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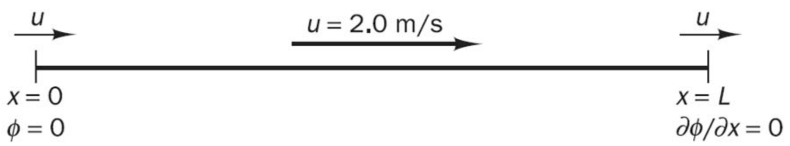
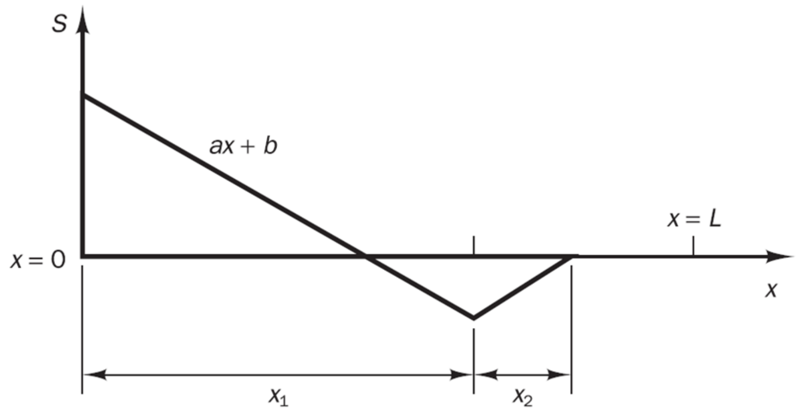
Worked example of transient C-D using QUICK

- Example 8.3

Consider convection and diffusion in the one-dimensional domain sketched in Figure 8.7. Calculate the transient temperature field if the initial temperature is zero everywhere and the boundary conditions are $\phi = 0$ at $x = 0$ and $\partial\phi/\partial x = 0$ at $x = L$. The data are $L = 1.5$ m, $u = 2$ m/s, $\rho = 1.0$ kg/m^3 and $\Gamma = 0.03$ kg/m.s. The source distribution defined by Figure 8.8 applies at times $t > 0$ with $a = -200$, $b = 100$, $x_1 = 0.6$ m, $x_2 = 0.2$ m. Write a computer program to calculate the transient temperature distribution until it reaches a steady state using the implicit method for time integration and the Hayase *et al.* variant of the QUICK scheme for the convective and diffusive terms, and compare this result with the analytical steady state solution.





Worked example of transient C-D using QUICK

- Example 8.3

  - Transient C-D equation

$$\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\rho u\phi)}{\partial x} = \frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right) + S$$

$$\phi_w = \frac{6}{8}\phi_W + \frac{3}{8}\phi_P - \frac{1}{8}\phi_{WW}$$

$$\phi_e = \frac{6}{8}\phi_P + \frac{3}{8}\phi_E - \frac{1}{8}\phi_W$$

  - 45 point grid

  - Initial and boundary conditions

$u = 2.0\ \mathrm{m/s}$ $\quad F = \rho u = 2.0$

$\Delta x = 0.0333$ $\quad D = \Gamma/\Delta x = 0.9$

$$\phi_w = \phi_W + \frac{1}{8}(3\phi_P - 2\phi_W - \phi_{WW})$$

$$\phi_e = \phi_P + \frac{1}{8}(3\phi_E - 2\phi_P - \phi_W)$$

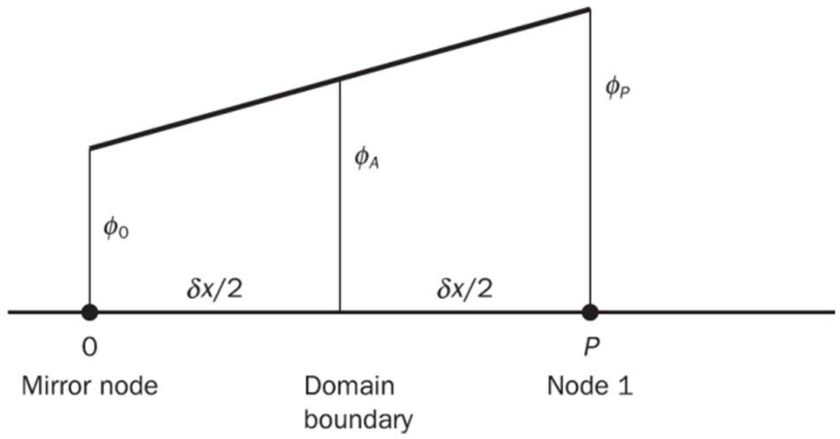
  - Implicit discretization

$$\frac{\rho(\phi_P - \phi_P^o)\Delta x}{\Delta t} + F_e\left[\phi_P + \frac{1}{8}(3\phi_E - 2\phi_P - \phi_W)\right] - F_w\left[\phi_W + \frac{1}{8}(3\phi_P - 2\phi_W - \phi_{WW})\right]$$

$$= D_e(\phi_E - \phi_P) - D_w(\phi_P - \phi_W)$$

Worked example of transient C-D using QUICK

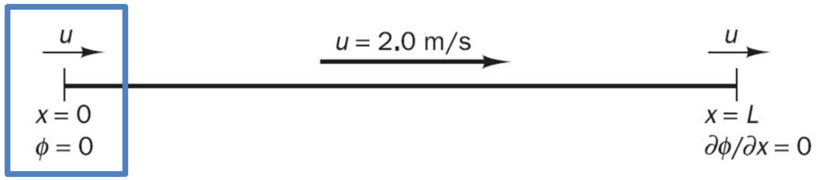
- Example 8.3
  - Modification for the boundary cells
    - For the first cell ⇒ mirror node approach

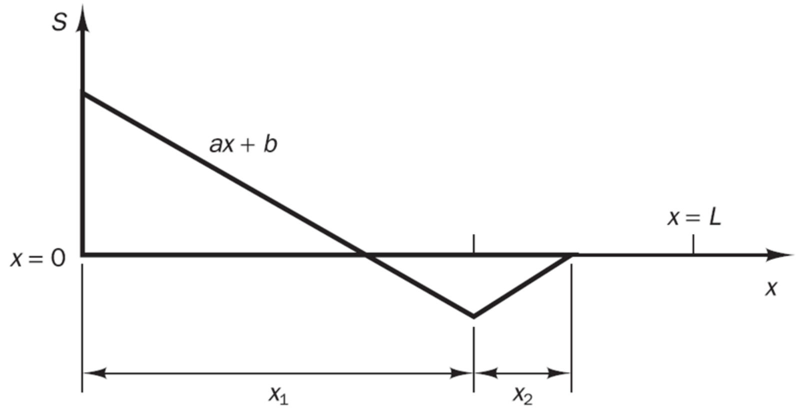


$$\phi_A = 0 \qquad \phi_0 = -\phi_P$$

$$\Gamma \frac{\partial \phi}{\partial x}\bigg|_A = \frac{D_A^*}{3}(9\phi_P - 8\phi_A - \phi_E) \qquad D_A^* = \Gamma/\Delta x$$

$$\frac{\rho(\phi_P - \phi_P^o)\Delta x}{\Delta t} + F_e\left[\phi_P + \frac{1}{8}(3\phi_E - \phi_P)\right] - F_A\phi_A = D_e(\phi_E - \phi_P) - \frac{D_A^*}{3}(9\phi_P - 8\phi_A - \phi_E)$$

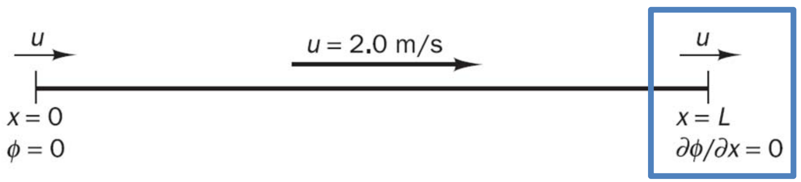


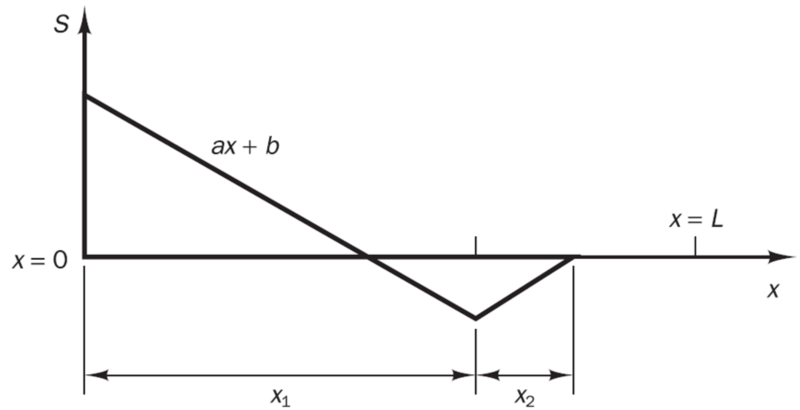


Worked example of transient C-D using QUICK

- Example 8.3
  - Modification for the boundary cells
    - For the last cell $\Rightarrow$ zero gradient boundary condition $\phi_B = \phi_P$

$$\frac{\rho(\phi_P - \phi_P^o)\Delta x}{\Delta t} + F_B\phi_P - F_w\left[\phi_W + \frac{1}{8}(3\phi_P - 2\phi_W - \phi_{WW})\right] = 0 - D_w(\phi_P - \phi_W)$$





Worked example of transient C-D using QUICK

- Example 8.3
  - General form

Iterative method (Hayase et al.)
Deferred correction

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + a_P^o\phi_P^o + S_u$$

$$a_P = a_W + a_E + a_P^o + (F_e - F_w) - S_P$$

$$a_P^o = \frac{\rho\Delta x}{\Delta t}$$

Node	a_W	a_E	S_P	S_u
1	0	$D_e + \frac{D_A^*}{3}$	$-\left(\frac{8}{3}D_A^* + F_A\right)$	$\left(\frac{8}{3}D_A^* + F_A\right)\phi_A + \frac{1}{8}F_e(\phi_P - 3\phi_E)$
2	$D_w + F_w$	D_e	0	$\boxed{\frac{1}{8}F_w(3\phi_P - \phi_W)} + \frac{1}{8}F_e(\phi_W + 2\phi_P - 3\phi_E)$
3–44	$D_w + F_w$	D_e	0	$\frac{1}{8}F_w(3\phi_P - 2\phi_W - \phi_{WW}) + \frac{1}{8}F_e(\phi_W + 2\phi_P - 3\phi_E)$
45	$D_w + F_w$	0	0	$\frac{1}{8}F_w(3\phi_P - 2\phi_W - \phi_{WW})$

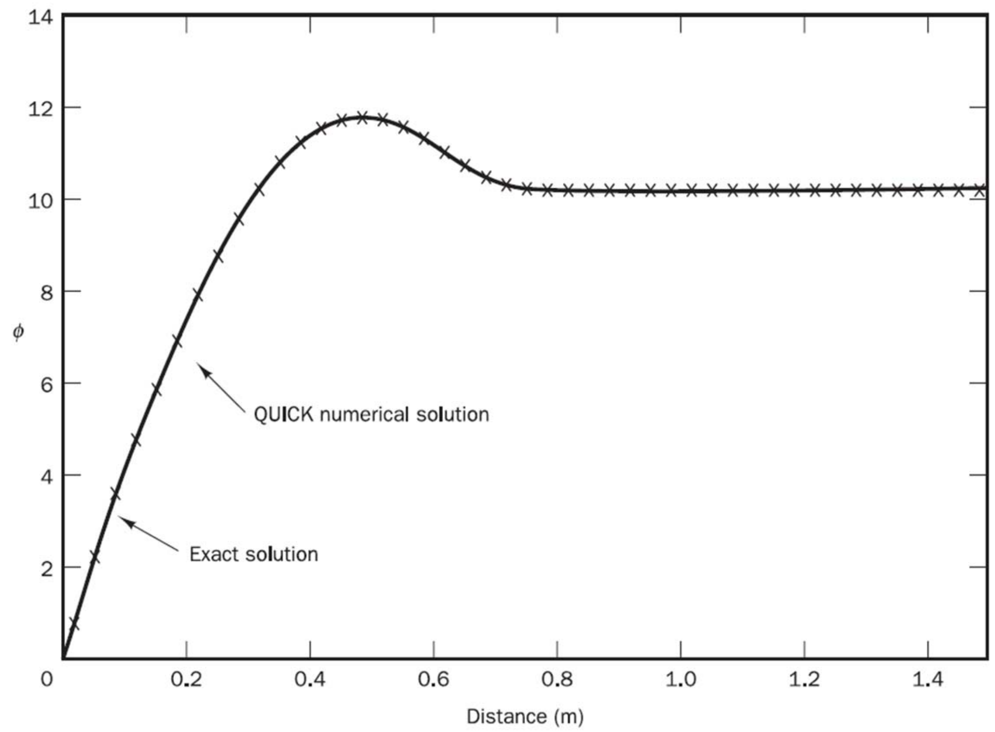
Worked example of transient C-D using QUICK

- Example 8.3
  - Numerical calculation
    - With time step: $\Delta t = 0.01$
      - Well within the stability limit for explicit schemes
      - Stable and reasonably accurate (3rd order accuracy in space) solutions
    - Numerical values of coefficients
    - Iterative solution procedure is required. (because of the deferred correction)

Node	a_W	a_E	a_P^o	***Total source***	S_P	a_P
1	0	1.2	3.33	$4.4\phi_A + 0.25(\phi_P - 3\phi_E) + 3.33\phi_P^o$	−4.4	8.93
2	2.9	0.9	3.33	$0.25(5\phi_P - 3\phi_E) + 3.33\phi_P^o$	0	7.13
3–44	2.9	0.9	3.33	$0.25(5\phi_P - \phi_W - \phi_{WW} - 3\phi_E) + 3.33\phi_P^o$	0	7.13
45	2.9	0	3.33	$0.25(3\phi_P - 2\phi_W - \phi_{WW}) + 3.33\phi_P^o$	0	6.23

Worked example of transient C-D using QUICK

- Example 8.3
  - Calculation results
    - Steady state solution: numerical results vs. analytical results



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Solution procedures for unsteady flow calculations

- Transient SIMPLE

  - Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

$$\frac{(\rho_P - \rho_P^o)}{\Delta t}\Delta V + [(\rho u A)_e - (\rho u A)_w] + [(\rho u A)_n - (\rho u A)_s] = 0$$

    - The pressure correction equation is derived from the continuity equation and should therefore contain terms representing its transient behavior.
    - For example, the equivalent of pressure correction equation (6.32) for a two-dimensional transient flow will take the form

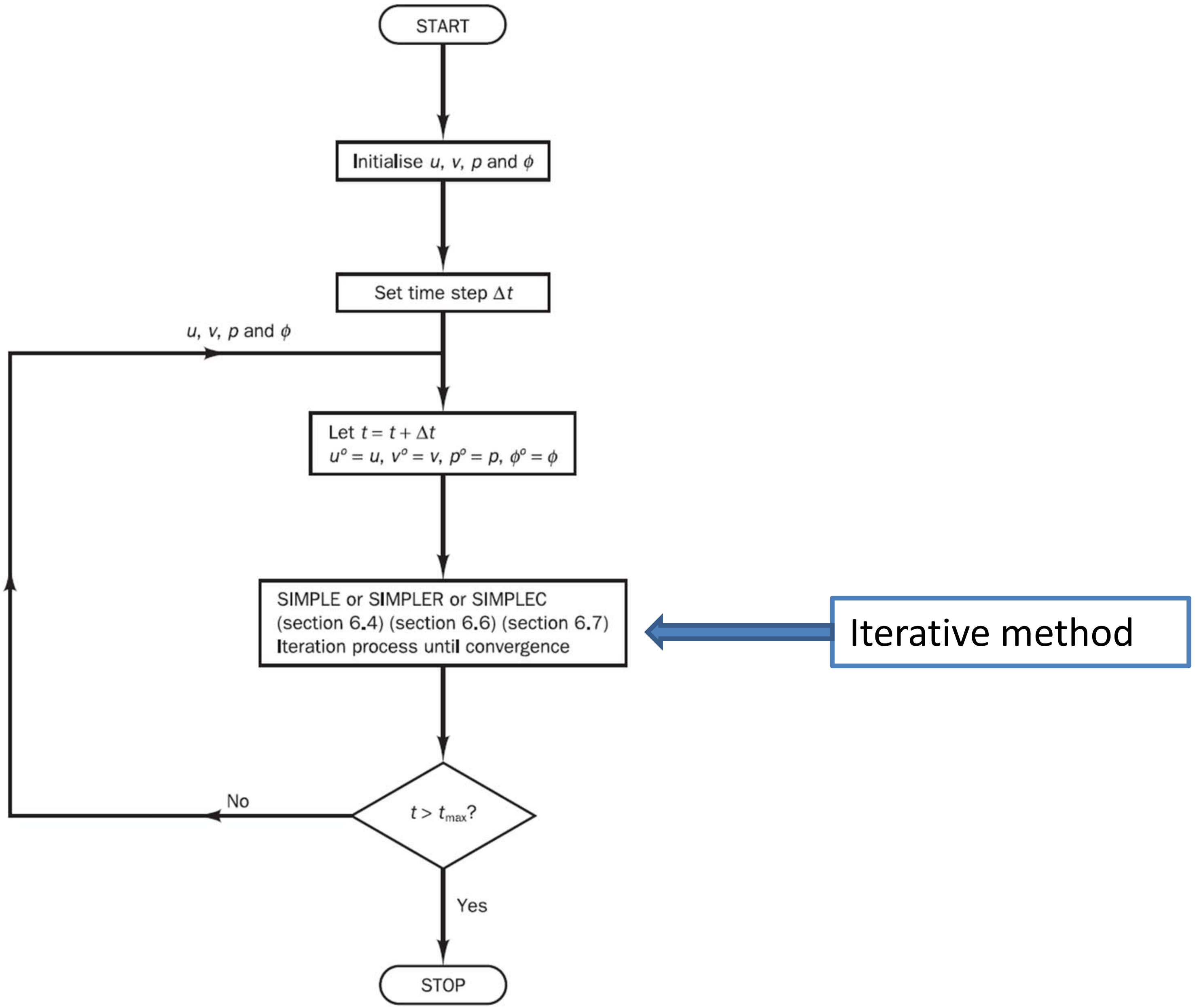
$$a_{I,J} p'_{I,J} = a_{I+1,J} p'_{I+1,J} + a_{I-1,J} p'_{I-1,J} + a_{I,J+1} p'_{I,J+1} + a_{I,J-1} p'_{I,J-1} + b'_{I,J}$$

$$a_{I,J} = a_{I+1,J} + a_{I-1,J} + a_{I,J+1} + a_{I,J-1}$$

$$b'_{I,J} = (\rho u^* A)_{i,J} - (\rho u^* A)_{i+1,J} + (\rho v^* A)_{I,j} - (\rho v^* A)_{I,j+1} + \boxed{\frac{(\rho_P^o - \rho_P)\Delta V}{\Delta t}}$$

Solution procedures for unsteady flow calculations

- Transient SIMPLE



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Pseudo-transient approach

- Pseudo-transient approach for steady state calculations
  - Under-relaxed form of the momentum equation

$$\frac{a_{i,J}}{\alpha_u} u_{i,J} = \sum a_{nb} u_{nb} + (p_{I-1,J} - p_{I,J}) A_{i,J} + b_{i,J} + \left[(1-\alpha_u) \frac{a_{i,J}}{\alpha_u} \right] u_{i,J}^{(n-1)}$$

Analogy

  - Transient momentum equation

$$\left(a_{i,J} + \frac{\rho^o_{i,J} \Delta V}{\Delta t} \right) u_{i,J} = \sum a_{nb} u_{nb} + (p_{I-1,J} - p_{I,J}) A_{i,J} + b_{i,J} + \frac{\rho^o_{i,J} \Delta V}{\Delta t} u^o_{i,J}$$

$$(1-\alpha_u) \frac{a_{i,J}}{\alpha_u} = \frac{\rho^o_{i,J} \Delta V}{\Delta t}$$

  - Alternatively steady state calculations may be interpreted as pseudo-transient solutions with spatially varying time steps.
  - The pseudo-transient approach is useful for situations in which governing equations give rise to stability problems, e.g. buoyant flows, highly swirling flows and compressible flows with shocks.

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Other transient schemes

- Other transient flow calculation procedures
  - MAC
  - SMAC
  - ICE
  - ICED-ALE

Central feature of the algorithm:
Poisson equation for the pressure

  - Kim and Benson (1992) compared the PISO method with the SMAC algorithms for the prediction of unsteady flows and reported that SMAC was more efficient, faster and more accurate than PISO.
  - The MAC/ICE class of methods are, however, mathematically complex and not widely used in general-purpose CFD procedures.