Chemostat performance analysis – example question

1) S_{min}, $\theta_x{}^{min}$, θ_x

$$S_{min} = K\frac{b}{\hat{Yq}-b} = (20\ mg\ BOD_L/L)\frac{0.15/d}{(0.42\ g\ VSS_a/g\ BOD_L)(20\ g\ BOD_L/g\ VSS_a - d) - 0.15/d}$$

$$= 0.36\ mg\ BOD_L/L$$

$$\theta_x^{min} = \frac{K + S^0}{S^0(\hat{Yq}-b) - bK} = \frac{(20 + 500)\ mg\ BOD_L/L}{(50\ mg\ BOD_L/L)(0.42\cdot 20/d - 0.15/d) - (20\ mg\ BOD_L/L)(0.15/d)}$$

$$= 0.126\ d$$

$$\theta_x = \theta = \frac{V}{Q} = \frac{2000 m^3}{1000\ m^3/d} = 2\ d$$

$$\frac{\theta_x}{\theta_x^{min}} = SF = \frac{2\ d}{0.126\ d} = 16 \qquad \text{(SF of 16 for washout)}$$

2) Effluent VSS, COD, BOD_L

Firstly, we need to determine the effluent substrate and active biomass concentrations:

$$S = K\frac{1 + b\theta_x}{\hat{Yq}\theta_x - (1 + b\theta_x)}$$

$$= (20\ mg\ BOD_L/L)\frac{1 + (0.15/d)(2\ d)}{(0.42\ g\ VSS_a/g\ BOD_L)(20\ g\ BOD_L/g\ VSS_a - d)(2\ d) - (1 + (0.15/d)(2\ d))}$$

$$= 1.7\ mg\ BOD_L/L$$

$$X_a = Y(S^0 - S)\frac{1}{1 + b\theta_x} = (0.42\ g\ VSS_a/g\ BOD_L)(500 - 1.7\ mg\ BOD_L/L)\frac{1}{1 + (0.15/d)(2\ d)}$$

$$= 161\ mg\ VSS_a/L$$

Calculate the effluent inert VSS concentration:

$$X_i = X_i^0 + X_a(1-f_d)b\theta_x$$

$$= 50 \ mg \ VSS_i/L + (161 \ mg \ VSS_a/L)(1-0.8g \ VSS_i/g \ VSS_a)(0.15/d)(2 \ d)$$

$$= 60 \ mg \ VSS_i/L$$

Now,

$$X_v = X_a + X_i = 161 + 60 = 221 \ mg \ VSS/L$$

Think about the composition of effluent COD & BOD$_L$:

Effluent COD = remaining substrate + SMP + all VSS (active biomass + inert)

Effluent BOD$_L$ = remaining substrate + SMP + active <u>and</u> biodegradable biomass

* Effluent bsCOD = Effluent sBOD$_L$ = remaining substrate + SMP

   Effluent bpCOD = all VSS

   Effluent pBOD$_L$ = active <u>and</u> biodegradable biomass

Calculate the effluent SMP

- let's first calculate the individual terms for Eqs. [3.38] & [3.39]

$$r_{ut} = -\frac{\hat{q}S}{K+S}X_a = \frac{dS}{dt} = -\frac{S^0-S}{\theta} = -\frac{(500-1.7) \ mg \ BOD_L/L}{2 \ d} = -249 \ mg \ BOD_L/L-d$$

$$\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta = 1.8 \cdot 161 \cdot 2 + 100 + 0.12 \cdot (-249) \cdot 2 = 620 \ mg \ BOD_L/L$$

$$4K_{UAP}k_1 r_{ut}\theta = 4 \cdot 100 \cdot 0.12 \cdot (-249) \cdot 2 = -23900 \ (mg \ BOD_L/L)^2$$

$$K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta = 85 + (0.1-0.09) \cdot 161 \cdot 2 = 88.2 \ mg \ BOD_L/L$$

$$4K_{BAP}k_2 X_a\theta = 4 \cdot 85 \cdot 0.09 \cdot 161 \cdot 2 = 9850 \ (mg \ BOD_L/L)^2$$

$$UAP = 620 + \frac{\sqrt{(620)^2 + 23900}}{2} = 9.5 \ mg \ BOD_L/L$$

$$BAP = \frac{88.2 + \sqrt{(88.2)^2 + 9850}}{2} = 22.3 \ mg \ BOD_L/L$$

$$SMP = UAP + BAP = 9.5 + 22.3 = 31.8 \ mg \ BOD_L/L$$

Biomass COD: recall that the COD value for a cell formula of $C_5H_7O_2N$ was 1.42 g COD/g cells

In sum,

$$Effluent \ COD = Substrate + SMP + Biomass \ COD$$

$$= 1.7 + 31.8 + (1.42 \ g \ COD/g \ VSS)X_v = 1.7 + 31.8 + 1.42 \cdot 221$$

$$= 1.7 + 31.8 + 313.8 = 347 \ mg \ COD/L$$

* Biomass accounts for most of COD – this COD can be removed by settling (but good settling property should be guaranteed)

* SMP account for most of soluble COD

$$Effluent \ BOD_L = Substrate + SMP + active \ \text{and} \ biodegradable \ biomass$$

$$= 1.7 + 31.8 + (1.42 \ g \ COD/g \ VSS) \cdot X_a \cdot f_d = 216 \ mg \ BOD_L/L$$

3) N and P

The N and P consumption rates,

$$r_N = (0.124 \ g \ N/g \ VSS) \cdot (0.42 \ g \ VSS/g \ BOD_L) \cdot (-249 \ mg \ BOD_L/L - d) \cdot \frac{1 + (1 - 0.8) \cdot 0.15 \cdot 2}{1 + 0.15 \cdot 2}$$

$$= -10.6 \ mg \ N/L - d$$

$$r_P = r_N \cdot 0.2 \ g \ P/g \ N = -10.6 \cdot 0.2 = -2.1 \ mg \ P/L - d$$

The effluent N and P concentrations

$$C_N = C_N^0 + r_N \theta = 50 \ mg \ N/L - (10.6 \ mg \ N/L - d) \cdot 2 \ d = 28.8 \ mg \ NH_4^+ - N/L$$

$$C_P = C_P^0 + r_P \theta = 10 \ mg \ P/L - (2.1 \ mg \ P/L - d) \cdot 2 \ d = 5.8 \ mg \ PO_4^{3-} - P/L$$

(the amount of nutrients did not limit the biological activity in the reactor)

4) O_2

The acceptor consumption in the reactor,

$$\frac{\Delta S_a}{\Delta t} = (1\ g\ O_2/g\ COD) \cdot (1000\ m^3/d)$$

$$\cdot\ [500 - 1.7 - 31.8 + 1.42(50 - 221)]\ mg\ COD/L \cdot 10^3\ L/m^3 \cdot 10^{-3} g/mg$$

$$= 2.24 \times 10^5\ g\ O_2/d$$

To support the acceptor consumption, O2 should be supplied to the reactor with a rate of:

$$R_{O_2} = 2.24 \times 10^5\ g\ O_2/d - (1000\ m^3/d) \cdot (6 - 2)\ mg/L \cdot 10^3\ L/m^3 \cdot 10^{-3}\ g/mg$$

$$= 2.20 \times 10^5\ g\ O_2/d$$

(O_2 supplied by the influent DO is very small compared to the O_2 requirement – aeration is essential)

5) Effect of hydrolysis

i) effluent particulate BOD, S_p

$$S_p = \frac{S_p^0}{1 + k_{hyd}\theta} = \frac{100\ mg\ COD/L}{1 + (0.2/d)(2\ d)} = 71\ mg\ COD/L$$

ii) effluent soluble BOD, S: no change, 1.7 mg BOD_L/L

iii) effective S^0 considering S_p, $S^{0'}$:

$$S^{0'} = S^0 + k_{hyd}S_p\theta = 500\ mg\ COD/L + (0.2/d)(71\ mg\ COD/L)(2\ d) = 528\ mg\ BOD_L/L$$

(error in the textbook!)

iv) Effluent VSS

$$X_a = Y(S^0 - S)\frac{1}{1 + b\theta_x} = 0.42 \cdot (528 - 1.7)\frac{1}{1 + 0.15 \cdot 2} = 170 \ mg \ VSS/L$$

(slight increase from 161 mg VSS/L without particulate BOD)

$$X_i = X_i^0 + X_a(1 - f_d)b\theta_x = 50 \ mg \ VSS/L + (170 \ mg \ VSS/L) \cdot (1 - 0.8) \cdot (0.15/d) \cdot (2 \ d)$$

$$= 60 \ mg \ VSS/L$$

(didn't change much – slight increase happened, but not enough to have increase in significant numbers)

$$X_v = X_a + X_i + S_p = (170 + 60) \ mg \ VSS/L + \frac{71 \ mg \ COD/L}{1.42 \ mg \ COD/mg \ VSS} = 280 \ mg \ VSS/L$$

(Assumed that the particulate COD has the same formula, $C_5H_7O_2N$, as the biomass)

v) SMP: let's skip the calculation and obtain value from the text:

SMP = 32.6 mg BOD_L/L

(slight increase from 31.8 mg BOD_L/L because of increased biomass – BAP increases)

vi) Effluent COD & BOD_L

$$Effluent \ COD = S + SMP + 1.42 \cdot X_v = 1.7 + 32.6 + 1.42 \cdot 280 = 432 \ mg \ COD/L$$

$$Effluent \ BOD_L = S + SMP + 1.42 \cdot f_d \cdot X_a + S_p = 1.7 + 32.7 + 1.42 \cdot 0.8 \cdot 170 + 71 =$$

$$= 299 \ mg \ BOD_L/L$$