Random Process
(Overview)

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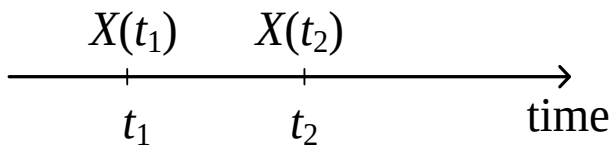
Random Process (1)

- Collection of random variables, which are indexed by time t
  - $\{X(t), t \in T\}$,
    - T is time domain of a system
    - $X(t)$ is a r.v. representing the state of system at time t
- describe the evolution through time of physical process



  - It is very useful to evaluate the average performance of the system

Random Process (2)

- Classification
  - Time
    - Countable time domain → discrete-time process
    - Uncountable time domain → continuous-time process
  - State space (the set of possible values that $X(t)$ may take on)
    - Countable state space → discrete-state process (or chain)
    - Uncountable state space → continuous-state process
  - Statistical dependency among random variables with different time index
    - If the state duration follows geometric or exponential distribution, it is a Markov process

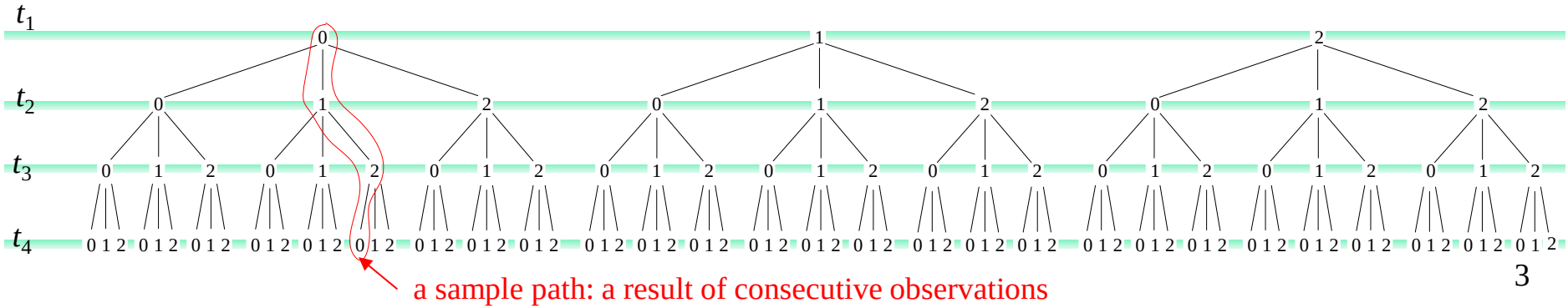
Example

- Barbershop example
  - Consider a barbershop (a system) with a barber (server) and several waiting chairs (queue)
  - Customer arrival rate and service time are given as system parameters
  - $X(t)$: the number of customers in the shop at time t

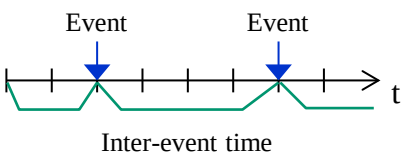
<Assumption>

  - One waiting chair → 0, 1, or 2 customers in the shop
  - Observe the number of customers in the shop only at four time instants t_1, t_2, t_3, t_4,

< Evolution of the process: 81 feasible sample paths >

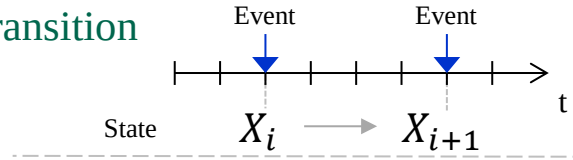


Classification by Statistical Dependency



Inter-event time

State transition



Stochastic process

$X(t)$: number of events up to t

$X(t)$: number of jobs in system at t

Markovian property

Counting process

Markov process

Inter-event times are independently and identically distributed

When current state is n, the next state is either $n+1$ or $n-1$

Renewal process

Birth & death process

When current state is n, the next state is only $n+1$

Inter-event times are exponentially distributed

Pure birth process

$\lambda(n)$ is always constant and independent from current state

Poisson process

* $\lambda(n)$: rate at which the process moves from current state to next state