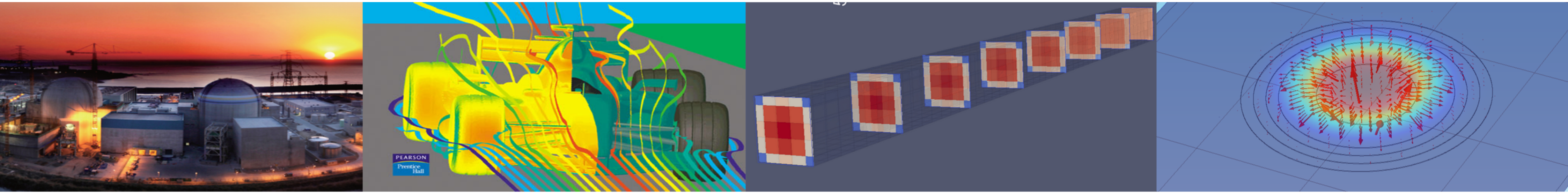

COMPUTATIONAL NUCLEAR THERMAL HYDRAULICS

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NuTHEL
Nuclear Thermal Hydraulic Engineering Lab.

Contents

- Introduction
- Conservation laws of fluid motion and boundary conditions
- Turbulence and its modeling
- The finite volume method for diffusion problems
- The finite volume method for convection-diffusion problems
- Solution algorithms for pressure-velocity coupling in steady flows
- Solution of discretized equations
- The finite volume method for unsteady flows

CHAPTER 6: SOLUTION ALGORITHMS FOR PRESSURE-VELOCITY COUPLING IN STEADY FLOWS

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- Introduction
- The staggered grid
- The momentum equations
- The SIMPLE algorithm
- Assembly of a complete method
- The SIMPLER algorithm
- The SIMPLEC algorithm
- The PISO algorithm
- General comments on SIMPLE, SIMPLER, SIMPLEC and PISO
- Worked examples of the SIMPLE algorithm
- Summary

Introduction

- The convection of a scalar variable ϕ depends on the magnitude and direction of the local velocity field.
- How to find flow field?
  - In the previous chapter, we assumed that the velocity field was somehow known.
  - In general the velocity field is, however, not known and emerges as part of the overall solution process along with all other flow variables.

- Governing equations (2D steady-state N-S eqs.)

$$\frac{\partial}{\partial x}(\rho uu) + \frac{\partial}{\partial y}(\rho vu) = \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\partial p}{\partial x} + S_u$$

$$\frac{\partial}{\partial x}(\rho uv) + \frac{\partial}{\partial y}(\rho vv) = \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) - \frac{\partial p}{\partial y} + S_v$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

Introduction

- New problems
  - Non-linearities
    - The convective terms of the momentum equations contain non-linear quantities.
    - All three equations are intricately coupled because every velocity component appears in each momentum equation and in the continuity equation.
  - The pressure-velocity linkage
    - The most complex issue to resolve is the role played by the pressure. It appears in both momentum equations, but there is evidently no (transport or other) equation for the pressure.
    - If the flow is incompressible the density is constant and hence by definition not linked to the pressure.
    - In this case coupling between pressure and velocity introduces a constraint in the solution of the flow field
      - If the correct pressure field is applied in the momentum equations the resulting velocity field should satisfy continuity.

  - Can be resolved by adopting an iterative solution strategy such as SIMPLE
    - Patankar and Spalding

- SIMPLE algorithm
  - The convective fluxes per unit mass *F* through cell faces are evaluated from so-called guessed velocity components.
  - A guessed pressure field is used to solve the momentum equations
  - A pressure correction equation, deduced from the continuity equation, is solved to obtain a pressure correction field, which is in turn used to update the velocity and pressure fields.
  - To start the iteration process we use initial guesses for the velocity and pressure fields.
  - The process is iterated until convergence of the velocity and pressure fields.

$$\frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho v u) = \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{\partial p}{\partial x} + S_u$$

$$\frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(\rho v v) = \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) - \frac{\partial p}{\partial y} + S_v$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

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If you stagger, you walk very unsteadily, for example because you are ill or drunk.

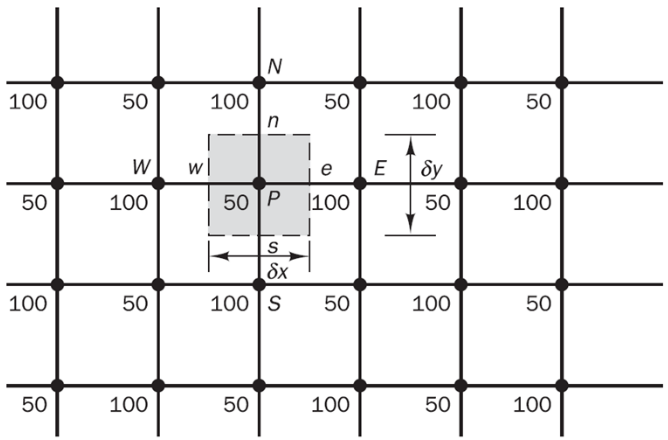
The staggered grid

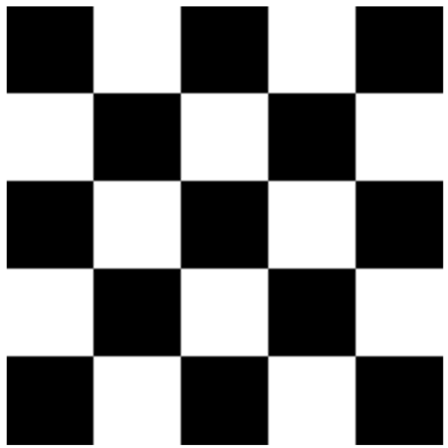
- Where to store the velocities?
- If the velocities and the pressures are both defined at the nodes of an ordinary CV a highly non-uniform pressure field can act like a uniform field in the discretized momentum equations.
  - Checker board problem
  - The pressure at the central node (*P*)
    - does not appear
  - dp/dx, dp/dy
    - Zero at all the nodal points even with the oscillations
    - Zero momentum source
  - This behaviour is obviously non-physical.

$$\frac{\partial p}{\partial x} = \frac{p_e - p_w}{\delta x} = \frac{\left(\frac{p_E + p_P}{2}\right) - \left(\frac{p_P + p_W}{2}\right)}{\delta x}$$

$$= \frac{p_E - p_W}{2\delta x}$$

Checker board pressure field

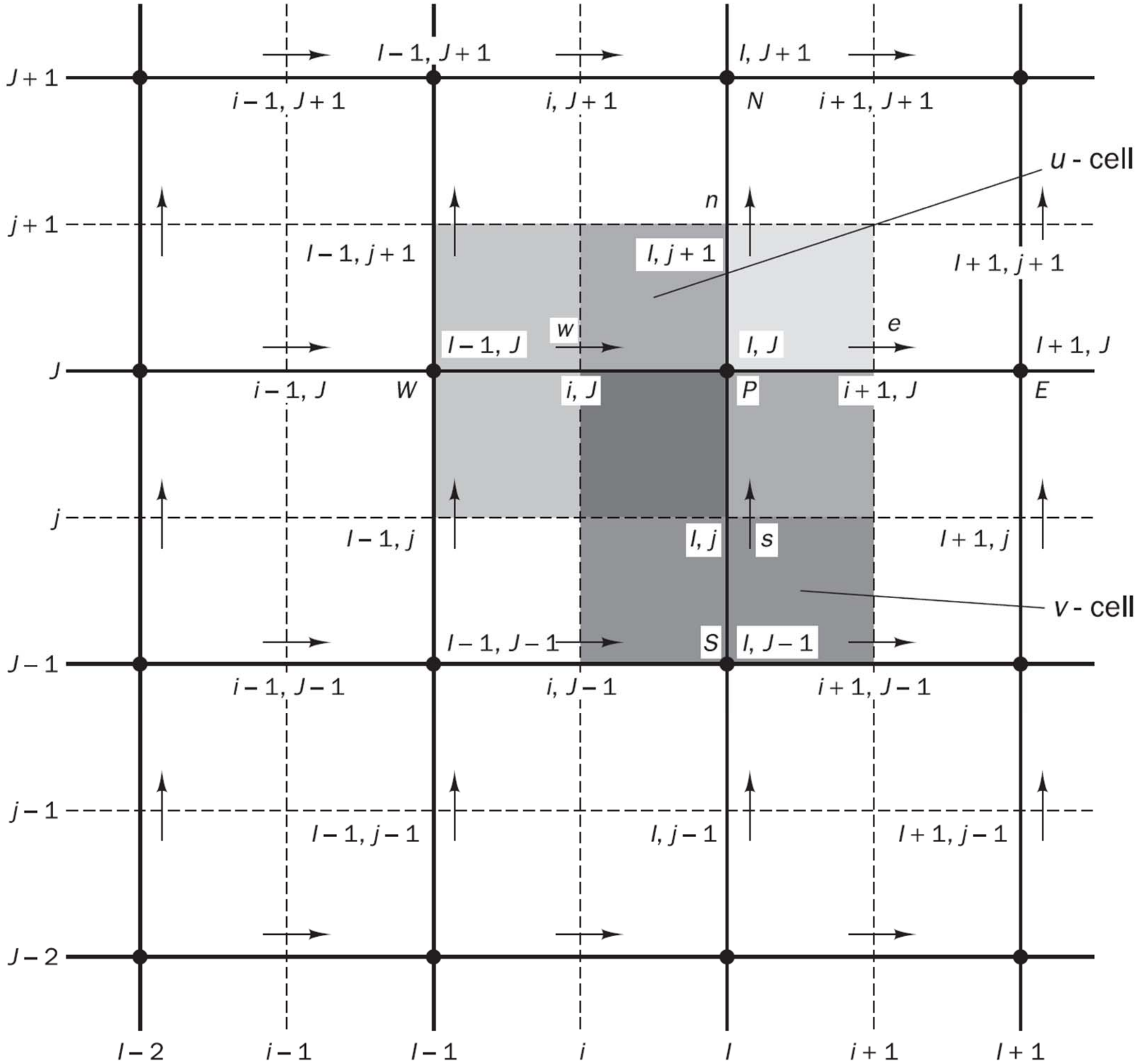




- Solutions
  - Use a staggered grid
  - Rhie-Chow interpolation

The staggered grid

- Harlow and Welch (1965)
  - Scalar variables
    - Pressure
    - Stored at •
  - Velocities
    - Stored at → and ↑
    - x-direction: →
    - y-direction: ↑
  - Scalar node
    - (I,J)
  - Velocity node
    - (i,J)
    - (I,j)



The staggered grid

- Harlow and Welch (1965)
  - Scalar variables
    - Pressure
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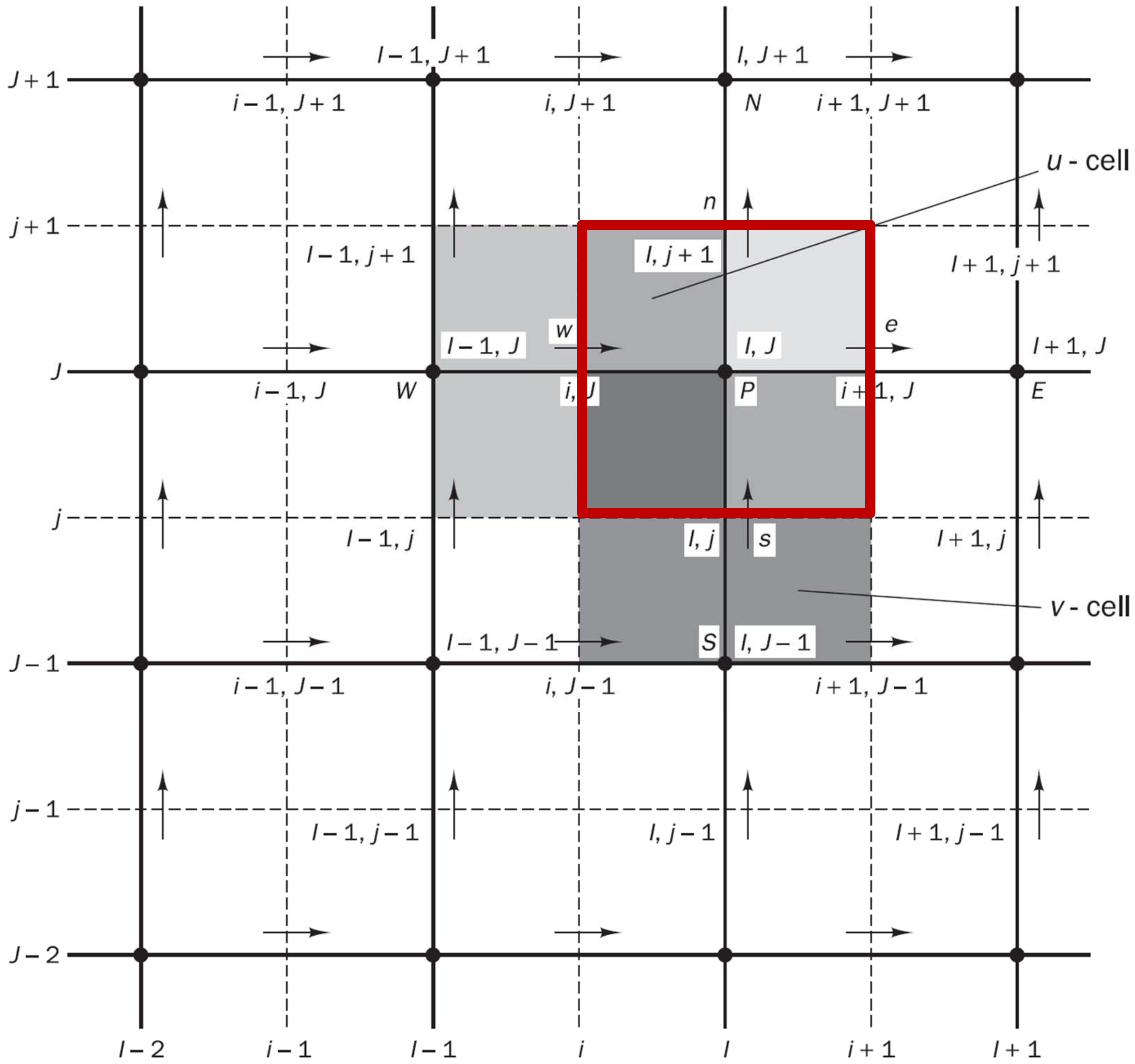

The staggered grid

- Harlow and Welch (1965)
  - Scalar variables
    - Pressure
    - Stored at •
  - Velocities
    - Stored at → and ↑
    - x-direction: →
    - y-direction: ↑
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    - (I,J)
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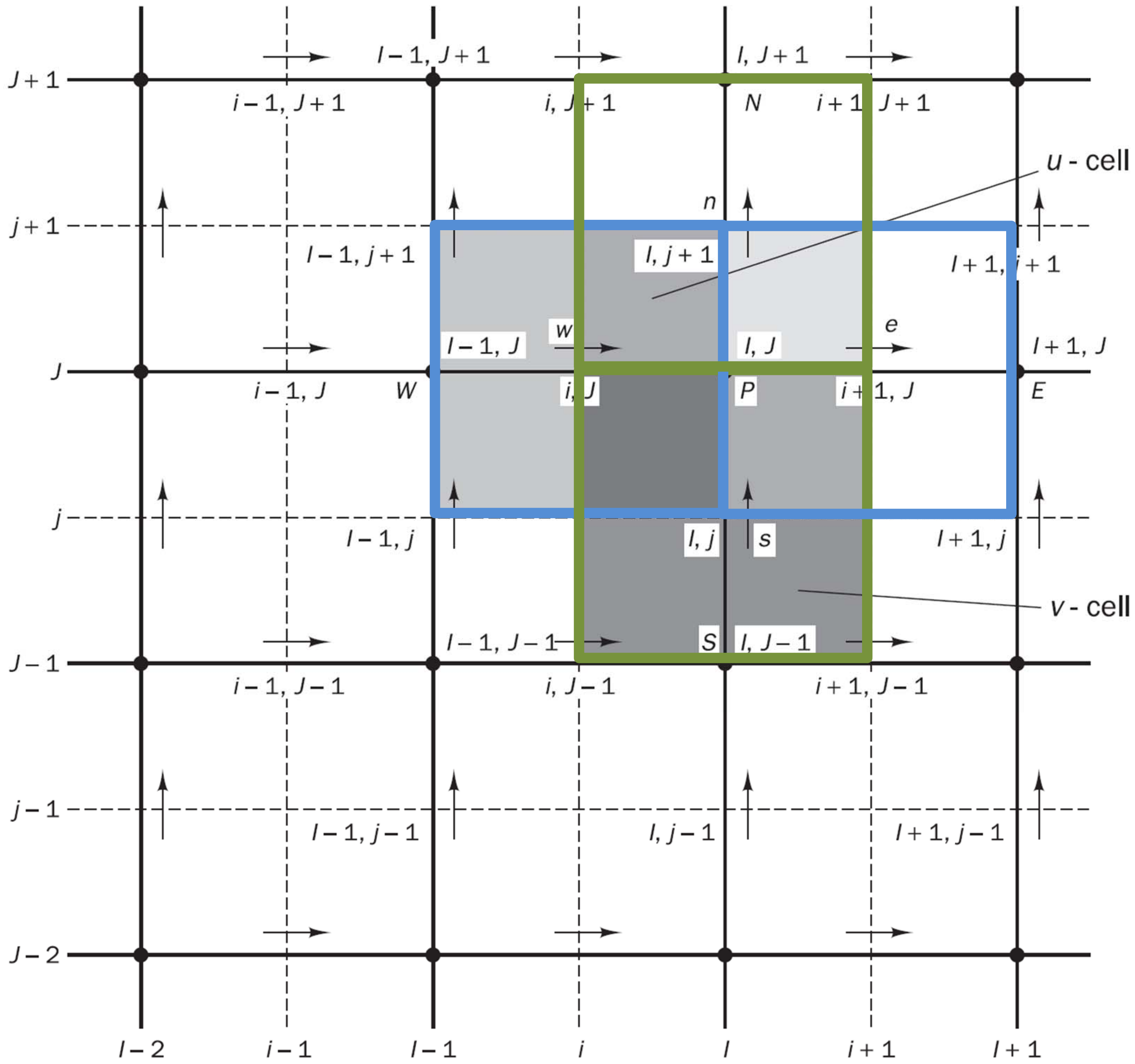
    - (i,J)
    - (I,j)

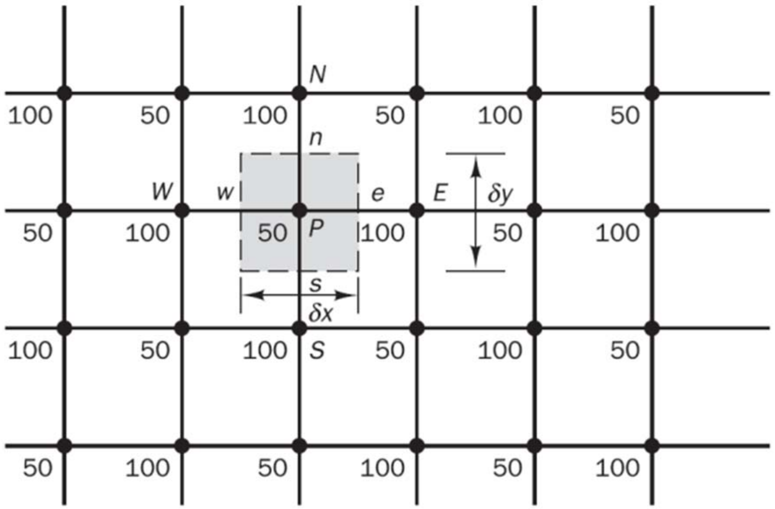


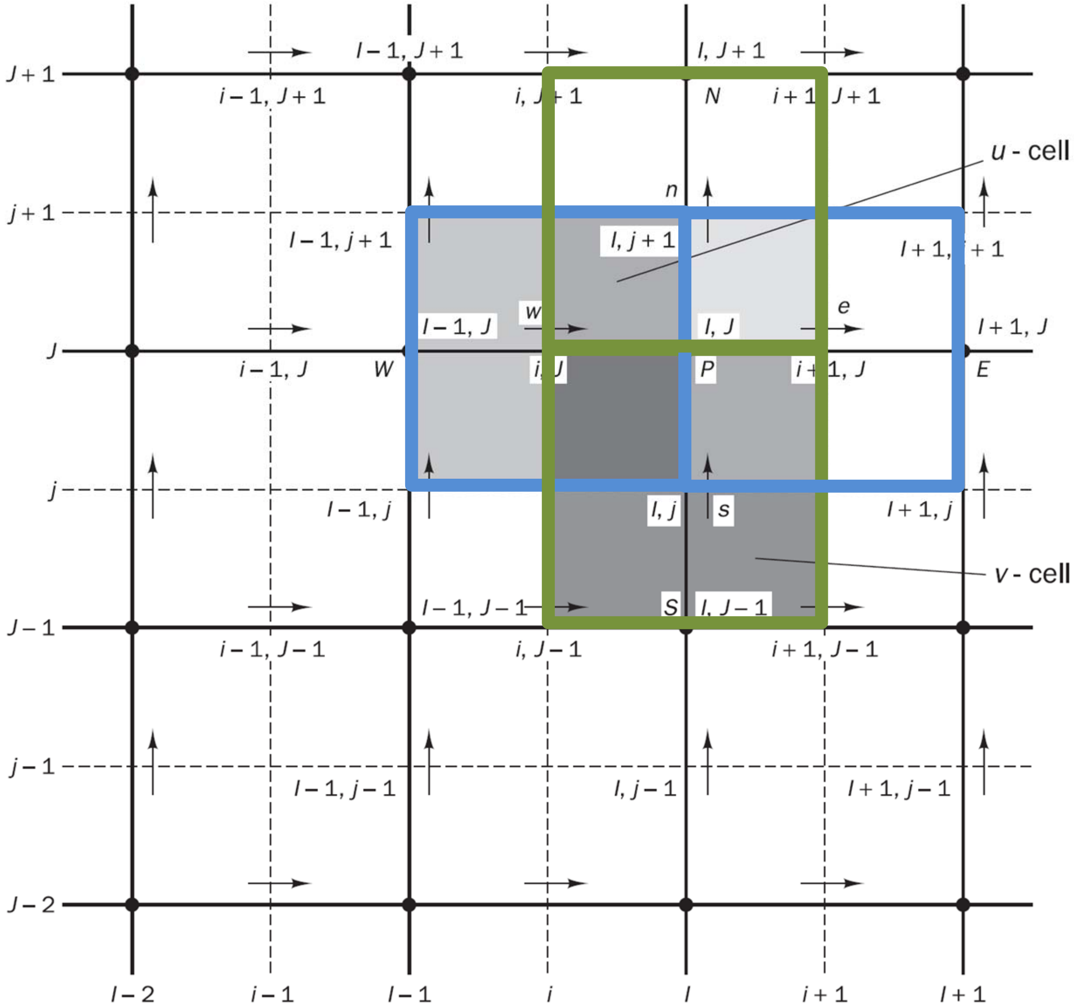
The staggered grid

- In the staggered grid arrangement
  - Pressure gradient
    - At (i,J) and (I,j)

$$\frac{\partial p}{\partial x} = \frac{p_P - p_W}{\delta x_u} \qquad \frac{\partial p}{\partial y} = \frac{p_P - p_S}{\delta y_v}$$

  - Advantages
    - No checkerboard problem
    - No interpolation to calculate velocities at cell faces





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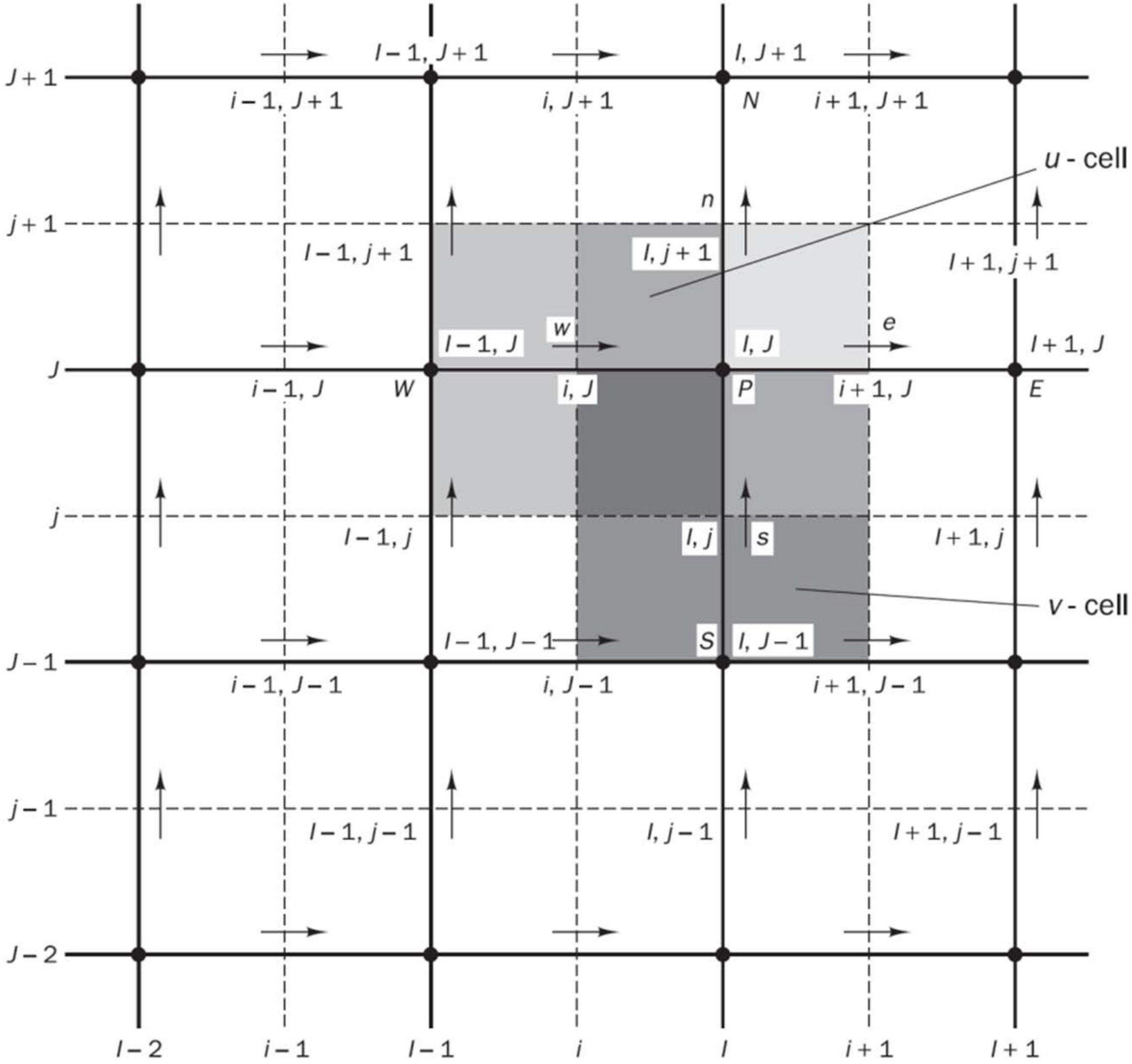
The momentum equations

- On staggered grid
  - Scalar grid: *I-1, I, I+1, …. , J-1, J, J+1, ….*
  - Velocity grid: *i-1, i, i+1, …. , j-1, j, j+1, ….*
  - Backward staggered grid

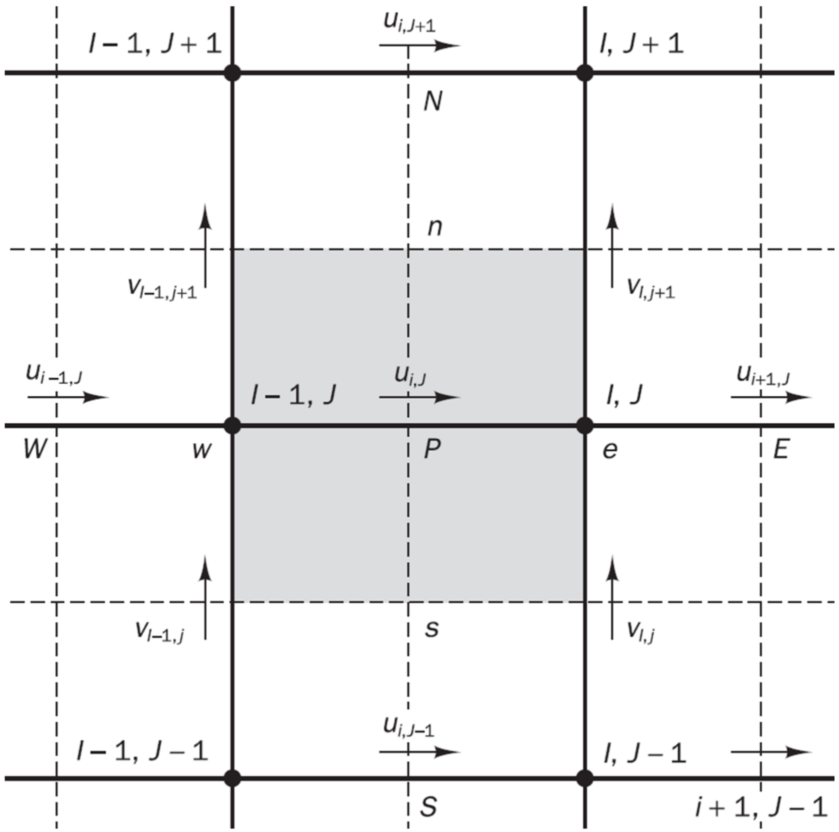
$$x_i = x_I - \frac{1}{2}\delta x_u \qquad y_j = y_J - \frac{1}{2}\delta y_v$$



The momentum equations

- Discretized momentum equation
  - x-directional momentum eq.

$$\frac{\partial}{\partial x}(\rho uu)+\frac{\partial}{\partial y}(\rho vu)=\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\partial p}{\partial x}+S_u$$



$$\int\nabla\cdot(\rho\vec{u}u)dV=\int(\rho\vec{u}u)\cdot\vec{n}dA=\sum_f(\rho\vec{u}u)\cdot\vec{A}$$

$$=(\rho uu)_e A_e-(\rho uu)_w A_w+(\rho vu)_n A_n-(\rho vu)_s A_s$$

$$=(\rho u)_e u_e A_e-(\rho u)_w u_w A_w+(\rho v)_n u_n A_n-(\rho v)_s u_s A_s$$

$$=F_e u_e A_e-F_w u_w A_w+F_n u_n A_n-F_s u_s A_s$$

$$F_e=(\rho u)_e=\frac{1}{2}[(\rho u)_E+(\rho u)_P]=\frac{1}{2}\left[\left(\frac{\rho_{ee}+\rho_e}{2}\right)u_E+\left(\frac{\rho_e+\rho_w}{2}\right)u_P\right]$$

$$=\frac{1}{2}\left[\left(\frac{\rho_{I+1,J}+\rho_{I,J}}{2}\right)u_{i+1,J}+\left(\frac{\rho_{I,J}+\rho_{I-1,J}}{2}\right)u_{i,J}\right]$$

The momentum equations

- Discretized momentum equation
  - x-directional momentum eq.

$$F_e = (\rho u)_e = \frac{1}{2}\left[\left(\frac{\rho_{I+1,J} + \rho_{I,J}}{2}\right)u_{i+1,J} + \left(\frac{\rho_{I,J} + \rho_{I-1,J}}{2}\right)u_{i,J}\right]$$

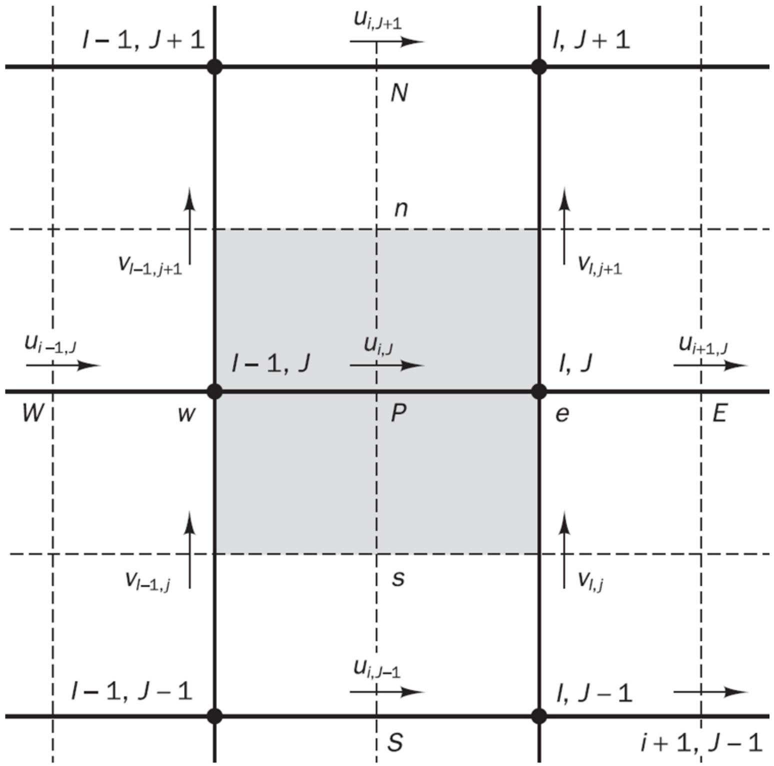
$$F_w = (\rho u)_w = \frac{1}{2}\left[\left(\frac{\rho_{I,J} + \rho_{I-1,J}}{2}\right)u_{i,J} + \left(\frac{\rho_{I-1,J} + \rho_{I-2,J}}{2}\right)u_{i-1,J}\right]$$

$$F_n = (\rho v)_n = \frac{1}{2}\left[(\rho v)_{nw} + (\rho u)_{ne}\right]$$

$$= \frac{1}{2}\left[\left(\frac{\rho_{I-1,J+1} + \rho_{I-1,J}}{2}\right)v_{I-1,j+1} + \left(\frac{\rho_{I,J+1} + \rho_{I,J}}{2}\right)v_{I,j+1}\right]$$

$$F_s = (\rho v)_s = \frac{1}{2}\left[(\rho v)_{sw} + (\rho u)_{se}\right]$$

$$= \frac{1}{2}\left[\left(\frac{\rho_{I-1,J} + \rho_{I-1,J-1}}{2}\right)v_{I-1,j} + \left(\frac{\rho_{I,J} + \rho_{I,J-1}}{2}\right)v_{I,j}\right]$$



The momentum equations

- Discretized momentum equation
  - x-directional momentum eq.

$$\int \nabla \cdot (\rho \vec{u} u) dV = \int (\rho \vec{u} u) \cdot \vec{n} dA = \sum_f (\rho \vec{u} u) \cdot \vec{A}$$

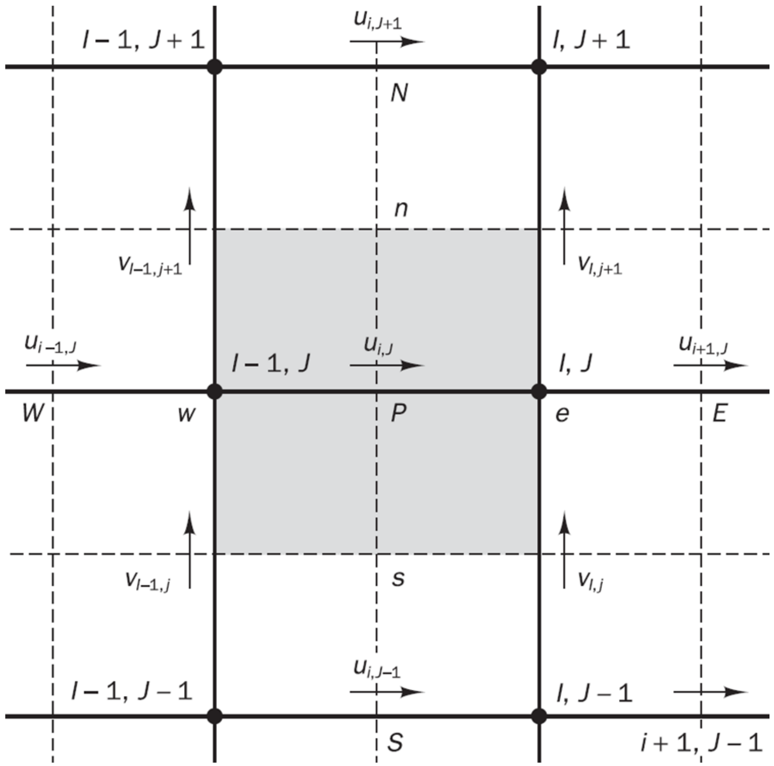
$$= (\rho uu)_e A_e - (\rho uu)_w A_w + (\rho vu)_n A_n - (\rho vu)_w A_w$$

$$= (\rho u)_e u_e A_e - (\rho u)_w u_w A_w + (\rho v)_n u_n A_n - (\rho v)_s u_s A_s$$

$$= F_e u_e A_e - F_w u_w A_w + F_n u_n A_n - F_s u_s A_s$$

$$u_e \quad u_w \quad u_n \quad u_s$$

- Dependent on the differencing method
  - upwind, hybrid, QUICK, TVD



The momentum equations

- Discretized momentum equation
  - x-directional momentum eq.

$$\frac{\partial}{\partial x}(\rho uu)+\frac{\partial}{\partial y}(\rho vu)=\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\partial p}{\partial x}+S_u$$

$$\int\nabla\cdot\mu(\nabla u)dV=\int\mu(\nabla u)\cdot\vec{n}dA=\sum_f(\mu\nabla u)\cdot\vec{A}$$

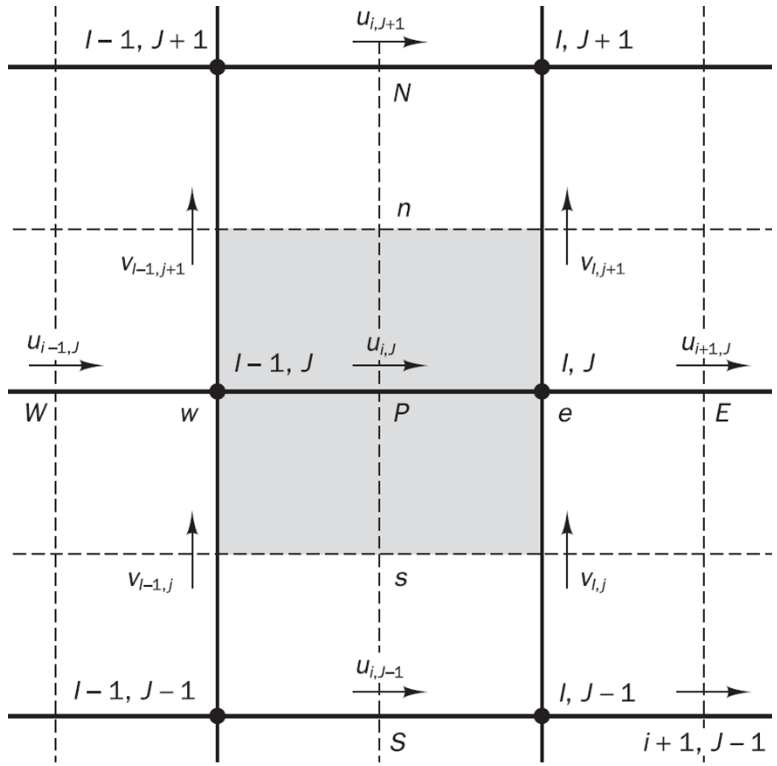
$$=\left(\mu\frac{du}{dx}\right)_e A_e-\left(\mu\frac{du}{dx}\right)_w A_w+\left(\mu\frac{du}{dy}\right)_n A_n-\left(\mu\frac{du}{dy}\right)_s A_s$$

$$=\left(\mu_e\frac{u_E-u_P}{\delta x_e}\right)A_e-\left(\mu_w\frac{u_P-u_W}{\delta x_w}\right)A_w$$

$$+\left(\mu_n\frac{u_N-u_P}{\delta x_n}\right)A_n-\left(\mu_s\frac{u_P-u_S}{\delta x_s}\right)_w A_s$$

$$=\left(\mu_e\frac{u_{i+1,J}-u_{i,J}}{x_{i+1}-x_i}\right)A_e-\left(\mu_w\frac{u_{i,j}-u_{i-1,J}}{x_i-x_{i-1}}\right)A_w$$

$$+\left(\mu_n\frac{u_{i,J+1}-u_{i,J}}{x_{i,J+1}-x_{i,J}}\right)A_n-\left(\mu_s\frac{u_{i,J}-u_{i,J-1}}{x_{i,J}-x_{i,J-1}}\right)A_s$$



The momentum equations

- Discretized momentum equation
  - x-directional momentum eq.

$$\int \nabla \cdot \mu(\nabla u) dV = \int \mu(\nabla u) \cdot \vec{n} dA = \sum_f (\mu \nabla u) \cdot \vec{A}$$

$$= \left(\mu_e \frac{u_{i+1,J} - u_{i,J}}{x_{i+1} - x_i} \right) A_e - \left(\mu_w \frac{u_{i,J} - u_{i-1,J}}{x_i - x_{i-1}} \right) A_w$$

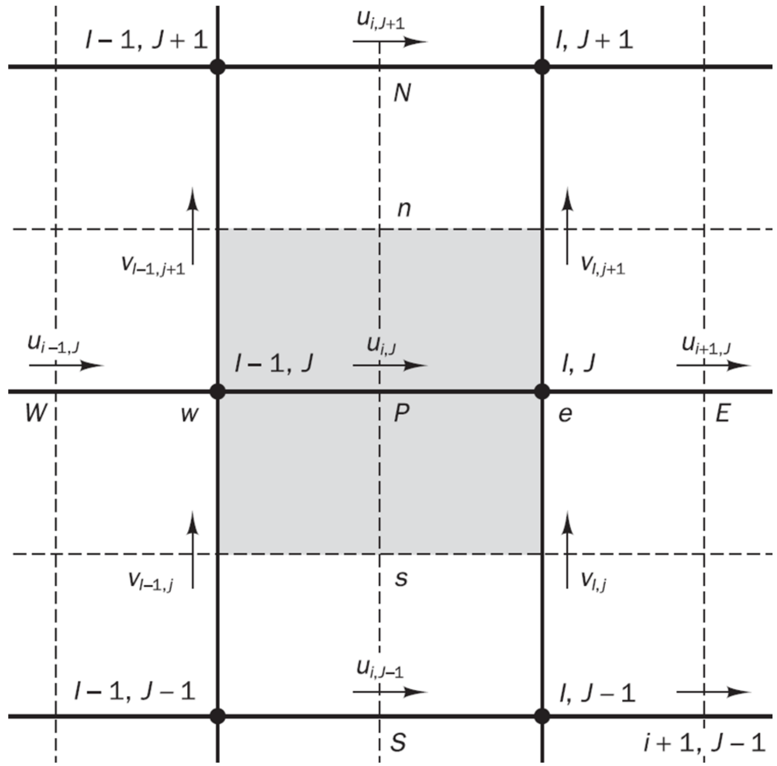
$$+ \left(\mu_n \frac{u_{i,J+1} - u_{i,J}}{x_{i,J+1} - x_{i,J}} \right) A_n - \left(\mu_s \frac{u_{i,J} - u_{i,J-1}}{x_{i,J} - x_{i,J-1}} \right) A_s$$

$$\mu_e = \mu_{I,J}$$

$$\mu_w = \mu_{I-1,J}$$

$$\mu_n = \frac{\mu_{I,J+1} + \mu_{I-1,J+1} + \mu_{I-1,J} + \mu_{I,J}}{4}$$

$$\mu_s = \frac{\mu_{I,J-1} + \mu_{I-1,J-1} + \mu_{I-1,J} + \mu_{I,J}}{4}$$



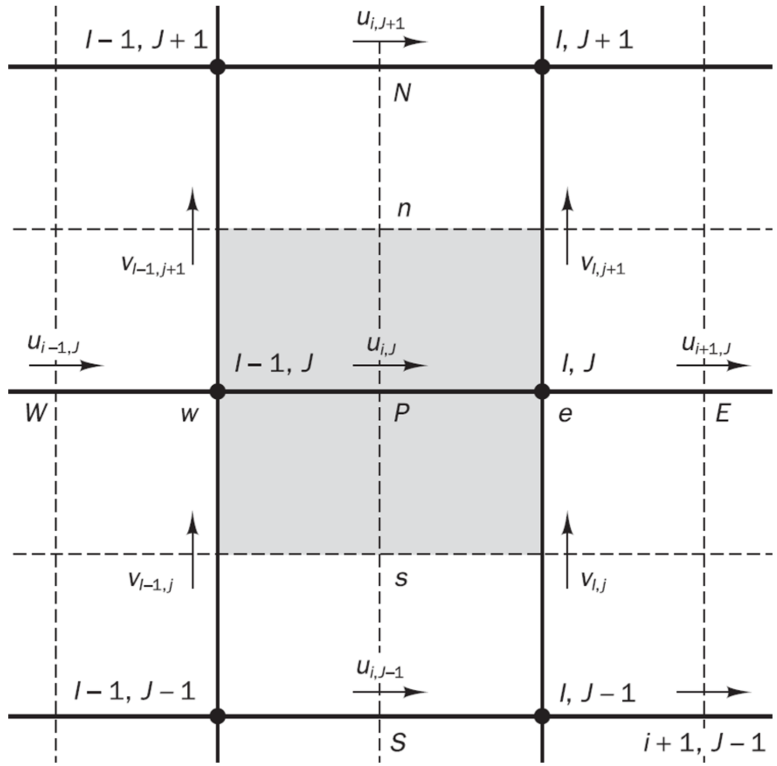
The momentum equations

- Discretized momentum equation
  - x-directional momentum eq.

$$\frac{\partial}{\partial x}(\rho uu)+\frac{\partial}{\partial y}(\rho vu)=\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\partial p}{\partial x}+S_u$$

$$\int\left(\frac{\partial p}{\partial x}\right)dV=\frac{p_e-p_w}{x_{I,J}-x_{I-1,J}}\Delta V=\frac{p_{I,J}-p_{I-1,J}}{x_{I,J}-x_{I-1,J}}\Delta V$$

$$\int S_u dV=S_u\Delta V$$



The momentum equations

- Discretized momentum equation
  - x-directional momentum eq.

$$\frac{\partial}{\partial x}(\rho uu) + \frac{\partial}{\partial y}(\rho vu) = \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\partial p}{\partial x} + S_u$$

$$F_e u_e A_e - F_w u_w A_w + F_n u_n A_n - F_s u_s A_s$$

$$\left(\mu_e \frac{u_{i+1,J} - u_{i,J}}{x_{i+1} - x_i}\right)A_e - \left(\mu_w \frac{u_{i,J} - u_{i-1,J}}{x_i - x_{i-1}}\right)A_w + \left(\mu_n \frac{u_{i,J+1} - u_{i,J}}{x_{i,J+1} - x_{i,J}}\right)A_n - \left(\mu_s \frac{u_{i,J} - u_{i,J-1}}{x_{i,J} - x_{i,J-1}}\right)A_s$$

$$\frac{p_{I,J} - p_{I-1,J}}{x_{I,J} - x_{I-1,J}}\Delta V \qquad S_u \Delta V$$

$$a_{i,J} u_{i,J} = \sum a_{nb} u_{nb} - \frac{p_{I,J} - p_{I-1,J}}{\delta x_u}\Delta V_u + \bar{S}\Delta V_u$$

$$a_{i,J} u_{i,J} = \sum a_{nb} u_{nb} + (p_{I-1,J} - p_{I,J})A_{i,J} + b_{i,J}$$

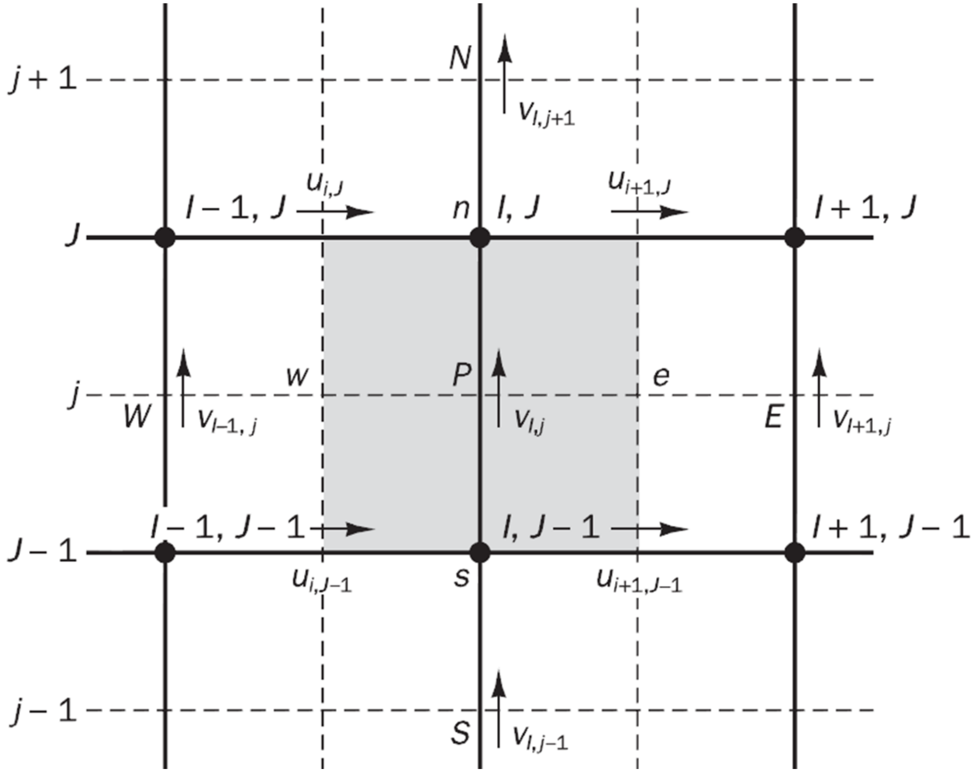
The momentum equations

- Discretized momentum equation
  - y-directional momentum eq.

$$\frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(\rho v v) = \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) - \frac{\partial p}{\partial y} + S_v$$

$$a_{I,j} v_{I,j} = \sum a_{nb} v_{nb} + (p_{I,J-1} - p_{I,J}) A_{I,j} + b_{I,j}$$

$$a_{i,J} u_{i,J} = \sum a_{nb} u_{nb} + (p_{I-1,J} - p_{I,J}) A_{i,J} + b_{i,J}$$



- F_e, F_w, F_n, F_s : include the velocity, non-linear part, use the previous iteration values
  - Iterative solution method is required.
- Given a pressure field *p*, discretized momentum equations of the form can be written for each *u*- and *v*-control volume and then solved to obtain the velocity fields.
- If the pressure field is correct the resulting velocity field will satisfy continuity.
- As the pressure field is unknown, we need a method for calculating pressure.

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The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Patankar and Spalding
  - Guess-and-correct procedure for the calculation of pressure on the staggered grid arrangement

  - Initiation of the SIMPLE calculation
    - Guess: p^*

  - Solve the discretized momentum equations

$$a_{i,J} u^*_{i,J} = \sum a_{nb} u^*_{nb} + (p^*_{I-1,J} - p^*_{I,J}) A_{i,J} + b_{i,J}$$

$$a_{I,j} v^*_{I,j} = \sum a_{nb} v^*_{nb} + (p^*_{I,J-1} - p^*_{I,J}) A_{I,j} + b_{I,j}$$

- Intermediate velocity
  - Two systems of equations
  - For u^* and v^*

- Pressure correction, velocity corrections

$$p = p^* + p'$$
$$u = u^* + u'$$
$$v = v^* + v'$$

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations

$$a_{i,J} u^*_{i,J} = \sum a_{nb} u^*_{nb} + (p^*_{I-1,J} - p^*_{I,J}) A_{i,J} + b_{i,J}$$

$$a_{i,J} u_{i,J} = \sum a_{nb} u_{nb} + \left(p_{I-1,J} - p_{I,J}\right) A_{i,J} + b_{i,J}$$

$$a_{i,J}(u_{i,J} - u^*_{i,J}) = \sum a_{nb}(u_{nb} - u^*_{nb}) + [(p_{I-1,J} - p^*_{I-1,J}) - (p_{I,J} - p^*_{I,J})] A_{i,J}$$

$$a_{I,j}(v_{I,j} - v^*_{I,j}) = \sum a_{nb}(v_{nb} - v^*_{nb}) + [(p_{I,J-1} - p^*_{I,J-1}) - (p_{I,J} - p^*_{I,J})] A_{I,j}$$

$$a_{i,J} u'_{i,J} = \sum a_{nb} u'_{nb} + (p'_{I-1,J} - p'_{I,J}) A_{i,J}$$

$$a_{I,j} v'_{I,j} = \sum a_{nb} v'_{nb} + (p'_{I,J-1} - p'_{I,J}) A_{I,j}$$

$$p = p^* + p'$$

$$u = u^* + u'$$

$$v = v^* + v'$$

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations

$$a_{i,J}u'_{i,J} = \sum a_{nb}u'_{nb} + (p'_{I-1,J} - p'_{I,J})A_{i,J}$$

$$a_{I,j}v'_{I,j} = \sum a_{nb}v'_{nb} + (p'_{I,J-1} - p'_{I,J})A_{I,j}$$

- For simplicity, the following two terms are dropped.

Omission of these terms Is the main approximation of SIMPLE

$$\sum a_{nb}u'_{nb} \qquad \sum a_{nb}v'_{nb}$$

$$a_{i,J}u'_{i,J} = \left(p'_{I-1,J} - p'_{I,J}\right)A_{i,J}$$

$$a_{I,j}v'_{I,j} = \left(p'_{I,J-1} - p'_{I,J}\right)A_{I,j}$$

STEP 1: Solve discretised momentum equations

$$a_{i,J}\, u^*_{i,J} = \Sigma a_{nb}u^*_{nb} + (p^*_{I-1,J} - p^*_{I,J})\, A_{i,J} + b_{i,J}$$

$$a_{I,j}\, v^*_{I,j} = \Sigma a_{nb}v^*_{nb} + (p^*_{I,J-1} - p^*_{I,J})\, A_{I,J} + b_{I,j}$$

Express u and v using p’

$$u'_{i,J} = d_{i,J}(p'_{I-1,J} - p'_{I,J})$$

$$v'_{I,j} = d_{I,j}(p'_{I,J-1} - p'_{I,J})$$

where $d_{i,J} = \dfrac{A_{i,J}}{a_{i,J}}$ and $d_{I,j} = \dfrac{A_{I,j}}{a_{I,j}}$

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations

$$u'_{i,J} = d_{i,J}(p'_{I-1,J} - p'_{I,J})$$
$$v'_{I,j} = d_{I,j}(p'_{I,J-1} - p'_{I,J})$$

$$u_{i,J} = u^*_{i,J} + d_{i,J}(p'_{I-1,J} - p'_{I,J})$$

$$v_{I,j} = v^*_{I,j} + d_{I,j}(p'_{I,J-1} - p'_{I,J})$$

$$u_{i+1,J} = u^*_{i+1,J} + d_{i+1,J}(p'_{I,J} - p'_{I+1,J})$$
$$v_{I,j+1} = v^*_{I,j+1} + d_{I,j+1}(p'_{I,J} - p'_{I,J+1})$$

STEP 1: Solve discretised momentum equations

$$a_{i,J}\, u^*_{i,J} = \Sigma a_{nb} u^*_{nb} + (p^*_{I-1,J} - p^*_{I,J})\, A_{i,J} + b_{i,J}$$
$$a_{I,j}\, v^*_{I,j} = \Sigma a_{nb} v^*_{nb} + (p^*_{I,J-1} - p^*_{I,J})\, A_{I,J} + b_{I,j}$$

Express u and v using p’

$$p = p^* + p'$$
$$u = u^* + u'$$
$$v = v^* + v'$$

- Solve the momentum equations with guessed pressure and guessed velocities for non-linear terms.
- The predicted velocity should satisfy continuity equation.

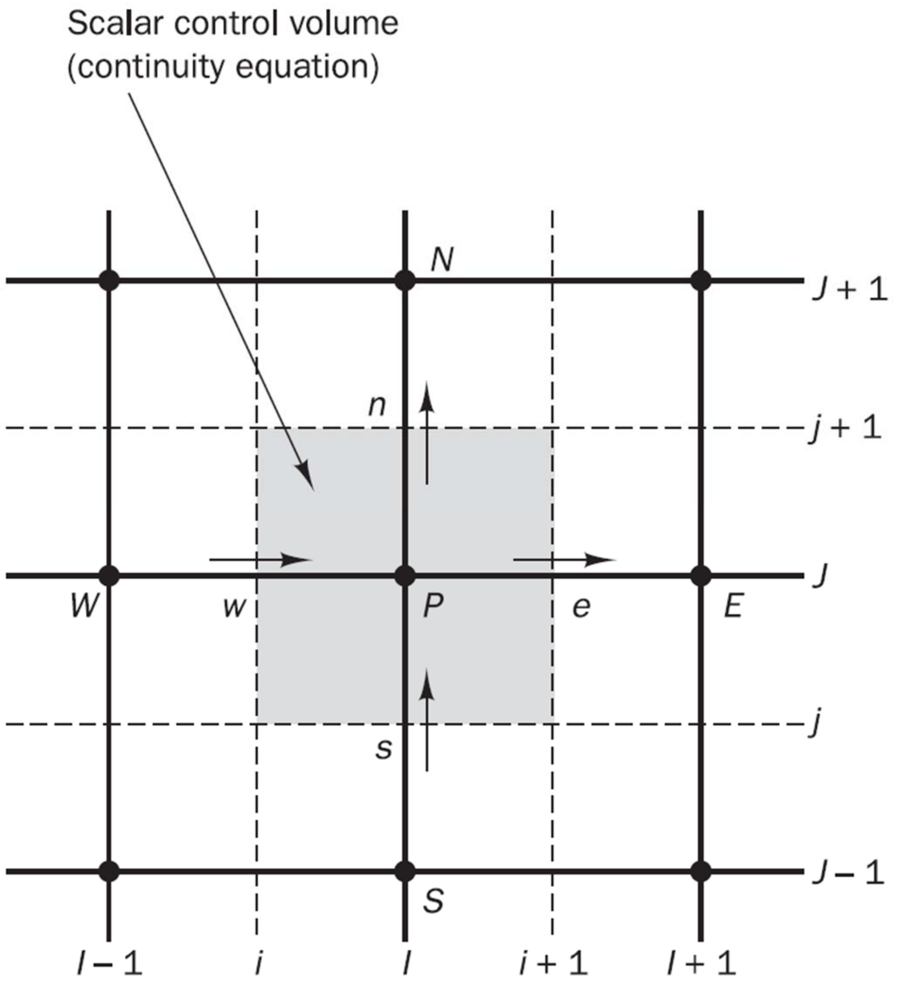
The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Continuity equation on scalar node

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

$$\sum_f \rho\vec{u}\cdot\vec{A} = \left[(\rho uA)_e - (\rho uA)_w\right] + \left[(\rho vA)_n - (\rho vA)_s\right]$$

$$= \left[(\rho uA)_{i+1,J} - (\rho uA)_{i,J}\right] + \left[(\rho vA)_{I,j+1} - (\rho vA)_{I,j}\right] = 0$$



STEP 1: Solve discretised momentum equations

$$a_{i,J}\, u^*_{i,J} = \Sigma a_{nb} u^*_{nb} + (p^*_{I-1,J} - p^*_{I,J})\, A_{i,J} + b_{i,J}$$

$$a_{I,j}\, v^*_{I,j} = \Sigma a_{nb} v^*_{nb} + (p^*_{I,J-1} - p^*_{I,J})\, A_{I,j} + b_{I,j}$$

Express u and v using p’

Derive the pressure correction eq.

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Continuity equation on scalar node

$$u_{i+1,J} = u^*_{i+1,J} + d_{i+1,J}(p'_{I,J} - p'_{I+1,J})$$
$$v_{I,j+1} = v^*_{I,j+1} + d_{I,j+1}(p'_{I,J} - p'_{I,J+1})$$

$$\left[(\rho u A)_{i+1,J} - (\rho u A)_{i,J}\right] + \left[(\rho v A)_{I,j+1} - (\rho v A)_{I,j}\right] = 0$$

$$\begin{aligned}
&[\rho_{i+1,J}A_{i+1,J}(u^*_{i+1,J} + d_{i+1,J}(p'_{I,J} - p'_{I+1,J})) \\
&\quad - \rho_{i,J}A_{i,J}(u^*_{i,J} + d_{i,J}(p'_{I-1,J} - p'_{I,J}))] \\
&+ [\rho_{I,j+1}A_{I,j+1}(v^*_{I,j+1} + d_{I,j+1}(p'_{I,J} - p'_{I,J+1})) \\
&\quad - \rho_{I,j}A_{I,j}(v^*_{I,j} + d_{I,j}(p'_{I,J-1} - p'_{I,J}))] = 0
\end{aligned}$$

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Continuity equation on scalar node
    - Rearrange

$$
\begin{aligned}
&[\rho_{i+1,J}A_{i+1,J}(u^*_{i+1,J} + d_{i+1,J}(p'_{I,J} - p'_{I+1,J})) \\
&\quad - \rho_{i,J}A_{i,J}(u^*_{i,J} + d_{i,J}(p'_{I-1,J} - p'_{I,J}))] \\
&+ [\rho_{I,j+1}A_{I,j+1}(v^*_{I,j+1} + d_{I,j+1}(p'_{I,J} - p'_{I,J+1})) \\
&\quad - \rho_{I,j}A_{I,j}(v^*_{I,j} + d_{I,j}(p'_{I,J-1} - p'_{I,J}))] = 0
\end{aligned}
$$

$$
\begin{aligned}
&[(\rho dA)_{i+1,J} + (\rho dA)_{i,J} + (\rho dA)_{I,j+1} + (\rho dA)_{I,j}]p'_{I,J} \\
&= (\rho dA)_{i+1,J}p'_{I+1,J} + (\rho dA)_{i,J}p'_{I-1,J} + (\rho dA)_{I,j+1}p'_{I,J+1} + (\rho dA)_{I,j}p'_{I,J-1} \\
&\quad + [(\rho u^* A)_{i,J} - (\rho u^* A)_{i+1,J} + (\rho v^* A)_{I,j} - (\rho v^* A)_{I,j+1}]
\end{aligned}
$$

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Continuity equation on scalar node
    - Cell pressure correction equation

$$[(\rho dA)_{i+1,J} + (\rho dA)_{i,J} + (\rho dA)_{I,j+1} + (\rho dA)_{I,j}]p'_{I,J}$$

$$= (\rho dA)_{i+1,J}p'_{I+1,J} + (\rho dA)_{i,J}p'_{I-1,J} + (\rho dA)_{I,j+1}p'_{I,J+1} + (\rho dA)_{I,j}p'_{I,J-1}$$

$$+ [(\rho u^* A)_{i,J} - (\rho u^* A)_{i+1,J} + (\rho v^* A)_{I,j} - (\rho v^* A)_{I,j+1}]$$

$$a_{I,J}p'_{I,J} = a_{I+1,J}p'_{I+1,J} + a_{I-1,J}p'_{I-1,J} + a_{I,J+1}p'_{I,J+1} + a_{I,J-1}p'_{I,J-1} + b'_{I,J}$$

$$a_{I,J} = a_{I+1,J} + a_{I-1,J} + a_{I,J+1} + a_{I,J-1}$$

$a_{I+1,J}$	$a_{I-1,J}$	$a_{I,J+1}$	$a_{I,J-1}$	$b'_{I,J}$
$(\rho dA)_{i+1,J}$	$(\rho dA)_{i,J}$	$(\rho dA)_{I,j+1}$	$(\rho dA)_{I,j}$	$(\rho u^* A)_{i,J} - (\rho u^* A)_{i+1,J} + (\rho v^* A)_{I,j} - (\rho v^* A)_{I,j+1}$

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Continuity equation on scalar node
    - Collect the cell pressure equations for all cells
    - System pressure correction equation

STEP 2: Solve pressure correction equation

$$a_{I,J}\, p'_{I,J} = a_{I-1,J}\, p'_{I-1,J} + a_{I+1,J}\, p'_{I+1,J} + a_{I,J-1}\, p'_{I,J-1} + a_{I,J+1}\, p'_{I,J+1} + b'_{I,J}$$

$$a_{I,J}p'_{I,J} = a_{I+1,J}p'_{I+1,J} + a_{I-1,J}p'_{I-1,J} + a_{I,J+1}p'_{I,J+1} + a_{I,J-1}p'_{I,J-1} + b'_{I,J}$$

$$\begin{pmatrix} \\ \cdots & -a_{I,J-1} & \cdots & -a_{I-1,J} & a_{I,J} & -a_{I+1,J} & \cdots & -a_{I,J-1} & \cdots \\ \\ \end{pmatrix} \begin{pmatrix} \vdots \\ p'_{I,J-1} \\ \vdots \\ p'_{I-1,J} \\ p'_{I,J} \\ p'_{I+1,J} \\ \vdots \\ p'_{I,J+1} \\ \vdots \end{pmatrix} = \begin{pmatrix} \\ b'_{I,J} \\ \\ \end{pmatrix}$$

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Solve the pressure correction equation
    - Pressure correction p' can be obtained.

$$p = p^* + p'$$

$$u = u^* + u'$$

$$v = v^* + v'$$

$$u_{i,J} = u^*_{i,J} + d_{i,J}(p'_{I-1,J} - p'_{I,J})$$

$$v_{I,j} = v^*_{I,j} + d_{I,j}(p'_{I,J-1} - p'_{I,J})$$

STEP 3: Correct pressure and velocities

$$p_{I,J} = p^*_{I,J} + p'_{I,J}$$

$$u_{i,J} = u^*_{i,J} + d_{i,J}\,(p'_{I-1,J} - p'_{I,J})$$

$$v_{I,j} = v^*_{I,j} + d_{I,j}\,(p'_{I,J-1} - p'_{I,J})$$

Not the final solution. Why?

$$a_{i,J}u'_{i,J} = \sum a_{nb}u'_{nb} + (p'_{I-1,J} - p'_{I,J})A_{i,J}$$

$$a_{I,j}v'_{I,j} = \sum a_{nb}v'_{nb} + (p'_{I,J-1} - p'_{I,J})A_{I,j}$$

$$a_{i,J}u'_{i,J} = \left(p'_{I-1,J} - p'_{I,J}\right)A_{i,J}$$

$$a_{I,j}v'_{I,j} = \left(p'_{I,J-1} - p'_{I,J}\right)A_{I,j}$$

$$F_e, F_w, F_n, F_s$$

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Under relaxation

$$p = p^* + p'$$

$$p^{new} = p^* + \alpha_p p'$$

STEP 3: Correct pressure and velocities

$$p_{I,J} = p^*_{I,J} + p'_{I,J}$$

$$u_{i,J} = u^*_{i,J} + d_{i,J}\,(p'_{I-1,J} - p'_{I,J})$$

$$v_{I,j} = v^*_{I,j} + d_{I,j}\,(p'_{I,J-1} - p'_{I,J})$$

- Pressure under-relaxation factor

$$0 < \alpha_p < 1$$

$$u^{new} = \alpha_u u + (1 - \alpha_u) u^{(n-1)}$$

$$v^{new} = \alpha_v v + (1 - \alpha_v) v^{(n-1)}$$

- u- and v- velocity under relaxation factors

- A correct choice of under-relaxation factors α is essential for cost-effective simulations.
- Too large a value of α may lead to oscillatory or even divergent iterative solutions, and a value which is too small will cause extremely slow convergence.
- Unfortunately, the optimum values of under-relaxation factors are flow dependent and must be sought on a case-by-case basis.

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Discretize momentum equations from the second iteration

$$a_{i,J}u_{i,J} = \sum a_{nb}u_{nb} + (p_{I-1,J} - p_{I,J})A_{i,J} + b_{i,J}$$

$$\frac{a_{i,J}}{\alpha_u}u_{i,J} = \sum a_{nb}u_{nb} + (p_{I-1,J} - p_{I,J})A_{i,J} + b_{i,J} + \left[(1-\alpha_u)\frac{a_{i,J}}{\alpha_u}\right]u_{i,J}^{(n-1)}$$

$$a_{I,j}v_{I,j} = \sum a_{nb}v_{nb} + (p_{I,J-1} - p_{I,J})A_{I,j} + b_{I,j}$$

$$\frac{a_{I,j}}{\alpha_v}v_{I,j} = \sum a_{nb}v_{nb} + (p_{I,J-1} - p_{I,J})A_{I,j} + b_{I,j} + \left[(1-\alpha_v)\frac{a_{I,j}}{\alpha_v}\right]v_{I,j}^{(n-1)}$$

The SIMPLE algorithm

- Semi-Implicit Method for Pressure Linked Equations
  - Pressure correction equation from the second iteration

$$a_{I,J}p'_{I,J} = a_{I+1,J}p'_{I+1,J} + a_{I-1,J}p'_{I-1,J} + a_{I,J+1}p'_{I,J+1} + a_{I,J-1}p'_{I,J-1} + b'_{I,J}$$

$a_{I+1,J}$	$a_{I-1,J}$	$a_{I,J+1}$	$a_{I,J-1}$	$b'_{I,J}$
$(\rho dA)_{i+1,J}$	$(\rho dA)_{i,J}$	$(\rho dA)_{I,j+1}$	$(\rho dA)_{I,j}$	$(\rho u^* A)_{i,J} - (\rho u^* A)_{i+1,J} + (\rho v^* A)_{I,j} - (\rho v^* A)_{I,j+1}$

$$d_{i,J} = \frac{A_{i,J}\alpha_u}{a_{i,J}}, \quad d_{i+1,J} = \frac{A_{i+1,J}\alpha_u}{a_{i+1,J}}, \quad d_{I,j} = \frac{A_{I,j}\alpha_v}{a_{I,j}}, \quad d_{I,j+1} = \frac{A_{I,j+1}\alpha_v}{a_{I,j+1}}$$

The SIMPLE algorithm