Topics in Ship Structures

11 Computational Fracture Mechanics

Reference :

Fracture Mechanics by T.L. Anderson Ch. 12

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Contents

General

- Numerical modeling has become an indispensable tool in fracture analysis, since relatively few practical problems have closed-form analytical solutions.

- Stress-intensity solutions for literally hundreds of configurations have been published, the majority of which were inferred from numerical models.

- Elastic-plastic analyses to compute the *J* crack-tip-opening displacement (CTOD) are also becoming relatively common..

- More efficient numerical algorithms have greatly reduced solution times in fracture problems. For example, the domain integral approach enables one to generate *K* and *J* solutions from finite element models with surprisingly coarse meshes.
- Commercial numerical analysis codes have become relatively user friendly, and many codes have incorporated fracture mechanics routines.

- This chapter will not turn the reader into an expert on computational fracture mechanics, but it should serve as an introduction to the subject.

Stress and Displacement Matching

❖ Stress Matching Method

- On the crack plane ($q = 0$), K_I is related to the stress normal to the crack plane as follows:

$$K_I = \lim_{r \to 0} [\sigma_{yy} \sqrt{2\pi r}] \quad (\theta = 0)$$

$$\boxed{\sigma_{xx} = \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}}$$

- The stress-intensity factor can be inferred by plotting the quantity in square brackets against distance from the crack tip, and extrapolating to r = 0

❖ Displacement Matching Method

- K_I can be estimated from a similar extrapolation of crack-opening displacement u_y.
- For plain strain

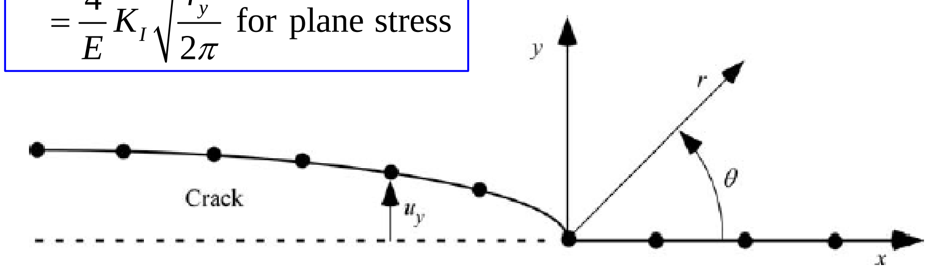
$$K_I = \lim_{r \to 0} \left[\frac{E u_y}{4(1-\nu^2)} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$

- For plain stress

$$K_I = \lim_{r \to 0} \left[\frac{E u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$

$$\boxed{\begin{aligned} u_y &= \frac{4}{E'} K_I \sqrt{\frac{r_y}{2\pi}} \text{ for plane strain} \\ &= \frac{4}{E} K_I \sqrt{\frac{r_y}{2\pi}} \text{ for plane stress} \end{aligned}}$$

Ch. 4.1



Local coordinate system for stresses and displacements at the crack tip in a finite element or boundary element model.

"Displacement Matching Method give more accurate estimates of K_I than Stress Matching Method because stresses are singular as $r \to 0$ but displacements are proportional to $\sqrt{r}$ near the crack tip"

Theoretical Background

❖ Similitude in Fatigue

- The generalized definition of J requires that the contour surrounding the crack tip be vanishingly small.

$$J = \lim_{\Gamma_o \to 0} \int_{\Gamma_o} \left[(w+T)\delta_{1i} - \sigma_{ij} \frac{\partial u_j}{\partial x_1} \right] n_i d\Gamma$$

- where *T* is the kinetic energy density. Various material behavior can be taken into account through the definition of *w*, the stress work.
- Consider an elastic-plastic material loaded under quasistatic conditions ($T = 0$). If thermal strains are present, the total strain is given by

$$\varepsilon_{ij}^{total} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} + \alpha \Theta \delta_{ij} = \varepsilon_{ij}^{m} + \varepsilon_{kk}^{t}$$

- where *a* is the coefficient of thermal expansion and Θ is the temperature relative to a strain-free condition. The superscripts *e*, *p*, *m*, and *t* denote elastic, plastic, mechanical, and thermal strains, respectively.
- The stress work is given by

$$w = \int_0^{\varepsilon_{kl}^m} \sigma_{ij} d\varepsilon_{ij}^m$$

Theoretical Background

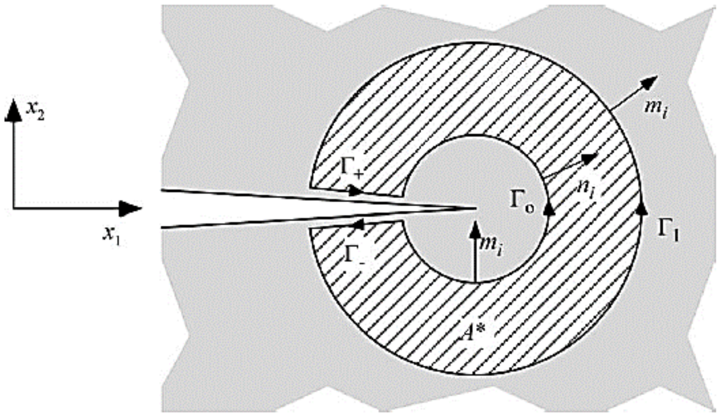
- It is not feasible to evaluate stresses and strains along a vanishingly small contour.

$$J = \lim_{\Gamma_o \to 0} \int_{\Gamma_o} \left[(w+T)\delta_{1i} - \sigma_{ij} \frac{\partial u_j}{\partial x_1} \right] n_i \, d\Gamma$$

- Let us construct a closed contour by connecting inner and outer contours, $\Gamma^* = \Gamma_1 + \Gamma_+ + \Gamma_- - \Gamma_o$.
- For quasistatic conditions, where $T = 0$,

$$J = \int_{\Gamma^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\delta_{1i} \right] q m_i \, d\Gamma - \cancel{\int_{\Gamma_+ + \Gamma_-} \sigma_{2j} \frac{\partial u_j}{\partial x_1} q \, d\Gamma}$$

$m_i = -n_i$ on Γ_o,
$m_1 = 0$ and $m_2 = \pm 1$ on Γ_- and Γ_+.



- q = arbitrary but smooth function that is equal to unity on Γ_o and zero on Γ_1.
- No crack-face tractions
  $\Rightarrow$ the second term becomes zero.

Inner and outer contours, which form a closed contour around the crack tip when connected by Γ_+ and Γ_-.

Theoretical Background

- Applying the divergence theorem

$$J=\int_{A^*}\frac{\partial}{\partial x_i}\left\{\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1}-w\delta_{1i}\right]q\right\}dA$$

$$=\int_{A^*}\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1}-w\delta_{1i}\right]\frac{\partial q}{\partial x_i}dA+\cancel{\int_{A^*}\left[\frac{\partial}{\partial x_i}\left(\sigma_{ij}\frac{\partial u_j}{\partial x_1}\right)-\frac{\partial w}{\partial x_1}\right]qdA}$$

$$\boxed{J=\int_{\Gamma^*}\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1}-w\delta_{1i}\right]qm_i\,d\Gamma}$$

$$\boxed{\iint_R\left(\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right)dxdy=\oint_C\left(F_1dx+F_2dy\right)}$$

$$\boxed{\frac{\partial}{\partial x_i}\left(\sigma_{ij}\frac{\partial u_j}{\partial x_1}\right)-\frac{\partial w}{\partial x_1}=0}$$

From (3.2)

when there are no body forces and w exhibits the properties of an elastic potential and divide w into elastic and plastic components.

$$\sigma_{ij}=\frac{\partial w}{\partial \varepsilon_{ij}} \qquad w=w^e+w^p=\int_0^{\varepsilon_{kl}^e}\sigma_{ij}d\varepsilon_{ij}^e+\int_0^{\varepsilon_{kl}^p}S_{ij}d\varepsilon_{ij}^p \qquad S_{ij}=\sigma_{ij}-\frac{1}{3}\sigma_{kk}\delta_{ij}$$

S_{ij} is the deviatoric component of the stress tensor

- For a linear or nonlinear elastic material under quasistatic conditions, in the absence of body forces, thermal strains, and crack-face tractions.

$$J=\int_{A^*}\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1}-w\delta_{1i}\right]\frac{\partial q}{\partial x_i}dA$$

- The q function is merely a mathematical device that enables the generation of an area integral, which is better suited to numerical calculations.

Finite Element Implementation

- The q function must be specified at all nodes within the area or volume of integration. The shape of the q function is arbitrary, as long as q has the correct values on the domain boundaries.
- In a plane stress or plane strain problem, for example, q = 1 at Γ_o, which is usually the crack tip, and q = 0 at the outer boundary.

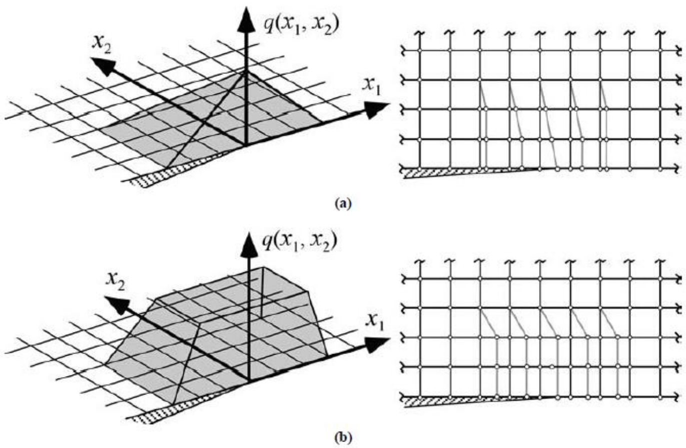
$$q(x_i) = \sum_{I=1}^{n} N_I q_I$$

where

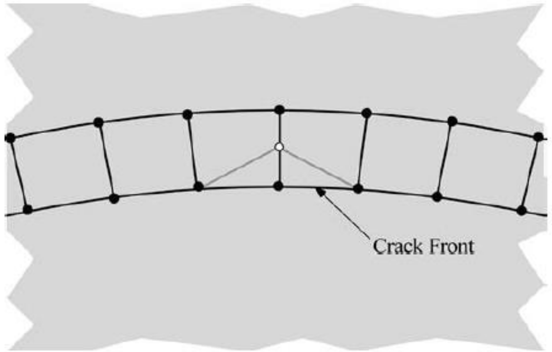
n = number of nodes per element

q_I = nodal values of q

N_I = element shape functions, which



Examples of q functions in two dimensions, with the corresponding virtual nodal displacements: (a) the pyramid function and (b) the plateau function



Definition of q in terms of a virtual nodal displacement along a three-dimensional crack front

Finite Element Implementation

$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\delta_{1i} \right] \frac{\partial q}{\partial x_i} dA$$

- The spatial derivatives of q are given by

$$\frac{\partial q}{\partial x_i} = \sum_{I=1}^{n} \sum_{k=1}^{2 \text{ or } 3} \frac{\partial N_I}{\partial \xi_k} \frac{\partial \xi_k}{\partial x_j} q_I$$

where ξ_i are the parametric coordinates for the element.

- In the absence of thermal strains, path-dependent plastic strains, and body forces within the integration volume or area, the discretized form of the domain integral is :

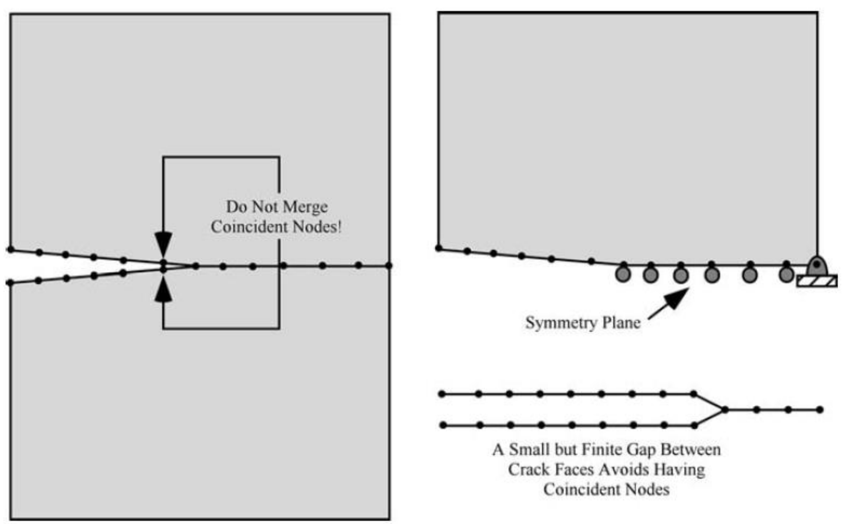
$$J = \sum_{A^* \text{ or } V^*} \sum_{p=1}^{m} \left\{ \left[\left(\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\delta_{1i} \right) \frac{\partial q}{\partial x_i} \right] \det\left(\frac{\partial x_j}{\partial \xi_k} \right) \right\}_p w_p$$

$$- \sum_{\text{crack faces}} \left(\sigma_{2j} \frac{\partial u_j}{\partial x_1} q \right) w$$

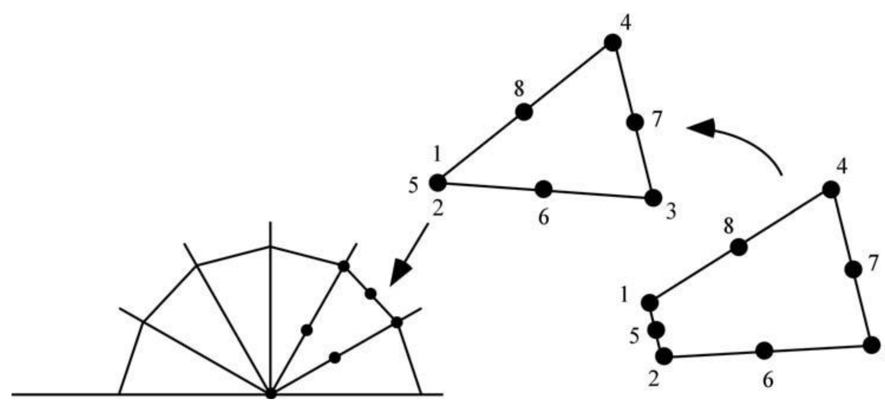
- m is the number of Gaussian points per element, and w_p and w are weighting factors. The quantities within $\{\ \}_p$ are evaluated at the Gaussian points.

Arrangement of nodes

- When both crack faces are modeled, there are usually matching nodes along each crack face. If these matching node pairs have identical coordinates, a small gap is maintained between the crack faces not to merge the nodes.
- At the crack tip, quadrilateral elements (in two-dimensional problems) are usually collapsed down to triangle, degeneration of a quadrilateral element into a triangle ⇒ exhibits a 1/r strain singularity, plastic singularity element.



Examples of nodes on the crack plane in two-dimensional finite element and boundary element models.



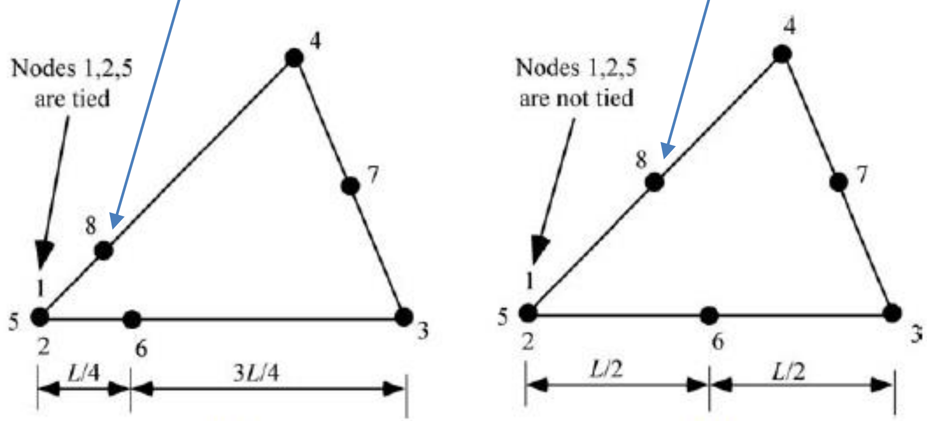
Degeneration of a quadrilateral element into a triangle at the crack tip.

Arrangement of nodes

❖Elastic singularity element and Plastic singularity element

¼ point node

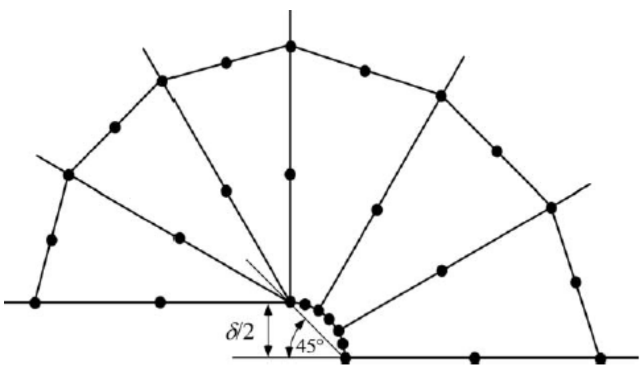
mid-side nodes



(a) (b)

$1/\sqrt{r}$ singularity,
Elastic singularity element

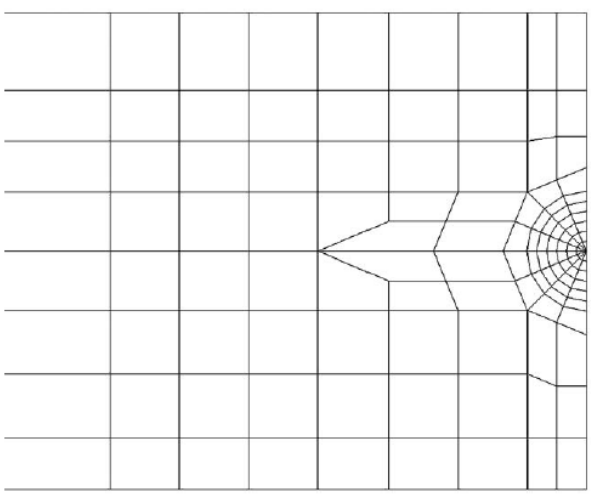
$1/r$ singularity,
Plastic singularity element



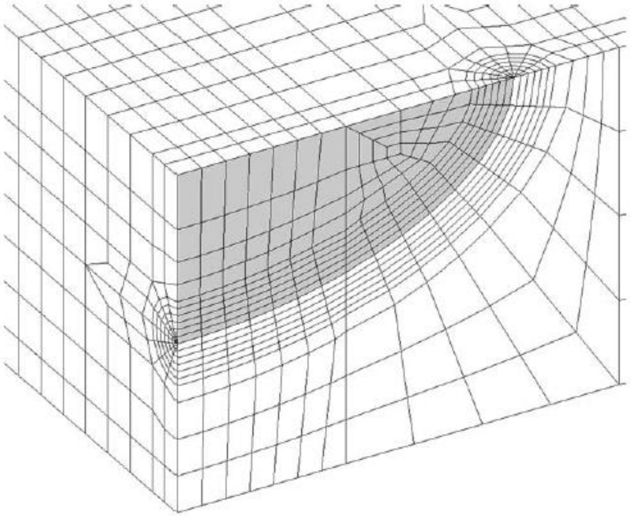
Deformed shape of plastic singularity elements. The crack-tip elements model blunting, and it is possible to measure CTOD.

Spider-web design

- For typical problems, the most efficient mesh design for the crack-tip region has proven to be the “spider-web” configuration, which consists of concentric rings of quadrilateral elements that are focused toward the crack tip.
- The elements in the innermost ring are degenerated to triangles.
- Since the crack tip region contains steep stress and strain gradients, the mesh refinement should be greatest at the crack-tip.
- The spider-web design facilitates a smooth transition from a fine mesh at the tip to a coarser mesh remote from the tip.
- This configuration results in a series of smooth, concentric integration domains (contours) for evaluating the J integral



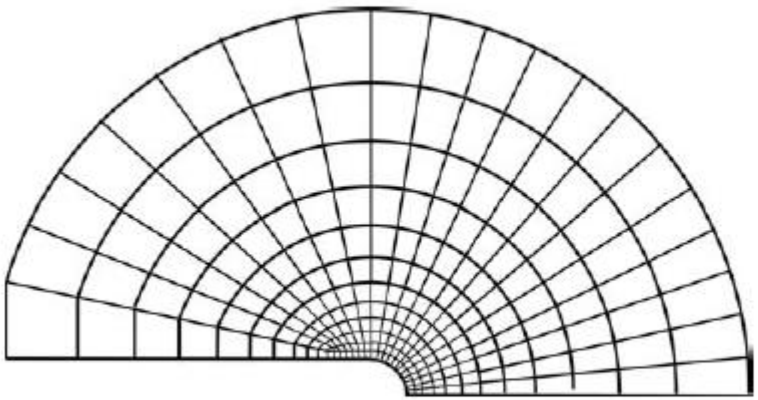
Half-symmetric two-dimensional model of an edge-cracked plate.



Quarter-symmetric three-dimensional model of a semielliptical surface crack in a flat plate.

Spider-web design

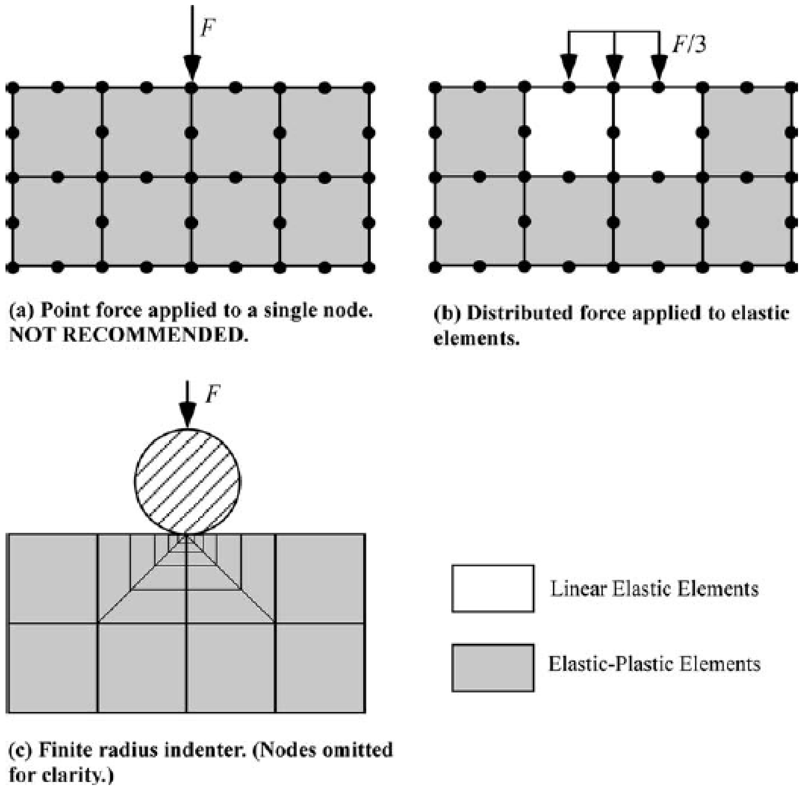
- Elastic analyses of stress intensity or energy release rate can be accomplished with relatively coarse meshes since modern methods, such as the domain integral approach, eliminate the need to resolve local cracktip fields accurately.
- The area and volume integrations in the newer approaches are relatively insensitive to mesh size for elastic problems. The mesh should include singularity elements at the crack tip, however, when the domain corresponds to the first ring of elements at the tip.
- If the domain is defined over a larger portion of the mesh, singularity elements are unnecessary because the crack tip elements contribute little to J.
- In a large-strain, nonlinear-geometry analysis, it is customary to begin with a finite radius at the crack tip. Note that the crack-tip elements are not collapsed to triangles in this case. Provided the CTOD after deformation is at least five times the initial value, the results should not be affected by the initial blunt notch.



Crack-tip region of a mesh for large strain analysis. Note that the initial crack-tip radius is finite and the crack-tip elements are not degenerated.

Application of force

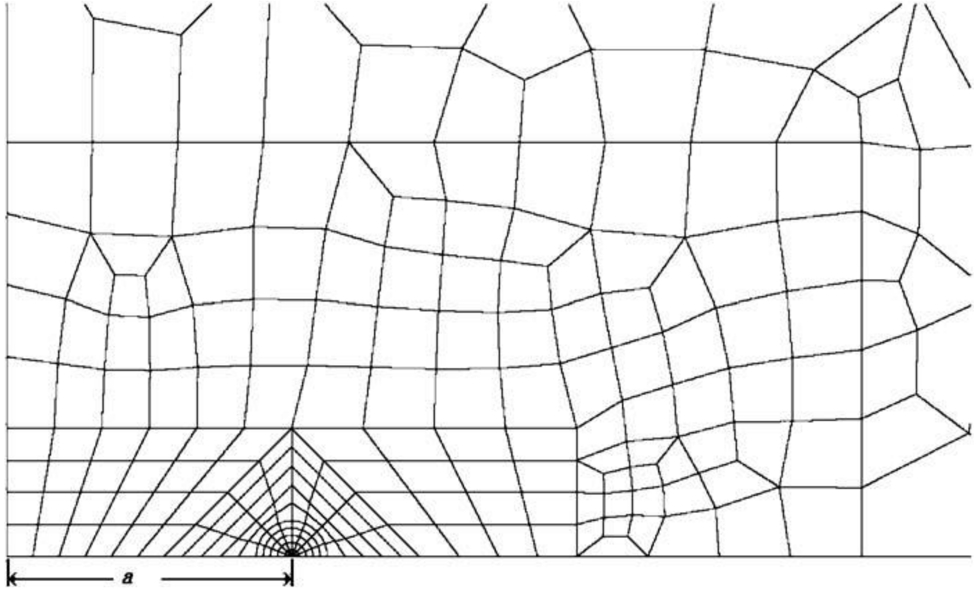
- Many problems require forces to be applied at the boundaries of the body.



Examples of improper (a) and proper (b and c) methods for applying a force to a boundary.

Convergence study model

- These analyses were performed on a through-thickness crack in a flat plate subject to either a remote membrane stress or a uniform crack face pressure. The plate width was 20 times the crack length, so the model approximated the so-called Griffith plate, where the width is infinite.



Close-up of the crack tip region of the baseline quarter-symmetric two-dimensional plane strain model used in the convergence study

Dimensionless stress-intensity factor

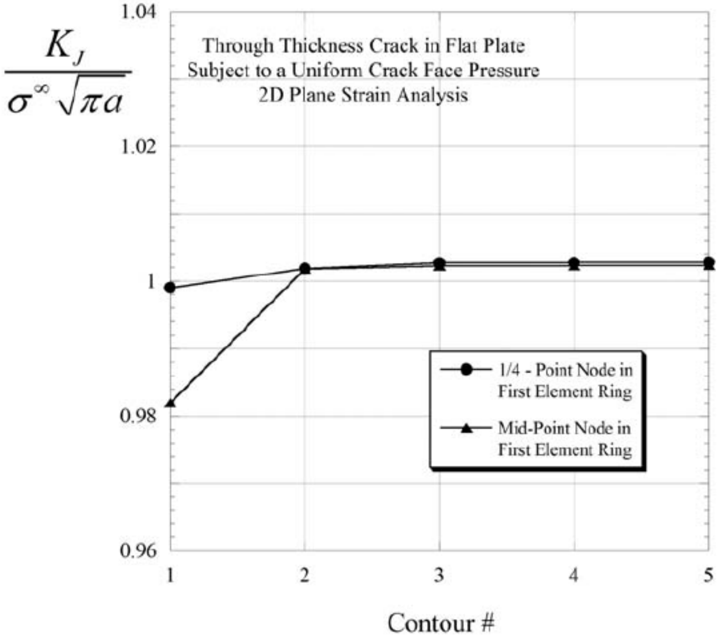
- The *J* integral was converted to the Mode I stress intensity factor

$$\mathcal{G} = \frac{K_I^2}{E'}$$

- The 1/4-point node location in the collapsed elements in the first ring improves the *J* estimate in the first contour but has little effect on the second and higher contours.
- The stress-intensity factor computed from the *J* integral is within 0.3% of the theoretical solution despite the fact that the baseline mesh is not particularly refined at the crack tip.

- *J*-integral method is an efficient way to compute *KI*, in that a high degree of mesh refinement is not required.

- 1/4-point node location elastic singularity elements are not necessary, provided the integration domain includes more than just the first ring of elements.



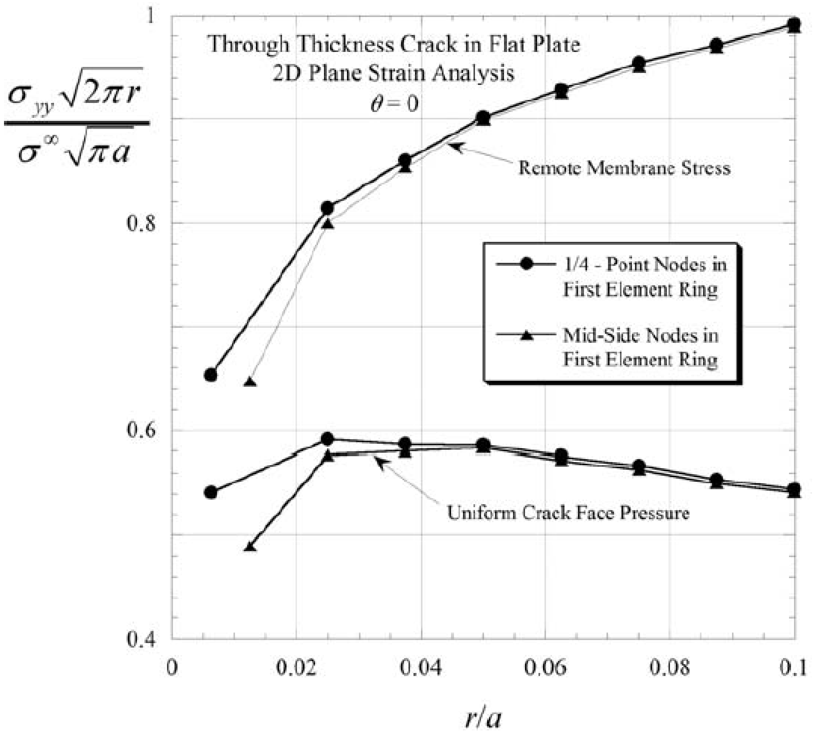
Dimensionless stress-intensity factor inferred from a *J*-integral analysis

Stress-matching vs. Displacement-matching method

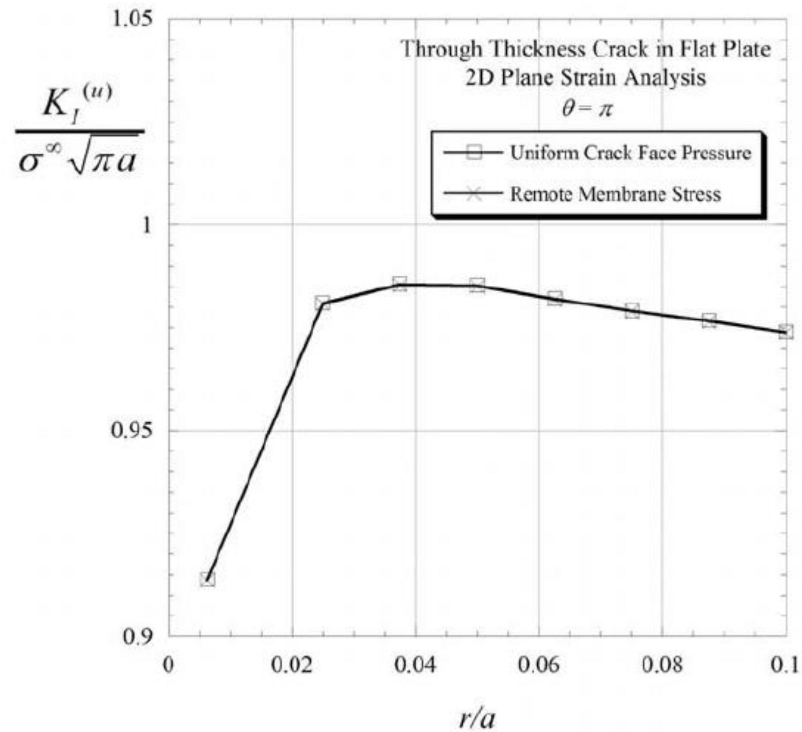
- Stress-Matching method vs. Displacement-matching method for Remote membrane stress and Uniform crack face pressure.

$$K_I = \lim_{r \to 0} [\sigma_{yy} \sqrt{2\pi r}] \quad (\theta = 0)$$

$$K_I = \lim_{r \to 0} \left[\frac{E u_y}{4(1-\nu^2)} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$



Dimensionless stress-intensity factor estimated from the normal stresses in front of the crack tip

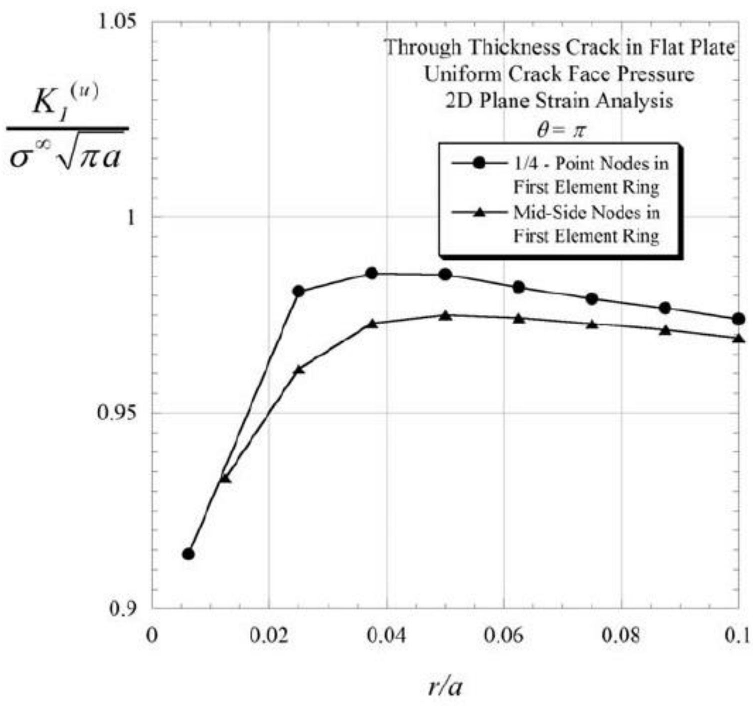


Dimensionless stress-intensity factor estimated from the displacements behind the crack tip.

Stress-matching vs. Displacement-matching method

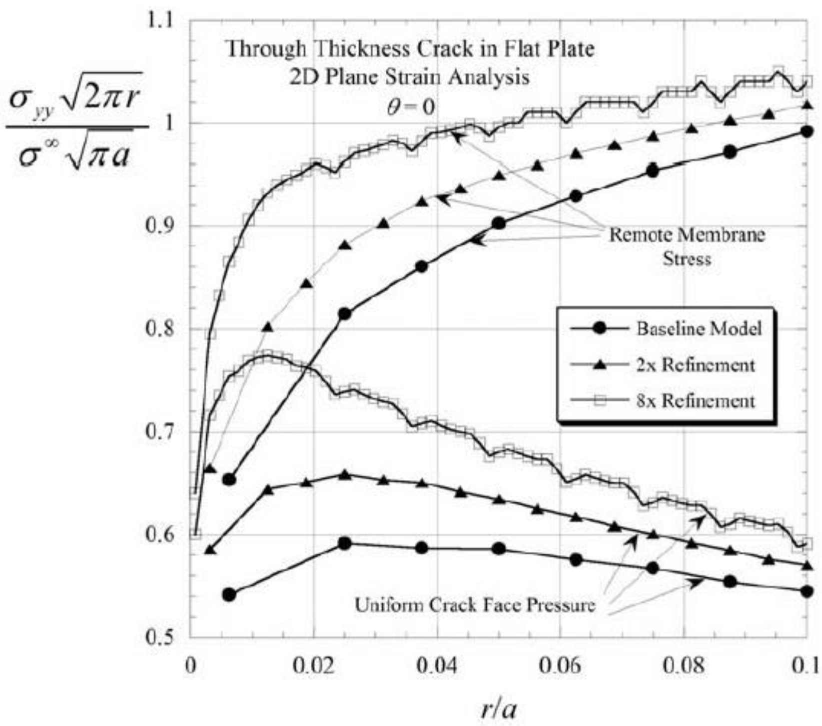
- Stress-Matching method vs. Displacement-matching method for Remote membrane stress and Uniform crack face pressure.

$$K_I = \lim_{r\to 0}\left[\frac{Eu_y}{4(1-\nu^2)}\sqrt{\frac{2\pi}{r}}\right] \quad (\theta=\pi)$$



Effect of elastic singularity elements at the crack tip on K_I estimated from opening displacements behind the crack tip in the baseline model.

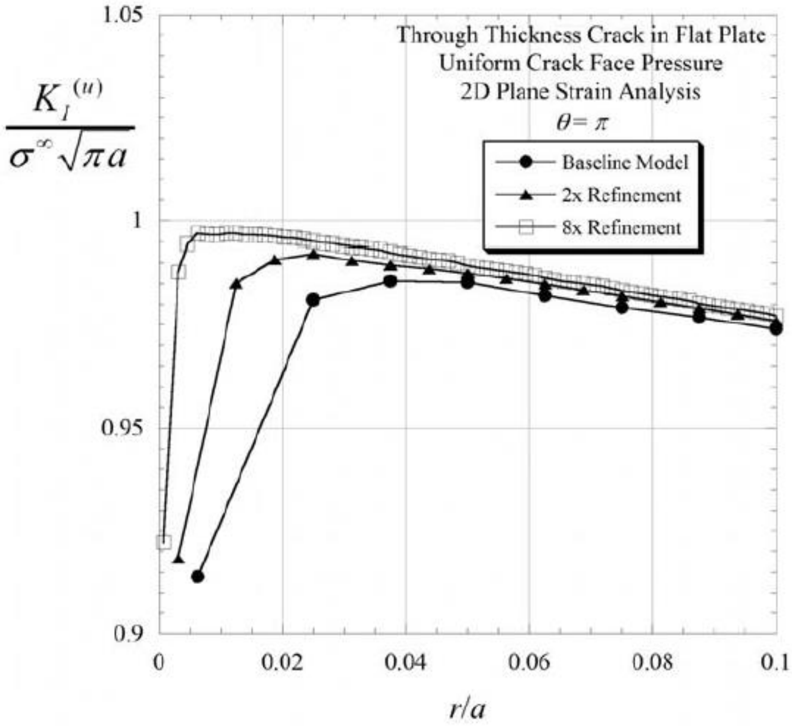
$$K_I = \lim_{r\to 0}[\sigma_{yy}\sqrt{2\pi r}] \quad (\theta=0)$$



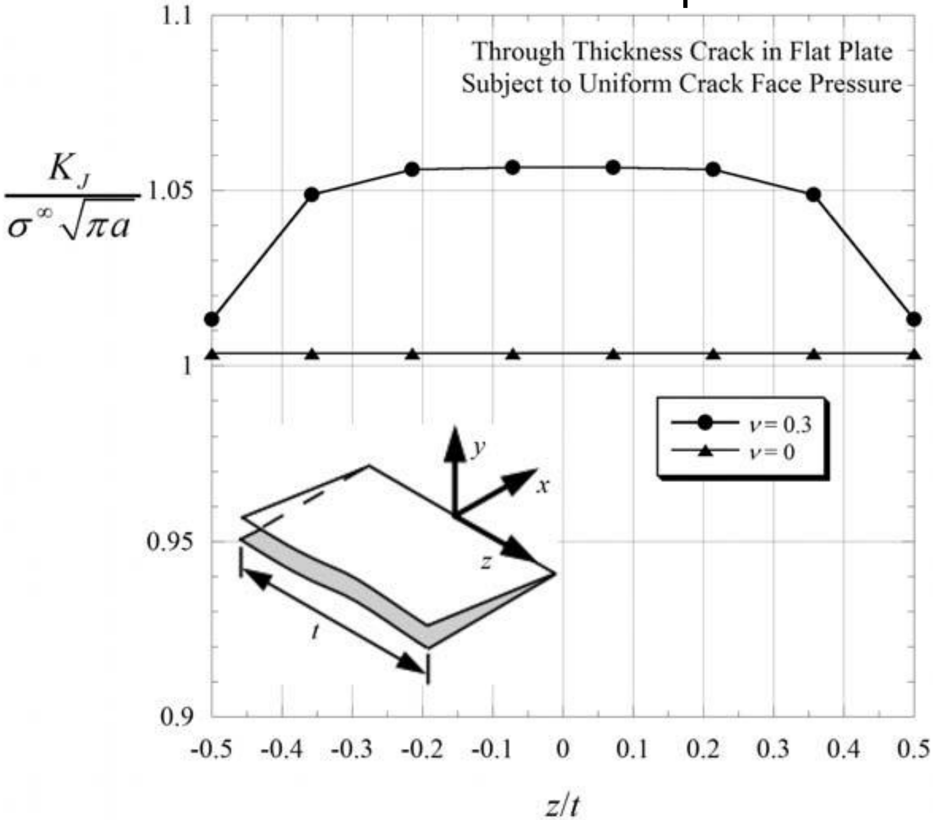
Effect of mesh refinement at the crack tip on K_I estimated from normal stresses in front of the crack tip..

Mesh refinement effect and Poisson ratio effect

ν= 0.3 : a real three-dimensional effect
ν = 0 : the three-dimensional model behaves like a two-dimensional plate



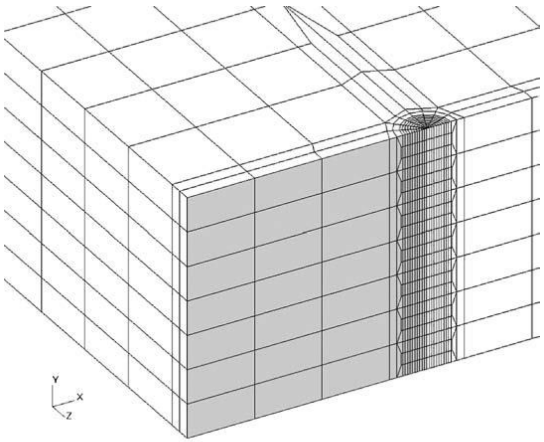
Effect of mesh refinement at the crack tip on KI estimated from opening displacements behind crack tip.



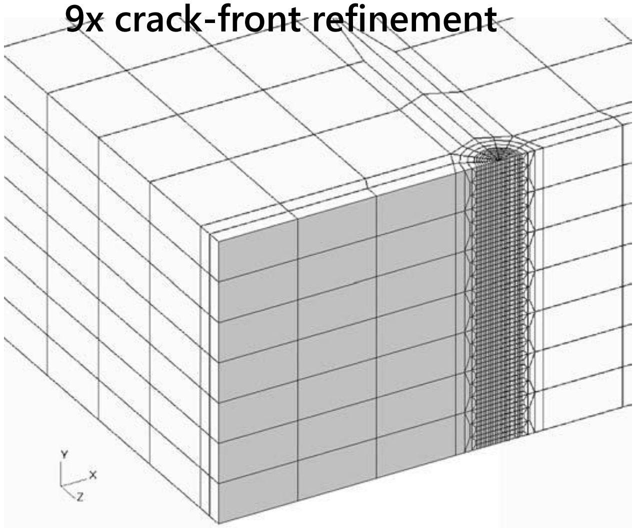
Through-thickness variation of the Mode I stress-intensity factor inferred from a *J*-integral analysis of the three-dimensional model

Mesh refinement effect

3x crack-front refinement



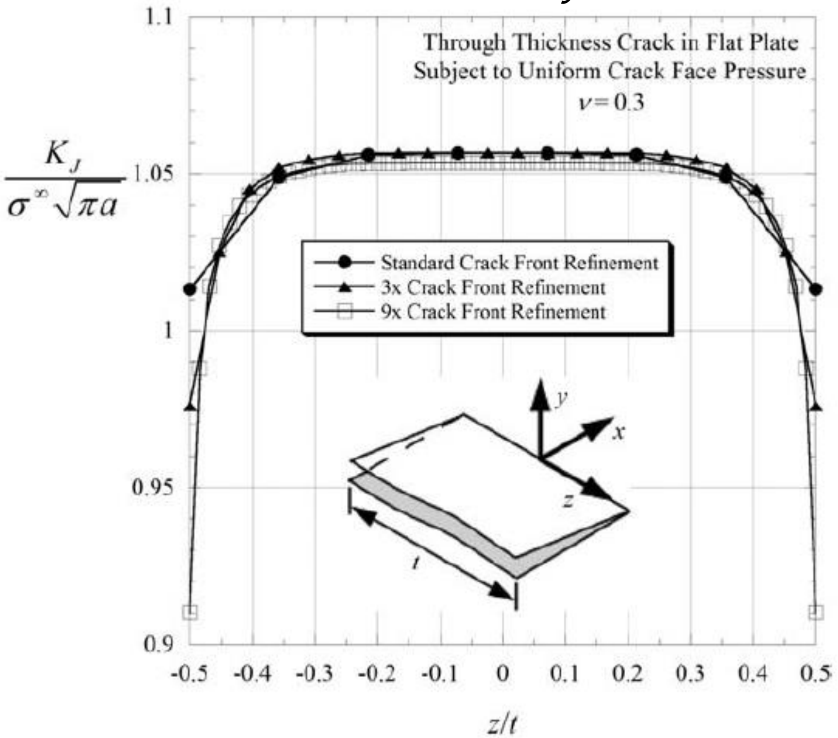
9x crack-front refinement



Refinement along the crack front of the three-dimensional model

The value on the free surface continually decreases with refinement along the crack front.

"The theoretical value of the J integral on the free surface of a three-dimensional body is zero."



Effect of crack-front refinement on the through-thickness variation of K_J

Summary

- In summary, the domain integral method is the most efficient means to infer stress-intensity factors solutions from finite element analysis.

- If one’s finite element software does not include such capabilities, the displacement-matching technique is an acceptable alternative, provided the mesh refinement is sufficient for convergence. The stress-matching method requires a very high level of mesh refinement, so it is not recommended.

- Finally, the level of mesh refinement required for convergence is problem-specific, since it depends on the geometry and loading.

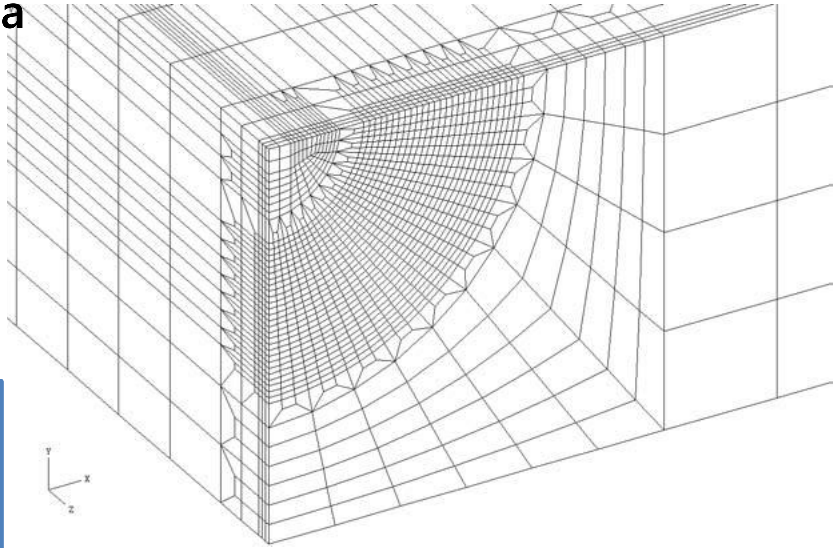
- The convergence results presented here are for purposes of illustration and should not be used as the sole basis for demonstrating convergence for a different problem.

Cell-type mesh

- Moving the crack tip in a focused mesh normally entails re-meshing.
- A new focused mesh with a slightly longer crack must be created. Re-meshing is appropriate for elastic problems because stress and strain are not history dependent.
- Crack growth by re-meshing is possible in principle, provided the prior plastic strain history is properly mapped onto the various models created at each step. However, this approach is highly cumbersome.
- A better alternative is to create a single mesh that accommodates crack growth. One such mesh configuration is the *cell mesh*.

- Three common methods to advance a crack in a cell mesh.

✓ Removing elements
✓ Release nodes
✓ Use cohesive elements



Cell-type mesh for analysis of crack growth in a semielliptical surface crack in a flat plate

Cell-type mesh

- Irrespective of the numerical crack growth strategy each increment of crack advance corresponds to the element size.
- For this reason the crack growth response in a finite element simulation is mesh dependent.
- In real materials, the crack growth response (e.g., the *J* resistance curve) depends on material length scales such as inclusion spacing. A finite element continuum model does not include microstructural features such as inclusions, so element size is the only available length scale to govern crack growth.
- Crack growth simulations usually need to be tuned to match experimental data. One of the key tuning parameters is the element size in the cell zone on the crack plane.