
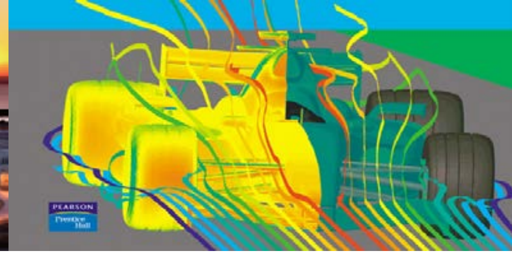
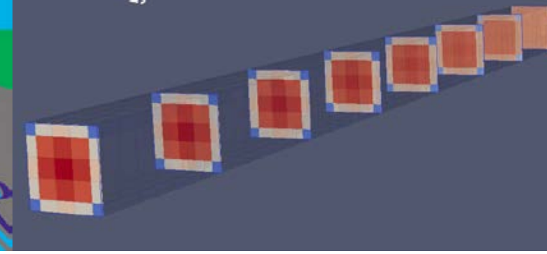
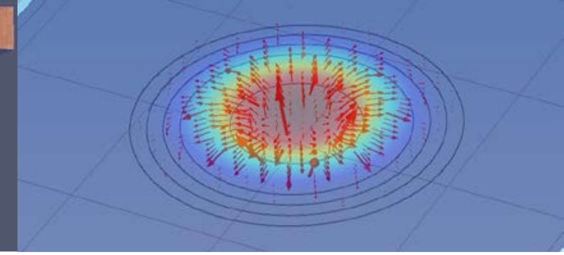

COMPUTATIONAL NUCLEAR THERMAL HYDRAULICS

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VERI LUX TAS MEA

TWO-FLUID MODEL SOLVER-2: SEMI-IMPLICIT SCHEME

Contents

Introduction

- Semi-implicit scheme or ICE (Implicit Continuous Eulerian)
  - Numerical algorithm for many nuclear thermal-hydraulic analysis codes
    - RELAP, MARS, SPACE, COBRA, CUPID
  - For the coupled scalar equations
    - Mass and energy equations cannot be decoupled even for an incompressible flow
    - Due to the phage change terms
    - Decoupling between the mass and energy equations
      - Instability may occur
      - Phage terms are very sensitive at the temperature variation

Governing equations

- Two-fluid model
  - Total 8 equations for 2D flow
    - 2 continuity equations
    - 4 momentum equations
    - 2 energy equations

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \boxed{\Gamma_g} \qquad \frac{\partial}{\partial t}(\alpha_l \rho_l) + \nabla \cdot (\alpha_l \rho_l \vec{u}_l) = \boxed{\Gamma_l}$$

$$\frac{\partial(\alpha_g \rho_g \vec{u}_g)}{\partial t} + \nabla \cdot \left(\alpha_g \rho_g \vec{u}_g \vec{u}_g\right) = -\alpha_g \nabla P + \nabla \cdot (\alpha_g \tau_g) + \nabla \cdot (\alpha_g \tau_g^t) + \alpha_g \rho_g \vec{g} + \vec{M}_g \qquad (17)$$

$$\frac{\partial(\alpha_l \rho_l \vec{u}_l)}{\partial t} + \nabla \cdot \left(\alpha_l \rho_l \vec{u}_l \vec{u}_l\right) = -\alpha_l \nabla P + \nabla \cdot (\alpha_l \tau_l) + \nabla \cdot (\alpha_l \tau_l^t) + \alpha_l \rho_l \vec{g} + \vec{M}_l \qquad (18)$$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g e_g) + \nabla \cdot (\alpha_g \rho_g e_g \vec{u}_g) = E_g^D - P \nabla \cdot (\alpha_g \vec{u}_g) + \boxed{S_{E,g}} - P \frac{\partial \alpha_g}{\partial t}$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l e_l) + \nabla \cdot (\alpha_l \rho_l e_l \vec{u}_l) = E_l^D - P \nabla \cdot (\alpha_l \vec{u}_l) + \boxed{S_{E,l}} - P \frac{\partial \alpha_l}{\partial t}$$

Governing equations

- Two-fluid model
  - Phage changer terms

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \boxed{\Gamma_g} \qquad \frac{\partial}{\partial t}(\alpha_l \rho_l) + \nabla \cdot (\alpha_l \rho_l \vec{u}_l) = \boxed{\Gamma_l}$$

$$\Gamma_g = \frac{H_{ig}(T_g - T^{sat}) + H_{il}(T_l - T^{sat})}{h_{gi} - h_{li}} = -\Gamma_l$$

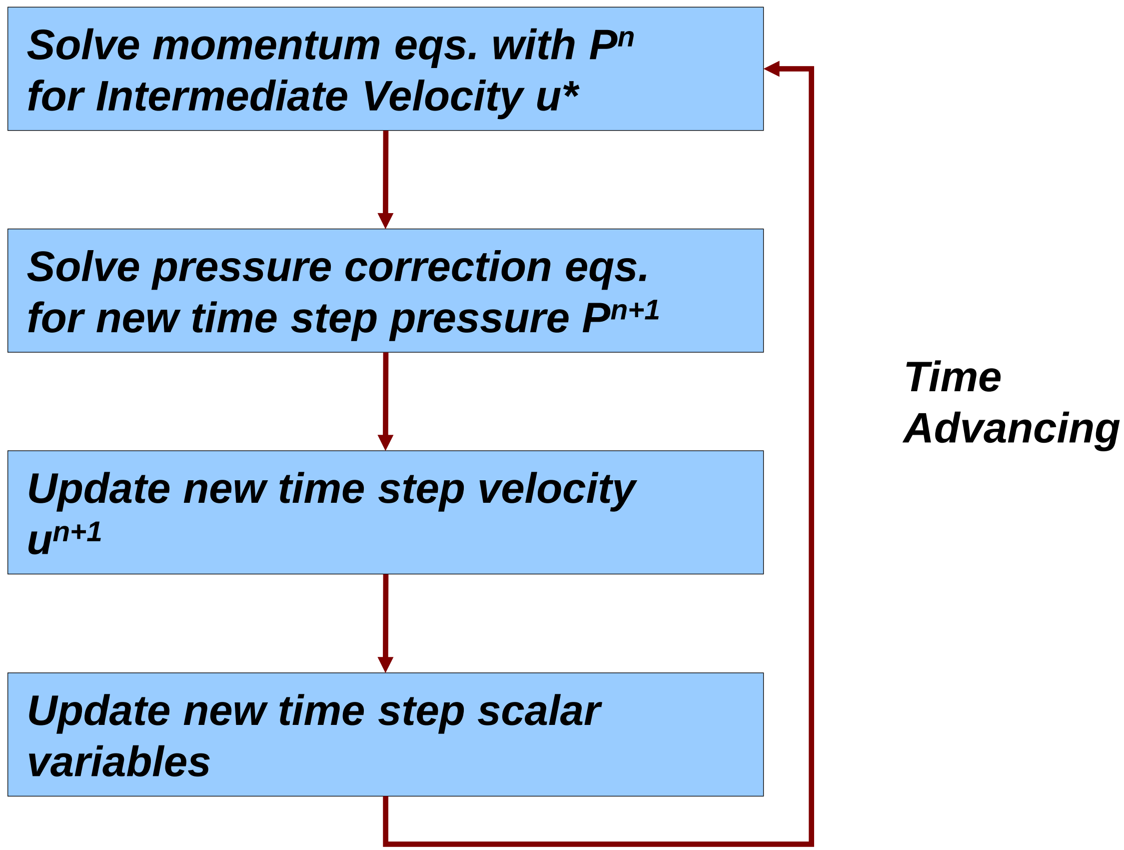
$$\frac{\partial}{\partial t}(\alpha_g \rho_g e_g) + \nabla \cdot (\alpha_g \rho_g e_g \vec{u}_g) = E_g^D - P\nabla \cdot (\alpha_g \vec{u}_g) + \boxed{S_{E,g}} - P\frac{\partial \alpha_g}{\partial t}$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l e_l) + \nabla \cdot (\alpha_l \rho_l e_l \vec{u}_l) = E_l^D - P\nabla \cdot (\alpha_l \vec{u}_l) + \boxed{S_{E,l}} - P\frac{\partial \alpha_l}{\partial t}$$

$$S_{E,g} = -\left(\frac{h_{li}}{h_{gi} - h_{li}}\right) H_{ig}(T^{sat} - T_g) - \left(\frac{h_{gi}}{h_{gi} - h_{li}}\right) H_{il}(T^{sat} - T_l)$$

Semi-implicit scheme for two-phase flow

- Numerical Procedure of ICE Scheme
  - Same with the two-phase SMAC



Semi-implicit scheme for two-phase flow

- Solution procedure for the momentum equations
  - Simplified virtual mass term
  - Implicit velocity for the interfacial drag

$$\frac{\partial}{\partial t}(\alpha_g \rho_g \vec{u}_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) = -\alpha_g \nabla P + \boxed{\nabla \cdot [\alpha_g (\tau_g + \tau_g^T)] + \vec{S}_{M,g}}$$

$$\vec{S}_{M,g} = \alpha_g \rho_g \vec{g} + \vec{M}_{ig} \qquad \vec{M}_{ig} = \Gamma_g \vec{u}_{gi} + \vec{M}_{drag,g} + \vec{M}_{vm,g} + \boxed{\vec{M}_{lift,g} + \vec{M}_{td,g} + \vec{M}_{wl,g}}$$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g \vec{u}_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) = -\alpha_g \nabla P + \Gamma_g \vec{u}_{gi} + \vec{M}_{drag,g} + \vec{M}_{vm,g} + \vec{S}'_{M,g}$$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g \vec{u}_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) = \alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \vec{u}_g \frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g)$$

$$= \alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \vec{u}_g \left[-\nabla \cdot (\alpha_g \rho_g \vec{u}_g) + \Gamma_g \right] + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g)$$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \Gamma_g$$

Semi-implicit scheme for two-phase flow

❖ Solution procedure for the momentum equations

$$\frac{\partial}{\partial t}(\alpha_g \rho_g \vec{u}_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) = -\alpha_g \nabla P + \Gamma_g \vec{u}_{gi} + \vec{M}_{drag,g} + \vec{M}_{vm,g} + \vec{S}'_{M,g}$$

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \vec{u}_g \left[-\nabla \cdot (\alpha_g \rho_g \vec{u}_g) + \Gamma_g \right] + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) = -\alpha_g \nabla P + \Gamma_g \vec{u}_{gi} + \vec{M}_{drag,g} + \vec{M}_{vm,g} + \vec{S}'_{M,g}$$

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) - \vec{u}_g \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \Gamma_g \vec{u}_{gi} - \Gamma_g \vec{u}_g - \alpha_g \nabla P + \vec{M}_{drag,g} + \vec{M}_{vm,g} + \vec{S}'_{M,g}$$

$$\Gamma_g = \Gamma_{EV} - \Gamma_{CD} \qquad \Gamma_g \vec{u}_{gi} - \Gamma_g \vec{u}_g = \Gamma_{EV} \vec{u}_l - \Gamma_{CD} \vec{u}_g - (\Gamma_{EV} - \Gamma_{CD}) \vec{u}_g = \Gamma_{EV} \vec{u}_l - \Gamma_{EV} \vec{u}_g$$

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) - \vec{u}_g \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \Gamma_{EV} (\vec{u}_l - \vec{u}_g) - \alpha_g \nabla P + \vec{M}_{drag,g} + \vec{M}_{vm,g} + \vec{S}'_{M,g}$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the momentum equations

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) - \vec{u}_g \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \Gamma_{EV} \left(\vec{u}_l - \vec{u}_g \right) - \alpha_g \nabla P + \boxed{\vec{M}_{drag,g} + \vec{M}_{vm,g}} + \vec{S}'_{M,g}$$

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) - \vec{u}_g \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \Gamma_{EV} \left(\vec{u}_l - \vec{u}_g \right) - \alpha_g \nabla P \boxed{- F_{gl} (\vec{u}_g - \vec{u}_l) + C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{\partial \left(\vec{u}_l - \vec{u}_g \right)}{\partial t}} + \vec{S}'_{M,g}$$

$$M_{drag,g} = -\frac{1}{8} A_i \rho_c C_D \, | \vec{u}_g - \vec{u}_l | \, (u_g - u_l) = -F_{gl} (\vec{u}_g - \vec{u}_l)$$

$$\vec{M}_{vm,g} = C_{vm} \alpha_g \alpha_l \rho_m \left[\left\{ \frac{\partial \vec{u}_l}{\partial t} + \cancel{\left(\vec{u}_l \cdot \nabla \right) \vec{u}_l} \right\} - \left\{ \frac{\partial \vec{u}_g}{\partial t} + \cancel{\left(\vec{u}_g \cdot \nabla \right) \vec{u}_g} \right\} \right]$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the momentum equations

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) - \vec{u}_g \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \Gamma_{EV} \left(\vec{u}_l - \vec{u}_g \right) - \alpha_g \nabla P$$

$$- F_{gl} (\vec{u}_g - \vec{u}_l) + C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{\partial \left(\vec{u}_l - \vec{u}_g \right)}{\partial t}$$

$$+ \vec{S}'_{M,g}$$

Implicit treatment of phase change and friction terms !

Finite Volume Discretization for intermediate velocity

$$\alpha_g \rho_g \int \frac{\partial \vec{u}_g}{\partial t} dV = \alpha_g \rho_g \frac{\left(\vec{u}_g^* - \vec{u}_g \right)}{\delta t} V$$

$$\int \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) dV = \int_S \alpha_g \rho_g \vec{u}_g \vec{u}_g \cdot d\vec{S} = \sum_f \alpha_g \rho_g \vec{u}_g \left(\vec{u}_g \cdot \vec{S} \right)_f$$

$$\int \vec{u}_g \nabla \cdot (\alpha_g \rho_g \vec{u}_g) dV = \vec{u}_g \int_S \alpha_g \rho_g \vec{u}_g \cdot d\vec{S} = \vec{u}_g \sum_f \alpha_g \rho_g \left(\vec{u}_g \cdot \vec{S} \right)_f$$

$$\alpha_g \int \nabla P dV = \alpha_g \nabla P V \qquad \int \Gamma_{Ev} \vec{u}_l dV = \Gamma_{Ev} \vec{u}_l^* V \qquad \int \Gamma_{Ev} \vec{u}_g dV = \Gamma_{Ev} \vec{u}_g^* V$$

$$\int F_{gl} (\vec{u}_g - \vec{u}_l) dV = F_{gl} (\vec{u}_g^* - \vec{u}_l^*) V$$

$$\int C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{\partial \left(\vec{u}_l - \vec{u}_g \right)}{\partial t} dV = C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{(\vec{u}_l^* - \vec{u}_g^*)}{\delta t} V - C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{(\vec{u}_l - \vec{u}_g)}{\delta t} V$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the momentum equations

$$\alpha_g \rho_g \frac{\partial \vec{u}_g}{\partial t} + \nabla \cdot (\alpha_g \rho_g \vec{u}_g \vec{u}_g) - \vec{u}_g \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \Gamma_{EV} \left(\vec{u}_l - \vec{u}_g \right) - \alpha_g \nabla P$$
$$- F_{gl} (\vec{u}_g - \vec{u}_l) + C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{\partial \left(\vec{u}_l - \vec{u}_g \right)}{\partial t}$$
$$+ \vec{S}'_{M,g}$$

Finite Volume Discretization for intermediate velocity

$$\alpha_g \rho_g \frac{\left(\boxed{\vec{u}_g^*} - \vec{u}_g \right)}{\delta t} V + \sum_f \alpha_g \rho_g \vec{u}_g \left(\vec{u}_g \cdot \vec{S} \right)_f - \vec{u}_g \sum_f \alpha_g \rho_g \left(\vec{u}_g \cdot \vec{S} \right)_f =$$
$$\alpha_g \nabla P V + \Gamma_{EV} \boxed{\vec{u}_l^*} V - \Gamma_{EV} \boxed{\vec{u}_g^*} V - F_{gl} (\boxed{\vec{u}_g^*} - \boxed{\vec{u}_l^*}) V$$
$$+ C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{(\boxed{\vec{u}_l^*} - \boxed{\vec{u}_g^*})}{\delta t} V - C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{(\vec{u}_l - \vec{u}_g)}{\delta t} V + \vec{S}'_{M,g} V$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the momentum equations

$$\alpha_g \rho_g \frac{\left(\vec{u}_g^* - \vec{u}_g\right)}{\delta t} V + \sum_f \alpha_g \rho_g \vec{u}_g \left(\vec{u}_g \cdot \vec{S}\right)_f - \vec{u}_g \sum_f \alpha_g \rho_g \left(\vec{u}_g \cdot \vec{S}\right)_f =$$

$$\alpha_g \nabla P V + \Gamma_{Ev} \vec{u}_l^* V - \Gamma_{Ev} \vec{u}_g^* V - F_{gl} (\vec{u}_g^* - \vec{u}_l^*) V$$

$$+ C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{(\vec{u}_l^* - \vec{u}_g^*)}{\delta t} V - C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{(\vec{u}_l - \vec{u}_g)}{\delta t} V + \vec{S}'_{M,g} V$$

$$\alpha_l \rho_l \frac{\left(\vec{u}_l^* - \vec{u}_l\right)}{\delta t} V + \sum_f \alpha_l \rho_l \vec{u}_l \left(\vec{u}_l \cdot \vec{S}\right)_f - \vec{u}_l \sum_f \alpha_l \rho_l \left(\vec{u}_l \cdot \vec{S}\right)_f =$$

$$\alpha_l \nabla P V - \Gamma_{Cond} \vec{u}_l^* V + \Gamma_{Cond} \vec{u}_g^* V + F_{gl} (\vec{u}_g^* - \vec{u}_l^*) V$$

$$- C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{(\vec{u}_l^* - \vec{u}_g^*)}{\delta t} V + C_{gl}^{VM} \alpha_g \alpha_l \rho_{m,gl} \frac{(\vec{u}_l - \vec{u}_g)}{\delta t} V + \vec{S}'_{M,l} V$$

Two linear equations
with two unknowns

Semi-implicit scheme for two-phase flow

- Solution procedure for the momentum equations

$$\alpha_g\rho_g\frac{\left(\vec{u}_g^{*}-\vec{u}_g\right)}{\delta t}V+\sum_f \alpha_g\rho_g\vec{u}_g\left(\vec{u}_g\cdot\vec{S}\right)_f-\vec{u}_g\sum_f \alpha_g\rho_g\left(\vec{u}_g\cdot\vec{S}\right)_f=$$
$$\alpha_g\nabla PV+\Gamma_{Ev}\vec{u}_l^{*}V-\Gamma_{Ev}\vec{u}_g^{*}V-F_{gl}\left(\vec{u}_g^{*}-\vec{u}_l^{*}\right)V$$
$$+C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gl}\frac{\left(\vec{u}_l^{*}-\vec{u}_g^{*}\right)}{\delta t}V-C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gl}\frac{\left(\vec{u}_l-\vec{u}_g\right)}{\delta t}V+\vec{S}'_{M,g}V$$

$$\begin{aligned}&\left[\left(\alpha_g\rho_g+C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gl}\right)V\delta t^{-1}+F_{gl}V+\Gamma_{Ev}V\right]\vec{u}_g^{*}\\&+\left[-C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gd}V\delta t^{-1}-F_{gl}V-\Gamma_{Ev}V\right]\vec{u}_l^{*}\end{aligned}=\vec{b}_1$$

$$\alpha_l\rho_l\frac{\left(\vec{u}_l^{*}-\vec{u}_l\right)}{\delta t}V+\sum_f \alpha_l\rho_l\vec{u}_l\left(\vec{u}_l\cdot\vec{S}\right)_f-\vec{u}_l\sum_f \alpha_l\rho_l\left(\vec{u}_l\cdot\vec{S}\right)_f=$$
$$\alpha_l\nabla PV-\Gamma_{Cond}\vec{u}_l^{*}V+\Gamma_{Cond}\vec{u}_g^{*}V+F_{gl}\left(\vec{u}_g^{*}-\vec{u}_l^{*}\right)V$$
$$-C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gl}\frac{\left(\vec{u}_l^{*}-\vec{u}_g^{*}\right)}{\delta t}V+C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gl}\frac{\left(\vec{u}_l-\vec{u}_g\right)}{\delta t}V++\vec{S}'_{M,l}V$$

$$\begin{aligned}&\left[\left(-C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gl}\right)V\delta t^{-1}-F_{gl}V-\Gamma_{Cond}V\right]\vec{u}_g^{*}\\&+\left[\alpha_l\rho_l+C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gd}V\delta t^{-1}+F_{gl}V+\Gamma_{Cond}V\right]\vec{u}_l^{*}\end{aligned}=\vec{b}_2$$

Two linear equations
with two unknowns

Semi-implicit scheme for two-phase flow

❖ Solution procedure for the momentum equations

$$\begin{aligned}&\left[\left(\alpha_g\rho_g + C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gl}\right)V\delta t^{-1} + F_{gl}V + \Gamma_{Cond}V\right]\underline{u}_g^* \\ &+\left[-C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gd}V\delta t^{-1} - F_{gl}V - \Gamma_{Ev}V\right]\underline{u}_l^* = \underline{b}_1\end{aligned}$$

$$\begin{aligned}&\left[\left(-C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gl}\right)V\delta t^{-1} - F_{gl}V - \Gamma_{Cond}V\right]\underline{u}_g^* \\ &+\left[\alpha_l\rho_l + C_{gl}^{VM}\alpha_g\alpha_l\rho_{m,gd}V\delta t^{-1} + F_{gl}V + \Gamma_{Ev}V\right]\underline{u}_l^* = \underline{b}_2\end{aligned}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} u_g^* \\ u_l^* \end{pmatrix} = \begin{pmatrix} b_{1,x} \\ b_{2,x} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} v_g^* \\ v_l^* \end{pmatrix} = \begin{pmatrix} b_{1,y} \\ b_{2,y} \end{pmatrix}$$

$$\begin{pmatrix} u_g^* \\ u_l^* \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1}\begin{pmatrix} b_{1,x} \\ b_{2,x} \end{pmatrix}$$

$$\begin{pmatrix} v_g^* \\ v_l^* \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1}\begin{pmatrix} b_{1,y} \\ b_{2,y} \end{pmatrix}$$

Repeat the same procedure
for every cell
for intermediate velocity !

Semi-implicit scheme for two-phase flow

- Solution procedure for the momentum equations

Relation between u* and u^{n+1}

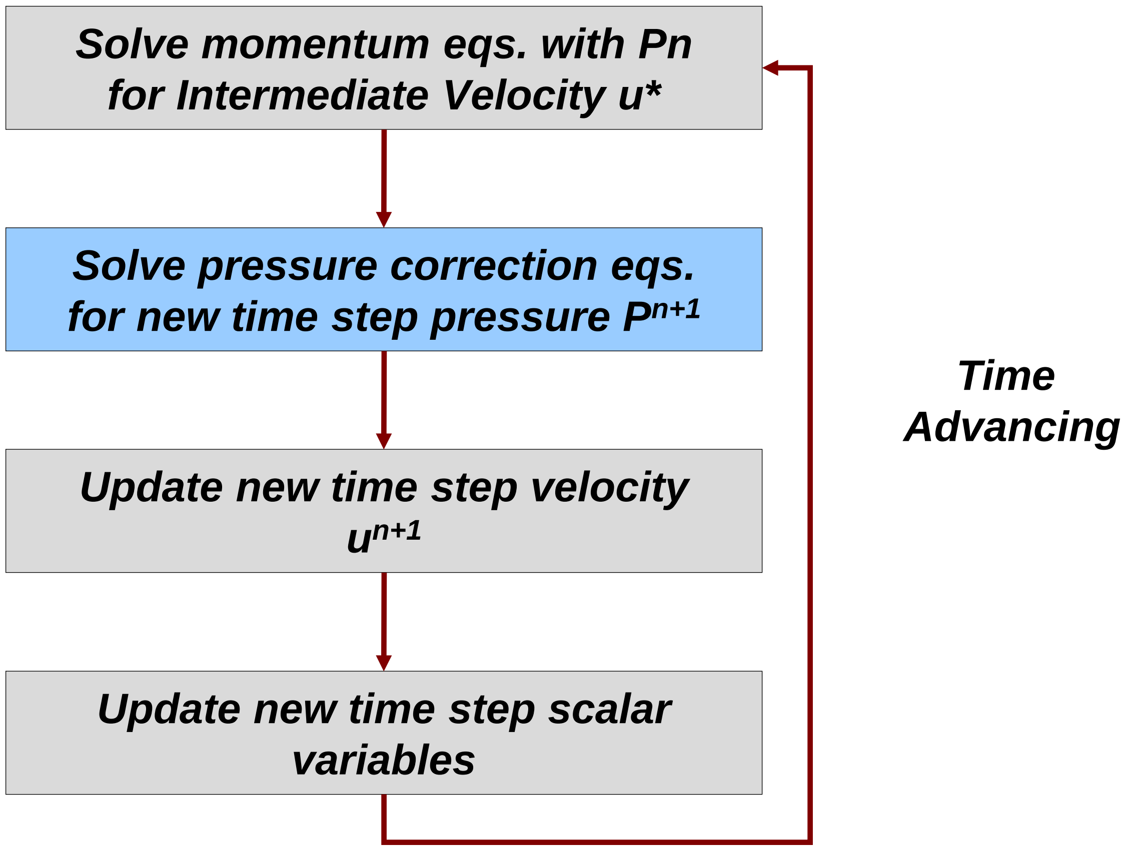
$$\begin{pmatrix} \vec{u}_g^{\,*} \\ \vec{u}_l^{\,*} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} \begin{pmatrix} \vec{b}_1 \\ \vec{b}_2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} \begin{pmatrix} \vec{c}_1 \\ \vec{c}_2 \end{pmatrix} - \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} \begin{pmatrix} \alpha_g \\ \alpha_l \end{pmatrix} \nabla P^n V$$

$$\begin{pmatrix} \vec{u}_g^{\,n+1} \\ \vec{u}_l^{\,n+1} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} \begin{pmatrix} \vec{b}_1 \\ \vec{b}_2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} \begin{pmatrix} \vec{c}_1 \\ \vec{c}_2 \end{pmatrix} - \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} \begin{pmatrix} \alpha_g \\ \alpha_l \end{pmatrix} \nabla P^{n+1} V$$

$$\begin{pmatrix} \vec{u}_g^{\,n+1} - \vec{u}_g^{\,*} \\ \vec{u}_l^{\,n+1} - \vec{u}_l^{\,*} \end{pmatrix} = -\begin{pmatrix} A_g \\ A_l \end{pmatrix} \nabla \left(P^{n+1} - P^n \right) = \begin{pmatrix} A_g \\ A_l \end{pmatrix} \nabla \delta P$$

$$\vec{u}_g^{\,n+1} = \vec{u}_g^{\,*} - A_g \nabla \delta P$$
$$\vec{u}_l^{\,n+1} = \vec{u}_l^{\,*} - A_l \nabla \delta P$$

Semi-implicit scheme for two-phase flow



Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - SMAC: derived from the continuity equations
  - ICE: derived from four scalar equations (2 continuity and 2 energy)
  - Discretized gas continuity equation

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \boxed{\Gamma_g} \qquad \frac{\partial}{\partial t}(\alpha_l \rho_l) + \nabla \cdot (\alpha_l \rho_l \vec{u}_l) = \boxed{\Gamma_l}$$

$$\int \frac{\partial}{\partial t}(\alpha_g \rho_g) dV = \alpha_g \int \frac{\partial \rho_g}{\partial t} dV + \rho_g \int \frac{\partial \alpha_g}{\partial t} dV = \frac{\alpha_g \delta \rho_g + \rho_g \delta \alpha_g}{\Delta t} V_{cell}$$

$$\int \nabla \cdot (\alpha_g \rho_g \vec{u}_g^{n+1}) dV = \int_S (\alpha_g \rho_g \vec{u}_g^{n+1}) \cdot d\vec{S} = \sum_f \left(\alpha_g \rho_g\right)_f \left(\vec{u}_g^{n+1} \cdot \vec{S}\right)_f$$

$$\int \Gamma_g dV = \Gamma_g V_{cell}$$

$$\Gamma_g = \frac{H_{ig}(T_g - T^{sat}) + H_{il}(T_l - T^{sat})}{h_{gi} - h_{li}} = -\Gamma_l \qquad \int \Gamma_g dV = \frac{H_{ig}(T_g^{n+1} - T_{sat}^{n+1}) + H_{il}(T_l^{n+1} - T_{sat}^{n+1})}{h_{gi} - h_{li}} V_{cell}$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Primary variables $\alpha_g \;\; \alpha_l \;\; e_g \;\; e_l \;\; P$
  - Unknowns $\rho_g^{n+1}, T_g^{n+1}, T_l^{n+1}, T_{sat}^{n+1}$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla \cdot (\alpha_g \rho_g \vec{u}_g) = \Gamma_g \qquad \frac{\partial}{\partial t}(\alpha_l \rho_l) + \nabla \cdot (\alpha_l \rho_l \vec{u}_l) = \Gamma_l$$

$$\int \frac{\partial}{\partial t}(\alpha_g \rho_g) dV = \alpha_g \int \frac{\partial \rho_g}{\partial t} dV + \rho_g \int \frac{\partial \alpha_g}{\partial t} dV = \frac{\boxed{\alpha_g \delta \rho_g + \rho_g \delta \alpha_g}}{\Delta t} V_{cell}$$

$$\int \nabla \cdot (\alpha_g \rho_g \vec{u}_g^{n+1}) dV = \int_S (\alpha_g \rho_g \vec{u}_g^{n+1}) \cdot d\vec{S} = \sum_f (\alpha_g \rho_g)_f \left(\boxed{\vec{u}_g^{n+1}} \cdot \vec{S} \right)_f$$

$$\int \Gamma_g dV = \Gamma_g V_{cell}$$

$$\Gamma_g = \frac{H_{ig}(T_g - T^{sat}) + H_{il}(T_l - T^{sat})}{h_{gi} - h_{li}} = -\Gamma_l \qquad \int \Gamma_g dV = \frac{H_{ig} \boxed{(T_g^{n+1} - T_{sat}^{n+1})} + H_{il} \boxed{(T_l^{n+1} - T_{sat}^{n+1})}}{h_{gi} - h_{li}} V_{cell}$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Primary variables $\alpha_g \ \alpha_l \ e_g \ e_l \ P$
  - Unknowns $\rho_g^{n+1}, T_g^{n+1}, T_l^{n+1}, T_{sat}^{n+1}$

$$\rho_g^{n+1} = \rho_g^n + \left(\frac{\partial \rho_g}{\partial P}\right)^n \left(P^{n+1} - P^n\right) + \left(\frac{\partial \rho_g}{\partial e_g}\right)^n \left(e_g^{n+1} - e_g^n\right)$$

$$\rho_l^{n+1} = \rho_l^n + \left(\frac{\partial \rho_l}{\partial P}\right)^n \left(P^{n+1} - P^n\right) + \left(\frac{\partial \rho_l}{\partial e_l}\right)^n \left(e_l^{n+1} - e_l^n\right)$$

Linearization

$$T_g^{n+1} = T_g^n + \left(\frac{\partial T_g}{\partial P}\right)^n \left(P^{n+1} - P^n\right) + \left(\frac{\partial T_g}{\partial e_g}\right)^n \left(e_g^{n+1} - e_g^n\right)$$

$$T_l^{n+1} = T_l^n + \left(\frac{\partial T_l}{\partial P}\right)^n \left(P^{n+1} - P^n\right) + \left(\frac{\partial T_l}{\partial e_l}\right)^n \left(e_l^{n+1} - e_l^n\right)$$

$$T_{sat}^{n+1} = T_{sat}^n + \left(\frac{\partial T_{sat}}{\partial P}\right)^n \left(P^{n+1} - P^n\right)$$

Replace the secondary variables in the scalar eqs.

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Primary variables $\alpha_g \ \alpha_l \ e_g \ e_l \ P$
  - Unknowns $\rho_g^{n+1}, T_g^{n+1}, T_l^{n+1}, T_{sat}^{n+1}$
  - Discretized gas continuity equation

$$\frac{\alpha_g \delta\rho_g + \rho_g \delta\alpha_g}{\Delta t} V_{cell} + \sum_f (\alpha_g \rho_g)_f \left(\vec{u}_g^{n+1} \cdot \vec{S}\right)_f = \frac{H_{ig}(T_g^{n+1} - T_{sat}^{n+1}) + H_{il}(T_l^{n+1} - T_{sat}^{n+1})}{h_{gi} - h_{li}} V_{cell}$$

$$\rho_g^{n+1} = \rho_g^n + \left(\frac{\partial \rho_g}{\partial P}\right)^n \left(P^{n+1} - P^n\right) + \left(\frac{\partial \rho_g}{\partial e_g}\right)^n \left(e_g^{n+1} - e_g^n\right)$$

$$\frac{V}{\Delta t}\left\{\alpha_g\left[\left(\frac{\partial \rho_g}{\partial P}\right)\delta P + \left(\frac{\partial \rho_g}{\partial e_g}\right)\delta e_g\right] + \rho_g \delta\alpha_g\right\} + \sum_f (\alpha_g \rho_g)_f \left(\vec{u}_g^{n+1} \cdot \vec{S}\right)_f$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Primary variables $\alpha_g \;\; \alpha_l \;\; e_g \;\; e_l \;\; P$
  - Unknowns $\rho_g^{n+1}, T_g^{n+1}, T_l^{n+1}, T_{sat}^{n+1}$
  - Discretized gas continuity equation

$$\frac{\alpha_g \delta\rho_g + \rho_g \delta\alpha_g}{\Delta t} V_{cell} + \sum_f (\alpha_g \rho_g)_f \left(\vec{u}_g^{n+1} \cdot \vec{S}\right)_f = \frac{H_{ig}(T_g^{n+1} - T_{sat}^{n+1}) + H_{il}(T_l^{n+1} - T_{sat}^{n+1})}{h_{gi} - h_{li}} V_{cell}$$

$$T_g^{n+1} = T_g^n + \left(\frac{\partial T_g}{\partial P}\right)^n \left(P^{n+1} - P^n\right) + \left(\frac{\partial T_g}{\partial e_g}\right)^n \left(e_g^{n+1} - e_g^n\right) \qquad T_l^{n+1} = T_l^n + \left(\frac{\partial T_l}{\partial P}\right)^n \left(P^{n+1} - P^n\right) + \left(\frac{\partial T_l}{\partial e_l}\right)^n \left(e_l^{n+1} - e_l^n\right)$$

$$T_{sat}^{n+1} = T_{sat}^n + \left(\frac{\partial T_{sat}}{\partial P}\right)^n \left(P^{n+1} - P^n\right)$$

$$-\frac{1}{h_g^* - h_f^*} VH_{ig}\left[T_{sat} - T_g + \left(\frac{\partial T_{sat}}{\partial P}\right)\delta P - \left(\frac{\partial T_g}{\partial P}\right)\delta P - \left(\frac{\partial T_g}{\partial e_g}\right)\delta e_g\right]$$

$$-\frac{1}{h_g^* - h_f^*} VH_{il}\left[T_{sat} - T_l + \left(\frac{\partial T_{sat}}{\partial P}\right)\delta P - \left(\frac{\partial T_l}{\partial P}\right)\delta P - \left(\frac{\partial T_l}{\partial e_l}\right)\delta e_l\right]$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Primary variables $\alpha_g \;\; \alpha_l \;\; e_g \;\; e_l \;\; P$
  - Unknowns $\rho_g^{n+1}, T_g^{n+1}, T_l^{n+1}, T_{sat}^{n+1}$
  - Discretized gas continuity equation

$$\frac{\alpha_g \delta\rho_g - \rho_g \delta\alpha_g}{\Delta t} V_{cell} + \sum_f \left(\alpha_g \rho_g\right)_f \left(\vec{u}_g^{\,n+1} \cdot \vec{S}\right)_f = \frac{H_{ig}(T_g^{n+1} - T_{sat}^{n+1}) + H_{il}(T_l^{n+1} - T_{sat}^{n+1})}{h_{gi} - h_{li}} V_{cell}$$

$$\begin{aligned}
&\frac{V}{\Delta t}\left\{\alpha_g\left[\left(\frac{\partial \rho_g}{\partial P}\right)\boxed{\delta P} + \left(\frac{\partial \rho_g}{\partial e_g}\right)\boxed{\delta e_g}\right] + \rho_g \boxed{\delta \alpha_g}\right\} + \sum_f \left(\alpha_g \rho_g\right)_f \left(\boxed{\vec{u}_g^{\,n+1}} \cdot \vec{S}\right)_f = \\
&-\frac{1}{h_g^* - h_f^*} V H_{ig}\left(\frac{P_s}{P}\right)\left[T_{sat} - T_g + \left(\frac{\partial T_{sat}}{\partial P}\right)\boxed{\delta P} - \left(\frac{\partial T_g}{\partial P}\right)\boxed{\delta P} - \left(\frac{\partial T_g}{\partial e_g}\right)\boxed{\delta e_g}\right] \\
&-\frac{1}{h_g^* - h_f^*} V H_{il}\left[T_{sat} - T_l + \left(\frac{\partial T_{sat}}{\partial P}\right)\boxed{\delta P} - \left(\frac{\partial T_l}{\partial P}\right)\boxed{\delta P} - \left(\frac{\partial T_l}{\partial e_l}\right)\boxed{\delta e_l}\right]
\end{aligned}$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Primary variables $\alpha_g \;\; \alpha_l \;\; e_g \;\; e_l \;\; P$
  - Unknowns $\rho_g^{n+1}, T_g^{n+1}, T_l^{n+1}, T_{sat}^{n+1}$
  - Discretized gas continuity equation

$$\frac{V}{\Delta t}\left\{\alpha_g\left[\left(\frac{\partial \rho_g}{\partial P}\right)\boxed{\delta P}+\left(\frac{\partial \rho_g}{\partial e_g}\right)\boxed{\delta e_g}\right]+\rho_g\boxed{\delta \alpha_g}\right\}+\sum_f\left(\alpha_g\rho_g\right)_f\left(\boxed{\vec{u}_g^{\,n+1}}\cdot\vec{S}\right)_f =$$

$$-\frac{1}{h_g^*-h_f^*}VH_{ig}\left(\frac{P_s}{P}\right)\left[T_{sat}-T_g+\left(\frac{\partial T_{sat}}{\partial P}\right)\boxed{\delta P}-\left(\frac{\partial T_g}{\partial P}\right)\boxed{\delta P}-\left(\frac{\partial T_g}{\partial e_g}\right)\boxed{\delta e_g}\right]$$

$$-\frac{1}{h_g^*-h_f^*}VH_{il}\left[T_{sat}-T_l+\left(\frac{\partial T_{sat}}{\partial P}\right)\boxed{\delta P}-\left(\frac{\partial T_l}{\partial P}\right)\boxed{\delta P}-\left(\frac{\partial T_l}{\partial e_l}\right)\boxed{\delta e_l}\right]$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{pmatrix}\begin{pmatrix}\delta e_g \\ \delta e_l \\ \delta\alpha_g \\ \delta\alpha_l \\ \delta P\end{pmatrix}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5\end{pmatrix}+\sum_f\begin{pmatrix} c_{g,1} \\ \\ \\ \\ \end{pmatrix}\left(\vec{u}_g^{\,n+1}\cdot\vec{S}\right)_f+\sum_f\begin{pmatrix} c_{l,1} \\ \\ \\ \\ \end{pmatrix}\left(\vec{u}_l^{\,n+1}\cdot\vec{S}\right)_f$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Repeat the same procedure for the other scalar equations

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} \delta e_g \\ \delta e_l \\ \delta \alpha_g \\ \delta \alpha_l \\ \delta P \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} + \sum_f \begin{pmatrix} c_{g,1} \\ c_{g,2} \\ c_{g,3} \\ c_{g,4} \\ c_{g,5} \end{pmatrix} \left(\vec{u}_g^{n+1} \cdot \vec{S}\right)_f + \sum_f \begin{pmatrix} c_{l,1} \\ c_{l,2} \\ c_{l,3} \\ c_{l,4} \\ c_{l,5} \end{pmatrix} \left(\vec{u}_l^{n+1} \cdot \vec{S}\right)_f$$

Relation between u* and u^{n+1}

$$\vec{u}_g^{n+1} = \vec{u}_g^{*} - A_g \nabla \delta P$$
$$\vec{u}_l^{n+1} = \vec{u}_l^{*} - A_l \nabla \delta P$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Replace the next time step velocity to the intermediate velocity

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} \delta e_g \\ \delta e_l \\ \delta \alpha_g \\ \delta \alpha_l \\ \delta P \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} + \sum_f \begin{pmatrix} c_{g,1} \\ c_{g,2} \\ c_{g,3} \\ c_{g,4} \\ c_{g,5} \end{pmatrix} \left(\vec{u}_g^{\,n+1} \cdot \vec{S} \right)_f + \sum_f \begin{pmatrix} c_{l,1} \\ c_{l,2} \\ c_{l,3} \\ c_{l,4} \\ c_{l,5} \end{pmatrix} \left(\vec{u}_l^{\,n+1} \cdot \vec{S} \right)_f$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} \delta e_g \\ \delta e_l \\ \delta \alpha_g \\ \delta \alpha_l \\ \delta P \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} + \sum_f \begin{pmatrix} c_{g,1} \\ c_{g,2} \\ c_{g,3} \\ c_{g,4} \\ c_{g,5} \end{pmatrix} \left(\vec{u}_g^{\,*} \cdot \vec{S} - A_g \nabla \delta P \cdot \vec{S} \right)_f + \sum_f \begin{pmatrix} c_{l,1} \\ c_{l,2} \\ c_{l,3} \\ c_{l,4} \\ c_{l,5} \end{pmatrix} \left(\vec{u}_l^{\,*} \cdot \vec{S} - A_l \nabla \delta P \cdot \vec{S} \right)_f$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Derive the matrix form of the scalar equations

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} \delta e_g \\ \delta e_l \\ \delta \alpha_g \\ \delta \alpha_l \\ \delta P \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} + \sum_f \begin{pmatrix} c_{g,1} \\ c_{g,2} \\ c_{g,3} \\ c_{g,4} \\ c_{g,5} \end{pmatrix} \left(\vec{u}_g^* \cdot \vec{S} - A_g \nabla \delta P \cdot \vec{S} \right)_f + \sum_f \begin{pmatrix} c_{l,1} \\ c_{l,2} \\ c_{l,3} \\ c_{l,4} \\ c_{l,5} \end{pmatrix} \left(\vec{u}_l^* \cdot \vec{S} - A_l \nabla \delta P \cdot \vec{S} \right)_f$$

$$\sum_f \left(\nabla \delta P \cdot \vec{S} \right)_f \approx \sum_f \left(\frac{\delta P_i - \delta P_m}{\Delta x} \right) S_f$$

$$\mathbf{A} \begin{pmatrix} \delta e_g \\ \delta e_l \\ \delta \alpha_g \\ \delta \alpha_l \\ \delta P \end{pmatrix} = \mathbf{B} + \sum_f \mathbf{C_g} \vec{u}_g^* \cdot \vec{S} + \sum_f \mathbf{C_l} \vec{u}_l^* \cdot \vec{S} - \sum_f \mathbf{C_g} \left(\frac{\delta P_i - \delta P_m}{\Delta x} \right) S_f - \sum_f \mathbf{C_l} \left(\frac{\delta P_i - \delta P_m}{\Delta x} \right) S_f$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Derive the cell pressure equation

$$\mathbf{A}\begin{pmatrix}\delta e_g \\ \delta e_l \\ \delta \alpha_g \\ \delta \alpha_l \\ \delta P\end{pmatrix} = \mathbf{B} + \sum_f \mathbf{C_g}\vec{u}_g^* \cdot \vec{S} + \sum_f \mathbf{C_l}\vec{u}_l^* \cdot \vec{S} - \sum_f \mathbf{C_g}\left(\frac{\delta P_i - \delta P_m}{\Delta x}\right) S_f - \sum_f \mathbf{C_l}\left(\frac{\delta P_i - \delta P_m}{\Delta x}\right) S_f$$

$$\begin{pmatrix}\delta e_g \\ \delta e_l \\ \delta \alpha_g \\ \delta \alpha_l \\ \delta P\end{pmatrix} = \mathbf{BB} - \sum_f \mathbf{CC}_f (\delta P - \delta P_m)$$

$\Rightarrow$

Cell pressure equation

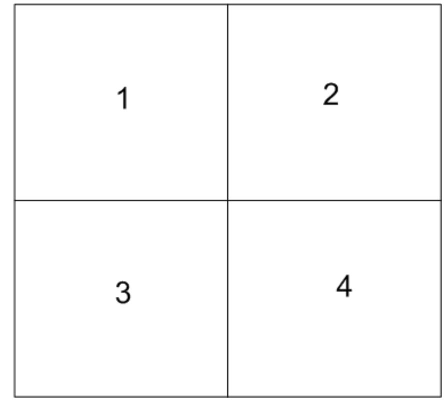
$$\delta P_i = \mathbf{BB}_5 - \sum_f (\mathbf{CC}_f)_5 (\delta P_i - \delta P_m)$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Cell pressure equation

$$\delta P_i = \mathbf{BB}_5 - \sum_f \left(\mathbf{CC}_f\right)_5 \left(\delta P_i - \delta P_m\right)$$

$$\left[1 + \sum_f \left(\mathbf{CC}_f\right)_5\right] \delta P_i - \sum_f \left(\mathbf{CC}_f\right)_5 \delta P_m = \mathbf{BB}_5$$



$$\left[1 + \sum_f \left(\mathbf{CC}_f\right)\right] \delta P_1 - \mathbf{CC}_{12} \delta P_2 - \mathbf{CC}_{13} \delta P_3 = \mathbf{BB}_1$$

$$\left[1 + \sum_f \left(\mathbf{CC}_f\right)\right] \delta P_2 - \mathbf{CC}_{21} \delta P_1 - \mathbf{CC}_{24} \delta P_4 = \mathbf{BB}_2$$

$$\left[1 + \sum_f \left(\mathbf{CC}_f\right)\right] \delta P_3 - \mathbf{CC}_{31} \delta P_1 - \mathbf{CC}_{34} \delta P_4 = \mathbf{BB}_3$$

$$\left[1 + \sum_f \left(\mathbf{CC}_f\right)\right] \delta P_4 - \mathbf{CC}_{42} \delta P_2 - \mathbf{CC}_{43} \delta P_2 = \mathbf{BB}_4$$

4 equations with 4 unknowns

$$\begin{pmatrix} \left[1 + \sum_f \left(\mathbf{CC}_f\right)\right]_1 & -\mathbf{CC}_{12} & -\mathbf{CC}_{13} & 0 \\ -\mathbf{CC}_{21} & \left[1 + \sum_f \left(\mathbf{CC}_f\right)\right]_2 & 0 & -\mathbf{CC}_{24} \\ -\mathbf{CC}_{31} & 0 & \left[1 + \sum_f \left(\mathbf{CC}_f\right)\right]_3 & -\mathbf{CC}_{34} \\ 0 & -\mathbf{CC}_{42} & -\mathbf{CC}_{43} & \left[1 + \sum_f \left(\mathbf{CC}_f\right)\right]_4 \end{pmatrix} \begin{pmatrix} \delta P_1 \\ \delta P_2 \\ \delta P_3 \\ \delta P_4 \end{pmatrix} = \begin{pmatrix} BB_1 \\ BB_2 \\ BB_3 \\ BB_4 \end{pmatrix}$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Cell pressure equation

Cell pressure equation

$$\left[1+\sum_f \left(\mathbf{CC}_f\right)_5\right]\delta P_i - \sum_f \left(\mathbf{CC}_f\right)_5 \delta P_m = \mathbf{BB}_5$$

System pressure equation: NxN matrix

$$\begin{pmatrix} \left[1+\sum_f (\mathbf{CC}_5)_f\right]_1 & \cdots & (\mathbf{CC}_5)_{1,m} & \cdots & (\mathbf{CC}_5)_{1,N} \\ \vdots & \ddots & & & \vdots \\ (\mathbf{CC}_5)_{i,1} & \cdots & \left[1+\sum_f (\mathbf{CC}_5)_f\right]_i & \cdots & (\mathbf{CC}_5)_{i,N} \\ \vdots & & & \ddots & \vdots \\ (\mathbf{CC}_5)_{N,1} & \cdots & (\mathbf{CC}_5)_{N,m} & \cdots & \left[1+\sum_f (\mathbf{CC}_5)_f\right]_N \end{pmatrix} \begin{pmatrix} \delta P_1 \\ \vdots \\ \delta P_i \\ \vdots \\ \delta P_N \end{pmatrix} = \begin{pmatrix} (\mathbf{BB}_5)_1 \\ \vdots \\ (\mathbf{BB}_5)_i \\ \vdots \\ (\mathbf{BB}_5)_N \end{pmatrix}$$

Solver $\Rightarrow$

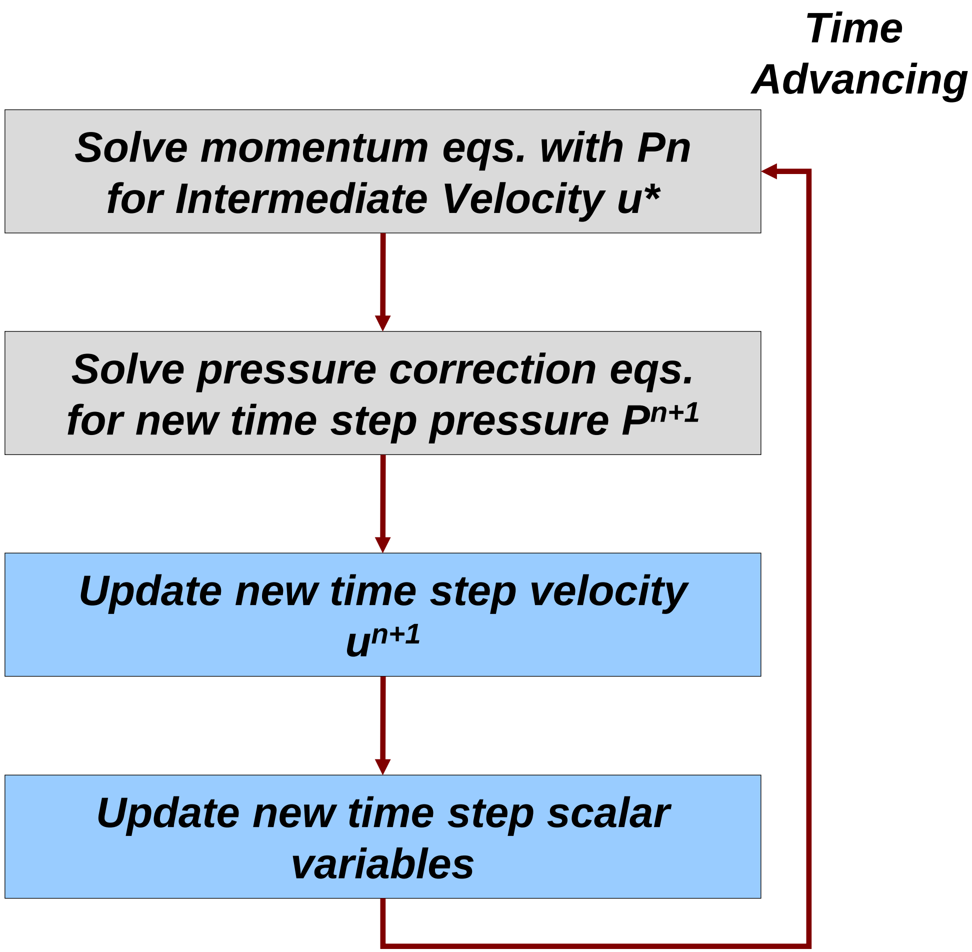
Obtain pressure correction

$$\delta P_i = P_i^{n+1} - P_i^n$$

$$P_i^{n+1} = P_i^n + \delta P_i$$

Semi-implicit scheme for two-phase flow

- Solution procedure for the pressure correction equations
  - Update



Relation between u* and u^{n+1}

$$\underline{u}_g^{n+1} = \underline{u}_g^{*} - A_g \nabla \delta P$$

$$\underline{u}_l^{n+1} = \underline{u}_l^{*} - A_l \nabla \delta P$$

Relation between u* and u^{n+1}

$$\begin{pmatrix} \delta e_g \\ \delta e_l \\ \delta \alpha_g \\ \delta \alpha_l \\ \delta P \end{pmatrix} = \mathbf{BB} - \sum_f \mathbf{CC}_f \left(\delta P - \delta P_m \right)$$

Semi-implicit scheme for two-phase flow

- Two-phase Flow Solvers
  - ICE (RELAP, SPACE, CUPID etc.)
  - SMAC (CUPID)
  - PISO and SIMPLE (CFD codes)
  - Fractional step method (NEPTUNE-CFD)

- Difficulties in Two-phase Analysis
  - Source terms
    - Modeling from experiments
  - Numerical instability
    - Density difference
    - Discontinuity
  - Application conditions
    - Wide range of application conditions