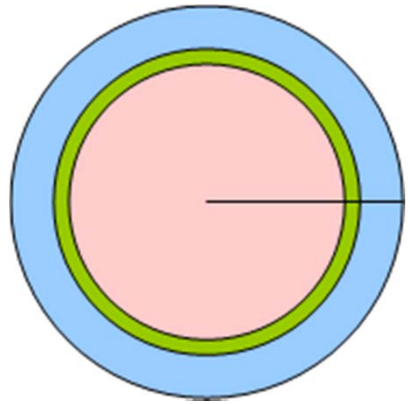
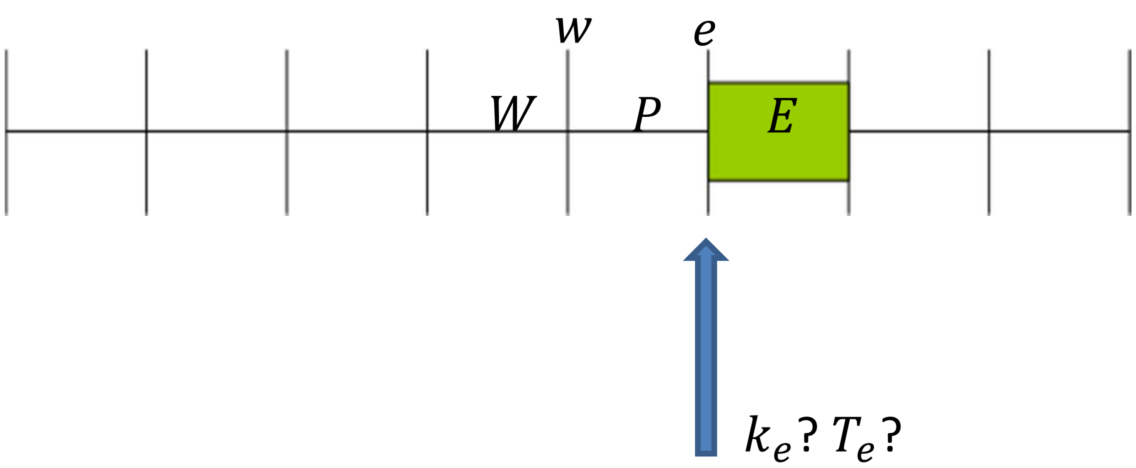
Homework Assignment

- HW #2
  - What you need to report
    - Discretized equations
    - Calculation conditions
    - Temperature profiles for three cases
    - Gap conductance





$$k\frac{T_e - T_P}{0.5\delta x}A_{eP} - k\frac{T_P - T_W}{\delta x}A_{wP} = qA\delta x$$

$$q''_{eP} = -k\frac{T_e - T_P}{\delta x_{eP}}A_{eP} = h_{g,e}A_{eP}(T_e - T_E)$$

$$T_e\left(h_{g,e}A_{eP} + \frac{k}{\delta x_{eP}}\right) = \left(\frac{k}{\delta x_{eP}}T_P + h_{g,e}T_E\right)$$

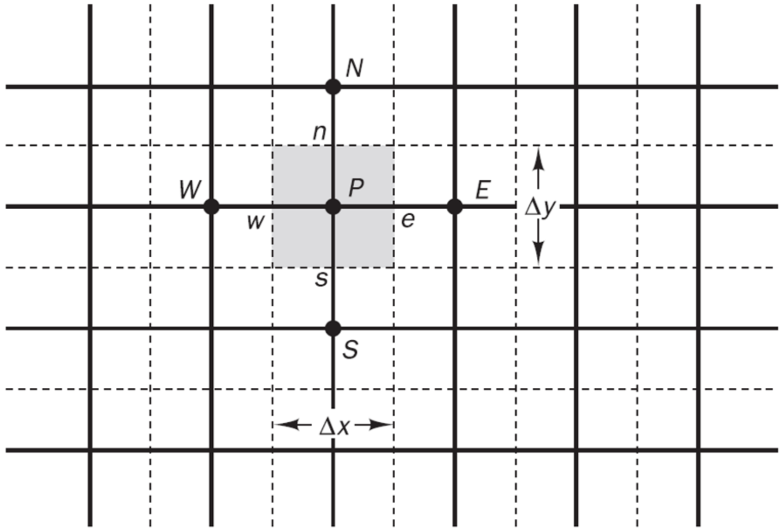
$$T_e = \frac{\left(\frac{k}{\delta x_{eP}}T_P + h_{g,e}T_E\right)}{\left(h_{g,e}A_{eP} + \frac{k}{\delta x_{eP}}\right)}$$

Contents

FVM for 2D diffusion problems

- Governing equation
  - 2D diffusion equation

$$\frac{\partial}{\partial x}\left(\Gamma\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial \phi}{\partial y}\right)+S_\phi=0$$



  - Integral form and discretization

$$\int_{\Delta V}\frac{\partial}{\partial x}\left(\Gamma\frac{\partial \phi}{\partial x}\right)\mathrm{d}x\,.\,\mathrm{d}y+\int_{\Delta V}\frac{\partial}{\partial y}\left(\Gamma\frac{\partial \phi}{\partial y}\right)\mathrm{d}x\,.\,\mathrm{d}y+\int_{\Delta V}S_\phi \mathrm{d}V=0$$

$$A_e=A_w=\Delta y \qquad A_n=A_s=\Delta x$$

$$\left[\Gamma_e A_e\left(\frac{\partial \phi}{\partial x}\right)_e-\Gamma_w A_w\left(\frac{\partial \phi}{\partial x}\right)_w\right]+\left[\Gamma_n A_n\left(\frac{\partial \phi}{\partial y}\right)_n-\Gamma_s A_s\left(\frac{\partial \phi}{\partial y}\right)_s\right]+\bar{S}\Delta V=0$$

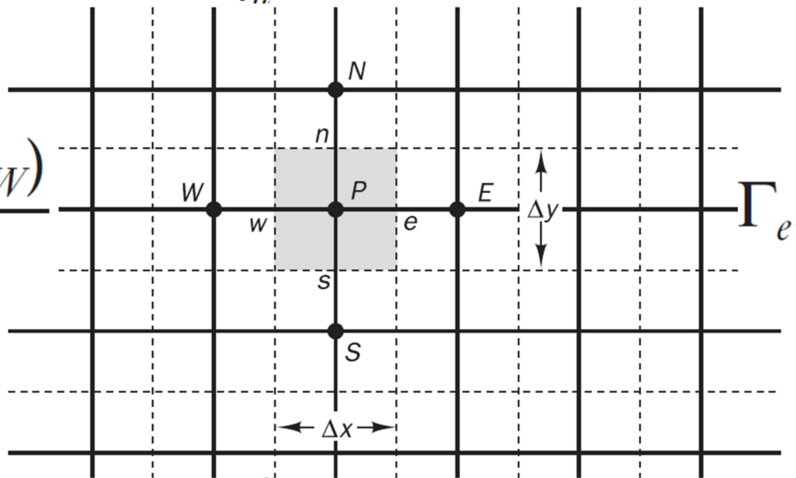
FVM for 2D diffusion problems

- Governing equation
  - Evaluation of fluxes across each face

$$\left[\Gamma_e A_e\left(\frac{\partial\phi}{\partial x}\right)_e - \Gamma_w A_w\left(\frac{\partial\phi}{\partial x}\right)_w\right] + \left[\Gamma_n A_n\left(\frac{\partial\phi}{\partial y}\right)_n - \Gamma_s A_s\left(\frac{\partial\phi}{\partial y}\right)_s\right] + \bar{S}\Delta V = 0$$

$$\Gamma_n A_n \frac{\partial\phi}{\partial y}\bigg|_n = \Gamma_n A_n \frac{(\phi_N - \phi_P)}{\delta y_{PN}}$$

$$\Gamma_w A_w \frac{\partial\phi}{\partial x}\bigg|_w = \Gamma_w A_w \frac{(\phi_P - \phi_W)}{\delta x_{WP}}$$



$$\Gamma_e A_e \frac{\partial\phi}{\partial x}\bigg|_e = \Gamma_e A_e \frac{(\phi_E - \phi_P)}{\delta x_{PE}}$$

$$\Gamma_s A_s \frac{\partial\phi}{\partial y}\bigg|_s = \Gamma_s A_s \frac{(\phi_P - \phi_S)}{\delta y_{SP}}$$

FVM for 2D diffusion problems

- Discretized equation

$$\Gamma_e A_e \frac{(\phi_E - \phi_P)}{\delta x_{PE}} - \Gamma_w A_w \frac{(\phi_P - \phi_W)}{\delta x_{WP}} + \Gamma_n A_n \frac{(\phi_N - \phi_P)}{\delta y_{PN}} - \Gamma_s A_s \frac{(\phi_P - \phi_S)}{\delta y_{SP}} + \bar{S}\Delta V = 0$$

$$\bar{S}\Delta V = S_u + S_p \phi_P$$

$$\left(\frac{\Gamma_w A_w}{\delta x_{WP}} + \frac{\Gamma_e A_e}{\delta x_{PE}} + \frac{\Gamma_s A_s}{\delta y_{SP}} + \frac{\Gamma_n A_n}{\delta y_{PN}} - S_p \right) \phi_P$$

$$= \left(\frac{\Gamma_w A_w}{\delta x_{WP}} \right) \phi_W + \left(\frac{\Gamma_e A_e}{\delta x_{PE}} \right) \phi_E + \left(\frac{\Gamma_s A_s}{\delta y_{SP}} \right) \phi_S + \left(\frac{\Gamma_n A_n}{\delta y_{PN}} \right) \phi_N + S_u$$

FVM for 2D diffusion problems

- Discretized equation
  - General form

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + a_S\phi_S + a_N\phi_N + S_u$$

a_W	a_E	a_S	a_N	a_P
$\frac{\Gamma_w A_w}{\delta x_{WP}}$	$\frac{\Gamma_e A_e}{\delta x_{PE}}$	$\frac{\Gamma_s A_s}{\delta y_{SP}}$	$\frac{\Gamma_n A_n}{\delta y_{PN}}$	$a_W + a_E + a_S + a_N - S_p$

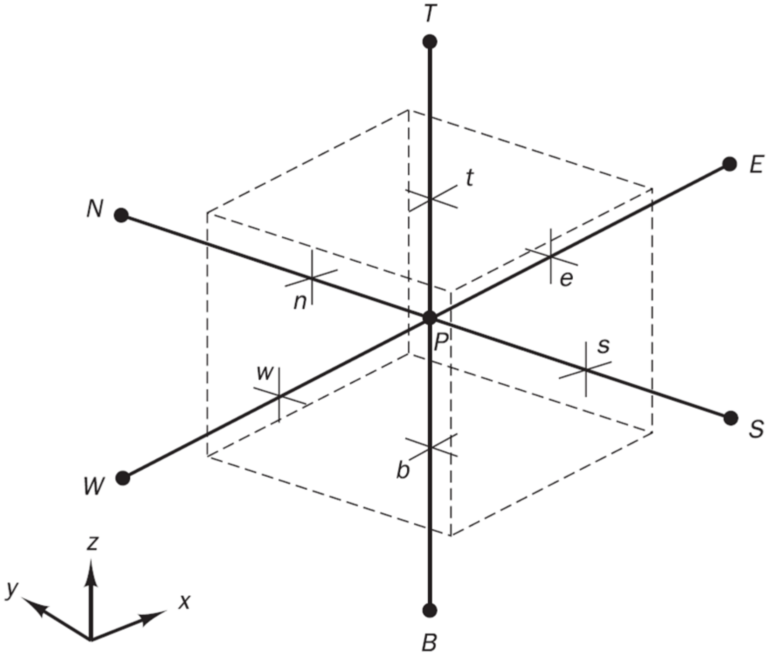
Contents

FVM for 3D diffusion problems

- Governing equation

$$\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\Gamma\frac{\partial\phi}{\partial z}\right)+S_\phi=0$$

  - Cell center: P
  - Face centers: e,w,s,n,t,b
  - Neighboring cell centers: E,W,S,N,T,B



- Discretization

$$\left[\Gamma_e A_e\left(\frac{\partial\phi}{\partial x}\right)_e-\Gamma_w A_w\left(\frac{\partial\phi}{\partial x}\right)_w\right]+\left[\Gamma_n A_n\left(\frac{\partial\phi}{\partial y}\right)_n\right.$$

$$\left.-\Gamma_s A_s\left(\frac{\partial\phi}{\partial y}\right)_s\right]+\left[\Gamma_t A_t\left(\frac{\partial\phi}{\partial z}\right)_t-\Gamma_b A_b\left(\frac{\partial\phi}{\partial z}\right)_b\right]+\bar{S}\Delta V=0$$

FVM for 3D diffusion problems

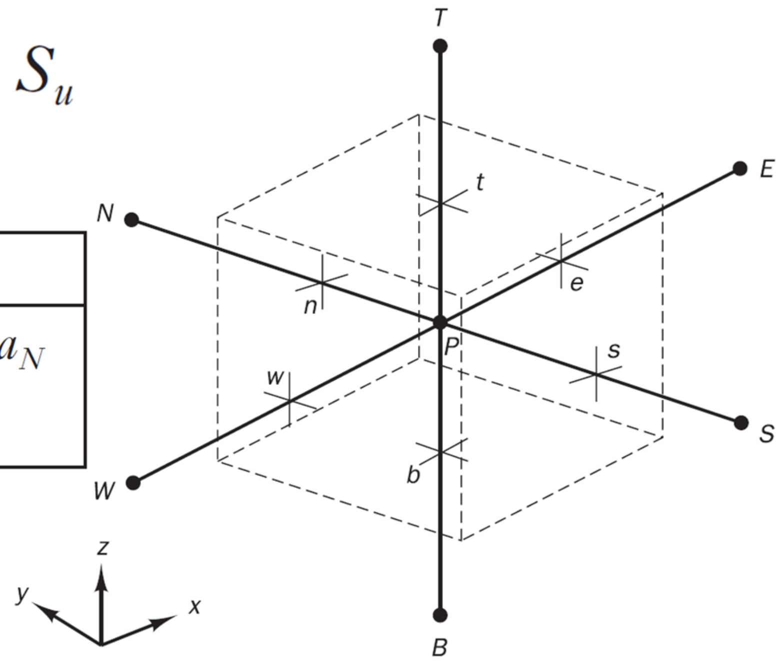
- Evaluation of the gradients and the general form

$$\left[\Gamma_e A_e \frac{(\phi_E - \phi_P)}{\delta x_{PE}} - \Gamma_w A_w \frac{(\phi_P - \phi_W)}{\delta x_{WP}}\right] + \left[\Gamma_n A_n \frac{(\phi_N - \phi_P)}{\delta y_{PN}} - \Gamma_s A_s \frac{(\phi_P - \phi_S)}{\delta y_{SP}}\right]$$

$$+ \left[\Gamma_t A_t \frac{(\phi_T - \phi_P)}{\delta z_{PT}} - \Gamma_b A_b \frac{(\phi_P - \phi_B)}{\delta z_{BP}}\right] + (S_u + S_P \phi_P) = 0$$

$$a_P \phi_P = a_W \phi_W + a_E \phi_E + a_S \phi_S + a_N \phi_N + a_B \phi_B + a_T \phi_T + S_u$$

a_W	a_E	a_S	a_N	a_B	a_T	a_P
$\frac{\Gamma_w A_w}{\delta x_{WP}}$	$\frac{\Gamma_e A_e}{\delta x_{PE}}$	$\frac{\Gamma_s A_s}{\delta y_{SP}}$	$\frac{\Gamma_n A_n}{\delta y_{PN}}$	$\frac{\Gamma_b A_b}{\delta z_{BP}}$	$\frac{\Gamma_t A_t}{\delta z_{PT}}$	$a_W + a_E + a_S + a_N + a_B + a_T - S_P$

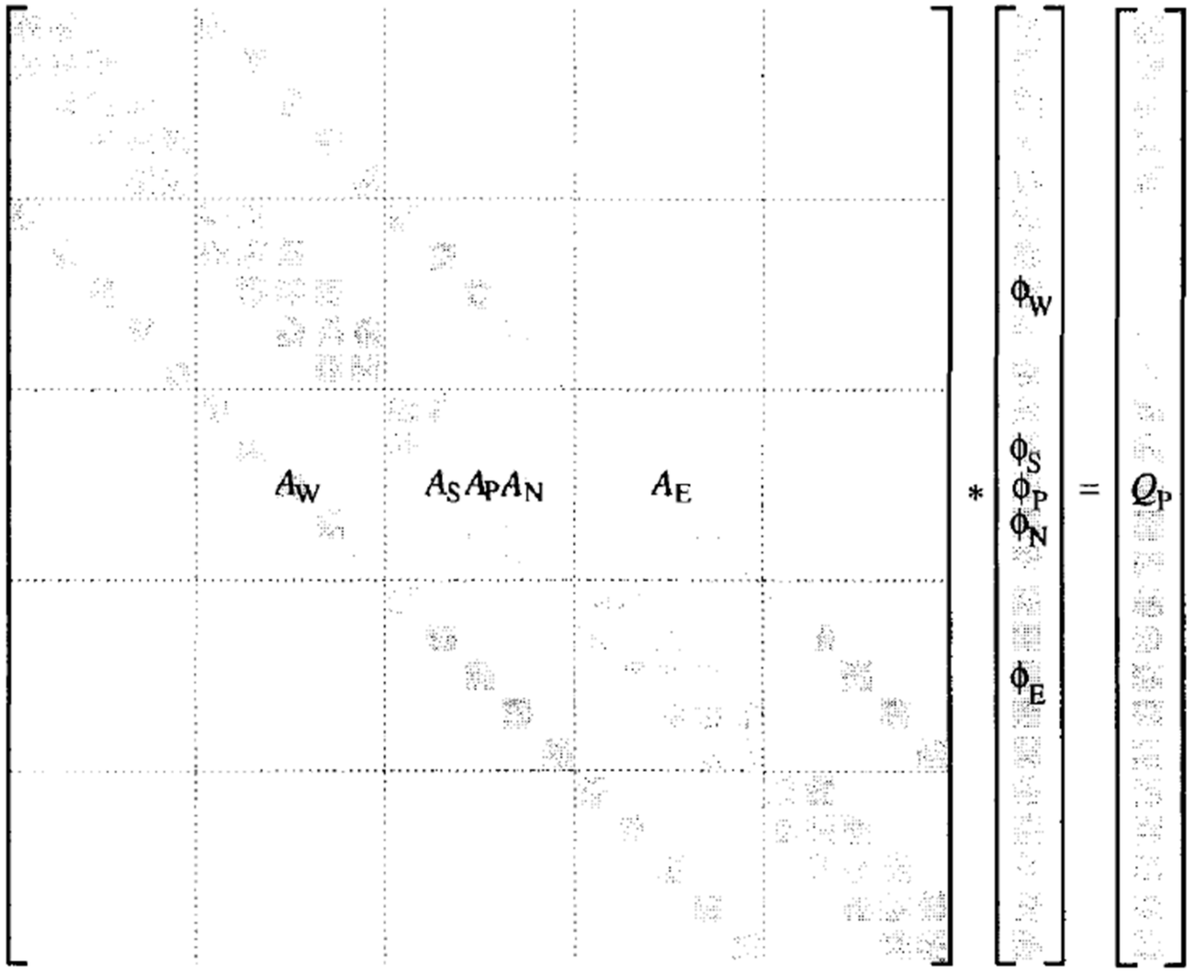


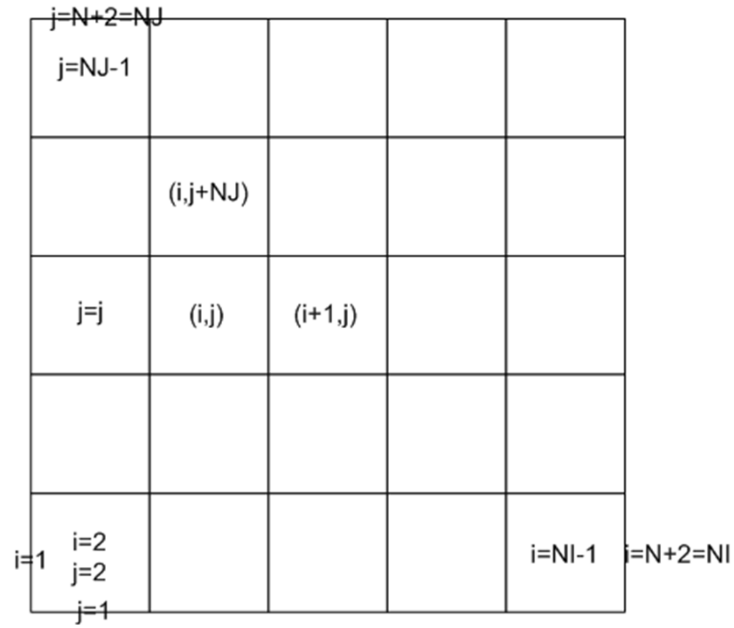
Numerical Procedure

- Numerical procedure
  - matrix equation [A]{f}={Q}
  - C.....DEFINE GRID IN X-DIRECTION
  - C.....DEFINE GRID IN Y-DIRECTION
  - C.....COORDINATES OF CELL CENTERS
  - C.....WORKING ARRAY FOR CONVERTING 2D INDICES TO 1D
  - C.....INITIALIZE FIELD VALUES (ZERO)
  - BOUNDARY VALUE SETTING
  - C.....CALCULATE ELEMENTS OF MATRIX [A]
    - AE(IJ)
    - AN(IJ)
    - AW(IJ)
    - AS(IJ)
    - AP(IJ)
    - VOL
    - Q(IJ)
  - C.....SOLVE EQUATION SYSTEM

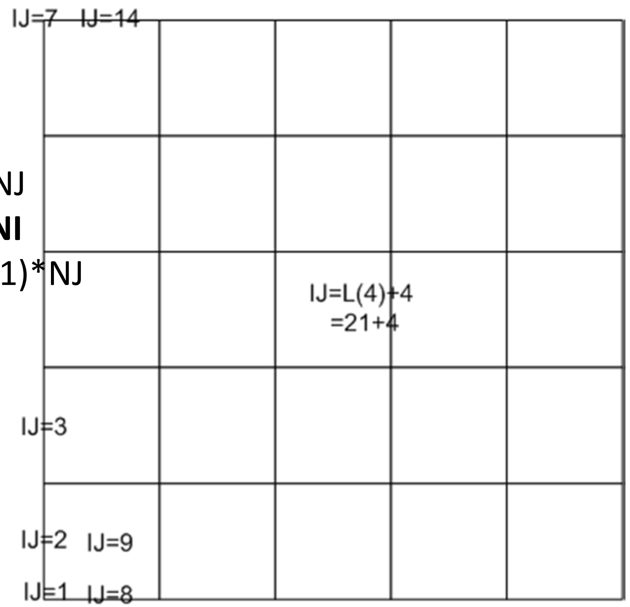
Numerical Procedure

- Matrix form
  - For 2D with 5×5 cells





NIJ=NI*NJ
DO I=1,NI
LI(I)=(I-1)*NJ
END DO



2D Conduction Equation in Unstructured Mesh

- Diffusion equation for regular hexagon
  - Triangular mesh
  - Orthogonal grid

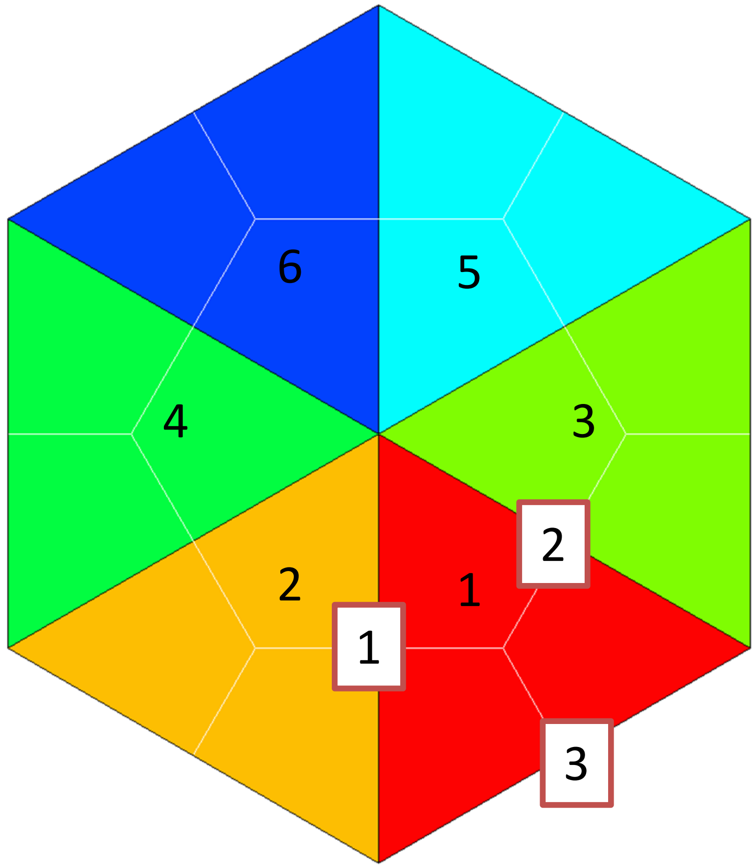
$$\int_{\Delta V} \frac{\partial}{\partial x}\left(\Gamma \frac{\partial \phi}{\partial x}\right) \mathrm{d}x \,.\, \mathrm{d}y + \int_{\Delta V} \frac{\partial}{\partial y}\left(\Gamma \frac{\partial \phi}{\partial y}\right) \mathrm{d}x \,.\, \mathrm{d}y + \int_{\Delta V} S_\phi \mathrm{d}V = 0$$

$$\int_S \vec{n} \cdot (\Gamma \nabla \phi) d\vec{S} = \sum_j (\Gamma \nabla \phi)_j S = \sum_j \Gamma_j \frac{\phi_k - \phi_i}{\delta_{ik}} S_j = 0$$

$$\Gamma_{1,1} \frac{\phi_2 - \phi_1}{\delta_{11}} S_{1,1} + \Gamma_{1,2} \frac{\phi_3 - \phi_1}{\delta_{12}} S_{1,2} + \Gamma_{1,3} \frac{\phi_{1,3} - \phi_1}{\delta_{13}} S_{1,3} = 0$$

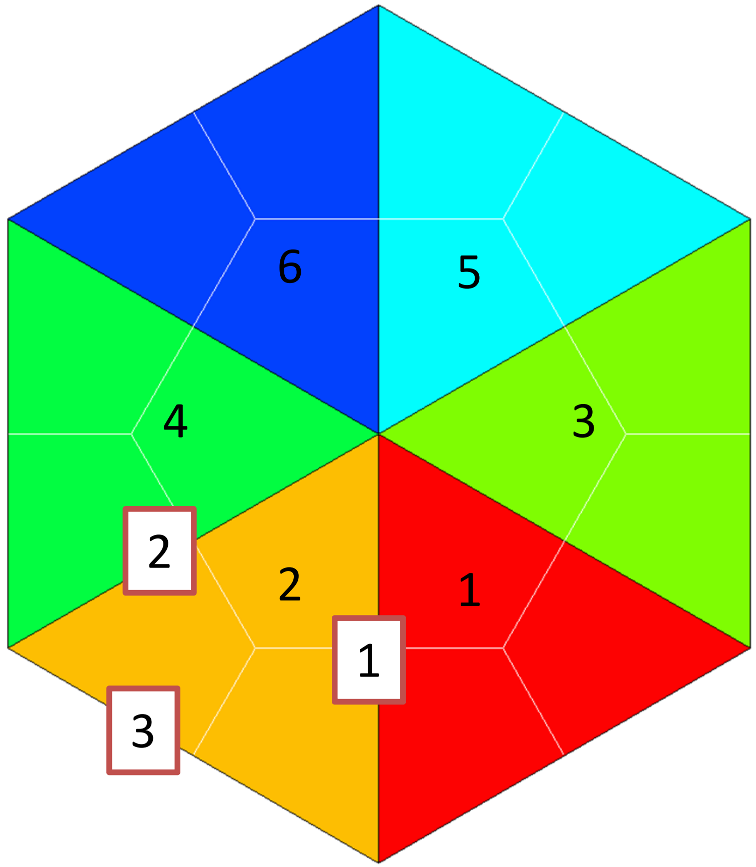
$$\vdots$$

$$\Gamma_{6,1} \frac{\phi_5 - \phi_6}{\delta_{6,1}} S_{6,1} + \Gamma_{6,2} \frac{\phi_4 - \phi_6}{\delta_{6,2}} S_{6,2} + \Gamma_{6,3} \frac{\phi_{6,3} - \phi_6}{\delta_{6,3}} S_{6,3} = 0$$



2D Conduction Equation in Unstructured Mesh

- Required grid information
  - Number of cells
  - Cell center coordinates: $xloc(i), yloc(i)$
  - Cell volume: $vol(i)$
  - Neighboring cell number: $neigh(i,j)$
  - Face characteristic: $nbcon(i,j)$
  - Distance between two cells: $dij(i,j)$
  - Surface area: $sa(i,j)$



- Mesh from "easymesh"
  - Generates two dimensional, unstructured, Delaunay and constrained Delaunay triangulations in general domains.

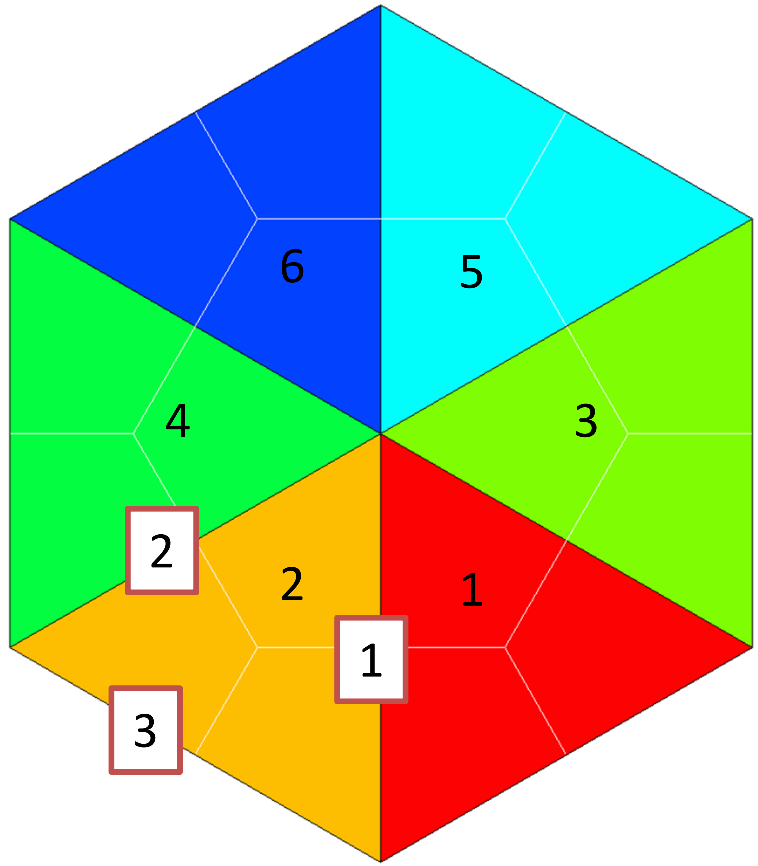
2D Conduction Equation in Unstructured Mesh

❖ Required grid information

i	xloc	yloc	vol	Face 수
1	1.155	0.5	0.433	3

i	j	jcell	itype	nx*sa	ny*sa	sa	dij	nx	ny
1	1	0	1	0.5	-0.8660	1.0	0.2887	0.5	-0.8660

xf(iface)	yf(iface)	1.0d0-rfac	dx1	dy1
1.2990	0.25	0	0.1443	-0.25



2D Conduction Equation in Unstructured Mesh

- Discretized equations

$$\Gamma_{1,1}\frac{\phi_2-\phi_1}{\delta_{11}}S_{1,1}+\Gamma_{1,2}\frac{\phi_3-\phi_1}{\delta_{12}}S_{1,2}+\Gamma_{1,3}\frac{\phi_{1,3}-\phi_1}{\delta_{13}}S_{1,3}=0$$

$$(\phi_2-\phi_1)+(\phi_3-\phi_1)+2(\phi_1-\phi_1)=0$$
$$2\phi_1-\phi_2-\phi_3=0$$

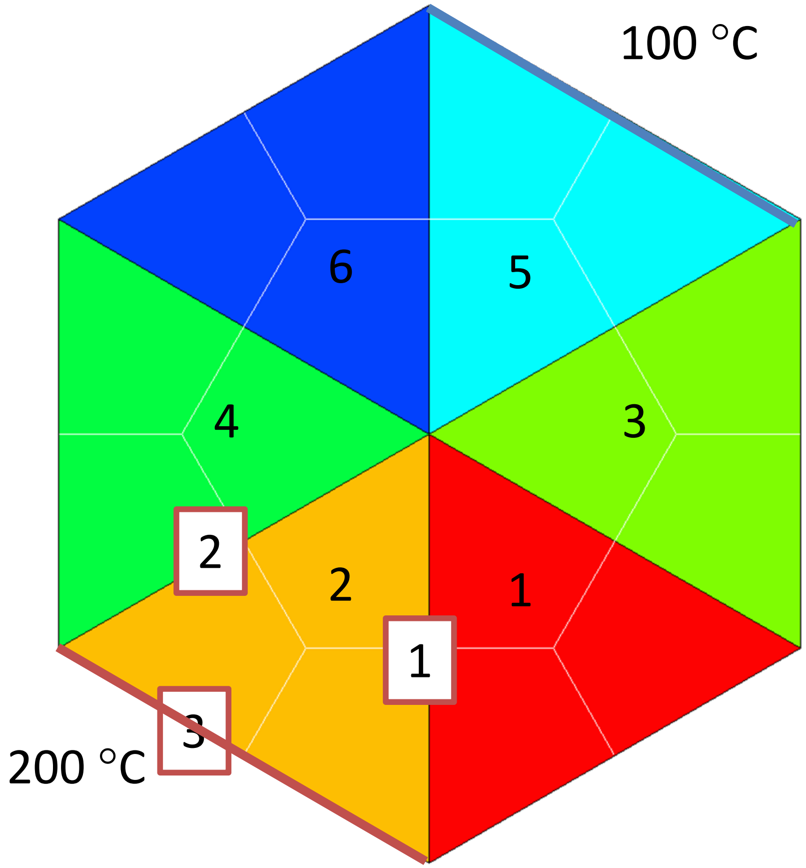
$$(\phi_1-\phi_2)+(\phi_4-\phi_2)+2(200-\phi_2)=0$$
$$4\phi_2-\phi_1-\phi_4=400$$

$$(\phi_1-\phi_3)+(\phi_5-\phi_3)+2(\phi_3-\phi_3)=0$$
$$2\phi_3-\phi_1-\phi_5=0$$

$$(\phi_2-\phi_4)+(\phi_6-\phi_4)+2(\phi_4-\phi_4)=0$$
$$2\phi_4-\phi_2-\phi_6=0$$

$$(\phi_3-\phi_5)+(\phi_6-\phi_5)+2(100-\phi_5)=0$$
$$4\phi_5-\phi_3-\phi_6=200$$

$$(\phi_4-\phi_6)+(\phi_5-\phi_6)+2(\phi_6-\phi_6)=0$$
$$2\phi_6-\phi_4-\phi_5=0$$



2D Conduction Equation in Unstructured Mesh

- Discretized equations

$$(\phi_2-\phi_1)+(\phi_3-\phi_1)+2(\phi_1-\phi_1)=0$$
$$2\phi_1-\phi_2-\phi_3=0$$

$$(\phi_1-\phi_2)+(\phi_4-\phi_2)+2(200-\phi_2)=0$$
$$4\phi_2-\phi_1-\phi_4=400$$

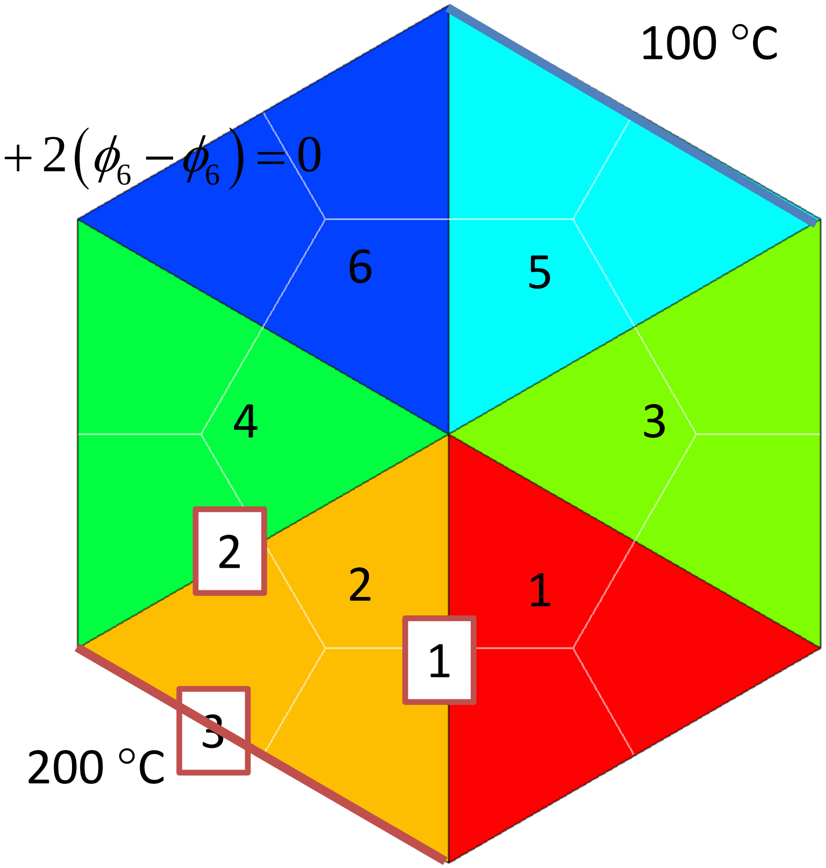
$$(\phi_1-\phi_3)+(\phi_5-\phi_3)+2(\phi_3-\phi_3)=0$$
$$2\phi_3-\phi_1-\phi_5=0$$

$$(\phi_2-\phi_4)+(\phi_6-\phi_4)+2(\phi_4-\phi_4)=0$$
$$2\phi_4-\phi_2-\phi_6=0$$

$$(\phi_3-\phi_5)+(\phi_6-\phi_5)+2(100-\phi_5)=0$$
$$4\phi_5-\phi_3-\phi_6=200$$

$$(\phi_4-\phi_6)+(\phi_5-\phi_6)+2(\phi_6-\phi_6)=0$$
$$2\phi_6-\phi_4-\phi_5=0$$

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 4 & 0 & -1 & 0 & 0 \\ -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & 0 & -1 \\ 0 & 0 & -1 & 0 & 4 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 400 \\ 0 \\ 0 \\ 200 \\ 0 \end{pmatrix}$$



2D Conduction Equation in Unstructured Mesh

- Discretized equations

$$(\phi_2-\phi_1)+(\phi_3-\phi_1)+2(\phi_1-\phi_1)=0$$
$$2\phi_1-\phi_2-\phi_3=0$$

$$(\phi_1-\phi_2)+(\phi_4-\phi_2)+2(200-\phi_2)=0$$
$$4\phi_2-\phi_1-\phi_4=400$$

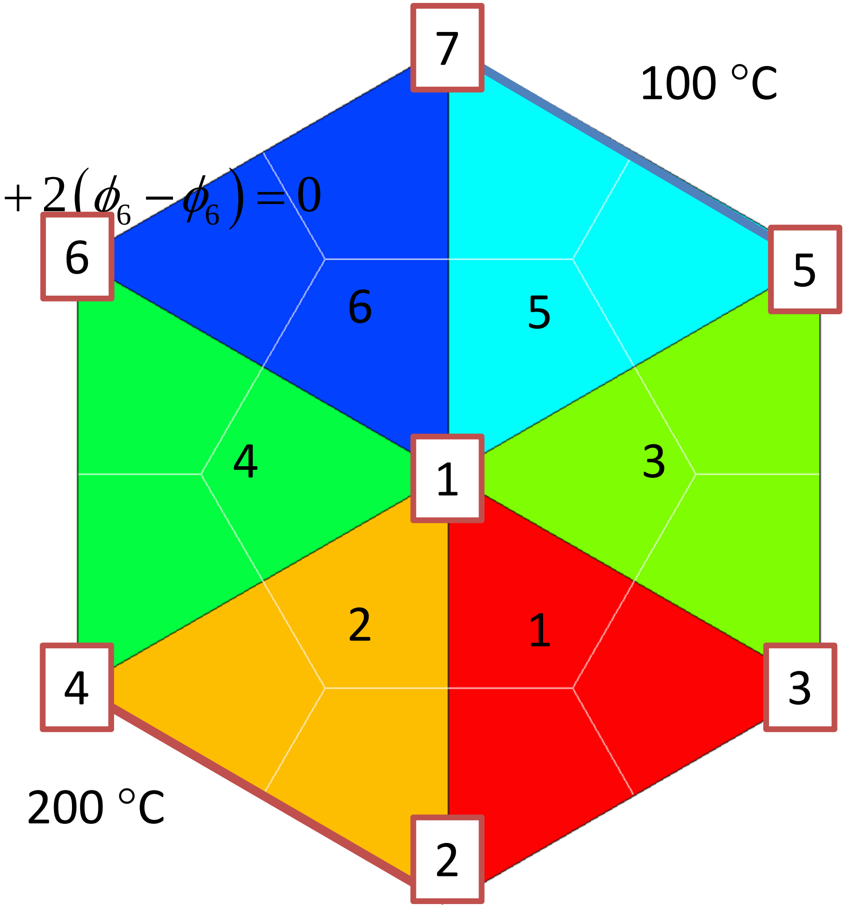
$$(\phi_1-\phi_3)+(\phi_5-\phi_3)+2(\phi_3-\phi_3)=0$$
$$2\phi_3-\phi_1-\phi_5=0$$

$$(\phi_2-\phi_4)+(\phi_6-\phi_4)+2(\phi_4-\phi_4)=0$$
$$2\phi_4-\phi_2-\phi_6=0$$

$$(\phi_3-\phi_5)+(\phi_6-\phi_5)+2(100-\phi_5)=0$$
$$4\phi_5-\phi_3-\phi_6=200$$

$$(\phi_4-\phi_6)+(\phi_5-\phi_6)+2(\phi_6-\phi_6)=0$$
$$2\phi_6-\phi_4-\phi_5=0$$

$$\begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 4 & 0 & -1 & 0 & 0 \\ -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & 0 & -1 \\ 0 & 0 & -1 & 0 & 4 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{pmatrix}\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix}=\begin{pmatrix} 0 \\ 400 \\ 0 \\ 0 \\ 200 \\ 0 \end{pmatrix}$$



2D Conduction Equation in Unstructured Mesh

❖ Calculation result

```
xnode=[0.866025 1.000000;
       0.866025 0.000000;
       1.732051 0.500000;
       0.000000 0.500000;
       1.732051 1.500000;
       0.000000 1.500000;
       0.866025 2.000000;];

cell_node=[1       2       3;
           4       2       1;
           1       3       5;
           4       1       6;
           1       5       7;
           6       1       7;];

trisurf(cell_node,xnode(:,1),xnode(:,2),0*xnode(:,1),'FaceColor', 'flat', ...
       'FaceVertexCData', T);
view(2),axis([0 2 0 2]),colorbar;
set(colorbar, 'ylim', [120 180]);
```