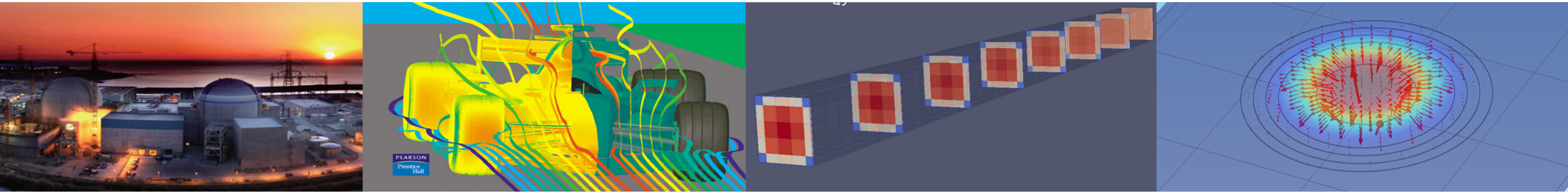

COMPUTATIONAL NUCLEAR THERMAL HYDRAULICS

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CHAPTER4.
THE FINITE VOLUME METHOD FOR DIFFUSION PROBLEMS

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Introduction

- General transport equation

$$\frac{\partial(\rho\phi)}{\partial t} + \mathrm{div}(\rho\phi\mathbf{u}) = \mathrm{div}(\Gamma \ \mathrm{grad}\ \phi) + S_\phi$$

  - Deleting the transient and convection terms,

$$\mathrm{div}(\Gamma \ \mathrm{grad}\ \phi) + S_\phi = 0$$

- Integral form

$$\int_{CV} \mathrm{div}(\Gamma \ \mathrm{grad}\ \phi)\mathrm{d}V + \int_{CV} S_\phi \mathrm{d}V = \boxed{}$$

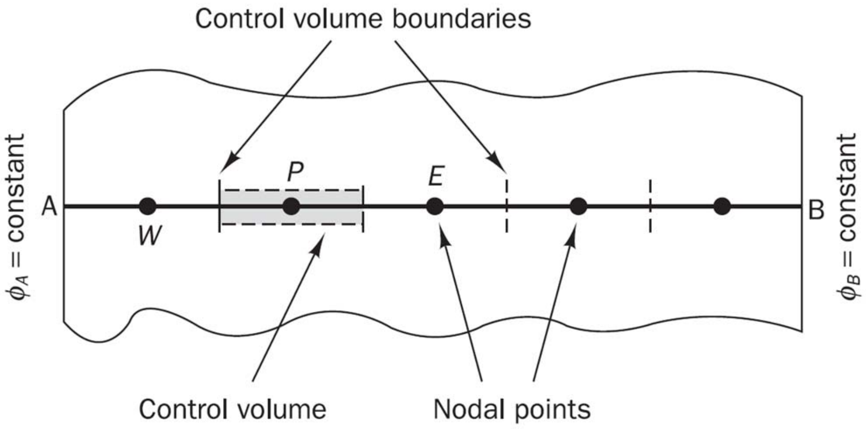
- From this 1D steady state diffusion equation, the discretized eqs. are introduced.
- The method is extended to 2D and 3D diffusion problems.

Contents

FVM for 1D steady state diffusion

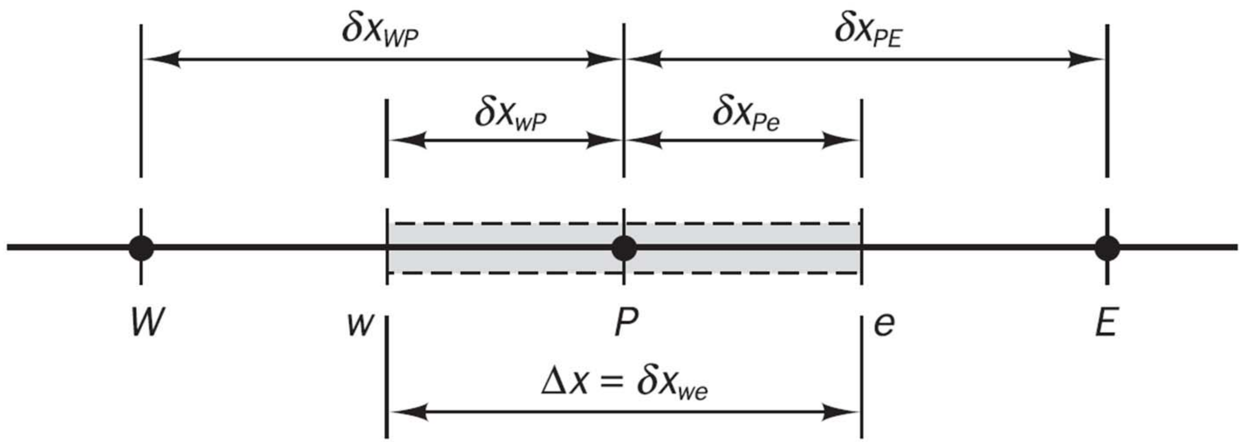
- 1D diffusion equation

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\Gamma\frac{\mathrm{d}\phi}{\mathrm{d}x}\right)+S=0$$



Control volume for FVM

- Step 1: grid generation



Usual convention of CFD methods

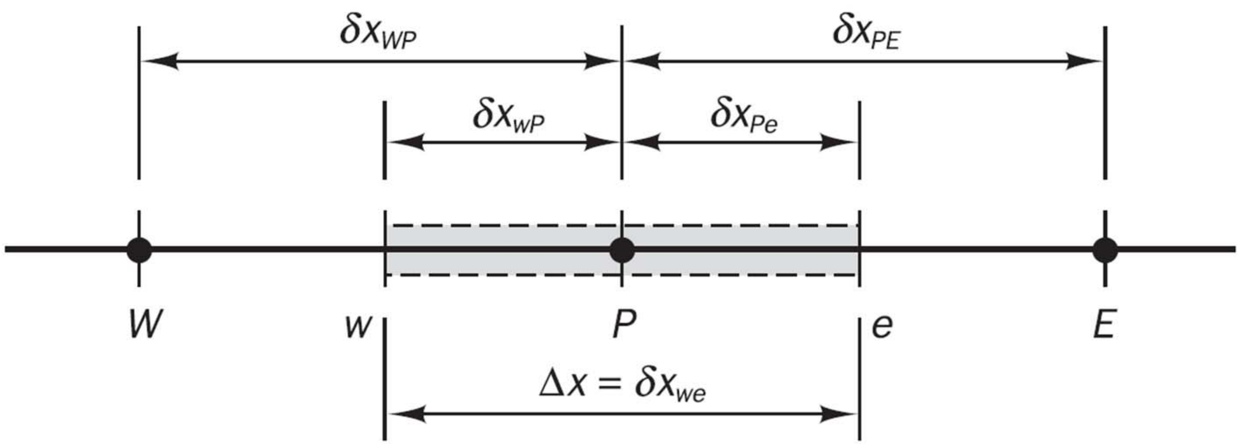
FVM for 1D steady state diffusion

- Step 2: discretization

$$\int_{\Delta V} \frac{\mathrm{d}}{\mathrm{d}x}\left(\Gamma \frac{\mathrm{d}\phi}{\mathrm{d}x}\right)\mathrm{d}V + \int_{\Delta V} S\,\mathrm{d}V = \int_A \mathbf{n}\cdot\left(\Gamma \frac{d\phi}{dx}\right)dA + \int_{\Delta V} S dV = \left(\Gamma A \frac{d\phi}{dx}\right)_e - \left(\Gamma A \frac{d\phi}{dx}\right)_w + \bar{S}\Delta V = 0$$

$$\int_A \mathbf{n}\cdot\left(\Gamma \frac{d\phi}{dx}\right)dA = \sum_f \left[n_x \cdot \left(\Gamma \frac{d\phi}{dx}\right)\cdot A\right]_f =$$

$$(n_x)_e = 1,\ (n_x)_w = -1$$



FVM for 1D steady state diffusion

- Step 2: discretization

$$\left(\Gamma A \frac{d\phi}{dx}\right)_e - \left(\Gamma A \frac{d\phi}{dx}\right)_w + \bar{S}\Delta V = 0$$

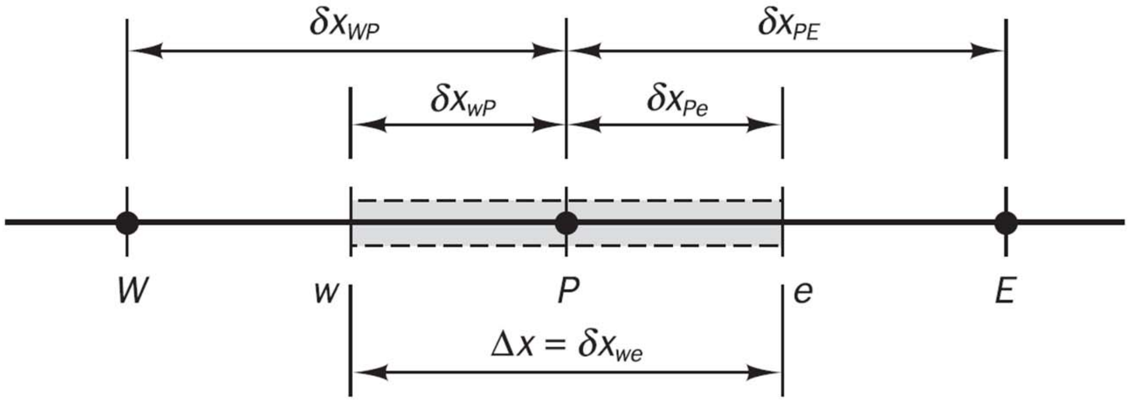
$$\frac{d\phi}{dx} \; ? \qquad (\Gamma)_e \; (\Gamma)_w \; ?$$

- Taylor series approximations

$$\phi(x+\Delta x) = \phi(x) + \left(\frac{\partial \phi}{\partial x}\right)_x \Delta x + \left(\frac{\partial^2 \phi}{\partial x^2}\right)_x \frac{\Delta x^2}{2} + \cdots$$

$$\frac{d\phi}{dx} \approx \frac{\phi(x+\Delta x) - \phi(x)}{\Delta x}$$

$$\left(\frac{d\phi}{dx}\right)_e \approx \qquad \left(\frac{d\phi}{dx}\right)_w \approx$$



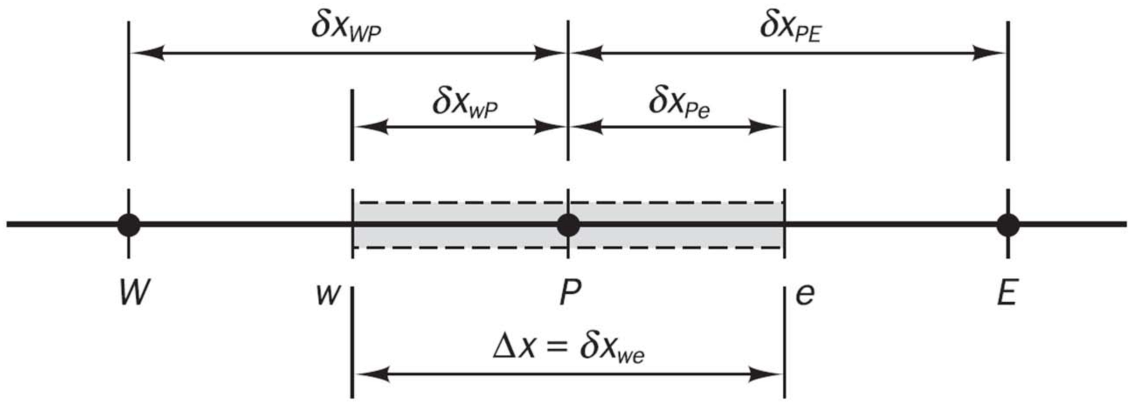
FVM for 1D steady state diffusion

- Step 2: discretization

$$\left(\Gamma A\frac{d\phi}{dx}\right)_e-\left(\Gamma A\frac{d\phi}{dx}\right)_w+\bar{S}\Delta V=0$$

$$\frac{d\phi}{dx}\ ? \qquad (\Gamma)_e\ (\Gamma)_w\ ?$$

- Interface properties (for uniform grid)



$\Gamma_w =$ $\qquad$ $\Gamma_e =$

$\Gamma_w =$ $\qquad$ $\Gamma_e =$

$\lambda_W =$ $\qquad$ $\lambda_E =$

FVM for 1D steady state diffusion

- Step 2: discretization

$$\left(\Gamma A\frac{d\phi}{dx}\right)_e-\left(\Gamma A\frac{d\phi}{dx}\right)_w+\bar{S}\Delta V=0$$

$$\left(\Gamma A\frac{d\phi}{dx}\right)_e=\Gamma_e A_e\left(\frac{\phi_E-\phi_P}{\delta x_{PE}}\right) \qquad \left(\Gamma A\frac{d\phi}{dx}\right)_w=\Gamma_w A_w\left(\frac{\phi_P-\phi_W}{\delta x_{WP}}\right) \qquad \bar{S}\Delta V=S_u+S_p\phi_p$$

$$\Gamma_e A_e\left(\frac{\phi_E-\phi_P}{\delta x_{PE}}\right)-\Gamma_w A_w\left(\frac{\phi_P-\phi_W}{\delta x_{WP}}\right)+S_u+S_p\phi_p=0$$

$$\boxed{}\,\phi_P=\boxed{}\,\phi_W+\boxed{}\,\phi_E+S_u$$

FVM for 1D steady state diffusion

- Step 2: discretization

$$\left(\frac{\Gamma_e}{\delta x_{PE}}A_e + \frac{\Gamma_w}{\delta x_{WP}}A_w - S_p\right)\phi_P = \left(\frac{\Gamma_w}{\delta x_{WP}}A_w\right)\phi_W + \left(\frac{\Gamma_e}{\delta x_{PE}}A_e\right)\phi_E + S_u$$

$$\boxed{a_P\phi_P = a_W\phi_W + a_E\phi_E + S_u}$$

a_W	a_E	a_P
$\frac{\Gamma_w}{\delta x_{WP}}A_w$	$\frac{\Gamma_e}{\delta x_{PE}}A_e$	$a_W + a_E - S_P$

FVM for 1D steady state diffusion

- Step 3: Solution of equations

$$\boxed{a_P\phi_P = a_W\phi_W + a_E\phi_E + S_u}$$

- Set up the discretized equation at each of the nodal points
- Modify the equation for the control volumes that are adjacent to the domain boundaries.
- Derive the system of linear algebraic equations
- Matrix solution techniques

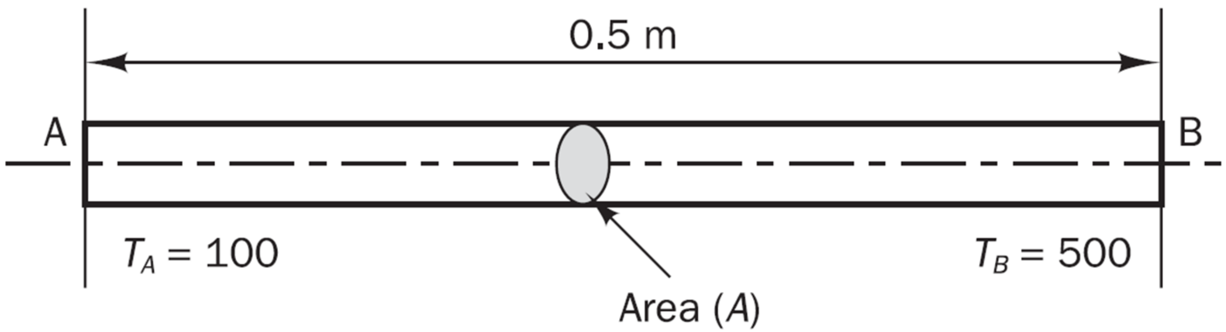
Contents

Worked examples: 1D steady state diffusion

- 1D Heat conduction equation

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(k\frac{\mathrm{d}T}{\mathrm{d}x}\right)+S=0$$

- Example 4.1
  - Consider the problem of source-free heat conduction in an insulated rod
  - Whose ends are maintained at constant temperatures of 100 °C and 500 °C respectively.
  - Calculate the steady state temperature in the rod.
  - Thermal conductivity k = 1000 W/mK
  - Cross-sectional area A =10×10^{-3} m^2



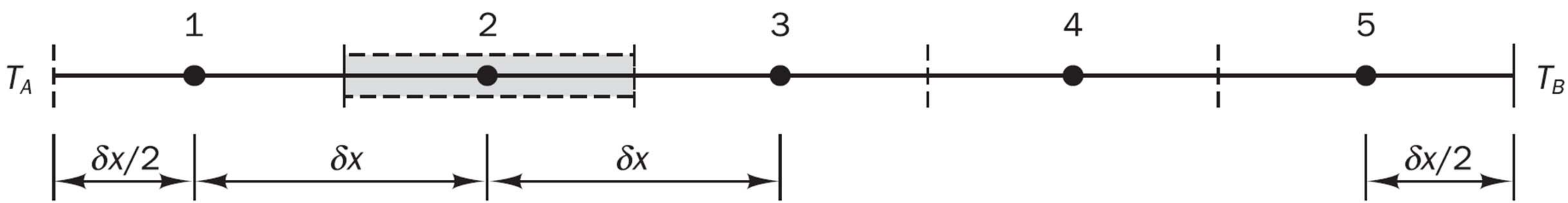
Worked examples: 1D steady state diffusion

- Example 4.1
  - Five control volumes (cells)
    - Cell center nodes: 5
    - Cell faces: 6 (4 internal faces, 2 boundary faces)

$$\left(\frac{\Gamma_e}{\delta x_{PE}}A_e+\frac{\Gamma_w}{\delta x_{WP}}A_w-S_p\right)\phi_P=\left(\frac{\Gamma_w}{\delta x_{WP}}A_w\right)\phi_W+\left(\frac{\Gamma_e}{\delta x_{PE}}A_e\right)\phi_E+S_u$$

$$\left(\frac{k_e}{\delta x_{PE}}A_e+\frac{k_w}{\delta x_{WP}}A_w\right)T_P=\left(\frac{k_w}{\delta x_{WP}}A_w\right)T_W+\left(\frac{k_e}{\delta x_{PE}}A_e\right)T_E$$

    - Constant thermal conductivity
    - Constant node spacing
    - Constant cross-sectional area
    - No source term



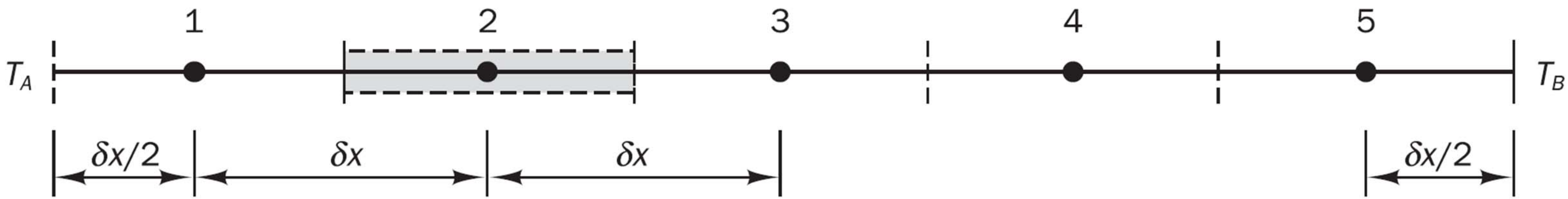
Worked examples: 1D steady state diffusion

- Example 4.1
  - Five control volumes (cells)

$$\left(\frac{k_e}{\delta x_{PE}}A_e + \frac{k_w}{\delta x_{WP}}A_w\right)T_P = \left(\frac{k_w}{\delta x_{WP}}A_w\right)T_W + \left(\frac{k_e}{\delta x_{PE}}A_e\right)T_E$$

$$a_P T_P = a_W T_W + a_E T_E$$

a_W	a_E	a_P
$\frac{k}{\delta x}A$	$\frac{k}{\delta x}A$	$a_W + a_E$



Worked examples: 1D steady state diffusion

- Example 4.1
  - For boundary #1

$$k_e A_e\left(\frac{T_E - T_P}{\delta x_{PE}}\right) - k_w A_w\left(\frac{T_P - T_W}{\delta x_{WP}}\right) = 0$$

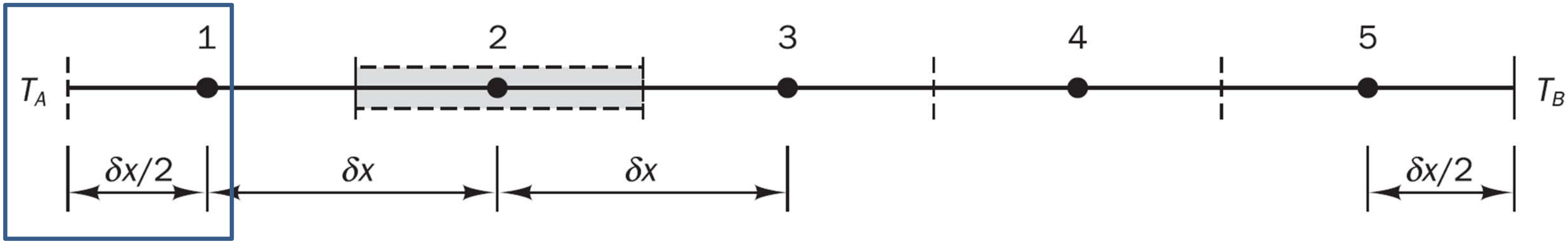
$$kA\left(\frac{T_E - T_P}{\delta x}\right) - \boxed{\quad} = 0$$

$$\left(\frac{k}{\delta x}A + \frac{2k}{\delta x}A\right)T_P = \boxed{\quad}$$

$$a_P T_P = a_W T_W + a_E T_E + S_u$$

a_W	a_E	a_P
$\frac{\Gamma_w}{\delta x_{WP}}A_w$	$\frac{\Gamma_e}{\delta x_{PE}}A_e$	$a_W + a_E - S_P$

a_W	a_E	a_P	S_P	S_u



Worked examples: 1D steady state diffusion

- Example 4.1
  - For boundary #5

$$k_e A_e\left(\frac{T_E - T_P}{\delta x_{PE}}\right) - k_w A_w\left(\frac{T_P - T_W}{\delta x_{WP}}\right) = 0$$

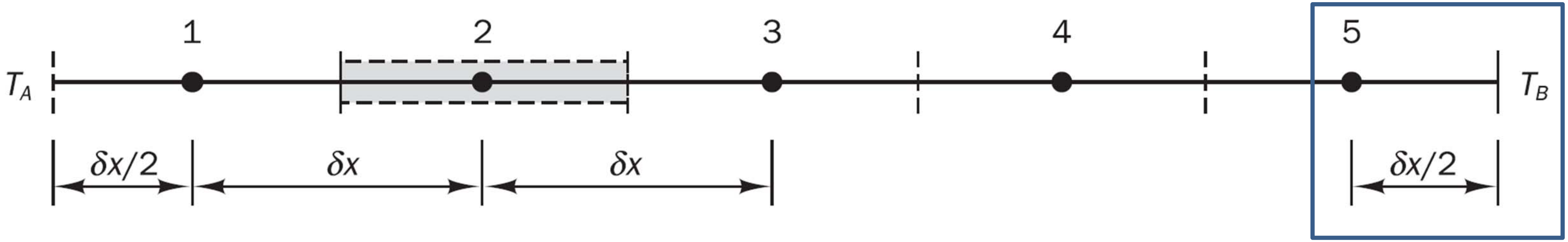
$$- kA\left(\frac{T_P - T_W}{\delta x}\right) = 0$$

$$\left(\frac{k}{\delta x}A + \frac{2k}{\delta x}A\right)T_P =$$

$$a_P T_P = a_W T_W + a_E T_E + S_u$$

a_W	a_E	a_P
$\frac{\Gamma_w}{\delta x_{WP}} A_w$	$\frac{\Gamma_e}{\delta x_{PE}} A_e$	$a_W + a_E - S_P$

a_W	a_E	a_P	S_P	S_u



Worked examples: 1D steady state diffusion

- Example 4.1
  - Systems of algebraic equations
    - Constant thermal conductivity (1000)
    - Constant node spacing (0.1)
    - Constant cross-sectional area (10×10^{-3})

$$kA/\delta x = 100$$

For cell #1 $$a_P T_P = a_W T_W + a_E T_E + S_u$$

a_W	a_E	a_P	S_P	S_u
0	$\frac{kA}{\delta x}$	$a_W + a_E - S_p$	$-\frac{2kA}{\delta x}$	$\frac{2kA}{\delta x}T_A$

For cell #2 ~#4 $$a_P T_P = a_W T_W + a_E T_E$$

a_W	a_E	a_P
$\frac{k}{\delta x}A$	$\frac{k}{\delta x}A$	$a_W + a_E$

For cell #5 $$a_P T_P = a_W T_W + a_E T_E + S_u$$

a_W	a_E	a_P	S_P	S_u
$\frac{kA}{\delta x}$	0	$a_W + a_E - S_p$	$-\frac{2kA}{\delta x}$	$\frac{2kA}{\delta x}T_B$

Worked examples: 1D steady state diffusion

- Example 4.1
  - Systems of algebraic equations

For cell #1 $$a_P T_P = a_W T_W + a_E T_E + S_u$$ $$300T_1 = 100T_2 + 200T_A$$

For cell #2 ~#4 $$a_P T_P = a_W T_W + a_E T_E$$ $$200T_2 = 100T_1 + 100T_3$$ $$200T_3 = 100T_2 + 100T_4$$ $$200T_4 = 100T_3 + 100T_5$$

For cell #5 $$a_P T_P = a_W T_W + a_E T_E + S_u$$ $$300T_5 = 100T_4 + 200T_B$$

Worked examples: 1D steady state diffusion

- Example 4.1
  - Systems of algebraic equations $\Rightarrow$ matrix form

$$300T_1 = 100T_2 + 200T_A$$

$$200T_2 = 100T_1 + 100T_3$$
$$200T_3 = 100T_2 + 100T_4$$
$$200T_4 = 100T_3 + 100T_5$$

$$300T_5 = 100T_4 + 200T_B$$

Linear solver
- Gaussian elimination
- LU decomposition
- TDMA
- ILU
- CGM
- BICG
- BICGSTAB
- Etc.

$$\begin{bmatrix} 300 & -100 & 0 & 0 & 0 \\ -100 & 200 & -100 & 0 & 0 \\ 0 & -100 & 200 & -100 & 0 \\ 0 & 0 & -100 & 200 & -100 \\ 0 & 0 & 0 & -100 & 300 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 200T_A \\ 0 \\ 0 \\ 0 \\ 200T_B \end{bmatrix}$$

Worked examples: 1D steady state diffusion

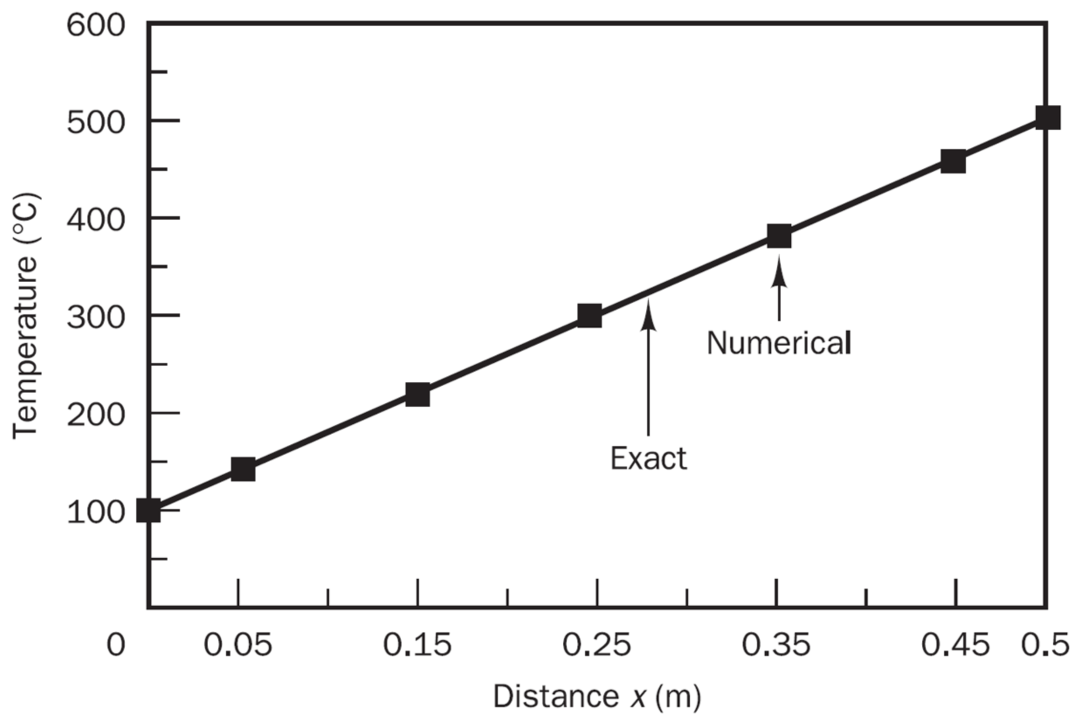
- Example 4.1

$$T_A = 100$$

$$T_B = 500$$

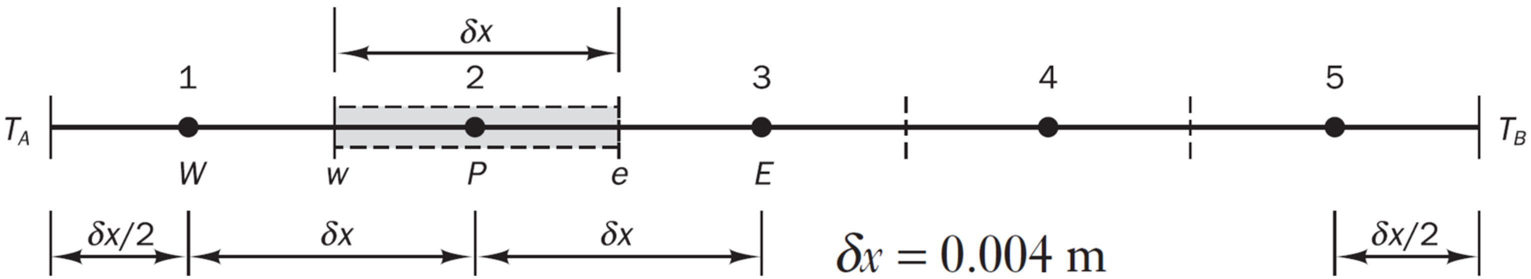
$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 140 \\ 220 \\ 300 \\ 380 \\ 460 \end{bmatrix}$$

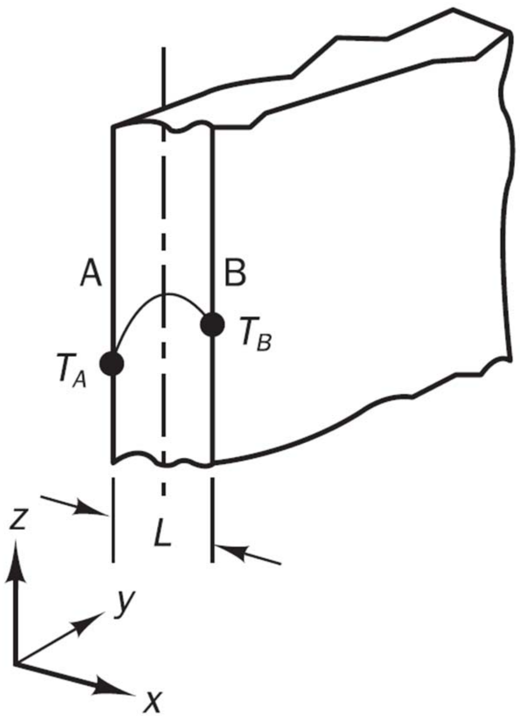
$$T = 800x + 100$$



Worked examples: 1D steady state diffusion

- Example 4.2
  - A problem that includes sources other than those arising from boundary conditions.
  - A large plate of thickness: L = 2 cm
  - Constant thermal conductivity: k = 0.5 W/m.K
  - Uniform heat generation: q = 1000 kW/m^3
  - The faces A and B are at temperatures: 100°C and 200°C respectively.
  - 1D problem
    - Dimensions in y- and z- are so large.





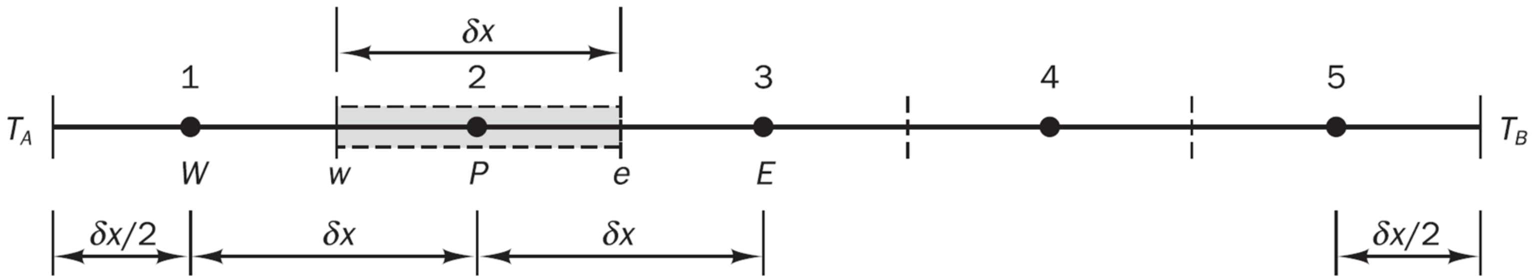
Worked examples: 1D steady state diffusion

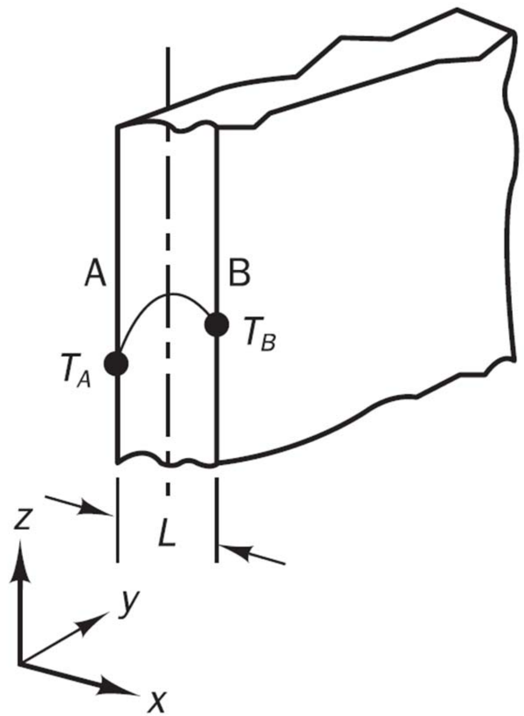
- Example 4.2
  - Integral form and discretization

$$\int_{\Delta V} \frac{\mathrm{d}}{\mathrm{d}x}\left(k\frac{\mathrm{d}T}{\mathrm{d}x}\right)\mathrm{d}V + \int_{\Delta V} q\,\mathrm{d}V = 0 \qquad \left[\left(kA\frac{\mathrm{d}T}{\mathrm{d}x}\right)_e - \left(kA\frac{\mathrm{d}T}{\mathrm{d}x}\right)_w\right] + q\Delta V = 0$$

$$\left[\boxed{\phantom{kA\frac{dT}{dx}}} - \boxed{\phantom{kA\frac{dT}{dx}}}\right] + \boxed{} = 0$$

$$\left(\frac{k_e A}{\delta x} + \frac{k_w A}{\delta x}\right)T_P = \left(\frac{k_w A}{\delta x}\right)T_W + \left(\frac{k_e A}{\delta x}\right)T_E + qA\delta x$$





Worked examples: 1D steady state diffusion

- Example 4.2
  - General form

$$\left(\frac{k_e A}{\delta x}+\frac{k_w A}{\delta x}\right)T_P=\left(\frac{k_w A}{\delta x}\right)T_W+\left(\frac{k_e A}{\delta x}\right)T_E+qA\delta x$$

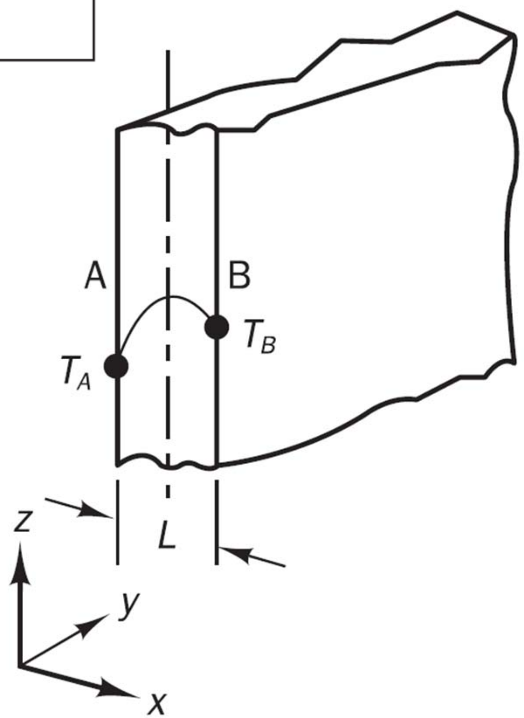
$$\boxed{a_P T_P = a_W T_W + a_E T_E + S_u}$$

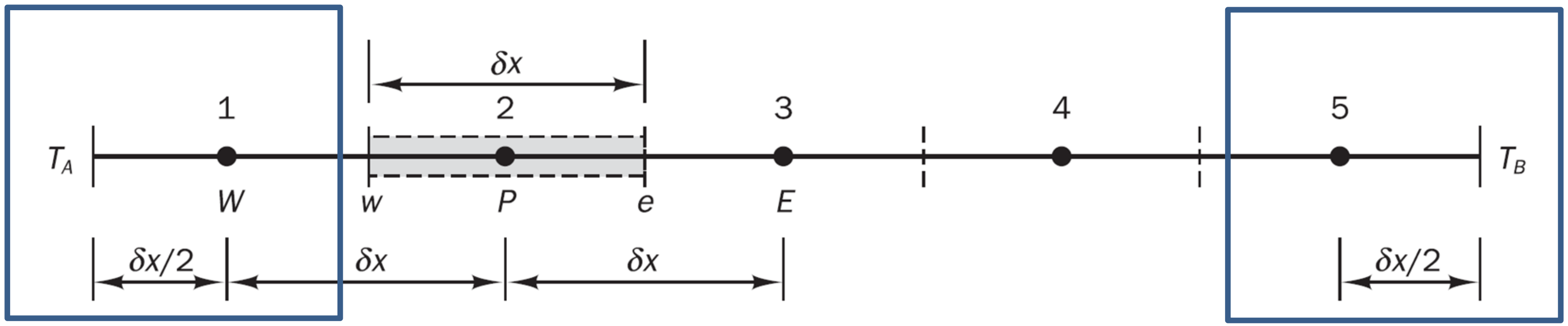
a_W	a_E	a_P	S_P	S_u

  - For boundaries

$$\left[k_e A\left(\frac{T_E-T_P}{\delta x}\right)-k_A A\left(\frac{T_P-T_A}{\delta x/2}\right)\right]+qA\delta x=0$$

$$\left[k_B A\left(\frac{T_B-T_P}{\delta x/2}\right)-k_w A\left(\frac{T_P-T_W}{\delta x}\right)\right]+qA\delta x=0$$





Worked examples: 1D steady state diffusion

- Example 4.2
  - For boundaries

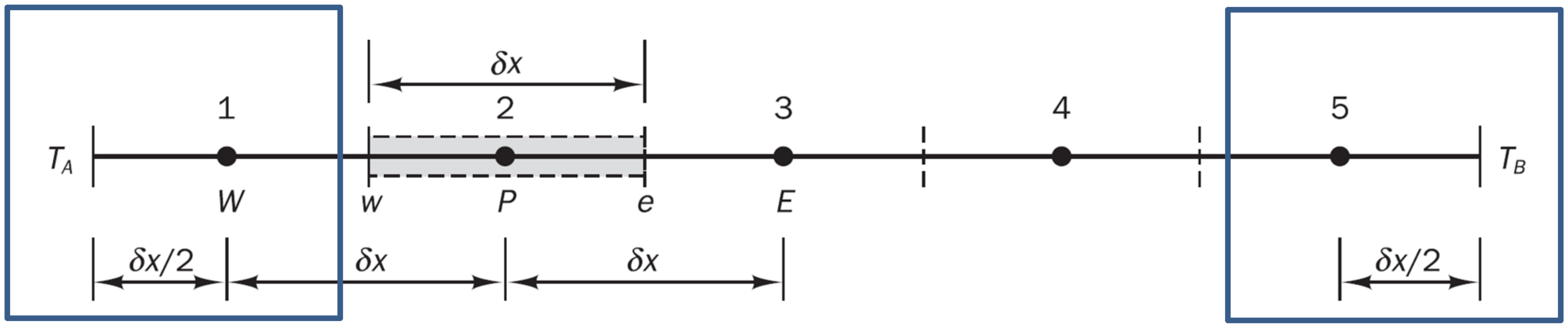
$$\left[k_e A\left(\frac{T_E - T_P}{\delta x}\right) - k_A A\left(\frac{T_P - T_A}{\delta x/2}\right)\right] + qA\delta x = 0$$

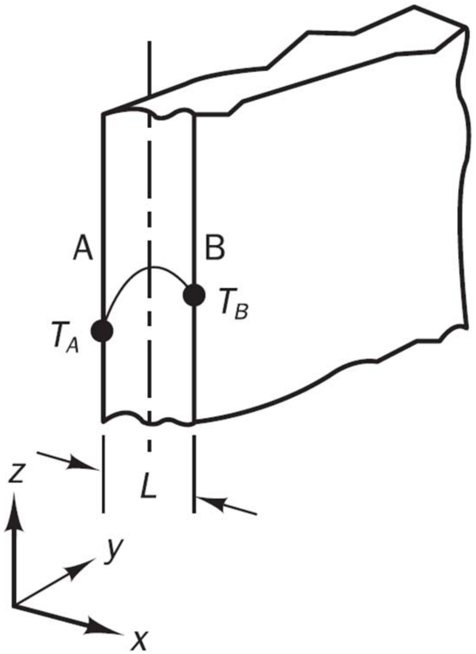
$$\left[k_B A\left(\frac{T_B - T_P}{\delta x/2}\right) - k_w A\left(\frac{T_P - T_W}{\delta x}\right)\right] + qA\delta x = 0$$

$$a_P T_P = a_W T_W + a_E T_E + S_u$$

a_W	a_E	a_P	S_P	S_u
0	$\frac{kA}{\delta x}$	$a_W + a_E - S_P$	$-\frac{2kA}{\delta x}$	$qA\delta x + \frac{2kA}{\delta x}T_A$

a_W	a_E	a_P	S_P	S_u
$\frac{kA}{\delta x}$	0	$a_W + a_E - S_P$	$-\frac{2kA}{\delta x}$	$qA\delta x + \frac{2kA}{\delta x}T_B$





Worked examples: 1D steady state diffusion

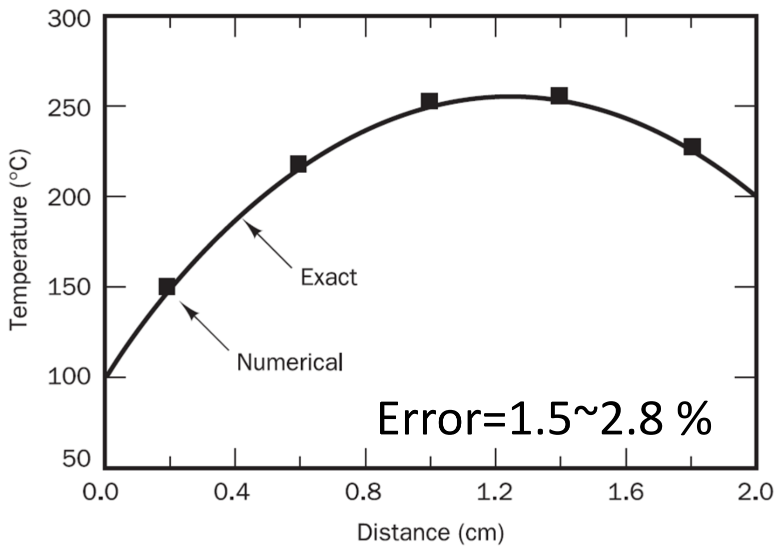
- Example 4.2
  - Five discretized equations $\Rightarrow$ matrix form

$$a_P T_P = a_W T_W + a_E T_E + S_u$$

$$A = 1,\ k = 0.5\ \mathrm{W/m.K} \quad q = 1000\ \mathrm{kW/m^3}$$

$$T_A = 100^\circ C \quad T_B = 200^\circ C$$

$$\begin{bmatrix} 375 & -125 & 0 & 0 & 0 \\ -125 & 250 & -125 & 0 & 0 \\ 0 & -125 & 250 & -125 & 0 \\ 0 & 0 & -125 & 250 & -125 \\ 0 & 0 & 0 & -125 & 375 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 29000 \\ 4000 \\ 4000 \\ 4000 \\ 54000 \end{bmatrix}$$



$$T = \left[\frac{T_B - T_A}{L} + \frac{q}{2k}(L - x) \right] x + T_A$$

Worked examples: 1D steady state diffusion

- Example 4.3
  - Cooling of a circular fin by means of convective heat transfer along its length.
  - Convection gives rise to a temperature-dependent heat loss or sink term
  - A cylindrical fin with uniform cross-sectional area *A*.
  - The base is at a temperature of 100°C (T_B)
  - The end is insulated.
  - The fin is exposed to an ambient temperature of 20°C.

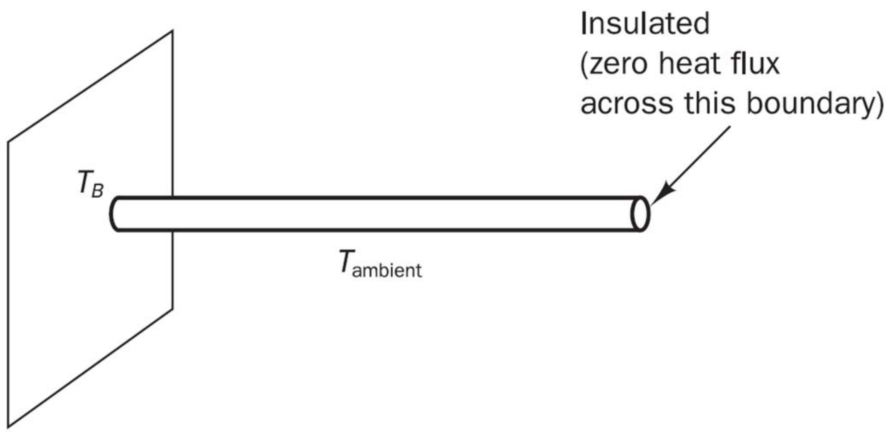
- Governing eq.

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - hP(T - T_\infty) = 0$$

  - *h:* the convective heat transfer coefficient
  - *P:* the perimeter
  - *k:* the thermal conductivity of the material
  - T_∞: the ambient temperature

Analytical solution

$$\frac{T - T_\infty}{T_B - T_\infty} = \frac{\cosh[n(L - x)]}{\cosh(nL)}$$



Worked examples: 1D steady state diffusion

- Example 4.3
  - Cooling of a circular fin by means of convective heat transfer along its length.
  - Convection gives rise to a temperature-dependent heat loss or sink term
  - A cylindrical fin with uniform cross-sectional area *A*.
  - The base is at a temperature of 100°C (T_B)
  - The end is insulated.
  - The fin is exposed to an ambient temperature of 20°C.

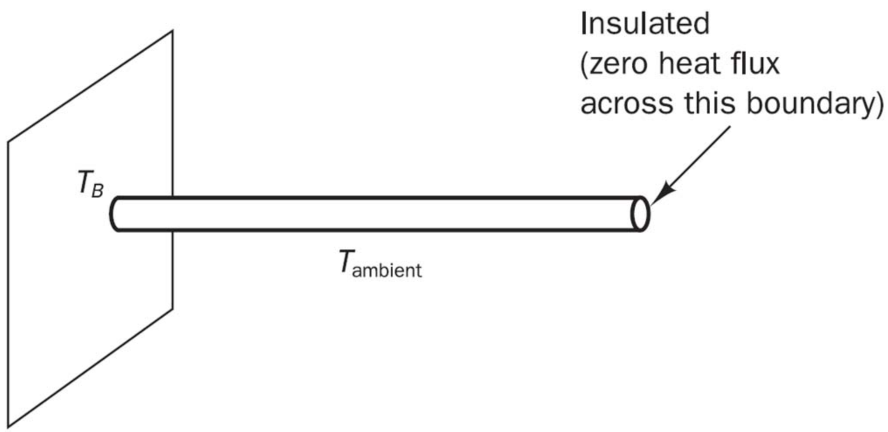
- Governing eq.

$$\boxed{\frac{d}{dx}\left(k\frac{dT}{dx}\right)-\frac{hP\delta x}{\delta x A}\left(T-T_\infty\right)=0}$$

  - *h:* the convective heat transfer coefficient
  - *P:* the perimeter
  - *k:* the thermal conductivity of the material
  - T_∞: the ambient temperature

Analytical solution

$$\frac{T-T_\infty}{T_B-T_\infty}=\frac{\cosh[n(L-x)]}{\cosh(nL)}$$



Worked examples: 1D steady state diffusion

- Data and meshes

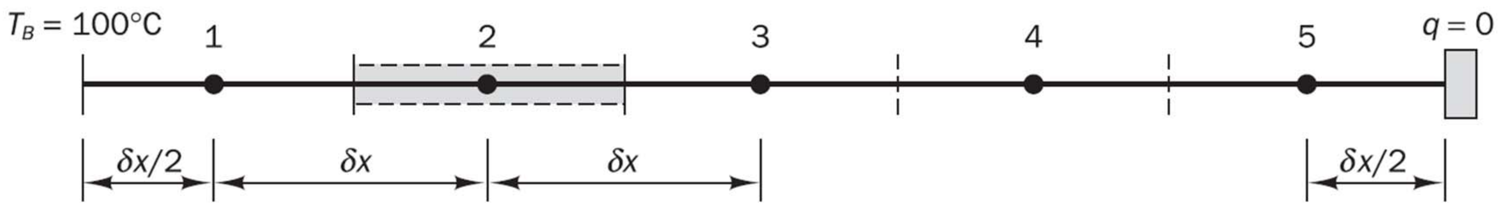
$$L = 1\ \text{m},\ hP/(kA) = 25/\text{m}^2$$

  - Uniform grid, divided into five control volumes $\delta x = 0.2\ \text{m}$

- Modified governing equation and its integral form

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right) - n^2(T - T_\infty) = 0 \text{ where } n^2 = hp/(kA)$$

$$\int_{\Delta V} \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)\mathrm{d}V - \int_{\Delta V} n^2(T - T_\infty)\mathrm{d}V = 0$$



Worked examples: 1D steady state diffusion

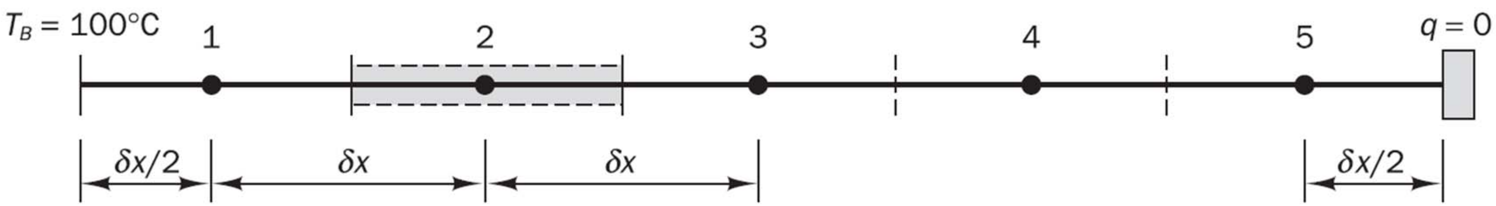
- Discretization

$$\left[\left(A\frac{\mathrm{d}T}{\mathrm{d}x}\right)_e - \left(A\frac{\mathrm{d}T}{\mathrm{d}x}\right)_w\right] - [n^2(T_P - T_\infty)A\delta x] = 0$$

- For mesh 2~4

$$\left[\left(\frac{T_E - T_P}{\delta x}\right) - \left(\frac{T_P - T_W}{\delta x}\right)\right] - [n^2(T_P - T_\infty)\delta x] = 0$$

$$\left(\frac{1}{\delta x} + \frac{1}{\delta x}\right)T_P = \left(\frac{1}{\delta x}\right)T_W + \left(\frac{1}{\delta x}\right)T_E + n^2\delta x T_\infty - n^2\delta x T_P$$



Worked examples: 1D steady state diffusion

- For mesh 2~4

$$a_P T_P = a_W T_W + a_E T_E + S_u$$

a_W	a_E	a_P	S_P	S_u
$\frac{1}{\delta x}$	$\frac{1}{\delta x}$	$a_W + a_E - S_P$	$-n^2\delta x$	$n^2 \delta x T_\infty$

- For mesh 1

$$\left[\left(\frac{T_E - T_P}{\delta x}\right) - \left(\frac{T_P - T_B}{\delta x/2}\right)\right] - [n^2(T_P - T_\infty)\delta x] = 0$$

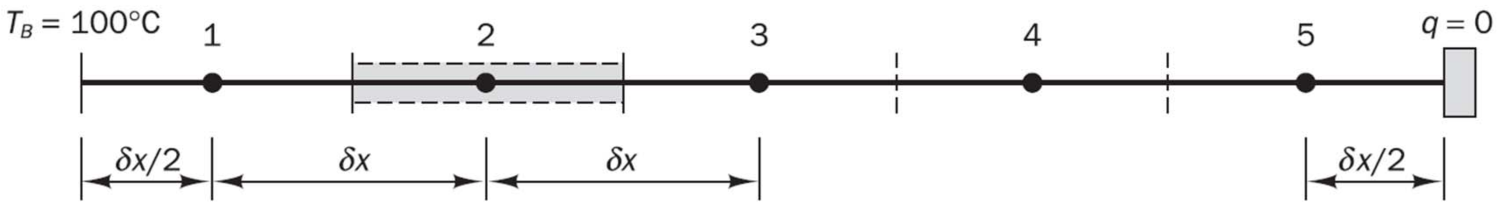
a_W	a_E	a_P	S_P	S_u
0	$\frac{1}{\delta x}$	$a_W + a_E - S_P$	$-n^2\delta x - \frac{2}{\delta x}$	$n^2 \delta x T_\infty + \frac{2}{\delta x} T_B$

- For mesh 5

$$\left[0 - \left(\frac{T_P - T_W}{\delta x}\right)\right] - [n^2(T_P - T_\infty)\delta x] = 0$$

a_W	a_E	a_P	S_P	S_u
$\frac{1}{\delta x}$	0	$a_W + a_E - S_P$	$-n^2\delta x$	$n^2 \delta x T_\infty$

$$\boxed{q = k\frac{T_{end} - T_5}{\delta x/2} = 0}$$



Worked examples: 1D steady state diffusion

- Matrix form of the algebraic equations

a_W	a_E	a_P	S_P	S_u
0	$\frac{1}{\delta x}$	$a_W + a_E - S_P$	$-n^2\delta x - \frac{2}{\delta x}$	$n^2\delta x T_\infty + \frac{2}{\delta x} T_B$

a_W	a_E	a_P	S_P	S_u
$\frac{1}{\delta x}$	$\frac{1}{\delta x}$	$a_W + a_E - S_P$	$-n^2\delta x$	$n^2\delta x T_\infty$

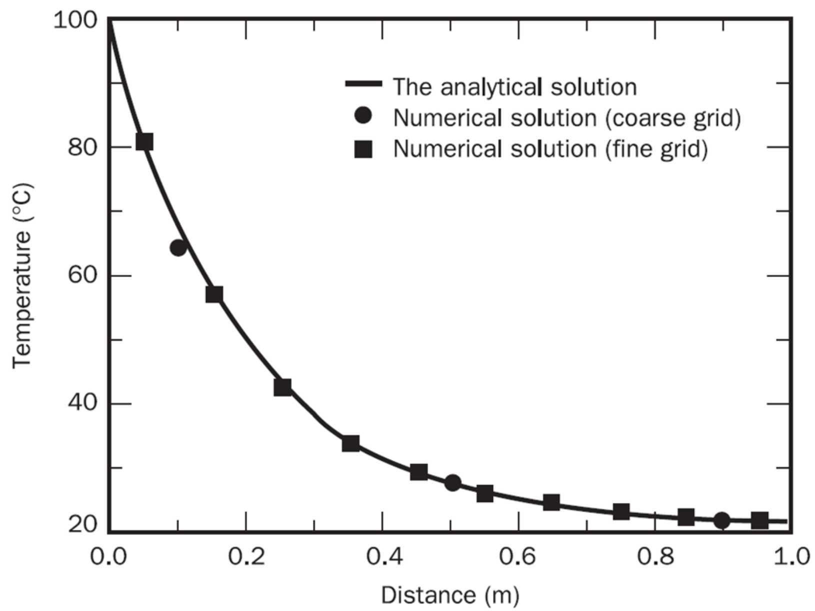
a_W	a_E	a_P	S_P	S_u
$\frac{1}{\delta x}$	0	$a_W + a_E - S_P$	$-n^2\delta x$	$n^2\delta x T_\infty$

$$n^2 = \frac{hP}{kA} = 25 \qquad \delta x = 0.2 \text{ m}$$

$$\begin{bmatrix} 20 & -5 & 0 & 0 & 0 \\ -5 & 15 & -5 & 0 & 0 \\ 0 & -5 & 15 & -5 & 0 \\ 0 & 0 & -5 & 15 & -5 \\ 0 & 0 & 0 & -5 & 10 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 1100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix} \qquad \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 64.22 \\ 36.91 \\ 26.50 \\ 22.60 \\ 21.30 \end{bmatrix}$$

Worked examples: 1D steady state diffusion

❖ Matrix form of the algebraic equations



- Error: max. 6.3 % with 5 meshes
- Error: max. 2.1 % with 10 meshes

HW#1

- Solve Example 3 by yourself
  - Derive the discretized equation
  - Write a code.
  - Find the analytical solution
  - Compare the calculation result with the solution
  - With 5, 10, 20 meshes

Homework Assignment

Check the unit of each parameter!

- HW #2
  - 1D conduction equation for nuclear fuel rod
  - Check the heat transfer area carefully!
  - Using FVM
    - Constant conductivity, k
      - For pellet
      - For cladding
    - Gap conductance
      - 1) Xenon conductivity
      - 2) Constant gap conductance value
      - 3) Gap conductance model
    - Flow condition
      - HTC: Dittus-Boelter
      - Fluid velocity

연료봉	
연료봉 재질(소결체)	UO_2
소결체 직경(공칭), cm(in)	0.819(0.3225)
소결체 길이, cm(in)	0.983(0.387)(농축우라늄)
소결체 밀도(공칭), (g/cm^3)	10.44
소결체 이론 밀도, (g/cm^3)	10.96
소결체 밀도(공칭), (이론 밀도 %)	95.25
적층(stack height) 밀도(공칭), (g/cm^3)	10.313
피복관 재질	ZIRLO
피복관 내경, cm(in)	0.836(0.329)
피복관 외경(공칭), cm(in)	0.950(0.374)

원자로 변수	신고리 3,4호기	System 80+	울진 3,4호기
전출력시 노심 평균 특성			
노심 총 열출력, MWt	3,983	3,914	2,815
노심 총 열출력, 10^6 Kcal/h(MBtu/h)	3,425(13,590)	3,367(13,360)	2,421(9,608)
평균 연료봉 에너지 저장비	0.975	0.975	0.975
고온 연료봉 에너지 저장비	0.975	0.975	0.975
1차계통 압력, kg/cm^{2}A(psia)	158(2,250)	158(2,250)	158(2,250)
원자로 입구 냉각재 온도, ℃(°F)	291(555)	291(556)	296(564.5)
원자로 출구 냉각재 온도, ℃(°F)	324(615)	324(615)	327(621)
노심 출구 평균 냉각재 온도, ℃(°F)	325(617)	325(617)	328(623)
노심 평균 엔탈피 상승, Kcal/kg(Btu/lbm)	46.7(84.1)	46.1(83.0)	45.3(81.5)
1차계통 최소 설계 유량, L/min(gpm)	1,689,000 (446,300)	1,683,000 (444,650)	1,249,000 (330,000)
노심 최대 설계 우회유량 (1차 계통 최소 설계 유량의 %)	3.0	3.0	3.0
노심 최소 설계 유량, L/min(gpm)	1,639,000 (432,900)	1,633,000 (431,300)	1,211,000 (320,000)
부수로 수력직경, cm(in)	1.264(0.498)	1.196(0.471)	1.264(0.498)
노심유로면적, m^2(ft^2)	5.825(62.7)	5.649(60.8)	4.293(46.21)
노심 평균 질량유속, million Kg/h-m^2(million lbm/h-ft^2)	12.60(2.58)	12.94(2.65)	12.45(2.55)
노심 평균 냉각재 유속, m/s(ft/s)	4.94(16.2)	5.10(16.7)	4.94(16.2)
노심 평균 연료봉 열속, Kcal/h-m^2(Btu/h-ft^2)	517,361(190,735)	497,200 (183,300)	497,859 (183,545)
총 열전달 면적, m^2(ft^2)	6,454(69,470)	6,592(70,960)	4,740(51,023)
연료봉 평균 선출력 생성률, W/cm(kW/ft)	179.2(5.46)	175.9(5.36)	172.6(5.26)
출력밀도, kW/L	100.5	98.4	96.6
연료봉 수	56,876	56,876	41,772

Homework Assignment

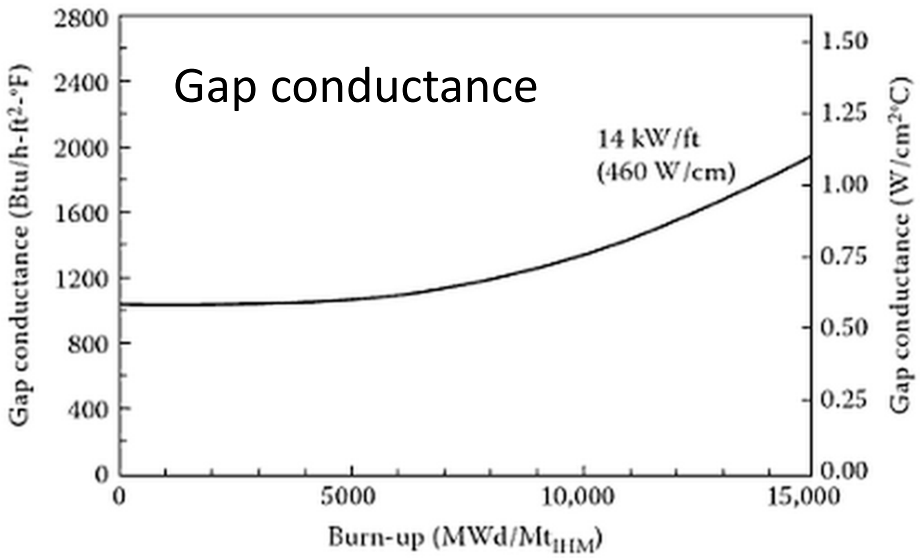
- HW #2
  - Gap conductance

$$h_g[W/m^{2\circ}C] = \frac{k_{gas}}{\delta_{eff}} + \frac{\sigma T_{fo}^3}{1/\varepsilon_f + 1/\varepsilon_c - 1}$$

$$q_{in} = q''_{g,fo} A_{in} = h_g A_{in} \left(T_{fo} - T_{gap}\right)$$

$$q_{out} = q''_{g,ci} A_{out} = h_g \left(\frac{A_{in}}{A_{out}}\right) A_{out} \left(T_{gap} - T_{ci}\right)$$

Check the unit of each parameter!



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- Non-linear dependency
- Start calculation with the gap conductance graph.
  - hg value from the graph
- With the calculated temperature, update hg.
  - Repeat the calculation
  - Update hg
  - Repeat until the solution converges.

Homework Assignment

- HW #2
  - What you need to report
    - Discretized equations
    - Calculation conditions
    - Temperature profiles for three cases
    - Gap conductance