (KCL) at each of the nodes of the circuit, except for the reference node.



(c)

<Node a>

$$i_s = \frac{v_a}{R_2} + \frac{v_a - v_b}{R_1}$$

<Node b>

$$\frac{v_a - v_b}{R_1} = \frac{v_b}{R_3}$$

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Example 4.2-1 *Node equations*

- Determine the value of the resistance R in the circuit shown in Figure 4.2-5a

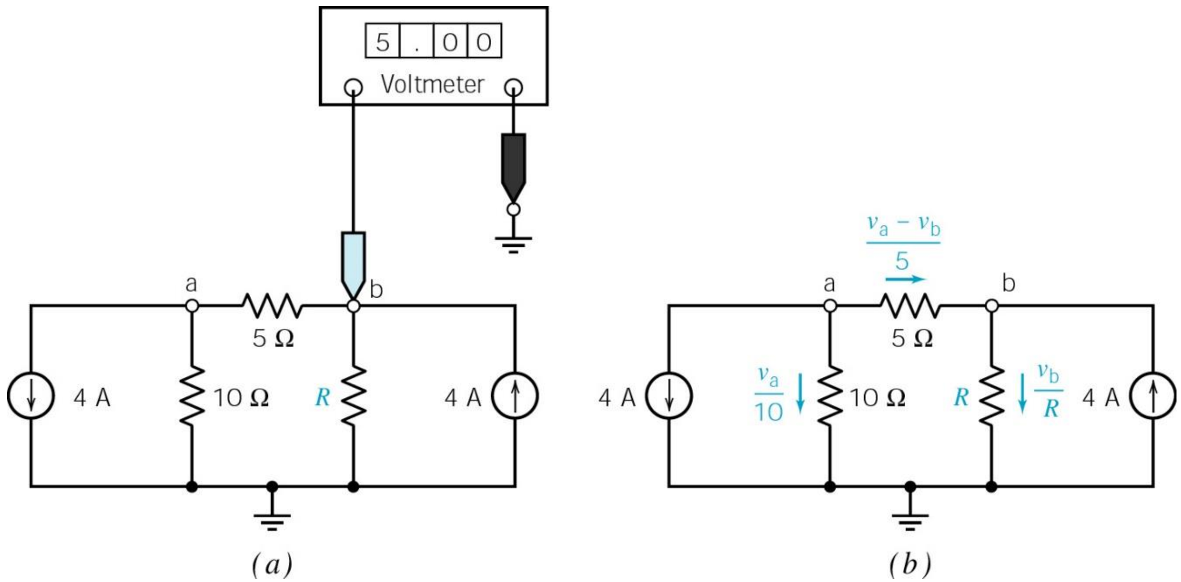
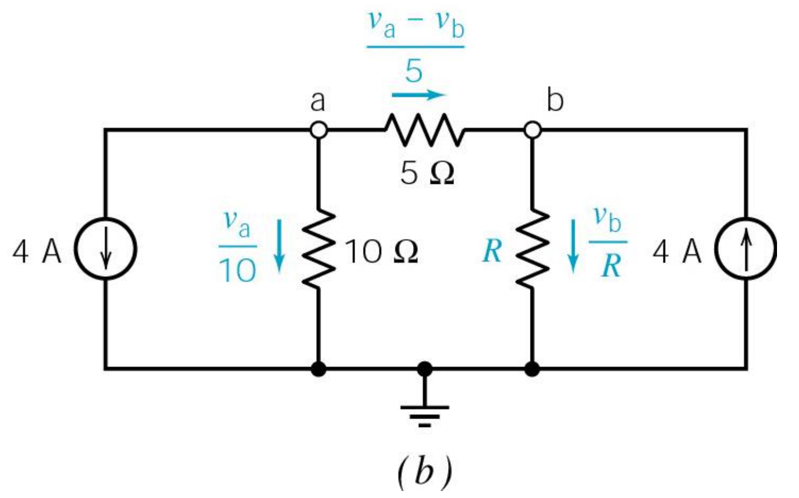


Figure 4.2-5 (p. 116)
(*a*) The circuit for Example 4.2-1. (*b*) The circuit after the resistor currents are expressed as functions of the node voltages.

Solution



1. Apply KCL at node a to obtain

$$4+\frac{v_a}{10}+\frac{v_a-v_b}{5}=0$$

2. Using v_b =5V, gives

$$4+\frac{v_a}{10}+\frac{v_a-5}{5}=0$$

3. Solving for v_a, we get

$$v_a=10(\mathrm{V})$$

4. Next, apply KCL at node b to obtain $-\left(\frac{v_a-v_b}{5}\right)+\frac{v_b}{R}-4=0$

5. Using v_a =-10V and v_b =5V gives $-\left(\frac{-10-5}{5}\right)+\frac{5}{R}-4=0$

6. Finally, solving for R gives $R=5(\Omega)$

Example 4.2-2 *Node equations*

- Obtain the node equations for the circuit in Figure 4.2-6.

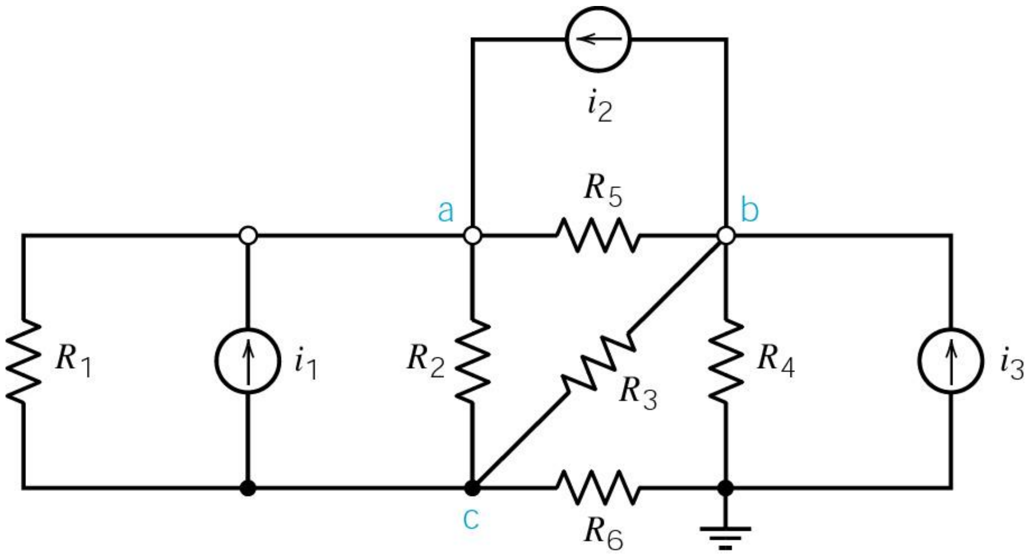


Figure 4.2-6 (p. 116)
The circuit for Example 4.2-2.

Solution

1. Apply KCL at node a to obtain

$$-\left(\frac{v_a - v_c}{R_1}\right) + i_1 - \left(\frac{v_a - v_c}{R_2}\right) + i_2 - \left(\frac{v_a - v_b}{R_5}\right) = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)v_a - \left(\frac{1}{R_5}\right)v_b - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_c = i_1 + i_2$$

2. Apply KCL at node b to obtain

$$-i_2 + \left(\frac{v_a - v_b}{R_5}\right) - \left(\frac{v_b - v_c}{R_3}\right) - \left(\frac{v_b}{R_4}\right) + i_3 = 0$$

$$-\left(\frac{1}{R_5}\right)v_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)v_b - \left(\frac{1}{R_3}\right)v_c = i_3 - i_2$$

3. Node equation at node c

$$-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_a - \left(\frac{1}{R_3}\right)v_b + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_6}\right)v_c = -i_1$$

Node Voltage Analysis Method with a Voltage Sources

- **Case 1**
  - The voltage source connects node q and the reference node (ground).
  - **Method:**
    - Set v_q equal to the source voltage accounting for the polarities and proceed to write the KCL at the remaining nodes.

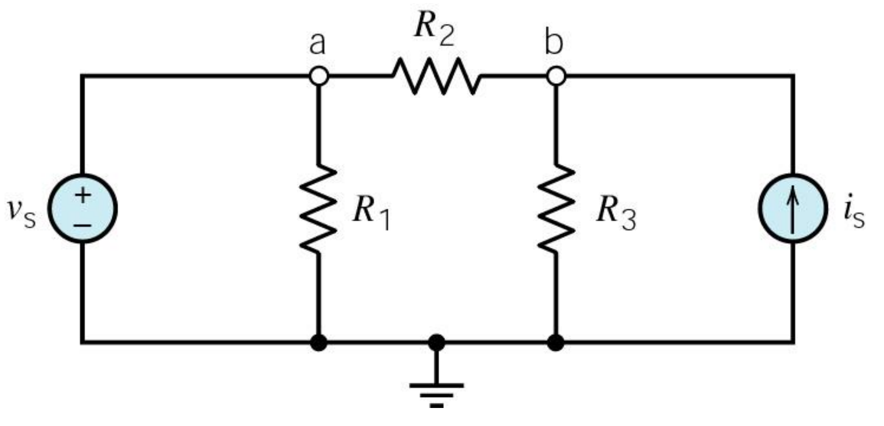


Figure 4.3-1 (p. 119)
Circuit with an independent voltage source and an independent current source.

Rules

- *Rule #1: Express currents in terms of node voltages: Count the number of unknown node voltages and the required number of KCL equations to solve.*
- *Rule #2: If a node voltage is known, use it.: Then the number of unknown is one less.*

Node Voltage Analysis Method with a Voltage Sources

- **Case 1**
  - The voltage source connects node q and the reference node (ground).

1. **at node a**

$$v_a = v_s$$

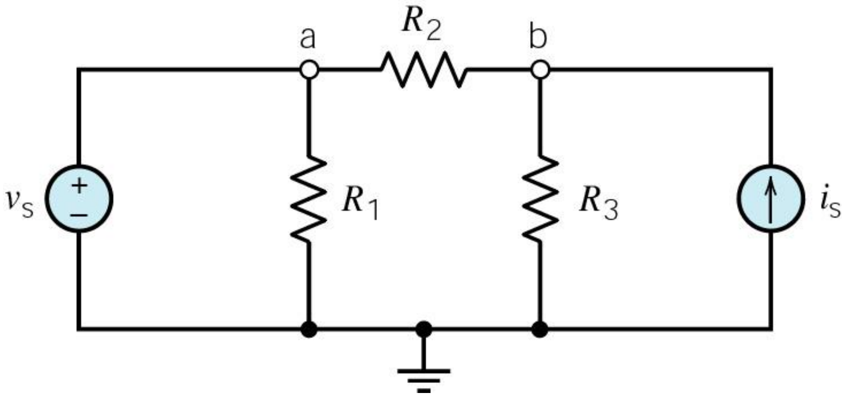
2. **at node b**

$$i_s = \frac{v_b}{R_3} + \frac{v_b - v_a}{R_2}$$

$$\xRightarrow{v_a = v_s} \quad i_s = \frac{v_b}{R_3} + \frac{v_b - v_s}{R_2}$$

3. **Solving for the unknown node voltage v_b**

$$v_b = \frac{R_2 R_3 i_s + R_3 v_s}{R_2 + R_3}$$



Circuit with an independent voltage source and an independent current source.

Node Voltage Analysis Method with a Voltage Sources

- **Case 2**
  - The voltage source lies between two nodes, a and b
  - **Method:**
    - Create **a(one)** supernode that incorporates a and b and equate the sum of all the currents into the supernode to zero.

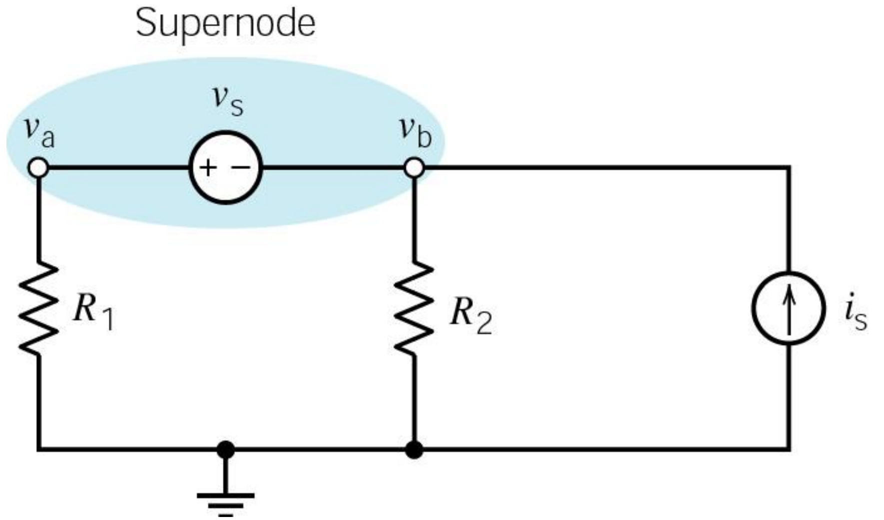


Figure 4.3-2 (p. 119)
Circuit with a supernode that incorporates v_a and v_b.

Node Voltage Analysis Method with a Voltage Sources

- **Case 2**
  - of a Supernode

1. **voltage source**

$$v_a - v_b = v_s \quad \Leftrightarrow \quad v_a = v_s + v_b$$

2. **At supernode**

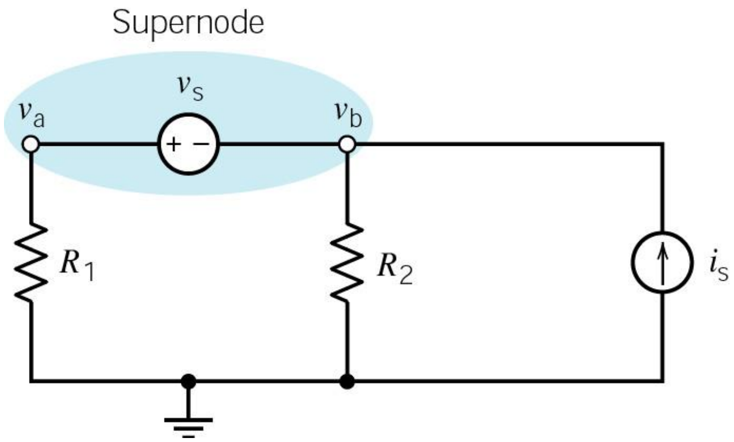
$$\frac{v_a}{R_1} + \frac{v_b}{R_2} = i_s$$

$$\xrightarrow{v_a = v_s + v_b} \quad \frac{v_s + v_b}{R_1} + \frac{v_b}{R_2} = i_s$$

3. **Solving for the unknown node voltage v_b**

$$v_b = \frac{R_1 R_2 i_s - R_2 v_s}{R_1 + R_2}$$

A supernode consists of two nodes connected by an independent or a dependent voltage source



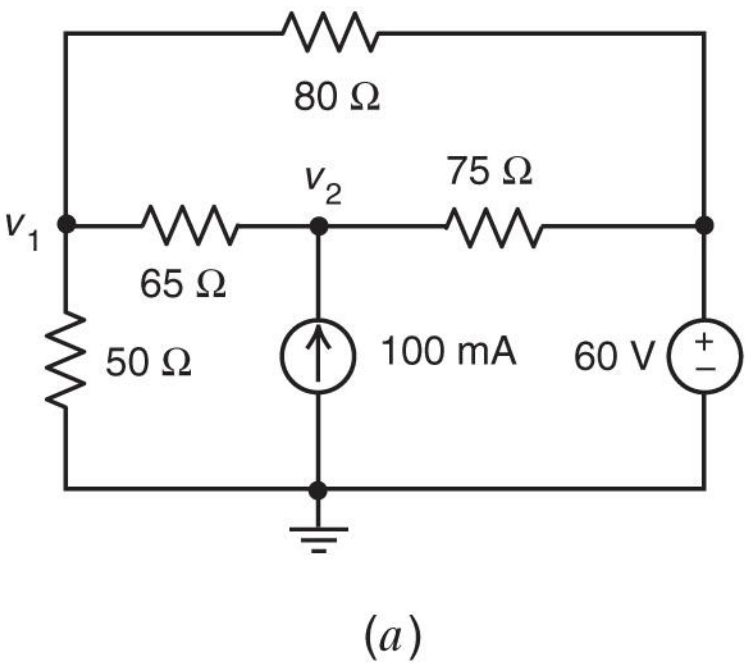
Circuit with a supernode that incorporates v_a and v_b.

Rules

- *Rule #1: Express currents in terms of node voltages: Count the number of unknown node voltages and the required number of KCL equations to solve.*
- *Rule #2: If a node voltage is known, use it.: Then the number of unknown is one less.*
- *Rule #3: Supernode makes the KCL equation simpler by removing the need to introduce another variable, a current through the voltage source whether it is an independent or a dependent source.*

Example 4.3-1 *Node Equations*

- Determine the node voltages, v1, v2, in the circuit shown in Figure 4.3-3a.



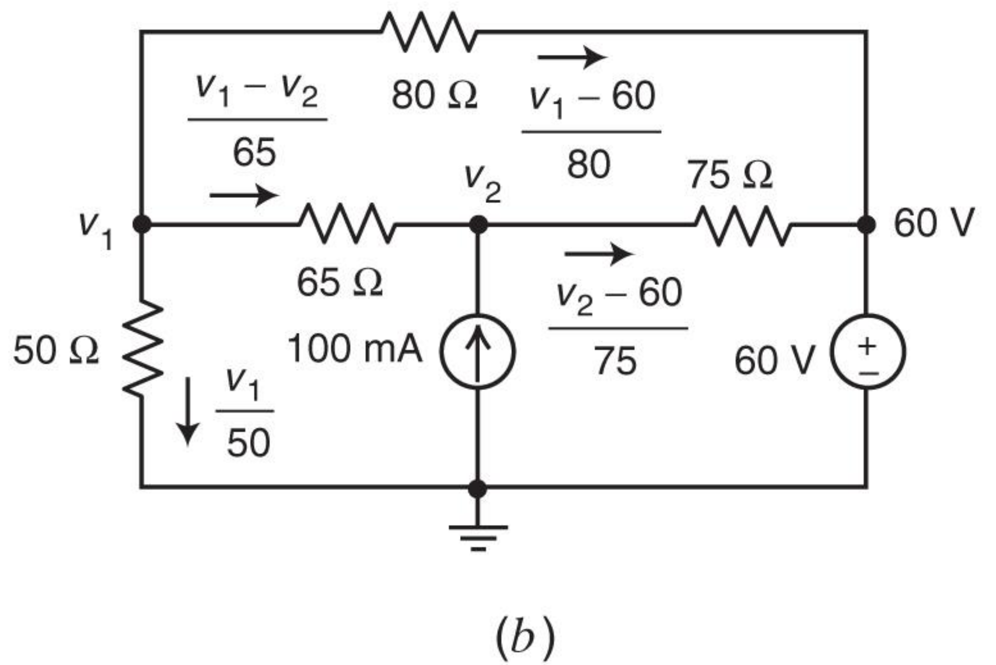


Figure 4.3-3 (p. 112)
The circuit considered in Example 4.3-1.

Solution

1. Apply KCL at node 1

$$\frac{v_1 - v_2}{65} + \frac{v_1}{50} + \frac{v_1 - 60}{80} = 0 \Rightarrow (\frac{1}{50} + \frac{1}{65} + \frac{1}{80})v_1 - (\frac{1}{65})v_2 = \frac{60}{80}$$

2. Apply KCL at node 2

$$\frac{v_2 - v_1}{65} + \frac{v_2 - 60}{75} = 0.1 \Rightarrow -(\frac{1}{65})v_1 + (\frac{1}{65} + \frac{1}{75})v_2 = 0.1$$

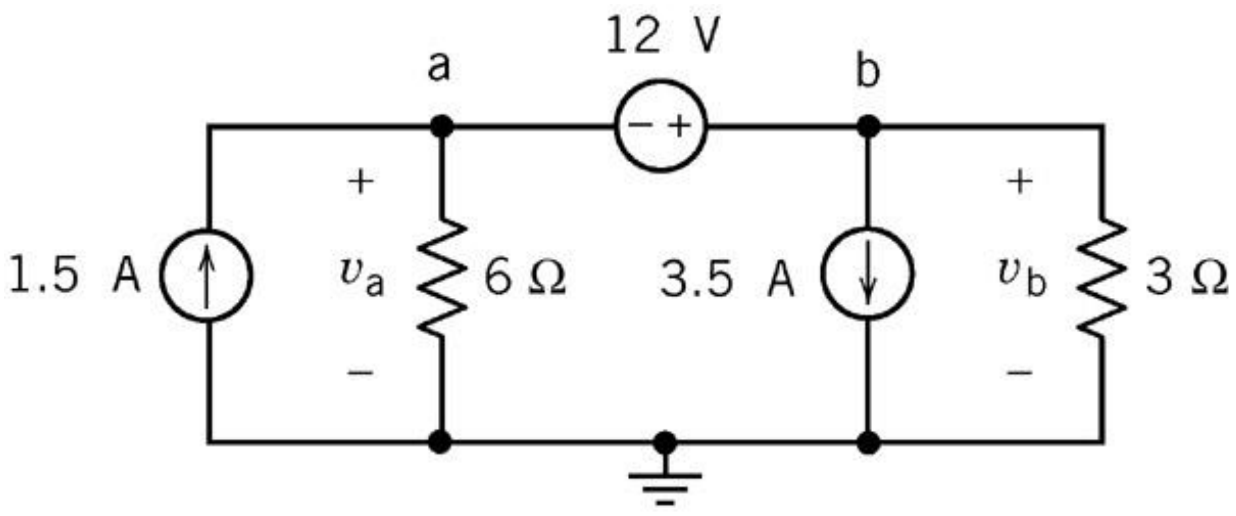
3. Organize these equations in matrix to write

$$\begin{bmatrix} \frac{1}{50} + \frac{1}{65} + \frac{1}{80} & -\frac{1}{65} \\ -\frac{1}{65} & \frac{1}{65} + \frac{1}{75} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{60}{80} \\ 0.1 \end{bmatrix}$$

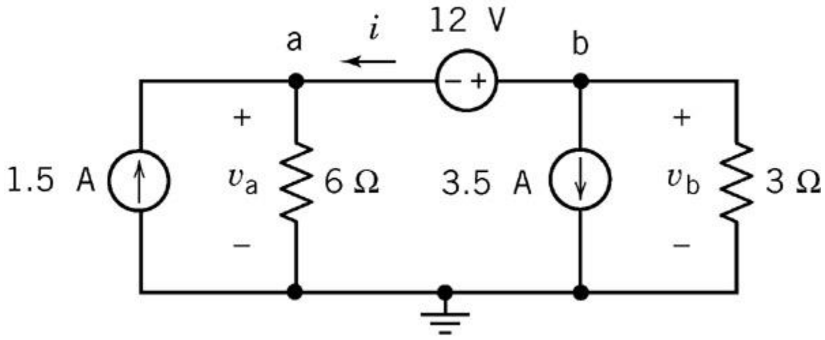
$$v_1 = 30.081\,V, v_1 = 47.990\,V$$

Example 4.3-2 *Supernode*

- Determine the values of the node voltages, v_a and v_b, for the circuit shown in Figure 4.3-4



Solution



$$v_b - v_a = 12 \quad \Rightarrow \quad v_b = v_a + 12$$

Method 1:

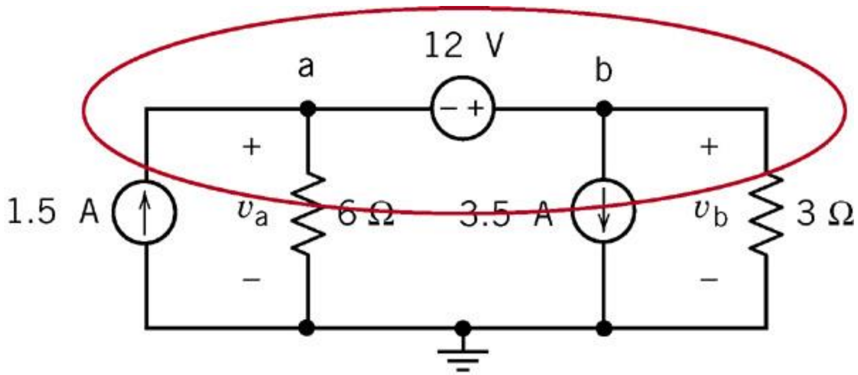
the KCL equation at node a $\quad 1.5 + i = \dfrac{v_a}{6}$

the KCL equation at node b $\quad i + 3.5 + \dfrac{v_b}{3} = 0$

Combining two equations

$$1.5 - \left(3.5 + \frac{v_b}{3}\right) = \frac{v_a}{6} \quad \Rightarrow \quad -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$$

Solution



Method 2:

Apply KCL to the supernode $1.5 = \frac{v_a}{6} + 3.5 + \frac{v_b}{3} \quad \Rightarrow \quad -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$

Applying KCL to the supernode is a shortcut for doing three things:
1. Labeling the voltage source current as i
2. Applying KCL at both nodes of the voltage source
3. Eliminating i from the KCL equations

In summary, $v_b - v_a = 12$

$$\frac{v_a}{6} + \frac{v_b}{3} = -2.0$$

Solving the equation

$$v_a = -12\text{V} \quad and \quad v_b = 0\text{V}$$

Example 4.3-3 *Node Equations for a Circuit Containing Voltage Sources*

- Determine the node voltages for the circuit shown in Figure 4.3-7.

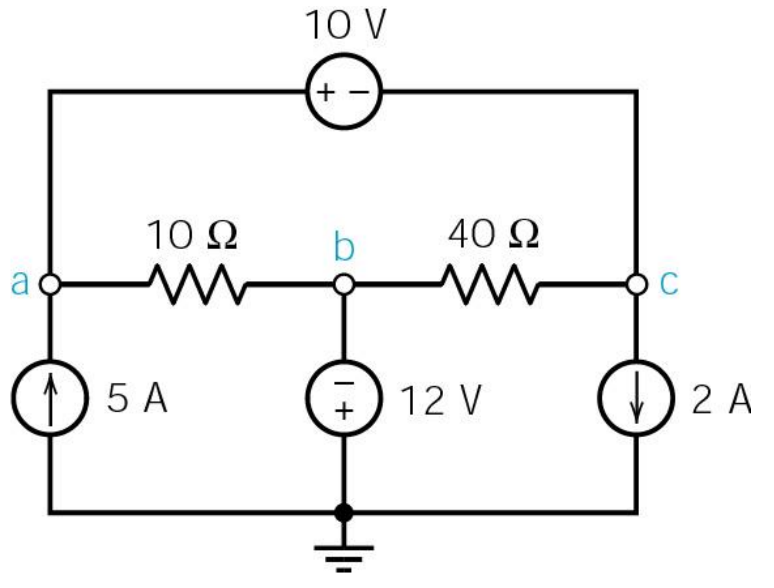


Figure 4.3-7 (p. 114)
The circuit for Example 4.3-3.

Solution

1. First notice that

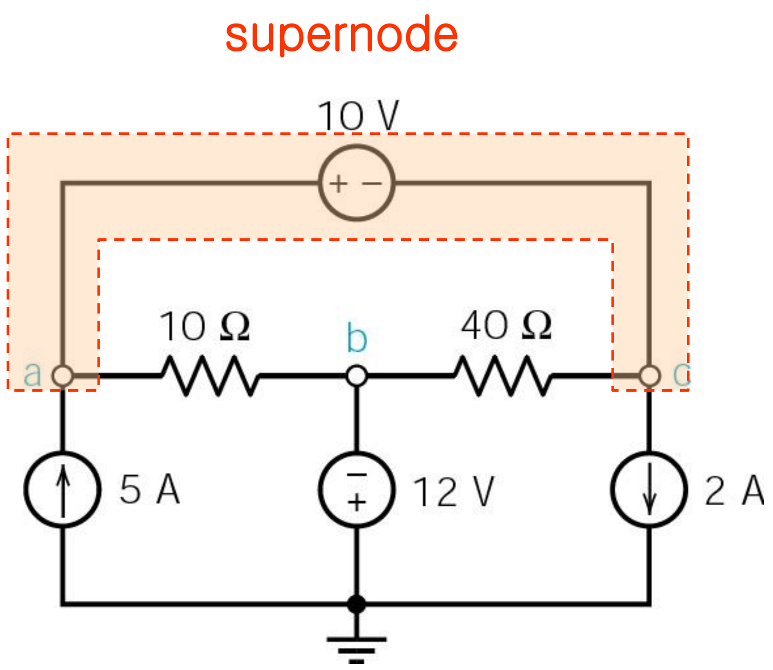
$$v_b = -12(\text{V})$$
$$v_a = v_c + 10$$

2. KCL equation for the supernode

$$\frac{v_a - v_b}{10} + 2 + \frac{v_c - v_b}{40} = 5$$
$$4v_a + v_c - 5v_b = 120$$

3. Using $\boldsymbol{v_a}$=$\boldsymbol{v_c}$+10 and $\boldsymbol{v_b}$=-12

$$4(v_c - 10) + v_c - 5(-12) = 120$$
$$v_c = 4(\text{V})$$



Node Voltage Analysis with Dependent Source

- **When a circuit contains a dependent source, the controlling current or voltage of that dependent source must be expressed as a function of the node voltage**

Rules

- *Rule #1: Express currents in terms of node voltages: Count the number of unknown node voltages and the required number of KCL equations to solve.*
- *Rule #2: If a node voltage is known, use it.: Then the number of unknown is one less.*
- *Rule #3: Supernode makes the KCL equation simpler by removing the need to introduce another variable, a current through the voltage source whether it is an independent or a dependent source.*
- *Rule #4: If there is a controlled source : Express its controlling source with node voltages. You probably have to do this first.*

Example 4.4-1 *Node Equations for a Circuit Containing a Dependent Source*

- Determine the node voltages for the circuit shown in Figure 4.4-1

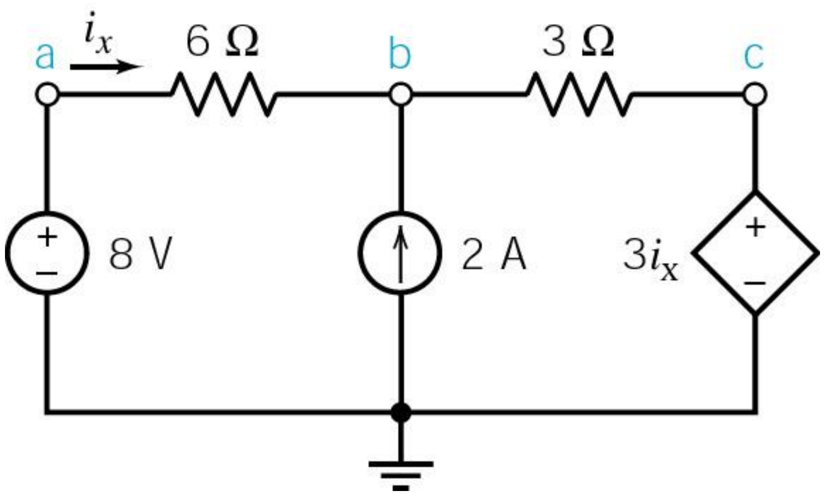


Figure 4.4-1 (p. 123)
A circuit with a CCVS.

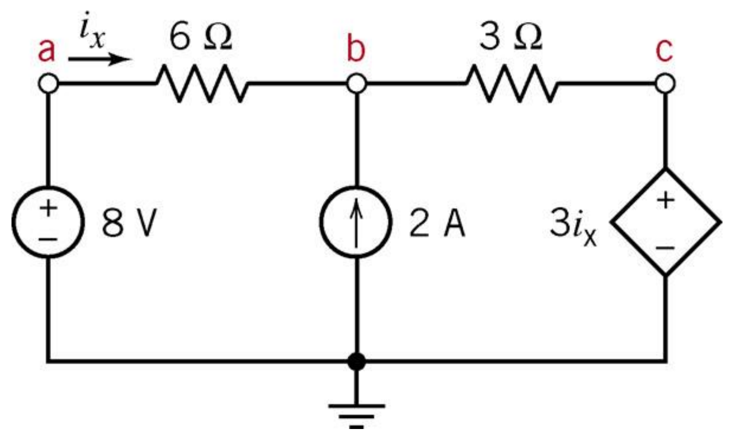
Rules

- *Rule #1: Express currents in terms of node voltages: Count the number of unknown node voltages and the required number of KCL equations to solve.*
- *Rule #2: If a node voltage is known, use it.: Then the number of unknown is one less.*
- *Rule #3: Supernode makes the KCL equation simpler by removing the need to introduce another variable, a current through the voltage source whether it is an independent or a dependent source.*
- *Rule #4: If there is a controlled source : Express its controlling source with node voltages. You probably have to do this first.*

Solution

1. Controlling current of the dependent source ($\boldsymbol{i_x}$)

$$i_x = \frac{v_a - v_b}{6}$$



2. Node voltage at node a

$$v_a = 8\text{V} \quad \Rightarrow \quad i_x = \frac{8 - v_b}{6}$$

3. Node voltage at node c

$$v_c = 3i_x = 3\left(\frac{8 - v_b}{6}\right) = 4 - \frac{v_b}{2}$$

4. Apply KCL at node b

$$\frac{8 - v_b}{6} + 2 = \frac{v_b - v_c}{3}$$

$$\frac{8 - v_b}{6} + 2 = \frac{v_b - \left(4 - \frac{v_b}{2}\right)}{3} = \frac{v_b}{2} - \frac{4}{3}$$

5. Solving for v_b and v_c

$$v_b = 7\text{V}$$

$$v_c = 4 - \frac{v_b}{2} = \frac{1}{2}\text{V}$$

Example 4.4-2

- Determine the node voltages for the circuit shown in Figure 4.4-2

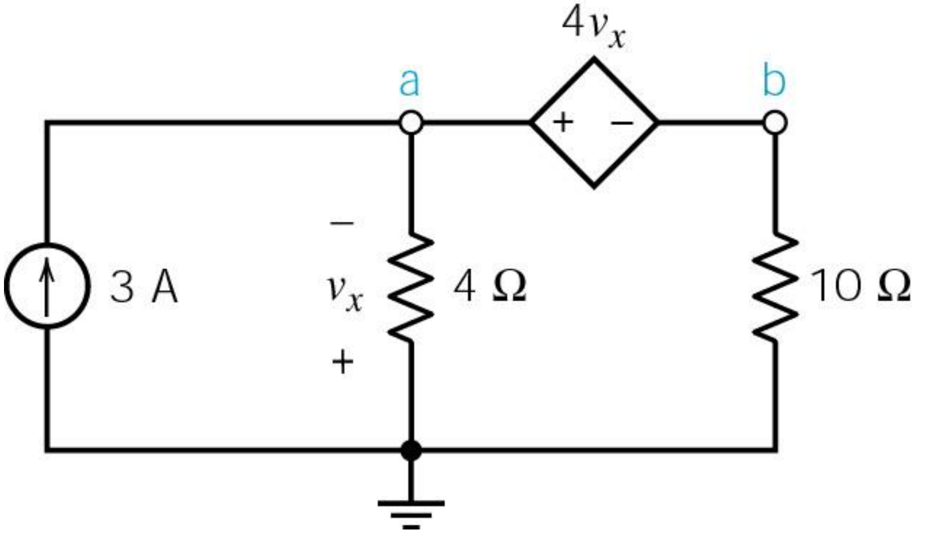


Figure 4.4-2 (p. 124)
A circuit with a VCVS.

Rules

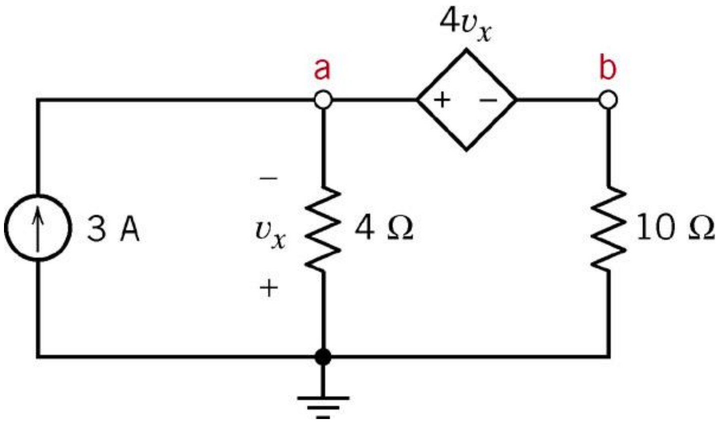
- *Rule #1: Express currents in terms of node voltages: Count the number of unknown node voltages and the required number of KCL equations to solve.*
- *Rule #2: If a node voltage is known, use it.: Then the number of unknown is one less.*
- *Rule #3: Supernode makes the KCL equation simpler by removing the need to introduce another variable, a current through the voltage source whether it is an independent or a dependent source.*
- *Rule #4: If there is a controlled source : Express its controlling source with node voltages. You probably have to do this first.*

Solution

1. Controlling voltage of the dependent source ($\boldsymbol{v_x}$)

$$v_x = -v_a$$

2. Node voltage at node a and b



$$v_a - v_b = 4v_x = 4(-v_a) = -4v_a$$
$$v_b = 5v_a$$

3. Apply KCL to the supernode

$$3 = \frac{v_a}{4} + \frac{v_b}{10}$$
$$3 = \frac{v_a}{4} + \frac{5v_a}{10} = \frac{3}{4}v_a$$

4. Solving for v_a and v_b

$$v_a = 4\text{V}$$
$$v_b = 5v_a = 20\text{V}$$

Example 4.4-3

- Determine the node voltages corresponding to nodes a and b for the circuit shown in Figure 4.4-3

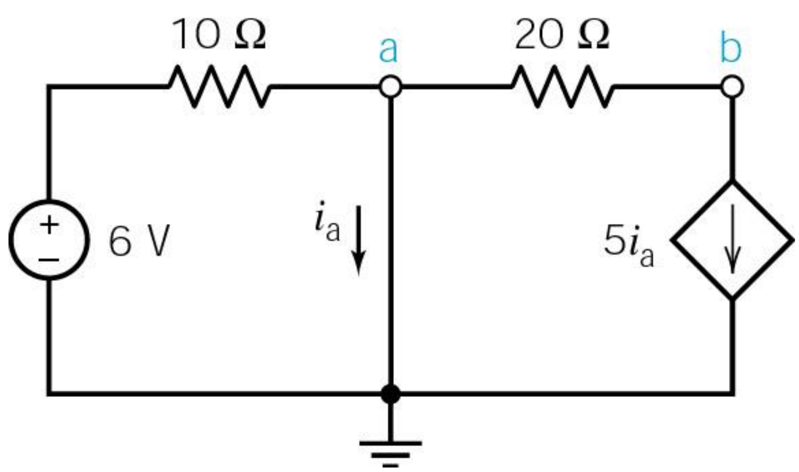


Figure 4.4-3 (p. 124)
A circuit with a CCCS.

Rules

- *Rule #1: Express currents in terms of node voltages: Count the number of unknown node voltages and the required number of KCL equations to solve.*
- *Rule #2: If a node voltage is known, use it.: Then the number of unknown is one less.*
- *Rule #3: Supernode makes the KCL equation simpler by removing the need to introduce another variable, a current through the voltage source whether it is an independent or a dependent source.*
- *Rule #4: If there is a controlled source : Express its controlling source with node voltages. You probably have to do this first.*

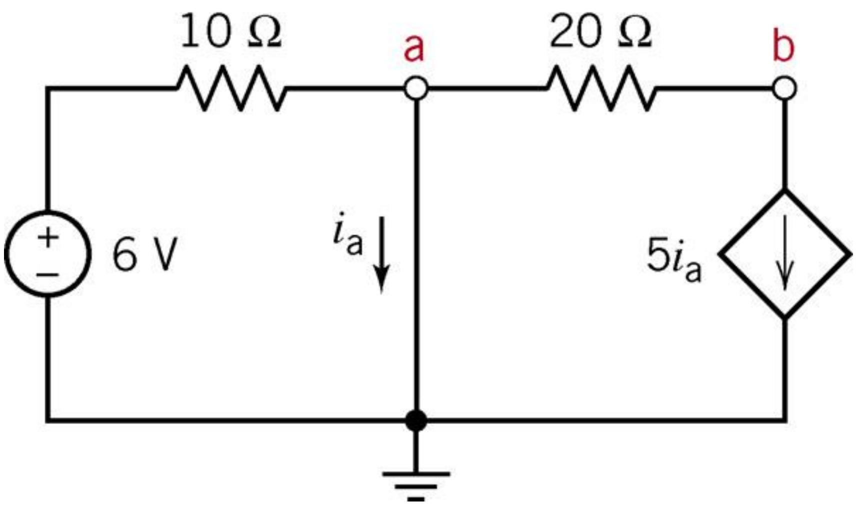
Solution

1. Controlling current of the dependent source (***i***$\boldsymbol{i_a}$)
   Apply KCL at node a

$$\frac{6-v_a}{10}=i_a+\frac{v_a-v_b}{20}$$

v_a =0

$$i_a=\frac{12+v_b}{20}$$



2. Apply KCL at node b

$$\frac{0-v_b}{20}=5i_a$$

$$\frac{0-v_b}{20}=5\left(\frac{12+v_b}{20}\right)$$

3. Solving for v_b $\quad v_b=-10\text{V}$

Mesh Current Analysis with Independent Voltage Source

- **A *mesh* is a loop that does not contain any other loops within it.**

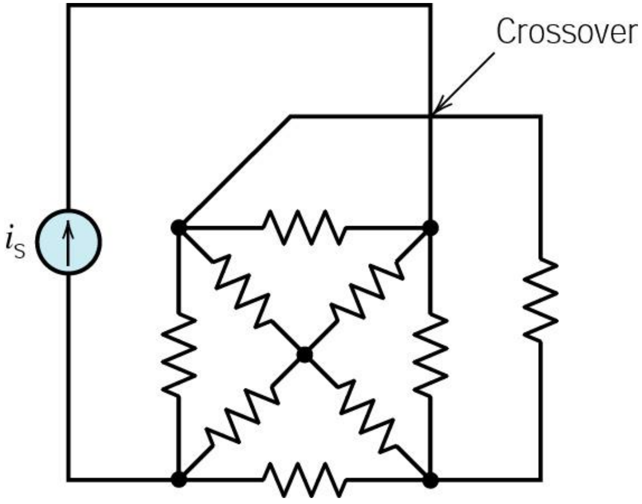


Figure 4.5-1 (p. 126)
Nonplanar circuit with a crossover.

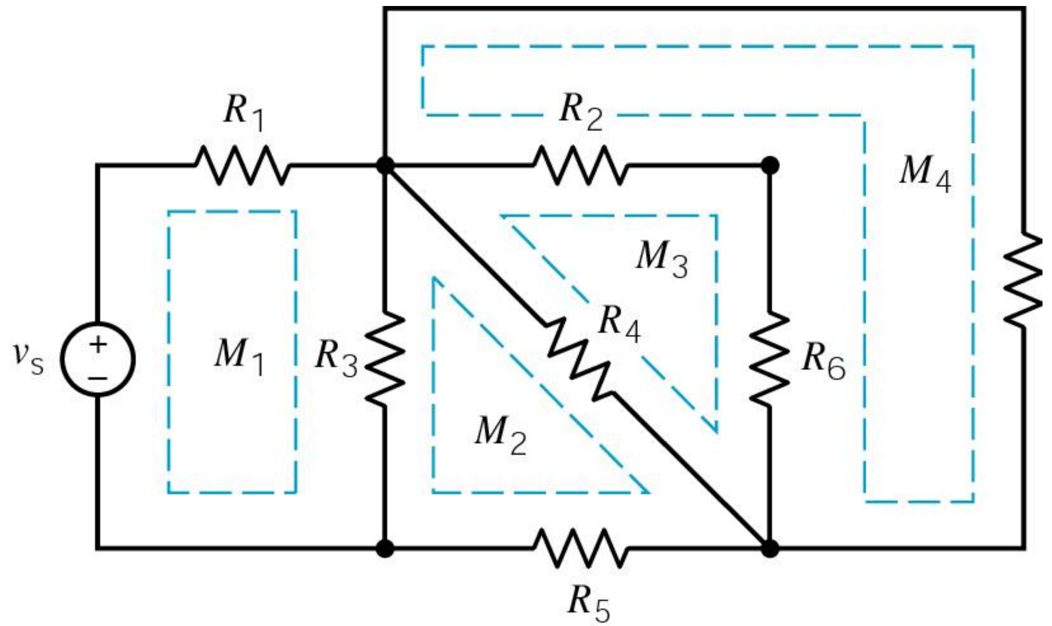


Figure 4.5-2 (p. 126)
Circuit with four meshes. Each mesh is identified by dashed lines.

Loop, Mesh, Mesh current

- **Loop**
  - A closed path drawn by starting at a node and tracing a path such that we return to the original node without passing an intermediate node more than once
- **Mesh**
  - A loop that does not contain any other loops within it.
- **Mesh current**
  - The current that flows through the elements constituting the mesh.

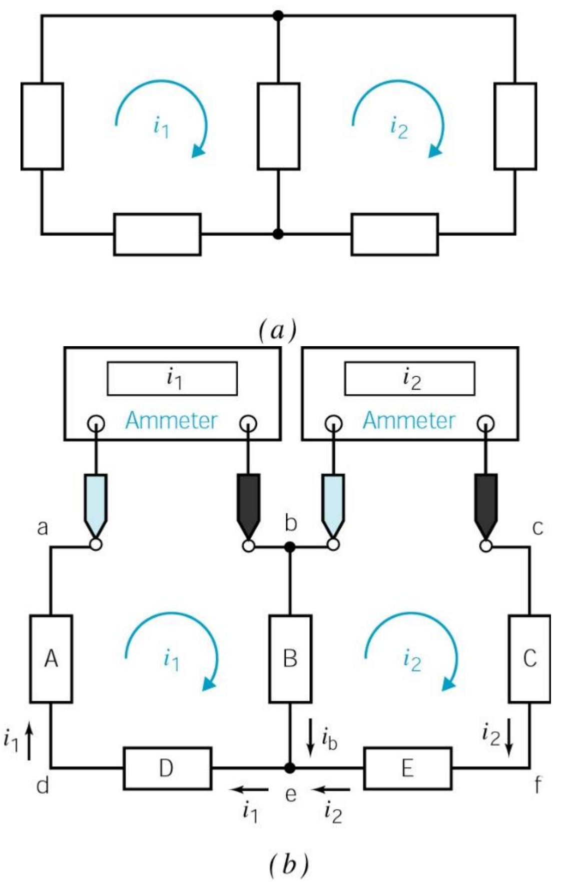


Figure 4.5-3 (p. 126)
(*a*) A circuit with two meshes. (*b*) Inserting ammeters to measure the mesh currents.

Mesh equations

- **To write a set of mesh equations, we do two things:**
  1. **Express element voltages as functions of the mesh currents.**
  2. **Apply Kirchhoff's voltage law (KVL) to each of the meshes of the circuit.**

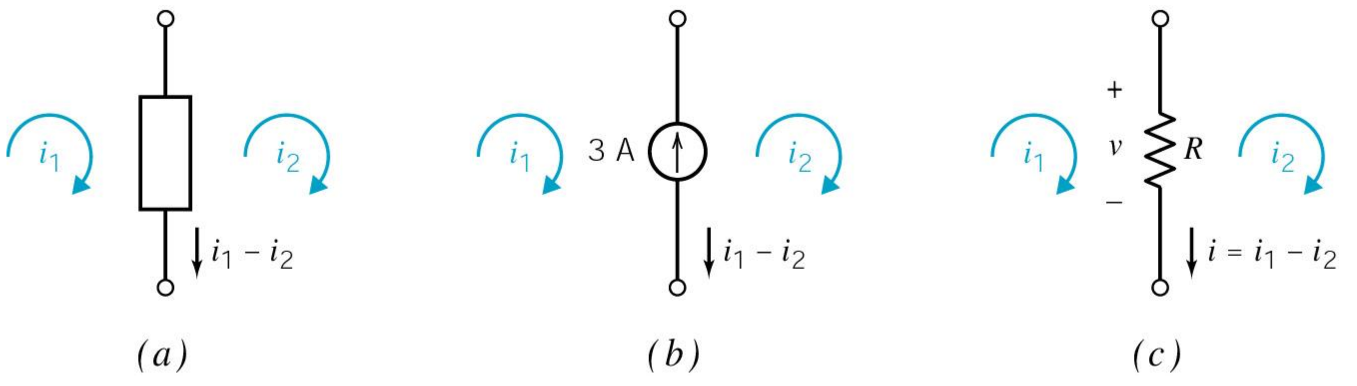
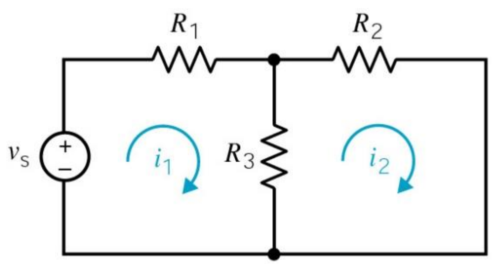


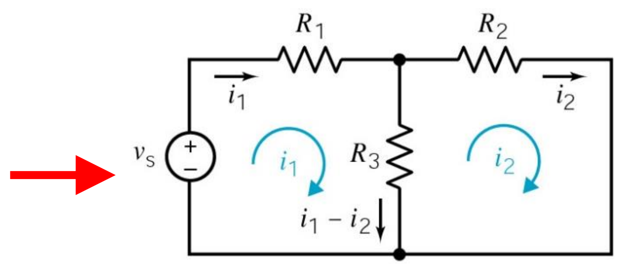
Figure 4.5-4 (p. 127)
Mesh currents, i_1 and i_2, and element current, $i_1 - i_2$, of a (*a*) generic circuit element, (*b*) current source, and (*c*) resistor.

Example of Mesh Current Analysis

- **Example of mesh current analysis**
  1. **Express element voltages as functions of the mesh currents.**



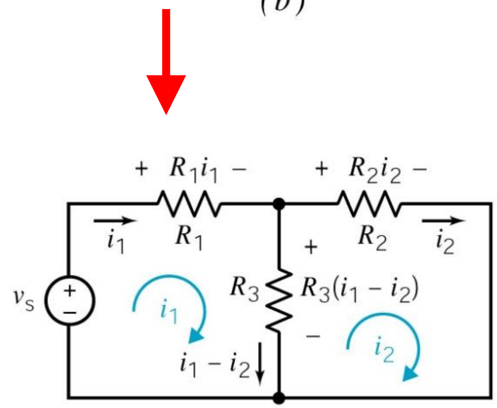
(a)



(b)

(*b*) The resistor currents expressed as functions of the mesh currents.

Figure 4.5-5 (p. 128)
(*a*) A circuit.

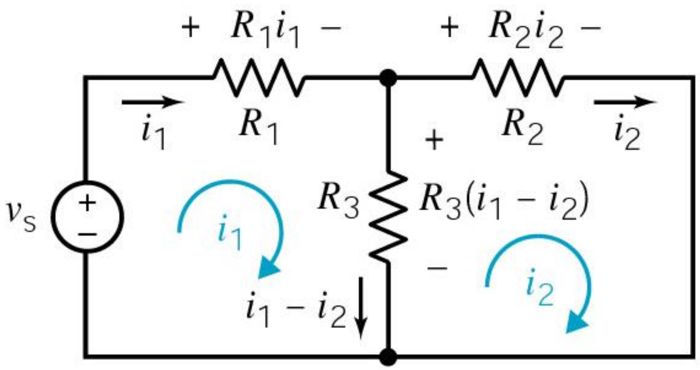


(c)

(*c*) The resistor voltages expressed as functions of the mesh currents.

Example of Mesh Current Analysis

- **Example of mesh current analysis**
  2. **Apply Kirchhoff's voltage law (KVL) to each of the meshes of the circuit.**



(c)

<Mesh 1>

$$-v_s + R_1 i_1 + R_3(i_1 - i_2) = 0$$

<Mesh 2>

$$-R_3(i_1 - i_2) + R_2 i_2 = 0$$

Mesh Current Analysis with Independent Voltage Source

- A circuit that contains only independent voltage sources and resistors results in a specific format of equations that can readily be obtained.

Mesh 1: $-v_s + R_1 i_1 + R_4(i_1 - i_2) = 0$

Mesh 2: $R_2 i_2 + R_5(i_2 - i_3) + R_4(i_2 - i_1) = 0$

Mesh 3: $R_5(i_3 - i_2) + R_3 i_3 + v_g = 0$

These equations can be written as

Mesh 1: $(R_1 + R_4)i_1 - R_4 i_2 = v_s$

Mesh 2: $-R_4 i_1 + (R_4 + R_2 + R_5)i_2 - R_5 i_3 = 0$

Mesh 3: $-R_5 i_2 + (R_3 + R_5)i_3 = -v_g$

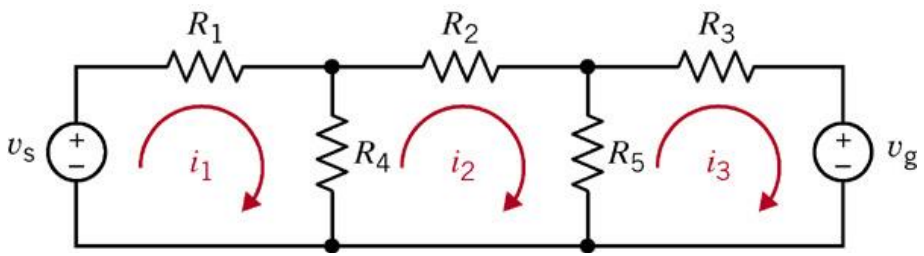


Figure 4.5-6 (p. 128)
Circuit with three mesh and two voltage sources

- The coefficient of the first mesh current for the first mesh:

  ➔ The sum of resistances in the loop

- The coefficient of the second mesh current for the first mesh:

  ➔The negative of the resistance common to meshes 1 and 2

Mesh Current Analysis with Independent Voltage Source

- In general, the equation for the *n*th mesh with independent voltage sources only is obtained as follows:

$$-\sum_{q=1}^{Q} R_k i_q + \sum_{j=1}^{P} R_j i_n = -\sum_{n=1}^{N} v_{Sn}$$

$$\text{where} \begin{cases} i_n : & n\text{th mesh current} \\ R_j : & \text{resistance around the } n\text{th mesh} \\ P : & \text{\# of resistors around the } n\text{th mesh} \\ i_q : & \text{mesh current in the adjacent mesh} \\ R_k : & \text{resistance in common with adjacent mesh} \\ Q : & \text{\# of adjacent meshes} \\ v_{Sn} : & \text{independent voltage source around the } n\text{th mesh} \end{cases}$$

  - All mesh currents flow clockwise.

Mesh Current Analysis with Independent Voltage Source

- The general matrix equation for the mesh current analysis for independent voltage sources present in a circuit is

$$\mathbf{R}\,\mathbf{i} = \mathbf{v_s}$$

$$\mathbf{i} = \begin{pmatrix} i_1 \\ i_2 \\ . \\ . \\ . \\ i_N \end{pmatrix} \qquad \mathbf{v_s} = \begin{pmatrix} v_{s1} \\ v_{s2} \\ . \\ . \\ . \\ v_{sN} \end{pmatrix}$$

$$R = \begin{bmatrix} (R_1 + R_4) & -R_4 & 0 \\ -R_4 & (R_2 + R_4 + R_5) & -R_5 \\ 0 & -R_5 & (R_3 + R_5) \end{bmatrix}$$

  - Where **R** is a symmetric matrix with a diagonal consisting of the sum of resistances in each mesh and the off-diagonal elements are the negative of sum of the resistances common to two meshes.

Cramer's rule

- For linear systems of three equations in three unknowns

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

- **Cramer's rule is**

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}, \quad x_3 = \frac{D_3}{D} \qquad (D \neq 0)$$

with the "determinant of the system" D given by $D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$D_1 = \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix}, \quad D_2 = \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}, \quad D_3 = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}$$

Mesh Current Analysis with Current and Voltage Source

- **Case 1**
  - A current source appears on the periphery of only one mesh, n
  - **Method:**
    - Equate the mesh current i_n to the current source current, accounting for the direction of the current source.

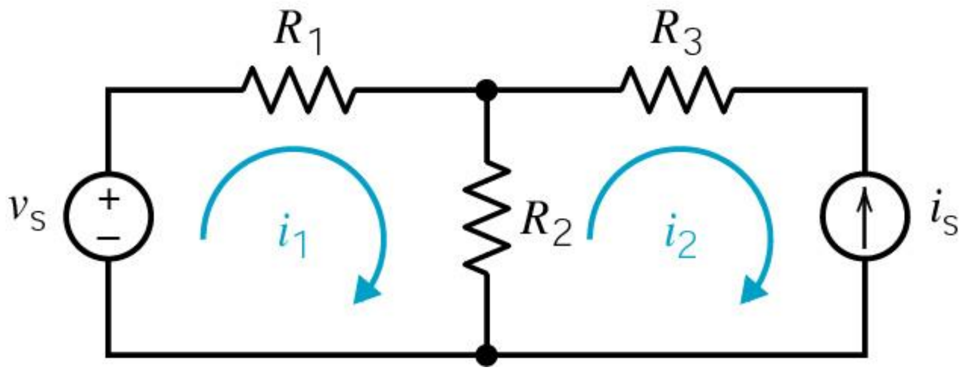


Figure 4.6-1 (p. 130)
Circuit with an independent voltage source and an independent current source.

Mesh Current Analysis with Current and Voltage Source

- **Case 1**
  - A current source appears on the periphery of only one mesh, *n*

1. **Mesh current**

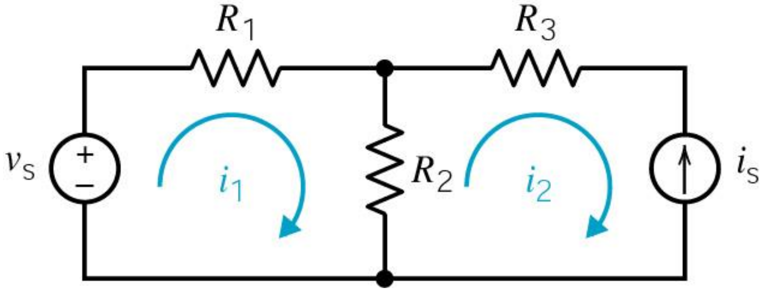
$$i_2 = -i_s$$

2. **KVL for the 1st mesh**

$$(R_1 + R_2)i_1 - R_2 i_2 = v_s$$

3. **Since $i_2=-i_s$,**

$$i_1 = \frac{v_s - R_2 v_s}{R_1 + R_2}$$



Circuit with an independent voltage source and an independent current source.

Mesh Current Analysis with Current and Voltage Source

- **Case 2-A**
  - A current source is common to two meshes
  - **Method:**
    - A. Assume a voltage v_{ab} across the terminals of the current source, write the KVL equations for the two meshes, and add them to eliminate v_{ab}
    - B. Create a supermesh as the periphery of the two meshes and write one KVL equation around the periphery of the supermesh. In addition, write the constraining equation for the two mesh currents in terms of the current source.

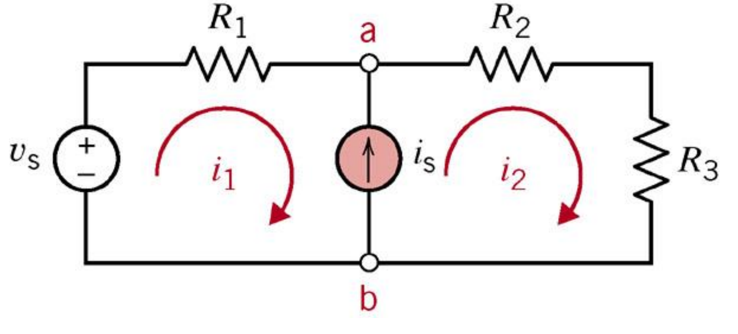


Figure 4.6-2 (p. 130)
Circuit with an independent current source common to both meshes.

Mesh Current Analysis with Current and Voltage Source

- **Case 2-A**
  - A current source is common to two meshes

1. **Mesh current**

$$i_2 - i_1 = i_s$$

2. **KVL at node a**

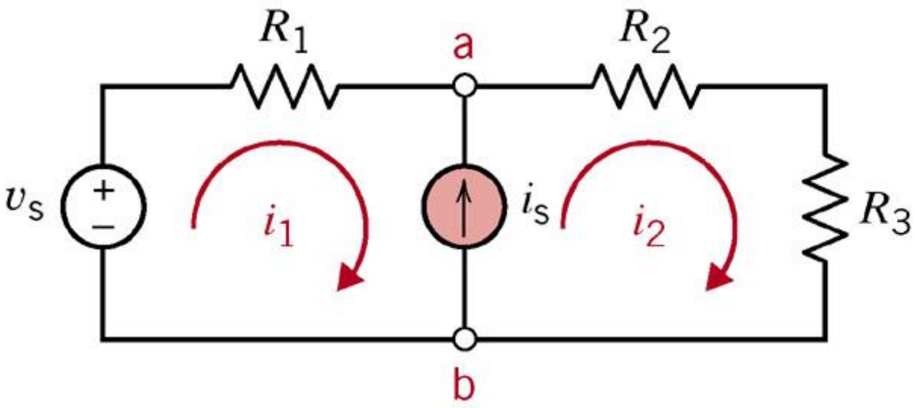
mesh 1: $R_1 i_1 + v_{ab} = v_s$

mesh 2: $(R_2 + R_3) i_2 - v_{ab} = 0$

$$\Longrightarrow \quad R_1 i_1 + (R_2 + R_3) i_2 = v_s$$

3. **Since $i_2 = i_s + i_1$**

$$R_1 i_1 + (R_2 + R_3)(i_s + i_1) = v_s$$



Circuit with an independent current source common to both meshes.

Example 4.6-1 *Mesh Equations*

- Consider the circuit of Figure 4.6-3 where R_1=R_2=1Ω and R_3=2Ω. Find the three mesh current.

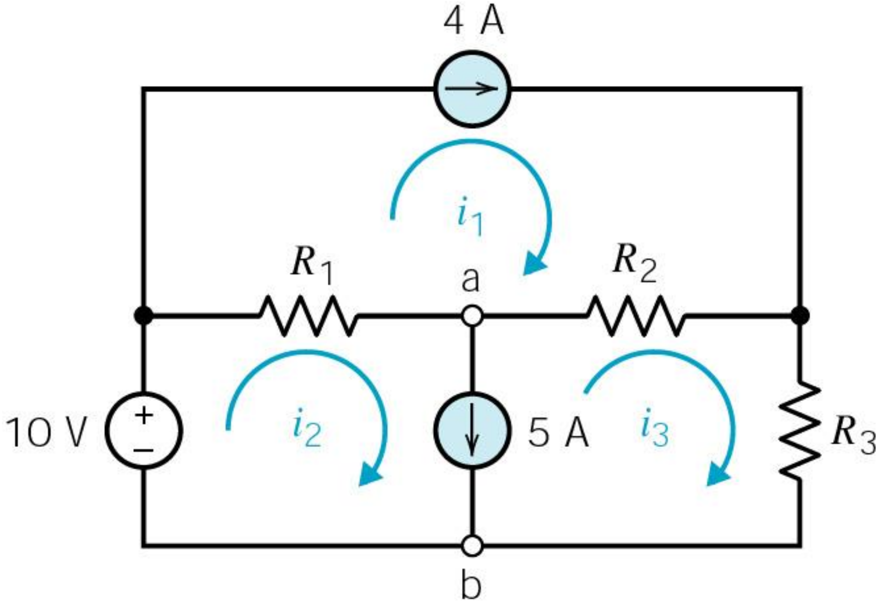


Figure 4.6-3 (p. 131)
Circuit with two independent current sources.

Solution

1. 4-A source $i_1 = 4$

2. 5-A source $i_2 - i_3 = 5$

3. KVL for mesh 2 and mesh 3

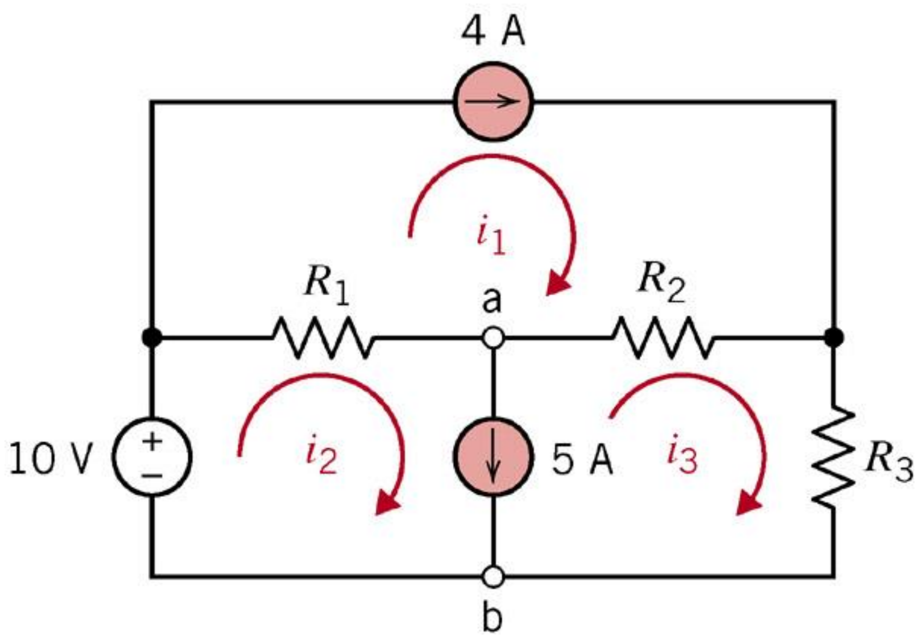
$$\text{mesh2}: \quad R_1(i_2 - i_1) + v_{ab} = 10$$
$$\text{mesh3}: \quad R_2(i_3 - i_1) + R_3 i_3 - v_{ab} = 0$$

4. Solve equations

$$R_1(i_2 - 4) + R_2(i_3 - 4) + R_3 i_3 = 10$$
$$R_1(5 + i_3 - 4) + R_2(i_3 - 4) + R_3 i_3 = 10$$

5. Using the values for the resistors, we obtain

$$i_3 = \frac{13}{4}\text{A} \quad \textit{and} \quad i_2 = 5 + i_3 = \frac{33}{4}\text{A}$$



Mesh Current Analysis with Current and Voltage Source

- **Case 2-B**
  - A current source is common to two meshes
  - **Method:**
    B. Create a supermesh as the periphery of the two meshes and write one KVL equation around the periphery of the supermesh. In addition, write the constraining equation for the two mesh currents in terms of the current source.

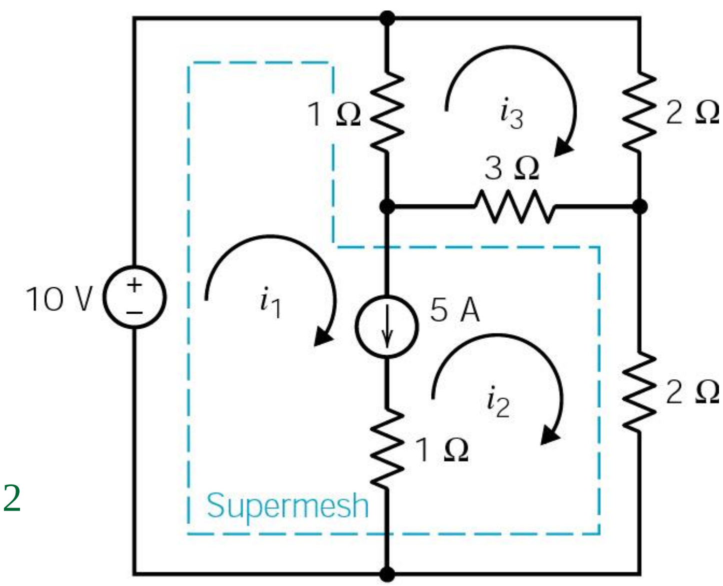


Figure 4.6-4 (p. 132)
Circuit with a supermesh that incorporates mesh 1 and mesh 2
The supermesh is indicated by the dashed line.

Mesh Current Analysis with Current and Voltage Source

- **Case 2-B**
  - A current source is common to two meshes

1. **supermesh**

$$-10+(i_1-i_3)+3(i_2-i_3)+2i_2=0$$

$$\Longrightarrow \quad i_1+5i_2-4i_3=10$$

2. **Mesh 3**

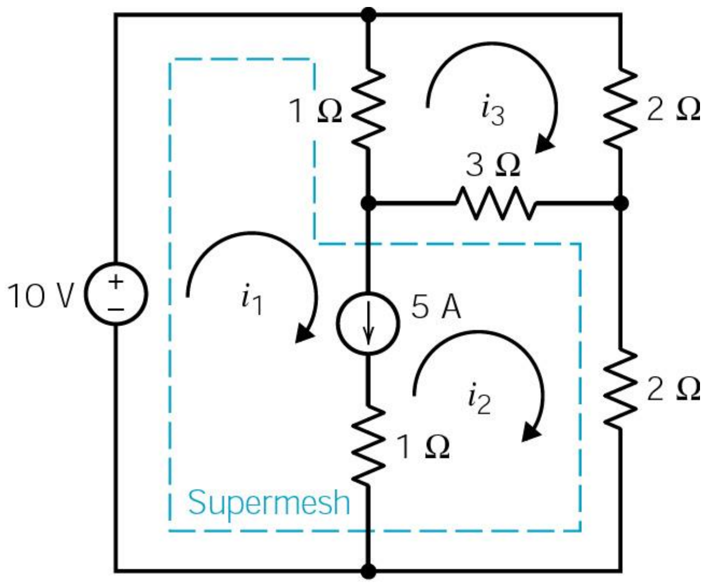
$$(i_3-i_1)+2i_3+3(i_3-i_2)=0$$

$$\Longrightarrow \quad -i_1-3i_2+6i_3=0$$

3. **Mesh current**

$$i_1-i_2=5$$

A supermesh is one larger mesh created from two meshes that have an independent or dependent current source in common.



Circuit with a supermesh that incorporates mesh 1 and mesh 2. The supermesh is indicated by the dashed line.

Example 4.6-2 *Supermesh*

- Determine the values of the mesh currents, i_1 and i_2, for the circuit shown in Figure 4.6-5

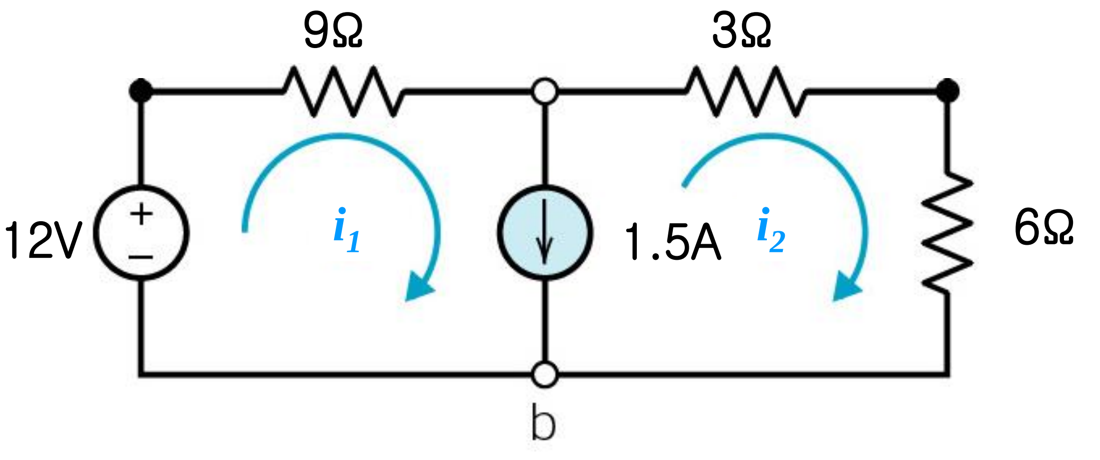
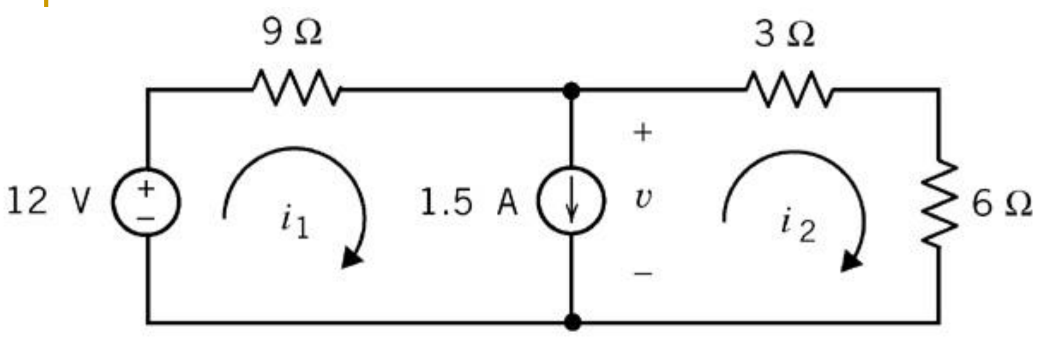


Figure 4.6-5 (p. 132)
The Circuit for example 4.6-2

Solution



$$i_1 - i_2 = 1.5 \quad \Rightarrow \quad i_1 = i_2 + 1.5$$

Method 1:

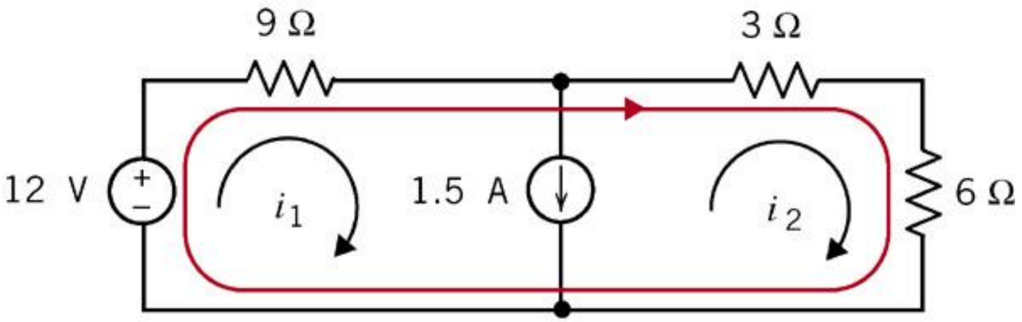
the KVL equation for mesh 1 $\quad 9i_1 + v - 12 = 0$

the KVL equation for mesh 2 $\quad 3i_2 + 6i_2 - v = 0$

Combining two equations

$$9i_1 + (3i_2 + 6i_2) - 12 = 0 \quad \Rightarrow \quad 9i_1 + 9i_2 = 12$$

Solution



Method 2:

Apply KVL to the supermesh to get

$$9i_1 + (3i_2 + 6i_2) - 12 = 0 \quad \Rightarrow \quad 9i_1 + 9i_2 = 12$$

Applying KVL to the supermesh is a shortcut for doing three things:

1. Labeling the current source voltage as v
2. Applying KVL to both meshes that contain the current source
3. Eliminating v from the KVL equations

In summary,

$$i_1 = i_2 + 1.5$$
$$9i_1 + 9i_2 = 12$$

Solving the equation

$$i_1 = 1.4167\text{A} \quad \textit{and} \quad i_2 = -83.3\text{mA}$$

Mesh Current Analysis with Dependent Sources

- **When a circuit contains a dependent source, the controlling current or voltage of that dependent source must be expressed as a function of the mesh currents.**

Example 4.7-1 *Mesh Equations and Dependent Sources*

- Consider the circuit shown in Figure 4.7-1a. Find the value of the voltage measured by the voltmeter.

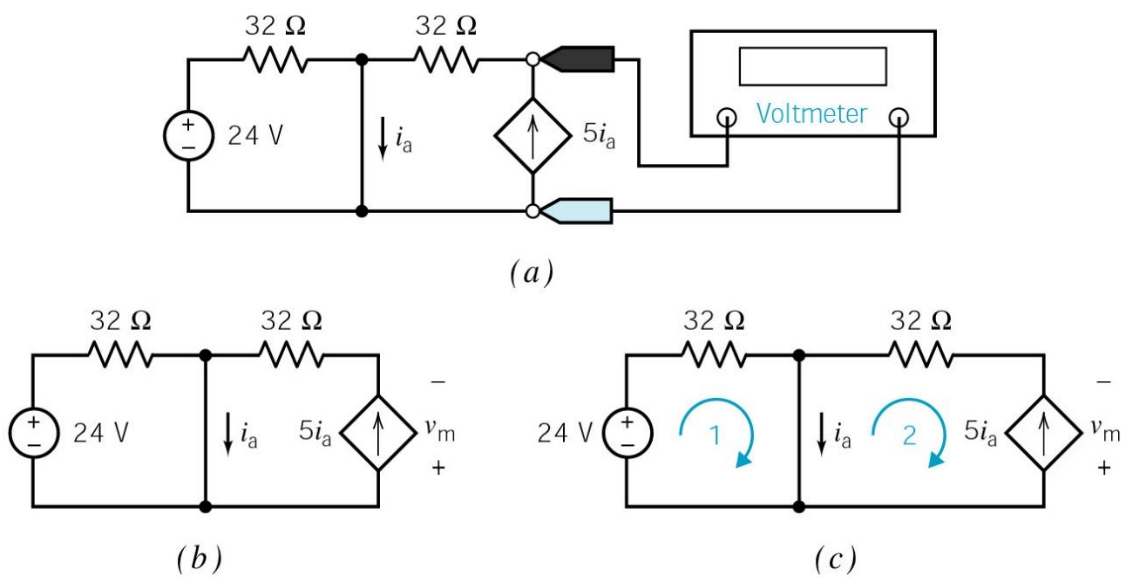
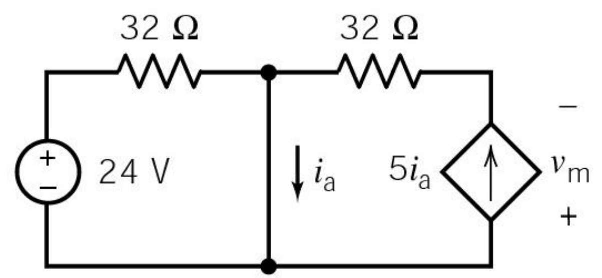


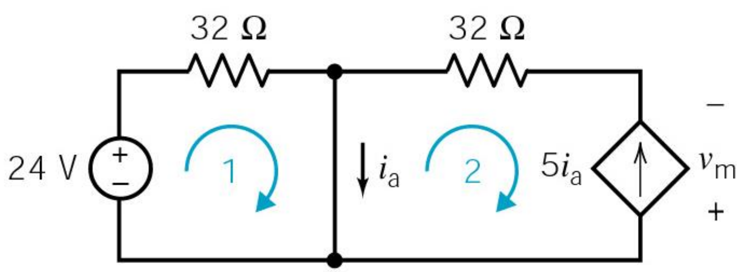
Figure 4.7-1 (p. 135)
(*a*) The circuit considered in Example 4.7-1. (*b*) The circuit after replacing the voltmeter by an open circuit. (*c*) The circuit after labeling the meshes.

Solution

- Fig. 4.7-1b (circuit after replacing the voltmeter)
  Fig. 4.7-1c (circuit after numbering the meshes)



(b)



(c)

1. Controlling current of the dependent source, i_a

$$i_a = i_1 - i_2$$

$$5i_a = -i_2$$

Solving for i_2 gives $$i_2 = \frac{5}{4} i_1$$

2. Apply KVL to mesh 1 to get

$$32i_1 - 24 = 0 \quad \Rightarrow \quad i_1 = \frac{3}{4}\,\text{A}$$

$$i_2 = \frac{5}{4}\left(\frac{3}{4}\right) = \frac{15}{16}\,\text{A}$$

3. Apply KVL to mesh 2 to get

$$32i_2 - v_m = 0 \quad \Rightarrow \quad v_m = 32i_2$$

4. Finally, $$v_m = 32\left(\frac{15}{16}\right) = 30\text{V}$$

Example 4.7-2 *Mesh Equations and Dependent Sources*

- Consider the circuit shown in Figure 4.7-2a. Find the value of the gain, *A*, of the CCVS

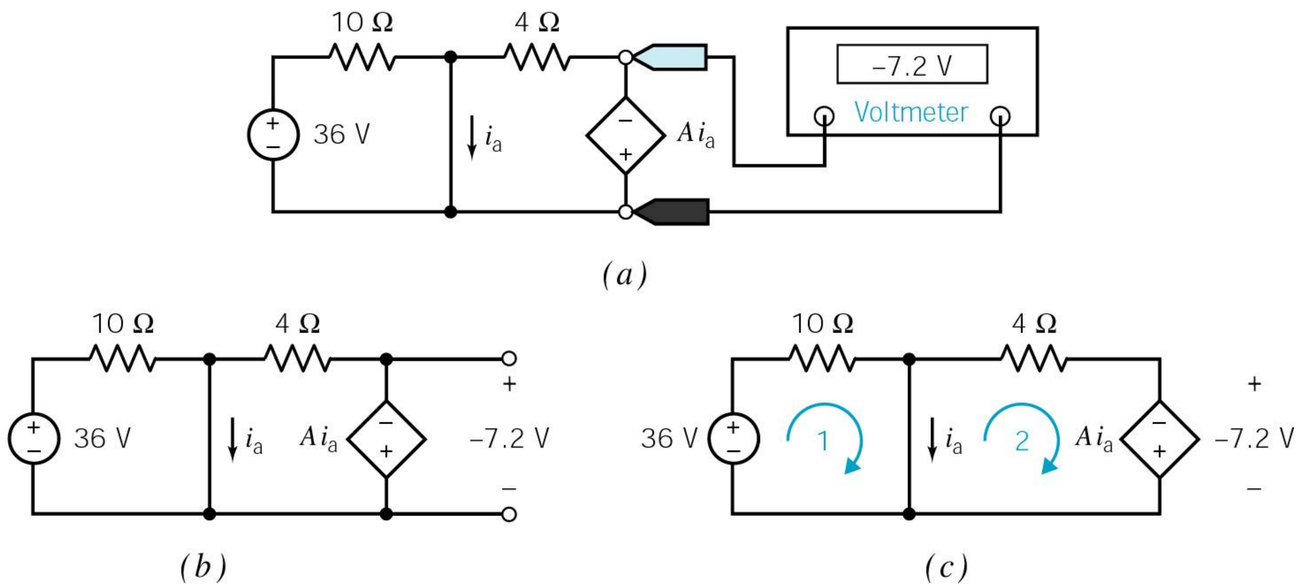


Figure 4.7-2 (p. 136)
(*a*) The circuit considered in Example 4.7-3. (*b*) The circuit after replacing the voltmeter by an open circuit. (*c*) The circuit after labeling the meshes.

Solution

1. Ai_a $\quad Ai_a = -(-7.2) = 7.2\text{V}$

2. Short-circuit current of the dependent source

$$i_a = i_1 - i_2$$

3. KVL to mesh 1

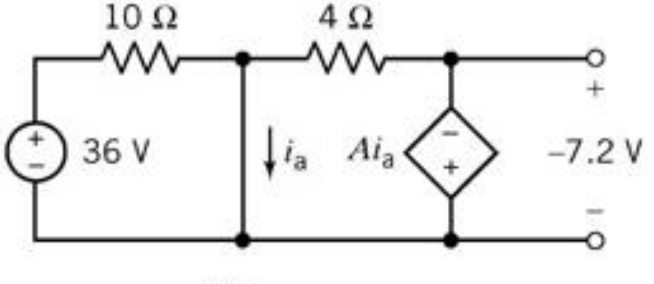
$$10i_1 - 36 = 0 \quad \Rightarrow \quad i_1 = 3.6\text{A}$$

4. KVL to mesh 2

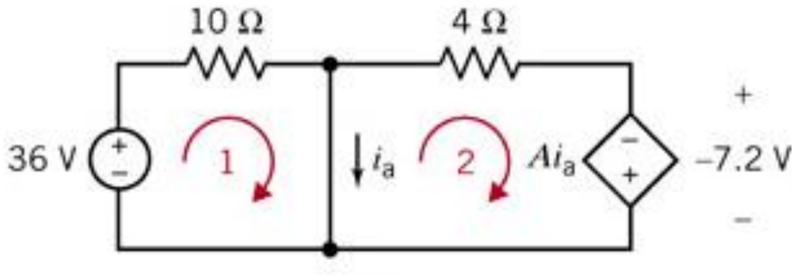
$$4i_2 + (-7.2) = 0 \quad \Rightarrow \quad i_2 = 1.8\text{A}$$

5. Finally,

$$A = \frac{Ai_a}{i_a} = \frac{Ai_a}{i_1 - i_2} = \frac{7.2}{3.6 - 1.8} = 4(\text{V/A})$$



(b)



(c)

Figure 4.7−2

The Node Voltage Method and Mesh Current Method Compared

- **The advantage of using node voltage method and mesh current method**
  - **Systematic procedure**
  - **Simultaneous equation**

- **In some cases one method is preferred over another.**
  - **The circuit contains only voltage sources: mesh current method**
  - **The circuit contains only current sources: node voltage method**
  - **The circuit has fewer nodes than meshes: node voltage method**
  - **The circuit has fewer meshs than nodes: mesh current method**
  - **You need to know several currents: mesh current method**
  - **You need to know several voltages: node voltage method**

Example 4.8-1 *Mesh Equations*

- Consider the circuit shown in Figure 4.8-1. Find the value of the resistance, R.

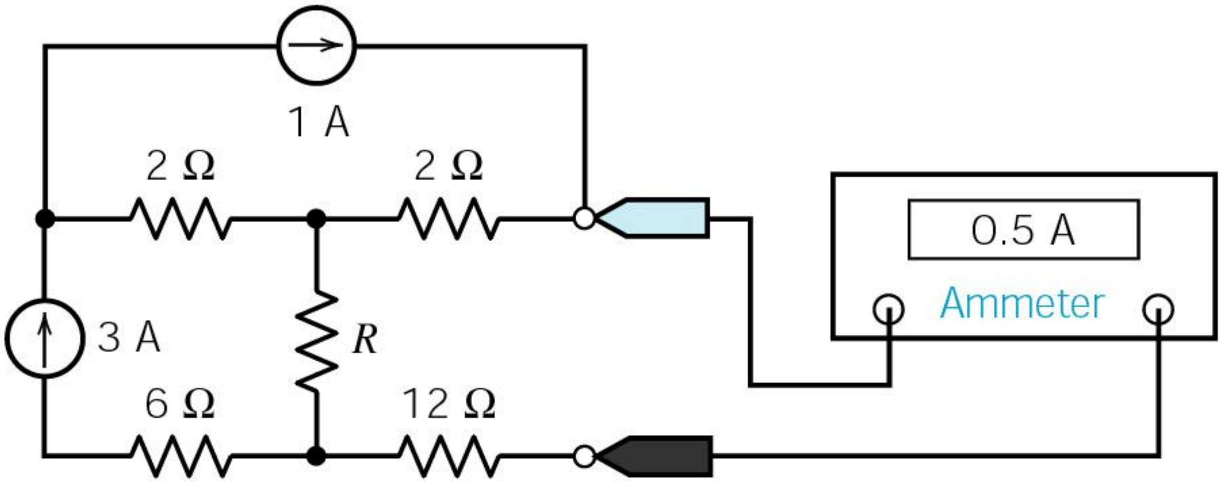


Figure 4.8-1 (p. 136)
The circuit considered in Example 4.8-1.

Solution

1. The mesh current i_1 is equal to the current in the 1-A current source.

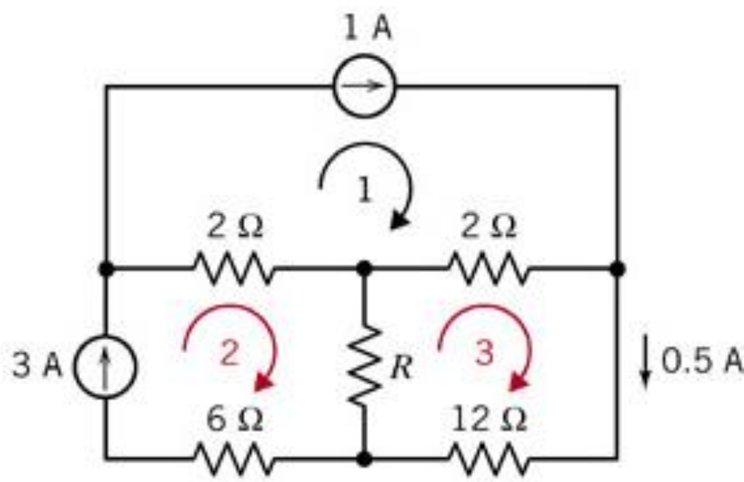
$$i_1 = 1\text{A}$$

2. The mesh current i_2 is equal to the current in the 3-A current source.

$$i_2 = 3\text{A}$$

3. The mesh current i_3 is equal to the current in the short circuit that replaced the ammeter

$$i_3 = 0.5\text{A}$$



4. Apply KVL to mesh 3

$$2(i_3 - i_1) + 12(i_3) + R(i_3 - i_2) = 0$$

5. Finally,

$$2(0.5-1) + 12(0.5) + R(0.5-3) = 0 \quad \Rightarrow \quad R = 2\Omega$$

Example 4.8-2 *Node Equations*

- Consider the circuit shown in Figure 4.8-3. Find the value of the resistance, R.

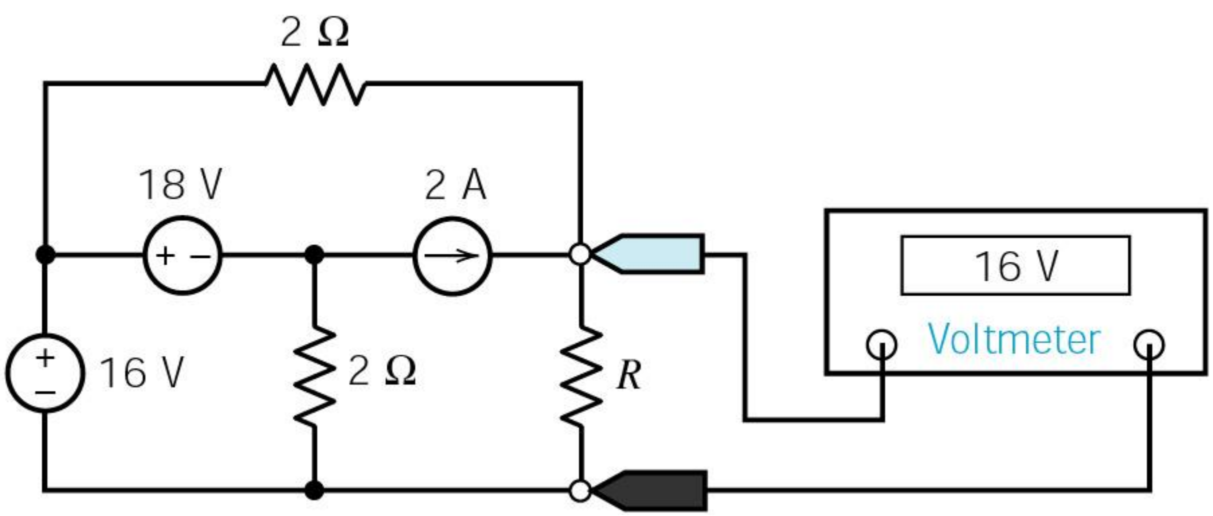


Figure 4.8-3 (p. 137)
The circuit considered in Example 4.8-2.

Solution

1. Node voltage at node 1

$$16 = v_1 - 0 \quad \Rightarrow \quad v_1 = 16\text{V}$$

2. Node voltage at node 2

$$18 = v_1 - v_2 \quad \Rightarrow \quad 18 = 16 - v_2 \quad \Rightarrow \quad v_2 = -2\text{V}$$

3. Node voltage at node 3

$$v_3 = 16\text{V}$$

4. Apply KCL at node 3

$$\frac{v_1 - v_3}{2} + 2 = \frac{v_3}{R}$$

5. Finally,

$$\frac{16-16}{2} + 2 = \frac{16}{R} \quad \Rightarrow \quad R = 8\Omega$$