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457.212 Statistics for Civil & Environmental Engineers
In-Class Material: Class 20
Point Parameter Estimation (2) (A&T: 6.1-6.2)

1. Statistical Inference

2. Point Estimation of Parameters

3. Point Estimation by "Method of Moments"

4. **Point Estimation by Method of "Maximum Likelihood Estimation (MLE)"**

   (a) Known as "best" and "efficient" (i.e. minimum variance of $\hat{\theta}$ for the same sample size)

   (b) Finds the values of parameters that ______________ the likelihood of the available data set.

   (c) Example: The available data set $\{x_1, x_2\} = \{-0.5, 0.5\}$

   Suppose we want to estimate the mean of the random variable X, μ_X.
   Which of the estimates $\hat{\mu}_X = 0$ and $\hat{\mu}_X = 100$ makes more sense to you?
   (and why?)

   (d) Consider a dataset: $\{x_1, x_2, ..., x_n\}$

   Suppose we know the quantity follows a certain type of distribution (e.g. *N*, *LN*) and it requires a distribution parameter θ. Its marginal PDF is denoted as $f_X(x;\theta)$

   Event E_1: $X = x_1$ ~ Probability $P(E_1) \propto f_X(x_1;\theta)$
   Event E_2: $X = x_2$ ~ Probability $P(E_2) \propto f_X(x_2;\theta)$
   …
   Event E_n: $X = x_n$ ~ Probability $P(E_n) \propto f_X(x_n;\theta)$

   Probability that we will get the dataset?

   $$P\left(\bigcap_{i=1}^{n} E_i\right) \propto \prod_{i=1}^{n} f_X(x_i;\theta) = L(x_1,...,x_n;\theta)$$

   $L(x_1,...,x_n;\theta)$: "______________" function ~ a function of the parameter θ that is proportional to the probability that we observe the given dataset.

   Want to find the value of θ that _____________ the likelihood function.

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(e) Point estimation by Method of Maximum Likelihood (MLE)

$$\hat{\theta}_{MLE} = \arg\max_{\theta} L(x_1, x_2, .., x_n; \theta)$$

Obtain the value of θ that maximizes the probability that the given data set would be observed.

(f) How? Solve

$$\frac{\partial L(x_1, ..., x_n; \theta)}{\partial \theta} =$$

Example 4: Based on the given data set $\{x_1, x_2, ..., x_n\}$, estimate the parameter of Exponential distribution by MLE.

(Hint: PDF $f_X(x;\nu) = \nu \exp(-\nu x)$)

(g) It is much more convenient to find a value that maximize the natural logarithm of the likelihood function, "log-likelihood function" $\ln L(x_1, x_2, ..., x_n; \theta)$. The solution of the following equation is the same as the one in (f).

$$\frac{\partial \ln L(x_1, ..., x_n; \theta)}{\partial \theta} = 0$$

Why the same result?

Why more convenient? (1) Derivative of product vs. summation
(2) Exponential functions
(3) Products of terms in each PDF

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Example 4 (Contd.): Estimate the parameter by MLE using the log-likelihood function

(h) Multiple distribution parameters (e.g. (λ, ζ) for LN, (μ, σ) for N)

Solve the system equation

$$\frac{\partial \ln L(x_1, \ldots, x_n; \theta_1)}{\partial \theta_1} = 0, \ \ldots, \ \frac{\partial \ln L(x_1, \ldots, x_n; \theta_m)}{\partial \theta_m} = 0$$

Example 5: Given $\{x_1, x_2, \ldots, x_n\}$

Find the point estimates on λ and ζ by MLE.

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Example 6: MLE estimates on μ and σ of a normal distribution.

Example 7: Tossing an unfair coin n times and observed "HEAD" x times.

Find the MLE estimate on the probability of "HEAD" each time.