

# Microbial kinetics in reactors I

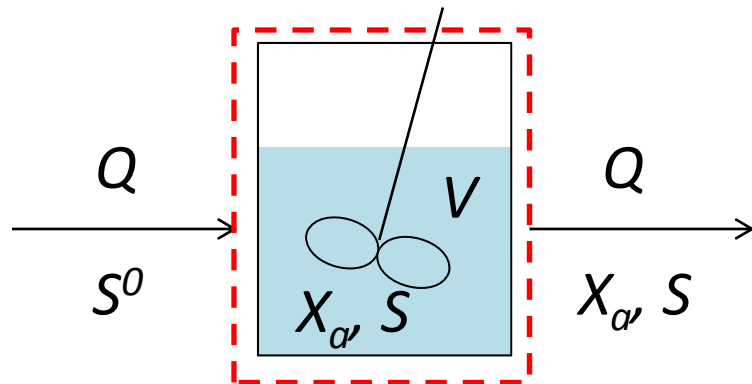
# Today's lecture

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- Master equations and key trends
- Including inert biomass
- Observed yield
- Soluble microbial products

# CSTR, Monod: Master equations

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Assumption:

- Steady state
- $X_a = 0$  in the influent  
(negligible influent biomass)

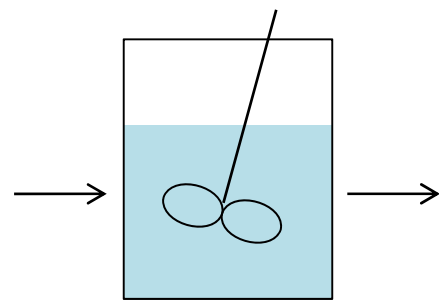
$$S = K \frac{1 + b\theta}{Y\hat{q}\theta - (1 + b\theta)}$$

$$X_a = Y \frac{S^0 - S}{1 + b\theta}$$

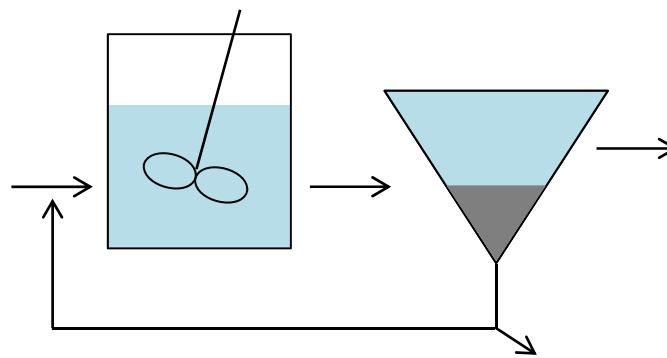
# HRT vs. SRT

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- HRT: **H**ydraulic **R**etention **T**ime; the average time the water stays in the system
- SRT: **S**olids **R**etention **T**ime (or mean cell residence time, MCRT); the average time the biomass stays in the system

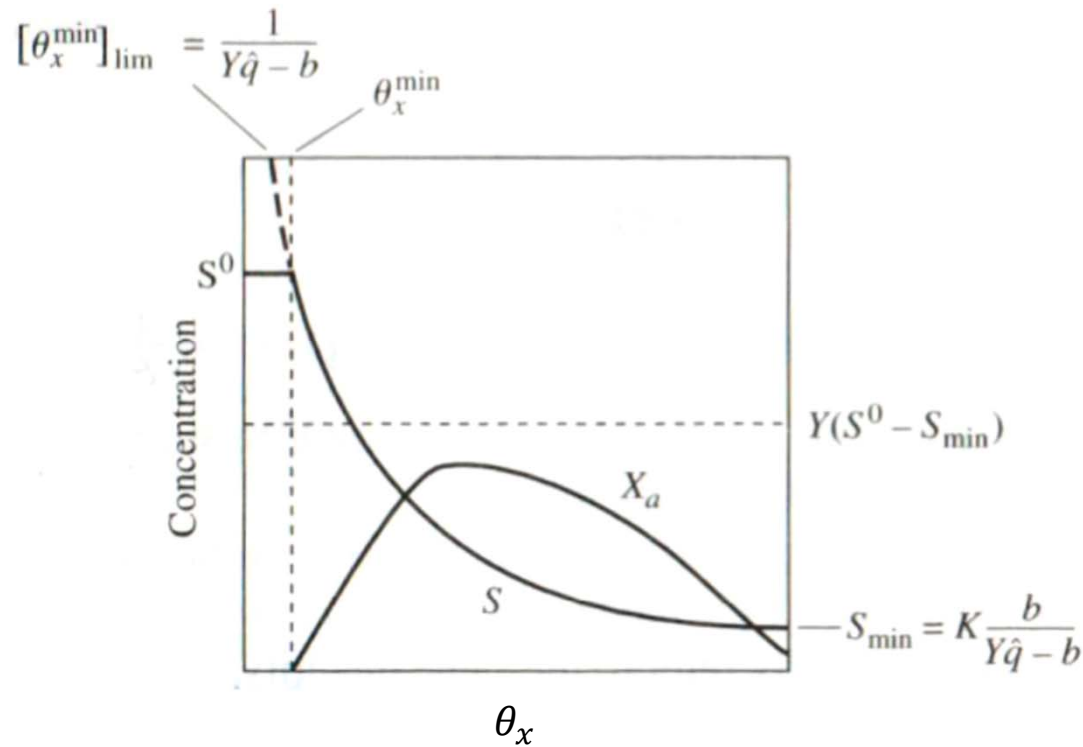


A chemostat:  $HRT = SRT$



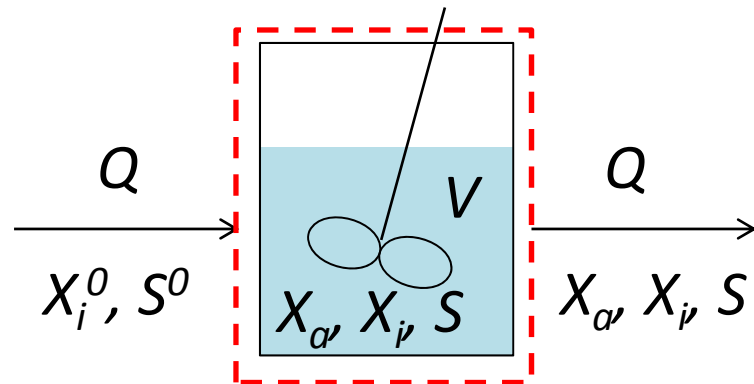
An activated sludge process with sludge return:  $HRT < SRT$

# $\theta_x$ vs. $S$ : key trends



- 1)  $\theta_x \leq \theta_{min}$ :  
washout
- 2)  $\theta_x \rightarrow \infty$ :  
 $S = S_{min}$
- 3) For  $\theta_{min} < \theta_x$ ,  $S$   
decreases with  
increase in  $\theta_x$ ,  
but  $X_a$  peaks at  
some point

# Including inert biomass

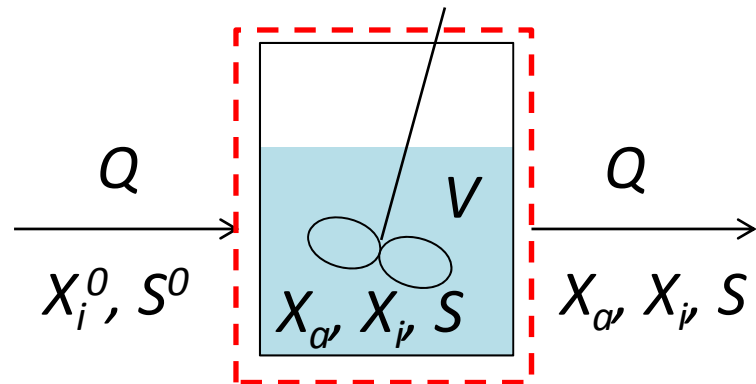


Influent contains some non-biodegradable, particulate organics: this is included when you measure VSS

Recall:

$$\left( \frac{1}{X_a} \frac{dX_a}{dt} \right)_{inert} = - \frac{1}{X_a} \frac{dX_i}{dt} = -(1 - f_d)b$$

# Including inert biomass



Solution for inert biomass:  $X_i = X_i^0 + X_a(1 - f_d)b\theta_x$

Solution for total VSS:

$$X_v = X_i + X_a = X_i^0 + Y(S^0 - S) \frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$

# Observed yield

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The VSS in a CSTR is calculated as:

$$X_v = X_i + X_a = X_i^0 + \underbrace{Y(S^0 - S)}_{\text{substrate utilized at the reactor}} \underbrace{\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}}_{\text{VSS gained at the reactor}}$$

Influent inert biomass
substrate utilized at the reactor
VSS gained at the reactor

The net yield in the CSTR is:  $Y \frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$

This value is more often called as **Observed Yield ( $Y_{obs}$ )**.



# Soluble microbial products (SMP)

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- Cell components that released during cell lysis, diffuse through the cell membrane, are lost during synthesis, or are excreted for some purpose
- Does not include intermediates of degradation pathway
- MW = 100s – 1000s
- Biodegradable

# Significance of SMP

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- Appear in all cases
- Constitute the majority of the effluent COD & BOD in many cases
- Can complex metals, foul membranes, & cause color or foaming

# Two types of SMP

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- UAP (substrate-**U**tilization-**A**ssociated **P**roducts):  
produced directly during substrate metabolism

$$r_{UAP} = -k_1 r_{ut}$$

$r_{UAP}$  = rate of UAP-formation [ $M_p L^{-3} T^{-1}$ ]  
 $k_1$  = UAP-formation coefficient [ $M_p M_s^{-1}$ ]

- BAP (**B**iomass-**A**ssociated **P**roducts):  
formed directly from biomass as part of  
maintenance and decay

$$r_{BAP} = k_2 X_a$$

$r_{BAP}$  = rate of BAP-formation [ $M_p L^{-3} T^{-1}$ ]  
 $k_2$  = BAP-formation coefficient [ $M_p M_x^{-1} T^{-1}$ ]

# SMP biodegradation

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- Assume both UAP and BAP follow Monod kinetics with different parameters:

$$r_{deg-UAP} = -\frac{\hat{q}_{UAP}UAP}{K_{UAP} + UAP}X_a$$

$$r_{deg-BAP} = -\frac{\hat{q}_{BAP}BAP}{K_{BAP} + BAP}X_a$$

# SMP in a CSTR (a Chemostat)

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- Steady-state mass balance for UAP & BAP:

$$0 = -k_1 r_{ut} V - \frac{\hat{q}_{UAP} UAP}{K_{UAP} + UAP} X_a V - Q \cdot UAP$$

$$0 = k_2 X_a V - \frac{\hat{q}_{BAP} BAP}{K_{BAP} + BAP} X_a V - Q \cdot BAP$$

Solution for UAP & BAP in Eqs. [3.38] & [3.39]