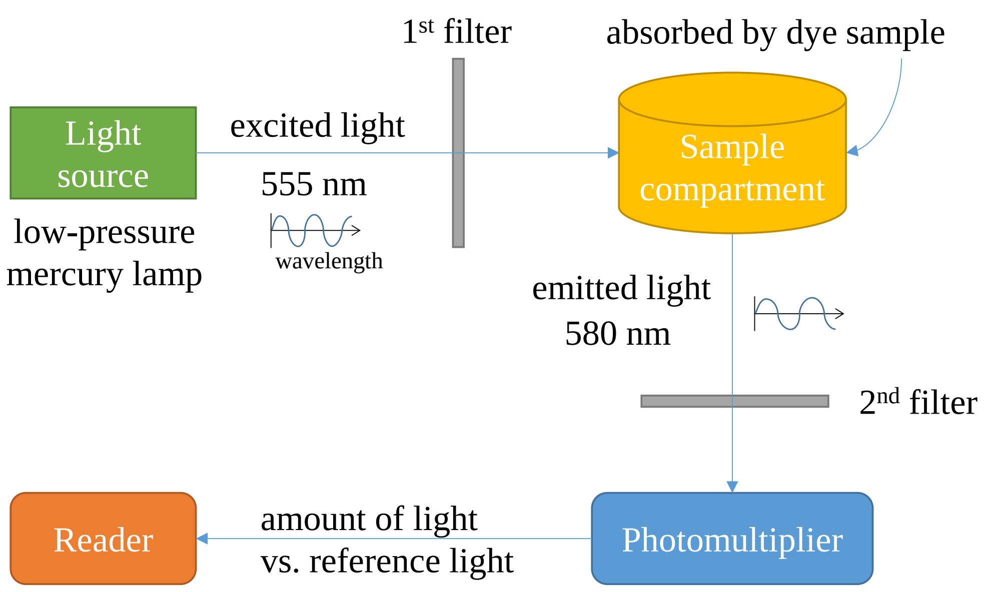
Chapter 7

Field Studies of Mixing in Rivers



Chapter 7 Field Studies of Mixing in Rivers

Contents

Objectives

- Classify stages of pollutant mixing in rivers
- Introduce methods for field survey and measurements for river mixing study
- Introduce data analysis methods
- Calculate dispersion coefficients from concentration data

7.1 Introduction

7.1.1 Governing Equations of Pollutants in Rivers

Mixing stages of pollutants in rivers:

Stage I: Three-dimensional mixing (vertical + lateral + longitudinal mixing)

Stage II: Two-dimensional mixing (lateral + longitudinal mixing)

Stage III: One-dimensional mixing (longitudinal mixing)

Stage I: Three-dimensional mixing

Two types of contaminant source

1) Effluent discharge through outfall structure

2) Accidental spill of slug of contaminant

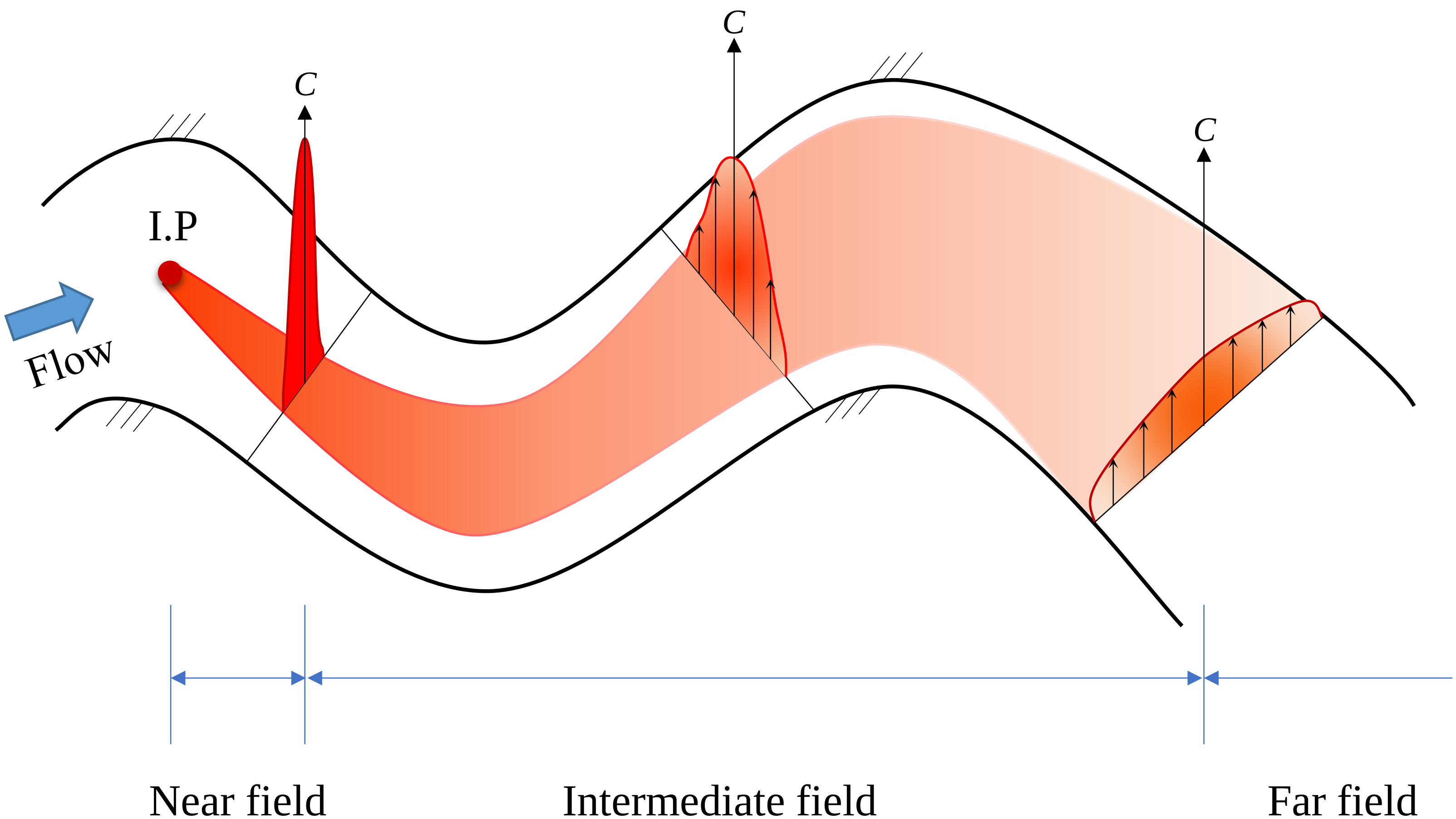
7.1 Introduction

$$\frac{\partial c}{\partial t}+u_x\frac{\partial c}{\partial x}+u_y\frac{\partial c}{\partial y}+u_z\frac{\partial c}{\partial z}=\frac{\partial}{\partial x}(\varepsilon_l\frac{\partial c}{\partial x})+\frac{\partial}{\partial y}(\varepsilon_t\frac{\partial c}{\partial y})+\frac{\partial}{\partial z}(\varepsilon_v\frac{\partial c}{\partial z}) \qquad (7.1)$$

where C = time-averaged concentration; t = time; u_x, u_y, u_z= velocity components; ε_l= longitudinal turbulent mixing coefficient; ε_t= transverse turbulent mixing coefficient; ε_v = vertical turbulent mixing coefficient

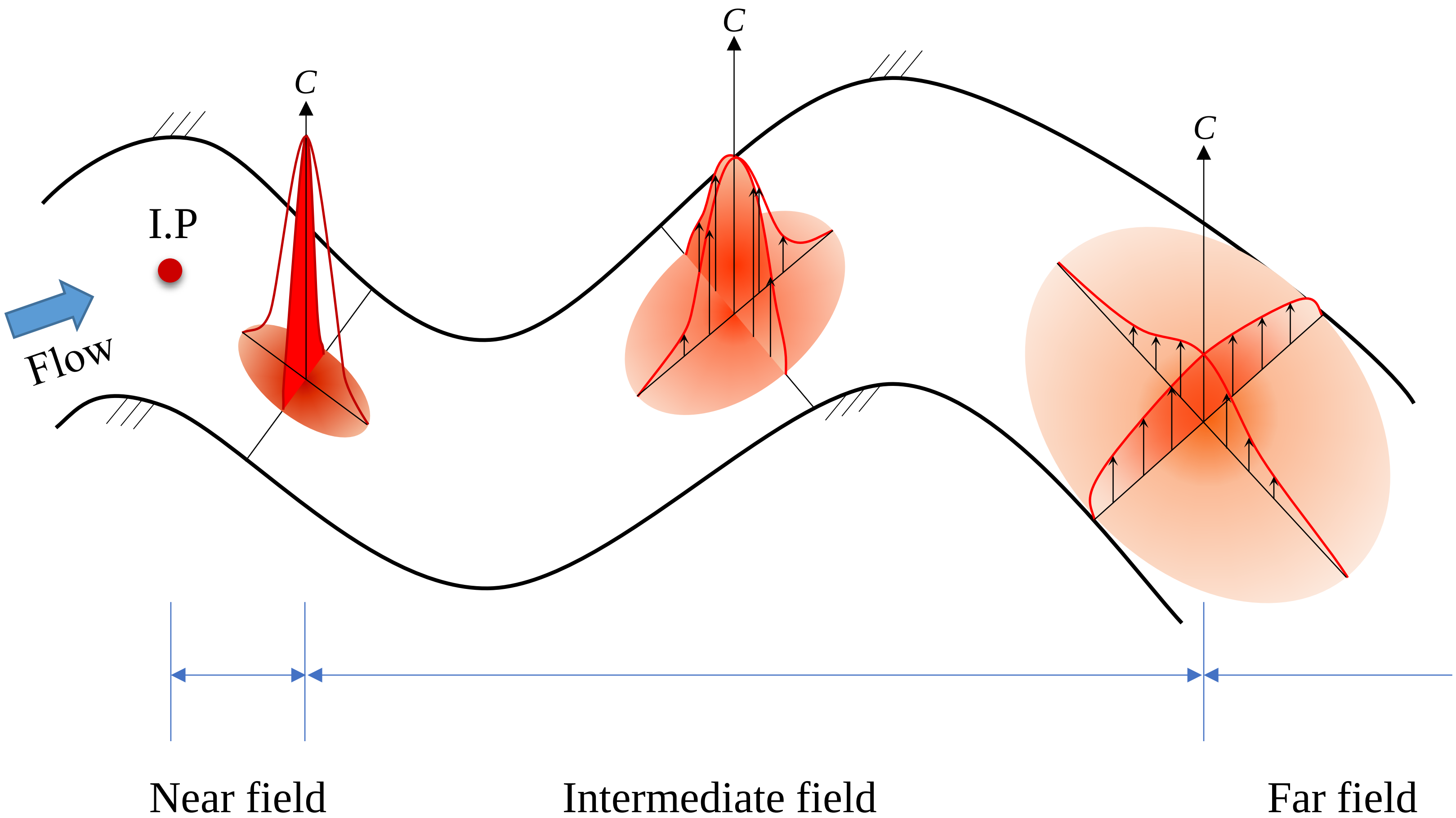
7.1 Introduction

a) Continuous Source



7.1 Introduction

b) Instantaneous Source



7.1 Introduction

Stage II: Two-dimensional mixing (longitudinal + lateral mixing)

~ Intermediate field mixing

~ Contaminant is mixed across the channel primarily by turbulent dispersion and spread longitudinally in the receiving stream.

→ apply 2D depth-averaged advection-dispersion equation for mixing in rivers

$$\frac{\partial \overline{c}}{\partial t} + u\frac{\partial \overline{c}}{\partial x} + v\frac{\partial \overline{c}}{\partial y} = \frac{\partial}{\partial x}\left(D_L \frac{\partial \overline{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_T \frac{\partial \overline{c}}{\partial y}\right) \qquad (7.2)$$

7.1 Introduction

7.1 Introduction

where $\overline{c}$ = depth-averaged concentration; u = depth-averaged longitudinal velocity; v = depth-averaged transverse velocity; $D_L = 2D$ longitudinal mixing coefficient; D_T = transverse mixing coefficient.

Stage III: Longitudinal dispersion

~ Far field mixing

~ Process of longitudinal shear flow dispersion erases any longitudinal concentration variations.

~ Apply 1D longitudinal dispersion model proposed by Taylor (Fischer et al., 1979)

7.1 Introduction

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}=\frac{\partial}{\partial x}\left(K\frac{\partial C}{\partial x}\right) \qquad (7.3)$$

where C = cross-sectional-averaged concentration; U = cross-sectional-averaged longitudinal velocity; $K = 1D$ longitudinal mixing coefficient.

7.1 Introduction

7.1.2 Field Studies on River Mixing

Field survey and measurements are needed to validate mixing predictions and to estimate the dispersion coefficients.

Field data include

- Bathymetric data: needed to make reliable mixing predictions especially when using numerical models
- Hydraulic data: local velocity and depth, discharge
- Concentration data: tracer concentration at sites below injection point

7.1 Introduction

Field studies should be designed and planned according to the mixing models that are investigated.

Field study for1D model: bathymetric and hydraulic data at each section; cross-sectional averaged concentration data $(C(x,t))$

Field study for 2D model: bathymetric and hydraulic data at each section; depth averaged concentration data $(\overline{c}(x,y,t))$

Field study for 3D model: bathymetric and hydraulic data at each section; local concentration data $(c(x,y,z,t))$

7.1 Introduction

Estimation of dispersion coefficients can be done by using

- change of variance of the tracer distribution
- routing method in which a concentration profile is routed from one site to the other site
- graphical methods using concentration profile or peak concentration

7.2 Field Survey and Experiments

7.2.1 Planning Field Work

Good planning is essential to get the most out of a field experiment minimizing the number of field crew and logistics.

Data needed to make reliable mixing predictions especially when using 2D numerical models are given below:

1) Topographic and bathymetric data
2) Hydraulic data
3) Concentration data

7.2 Field Survey and Experiments

Good radio communication between field parties is necessary so that sampling procedures can be changed during the course of an experiment.

Careful book keeping is essential to label each gaging, echo sounding, dye sampling with site, date, time, location and direction of travel.

It is necessary to carefully select sampling sites and to optimize sampling procedures.

7.2 Field Survey and Experiments

Field experimental procedure is given below:

1) Selecting sampling sites
2) Installing tag lines
3) Surveying channel bathymetry
4) Measuring velocity and discharge
5) Measuring dye concentration

7.2 Field Survey and Experiments

7.2.2 Selection of Sampling Sites

Sampling sections should be decided considering distances for vertical and transverse mixing.

1) 1D mixing test

The first sampling section should be located below L_t downstream of the injection point.

$$L_t = 0.1\frac{UW^2}{D_T}, \text{ centerline injection} \quad (7.4)$$

$$L_t = 0.4\frac{UW^2}{D_T}, \text{ side injection} \quad (7.5)$$

7.2 Field Survey and Experiments

2) 2D mixing test

The first sampling section should be located below L_v downstream of the injection point, and last sampling section located inside of L_t downstream of the injection point.

$$L_v = 0.35\frac{UH^2}{\varepsilon_v} \tag{7.6}$$

[Ex] For slowly meandering streams,

$$D_T = (0.3 \sim 0.9)HU^*$$

$$\frac{W}{H} \approx 50;\ \frac{U}{U^*} \approx 10$$

$$L_t \approx (50 \sim 150)W$$

7.2 Field Survey and Experiments

When ten or less sampling sections can be selected, the interval between sections can be 5~15 times of channel width.

Tag lines should be installed for accurate position fixing, which are stretched across the channel, in small rivers.

Tag lines are used to measure transverse distances, a tape for longitudinal distance, and a calibrated rod for vertical distance.

7.2 Field Survey and Experiments

7.2.3 Surveying River Bathymetry

Accurate estimates of river width, depth are needed to make reliable mixing calculations.

Channel topography and bathymetry are needed for calculation of dispersion coefficients and application of the 2D numerical model.

7.2 Field Survey and Experiments

Position fixing in rivers can be done by given methods:

i) Satellite position fixing system: use GPS; accuracy is only of the order of meters

ii) Remote shore-based systems: use Lidar

iii) Land-based surveying: gives excellent results but is very labor intensive

7.2 Field Survey and Experiments

Typical procedure of the land-based surveying can be as follow:

1) Install tag line at each sampling section

- Install GHP using wooden or steel pile at both banks of the channel
- Use GPS to measure position and datum of the GHP
- Need benchmarks along the bank whose position (x, y) and height above datum (z) are known

2) At each cross-section, measure water level using echo sounder or ADCP across the channel by boat

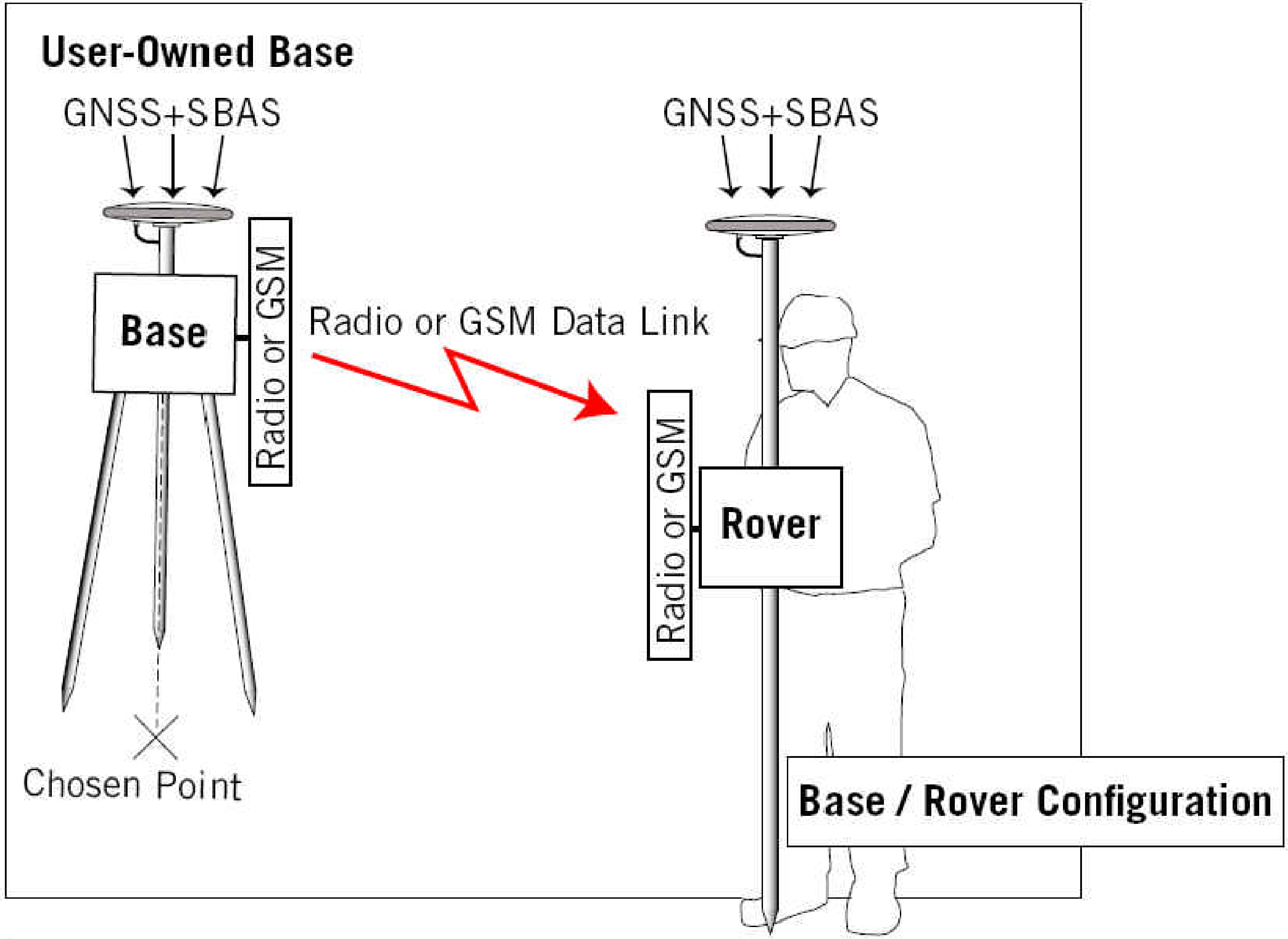
7.2 Field Survey and Experiments

3) In case tag lines are not possible, automatic position fixing systems can be installed in the boat which follows a random path thereby → require line of sight between the shore beacons and the boat

4) For surveying land in the floodplain area, use Real Time Kinematic-GPS (RTK-GPS)

- Set up the base station around the surveying field
- Yield absolute positions of 1~2 cm horizontal accuracy in real time with no post-processing
- The receiver with unknown position is called “Rover”

7.2 Field Survey and Experiments



7.2 Field Survey and Experiments

a) Installation of tag line

b) Measuring water level using ADCP

c) Using the boat

d) Surveying land using RTK-GPS

7.2 Field Survey and Experiments

7.2.4 Velocity Measurements

Accurate measurements of river discharge and velocity are essential for reliable mixing and mass balance calculations.

It is advisable during tracer tests to have one or more hydrological field parties undertake river gauging.

7.2 Field Survey and Experiments

7.2.4.1 ADV (Acoustic Doppler Velocimetry)

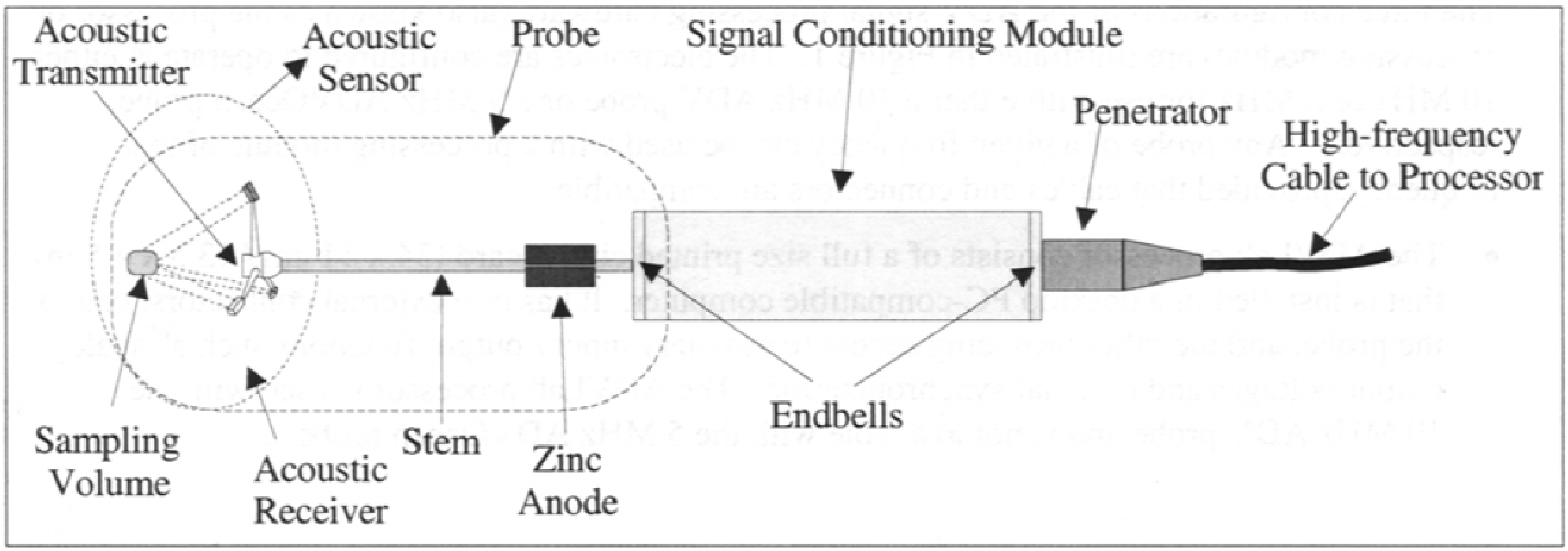
- Sends out a beam of acoustic waves at a fixed frequency from a transmitter probe.
- Waves bounce off of moving particulate matter in the water and three receiving probes "listen" for the change in frequency of the returned waves.
- Calculates the velocity of the water in the x, y, and z directions as following formulas.

7.2 Field Survey and Experiments

$$F_d = F_s \frac{V}{C_s} \qquad (7.7)$$

where F_d = change in received frequency (Doppler shift); F_s = frequency of transmitted sound; V = velocity of source relative to receiver; C = speed of sound.

7.2 Field Survey and Experiments



7.2 Field Survey and Experiments

7.2 Field Survey and Experiments

7.2.4.2 ADCP (Acoustic Doppler Current Profiler)

- Applied the Doppler principle by bouncing an ultrasonic sound pulse off small particles of sediment and other material.
- Transmits a pulse into the water column and then listens for the return echo from the acoustic backscatters in the water column.
- The sound is shifted one time (as perceived by the backscatter) and a second time (as perceived by the ADCP transducer). Because there are two Doppler shifts, the velocity of the water along the acoustic path can be computed from the following.

7.2 Field Survey and Experiments

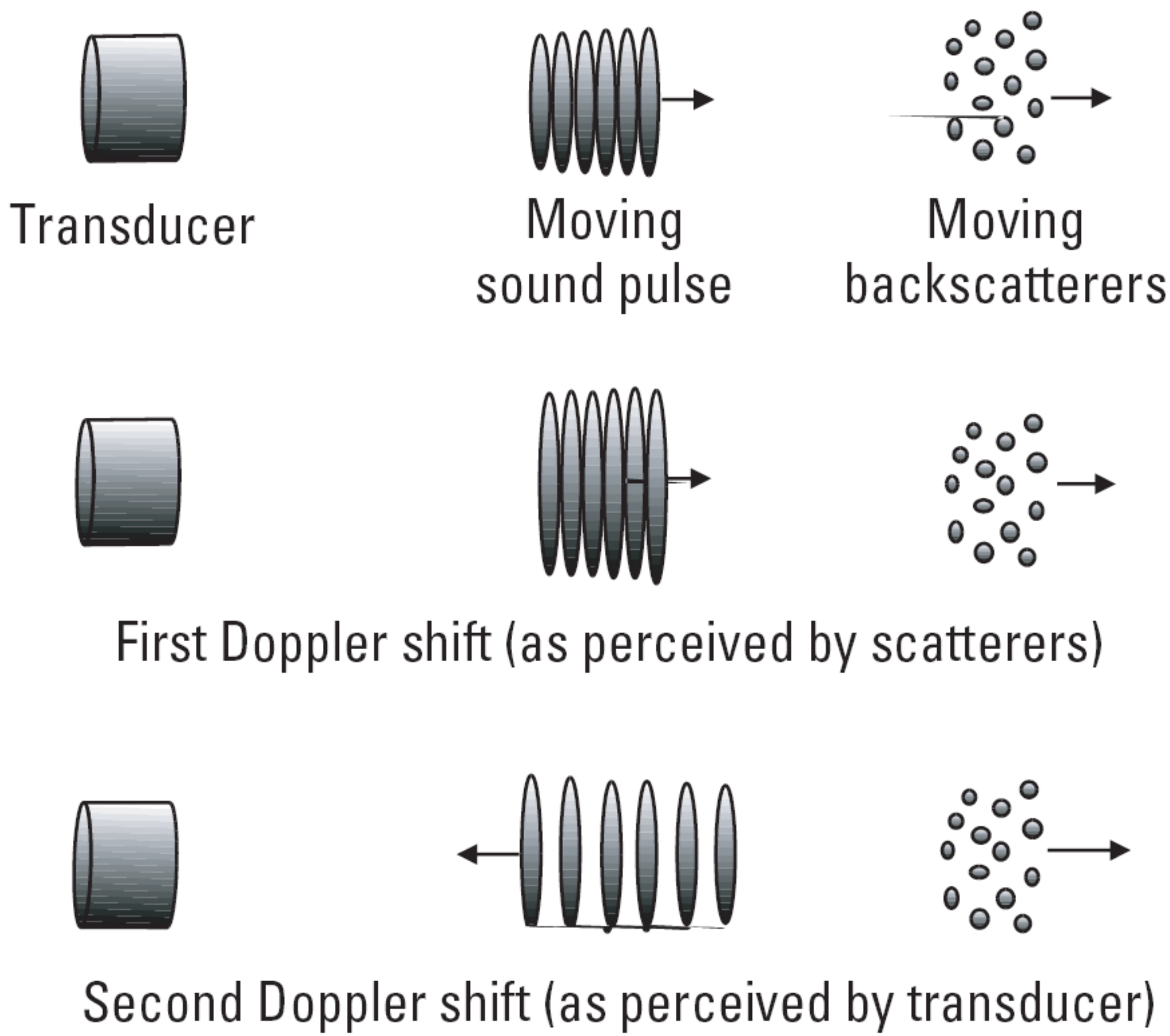
$$F_d = 2F_s \left(\frac{V}{C_S} \right) \tag{7.8}$$

- Computation of the real time discharge with ADCP deployed on a moving boat

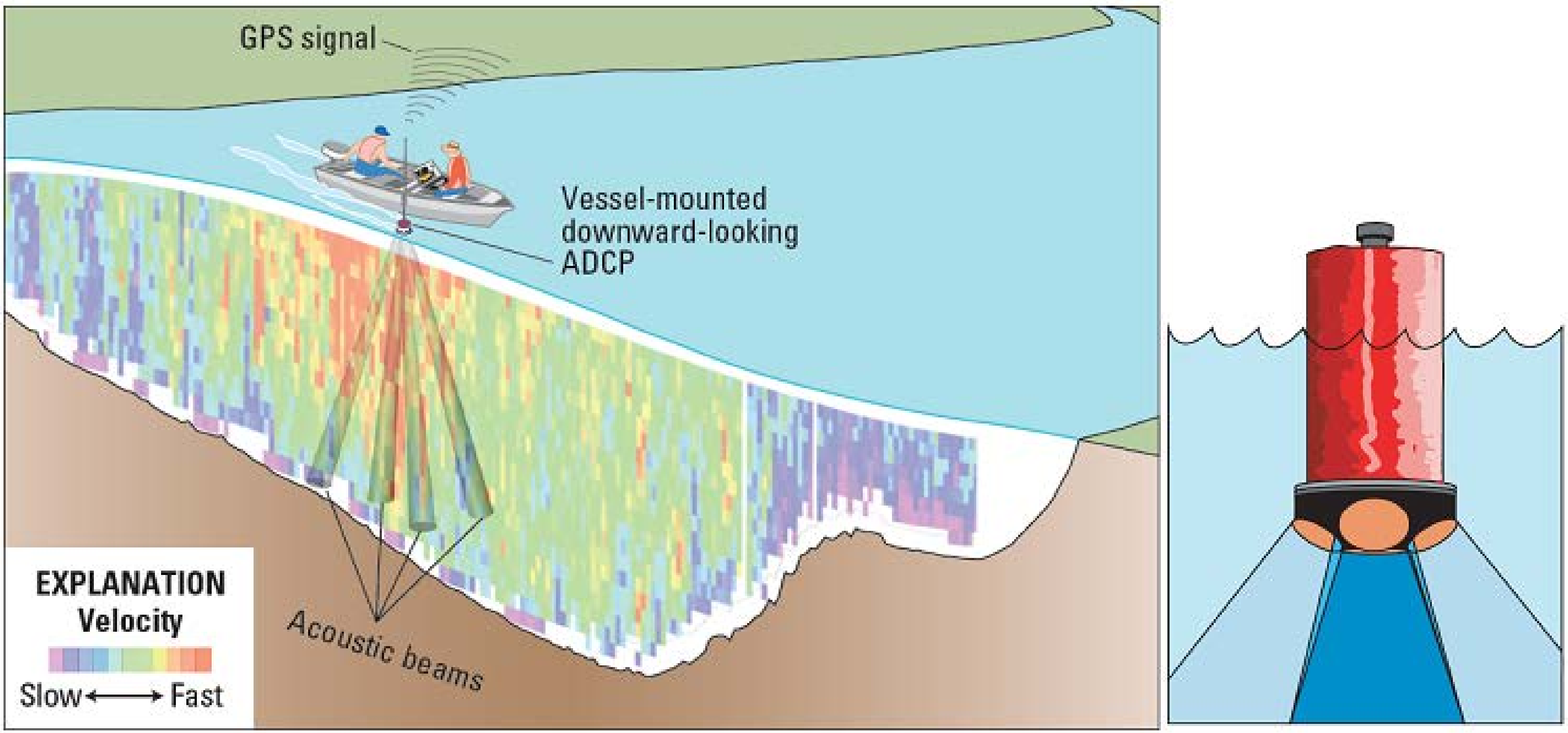
$$Q = Q_{LE} + Q_{Top} + Q_{Measured} + Q_{Bottom} + Q_{RE} \tag{7.9}$$

where Q_{LE} = discharge estimated for the unmeasured area near the left bank; Q_{Top} = discharge estimated for the top unmeasured area; $Q_{Measureed}$ = discharge measured directly by the ADCP; Q_{Bottom}= discharge estimated for the bottom unmeasured area; Q_{RE} = discharge estimated for the unmeasured area near the right bank.

7.2 Field Survey and Experiments



7.2 Field Survey and Experiments



7.2 Field Survey and Experiments

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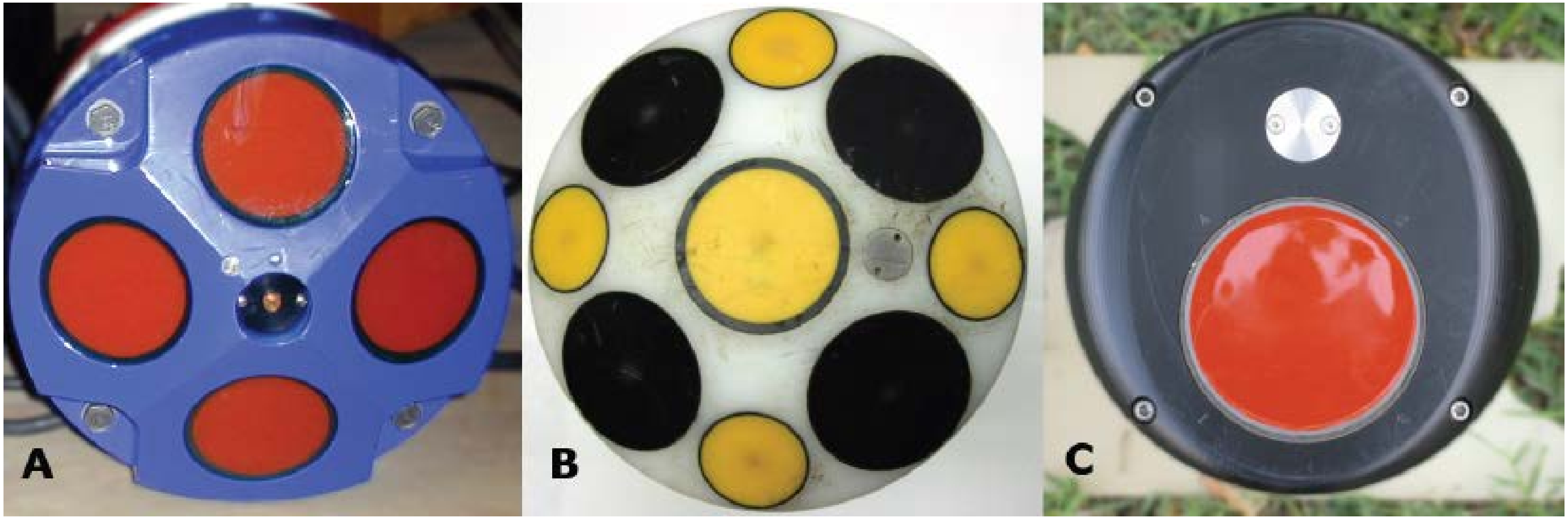
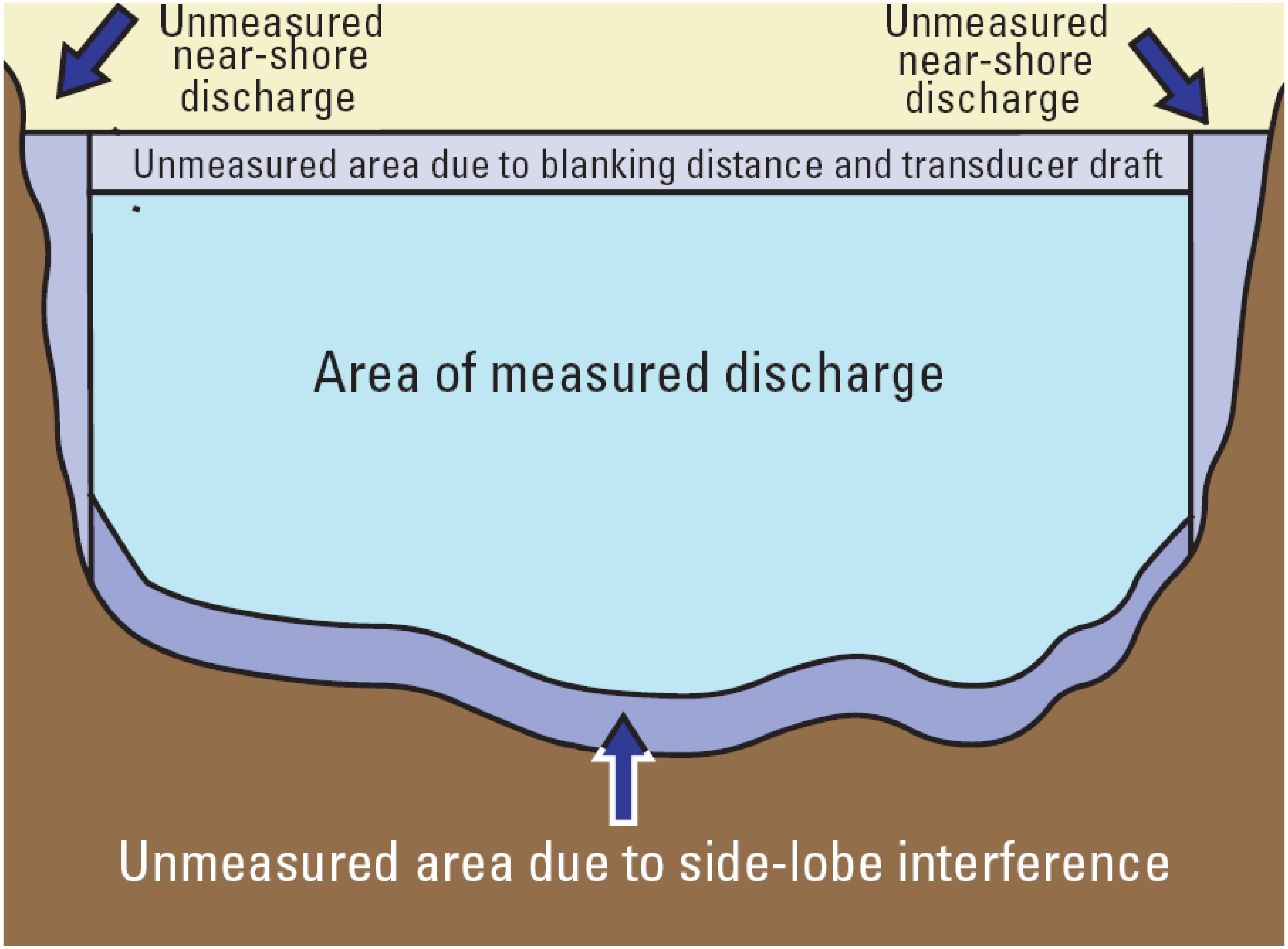
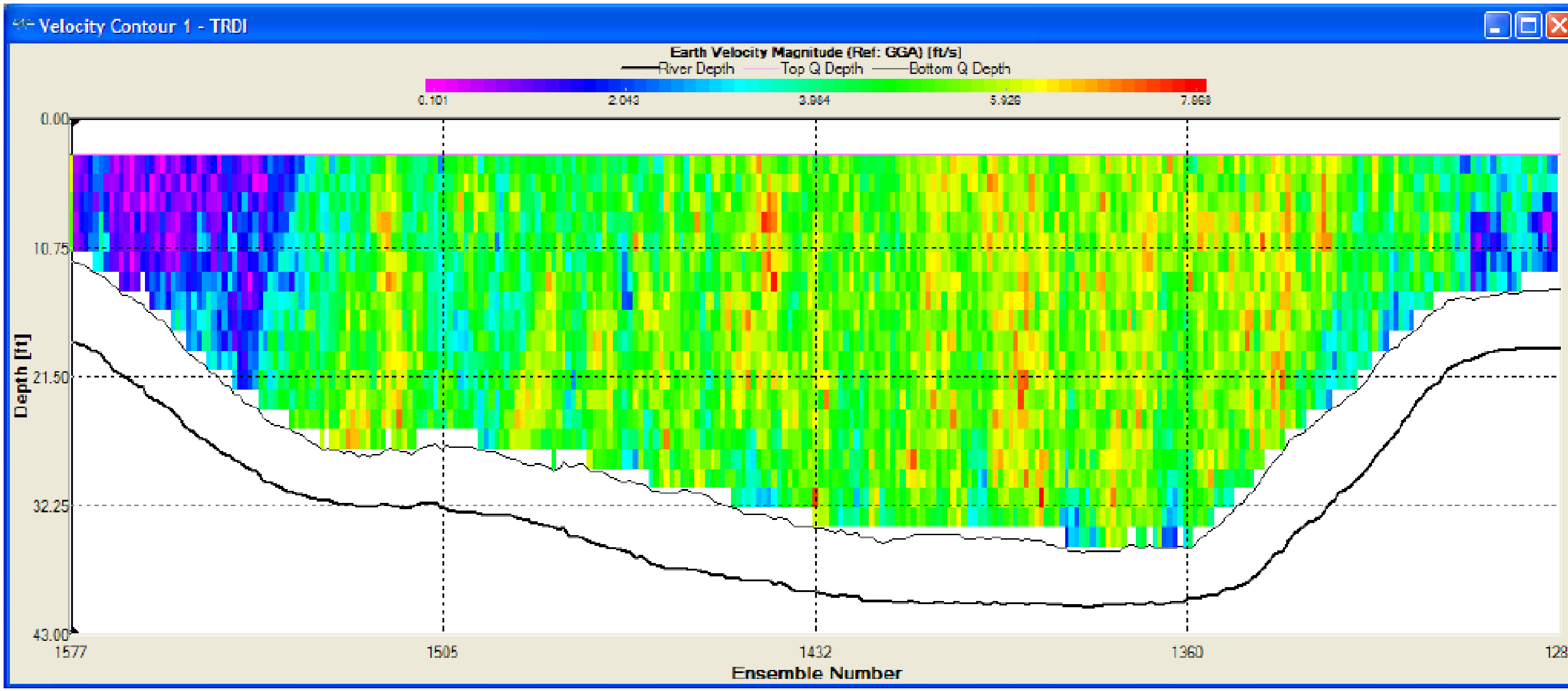


7.2 Field Survey and Experiments



7.2 Field Survey and Experiments



7.2 Field Survey and Experiments

7.2.4.3 Other Methods

(1) **Mechanical velocity meters**

- The principle of operation is based on the proportionality between the velocity of the water and the resulting angular velocity of the meter rotor.
- Velocity is determined by placing meter in stream and counting number of revolutions in a measured amount of time.

7.2 Field Survey and Experiments

7.2 Field Survey and Experiments

7.2 Field Survey and Experiments

(2) **Volumetric measurements**

- The most accurate way of measuring small discharges
- Observing the time it takes to fill a container of known capacity

7.2 Field Survey and Experiments

7.2 Field Survey and Experiments

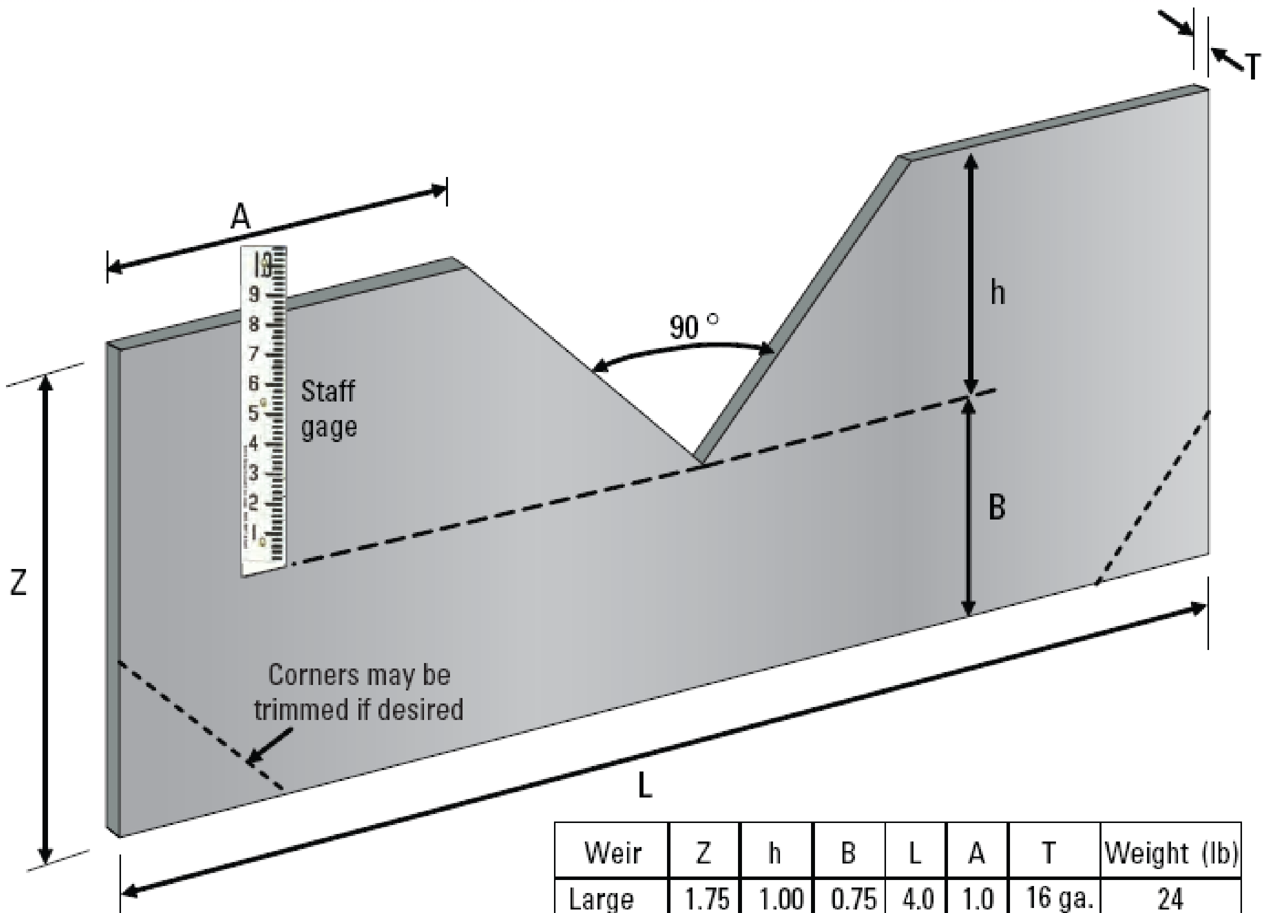
(3) **Portable weir plates**

- Used when depths are too shallow and velocity too low for a conventional velocity meter
- Discharge from 90 degree portable weir plates is computed using the formula.

$$Q = 2.49H^{2.48} \tag{7.10}$$

where H = height of water behind the notch.

7.2 Field Survey and Experiments



Weir	Z	h	B	L	A	T	Weight (lb)
Large	1.75	1.00	0.75	4.0	1.0	16 ga.	24
Medium	1.25	0.80	0.45	3.0	0.7	14 ga.	17
Small	0.75	0.47	0.28	2.0	0.53	10 ga.	8

7.2 Field Survey and Experiments

(4) **Float measurements**

- Distribute a number of floats uniformly over the stream width, noting the position of each with respect to the bank.
- The velocity of the float is equal to the distance between the cross sections divided by the time of travel.

7.2 Field Survey and Experiments

7.2 Field Survey and Experiments

7.2.5 Tracer Tests

7.2.5.1 Tracers

~ common method of measuring velocities and dispersion rates

~ can use two kinds of tracers:

i) Artificial tracers

ii) Natural (environmental) tracers

7.2 Field Survey and Experiments

1) Artificial tracer

• Two types of artificial tracers

i) Soluble tracer:

~ not affected by photochemical decay or adsorption

~ tracer input rate is controlled

ii) Floating tracer:

~ use surface floats or sub-surface drogues

~ used in the oceans to study advection and dispersion and in early laboratory dispersion studies

7.2 Field Survey and Experiments

~ disadvantage of only sampling surface currents which tend to be higher than the depth-averaged current

~ affected by the wind especially in wide river channels

~ difficult to use in natural rivers because of snags

~ travel faster than the dye and give advance warning to the sampling teams

- Commonly used dyes (Smart & Laidlaw, 1977)

i) Rhodamine WT

~ conservative

~ easy to handle and cheaper than Rhodamine B

~ readily measurable to very low levels

7.2 Field Survey and Experiments

ii) Rhodamine B

~ absorbed onto sediments

iii) Salt

~ detection limit is high, thus large amounts are required

iv) Radioactive tracers:

~ require specialist injection and detection equipment

~ arouses public disquiet

2) Conservative natural tracer

~ present in a tributary (Stallard, 1987)

~ STP effluents sometimes contain a suitable tracer such as EC, temperature.

7.2 Field Survey and Experiments

- Temperature

~ can be used to quantify mixing rates in rivers downstream from large cooling water discharges of power plants

~ non-conservative behavior in the far-field

- Sediment

~ especially used at junction of tributary

~ using the concentration of suspended sediment

- Conductivity

~ nearby inflow of effluents are needed

~ using the conductivity difference by effluent mixing

7.2 Field Survey and Experiments

7.2.5.2 Tracer Injection

~ can use two types of tracer injection:

i) Steady source

~ represent effluent discharge from STP or power plant

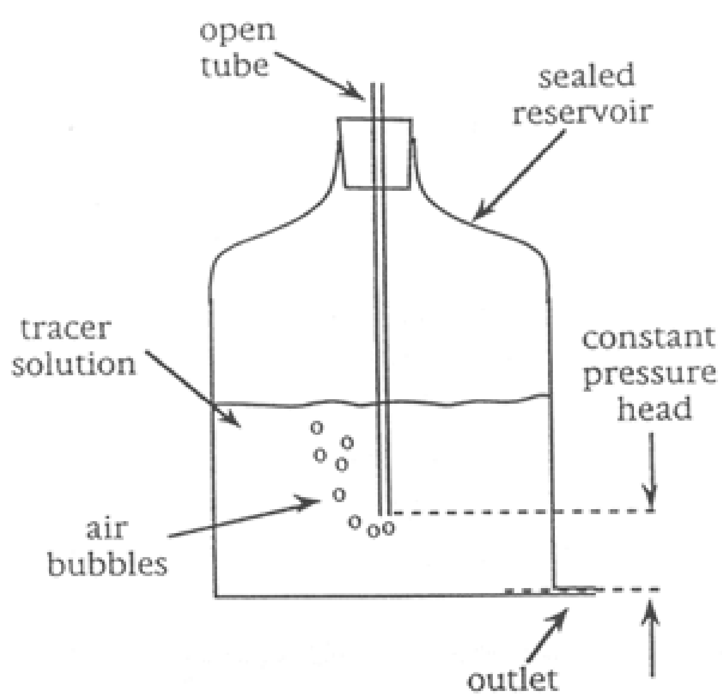
~ neglect longitudinal dispersion

~ do not give estimates of velocity

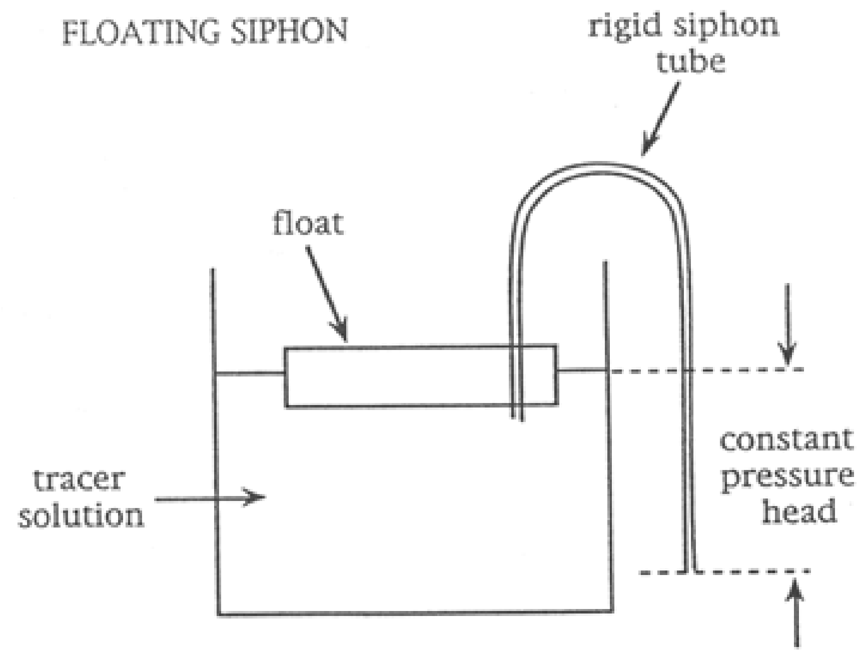
Use special apparatus for steady introduction of the tracer

- Marriott vessel
- Floating siphon
- Peristaltic pump

7.2 Field Survey and Experiments



a) Marrioot vessel



b) Floating siphon

7.2 Field Survey and Experiments

ii) Slug input

~ represent instantaneous injection of accidental spills

~ use simple apparatus for point or line sources

7.2.5.3 Instruments

~ can use two types of instruments for measurements of Rhodamine concentration:

1) Spectro-fluorometer

~ expensive, complex, delicate

7.2 Field Survey and Experiments

2) Filter fluorometer

~ moderately expensive, simple to use, robust in the field sensitivity comparable

- Turner 111
- Aminco Bowman fluoro / colorimeter
- YSI

7.2 Field Survey and Experiments

~ for natural tracer

i) Electrical conductivity meter

~ simple to use, commonly used for conductivity measurement

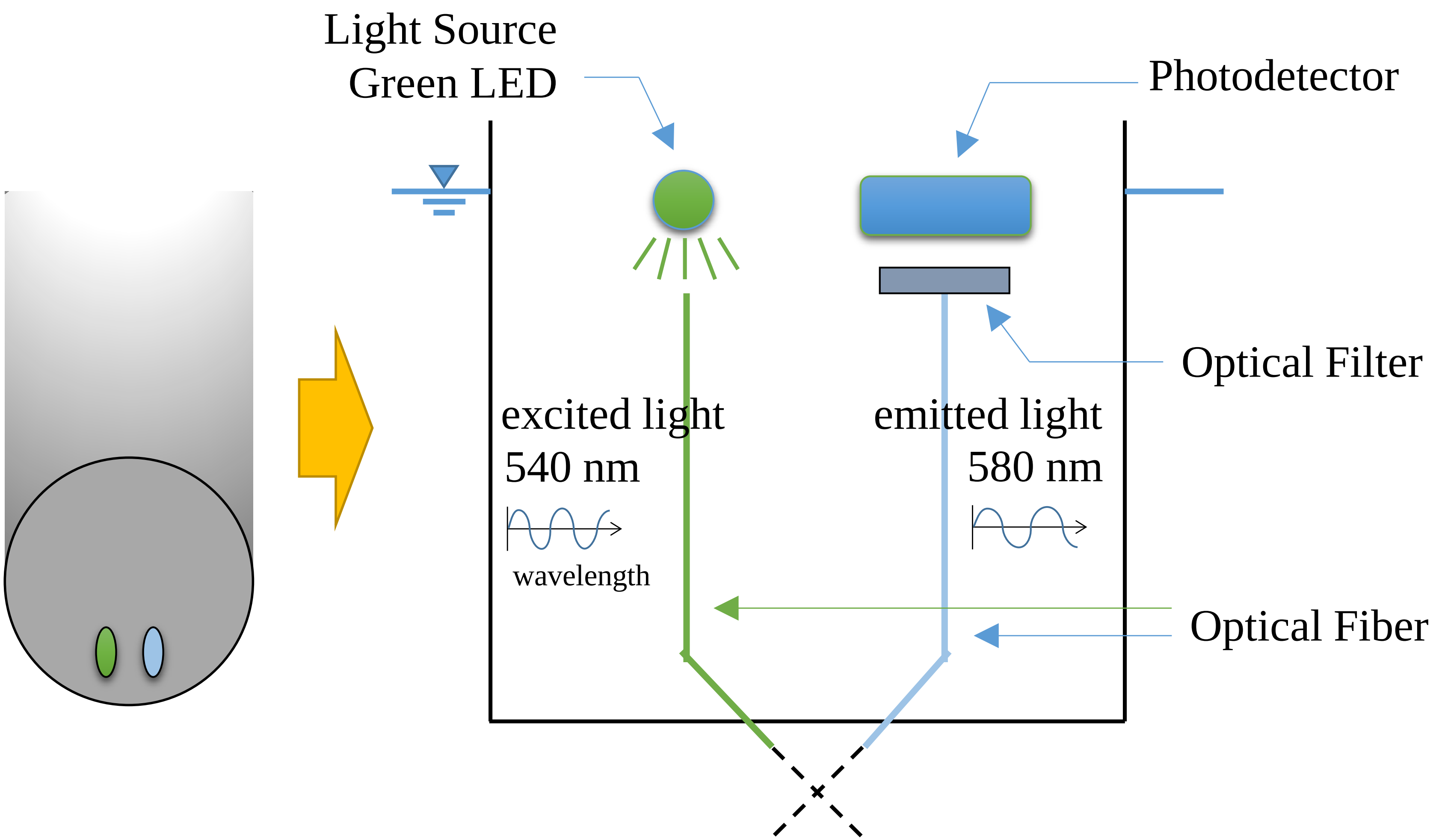
~ need temperature compensated instrument

ii) Thermistor

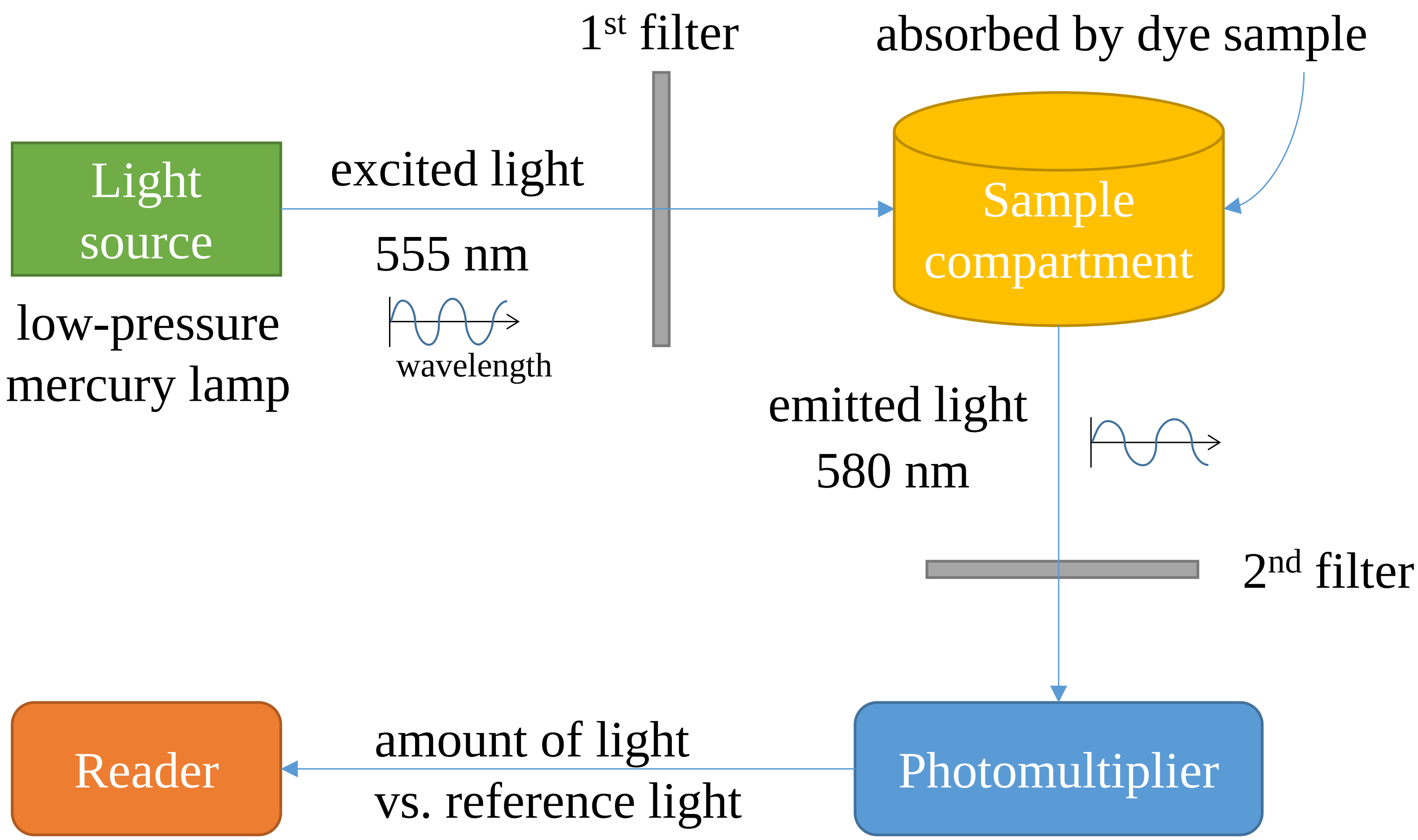
~ simple to use, commonly used for temperature measurement

~ high precision, typical range -90 to 130 °C

7.2 Field Survey and Experiments



7.2 Field Survey and Experiments



7.2 Field Survey and Experiments

7.2.5.4 Sampling of the tracer

~ can use two types of tracer sampling:

i) Sampling bottle

~ take many samples with time using sampling bottle

~ can use tubing and pump attached to the tag line in small-to-medium streams or take samples on the boat in large rivers

~ analyze concentration of each bottle at the base camp or in the laboratory

7.2 Field Survey and Experiments

ii) Flow-through type sensor

~ measure tracer concentration using flow-through type sensors in sites

~ Sensors can be fixed to the tag line in small streams

~ in large rivers, sensors are mounted in the boat along with position fixing system, GPS, and dragged in the river

7.2 Field Survey and Experiments

(1) Unsteady source

~ ensure that the maximum tracer concentration is well above the detection limit

~ there is a large enough change in concentration between sites to calculate the velocity and the dispersion coefficient

~ need to conduct a number of identical field tests and average the results to measure an ensemble average

~ when studying longitudinal dispersion, it is necessary to sample for long enough to measure the entire concentration versus time profile at each site and check for tracer loss

~ can be very time consuming because there are often long tails associated with tracer becoming trapped in dead zones

7.2 Field Survey and Experiments

(2) **Steady source**

~ is possible to collect replicate samples over a reasonably long time period and calculate time averaged concentration

~ when studying transverse dispersion, it is desirable to traverse the plume several times at the same transect and average the results

• Check for fluorometer drift

~ a number of discrete water samples should be collected from time to time, analyzed later in the laboratory to check the fluorometer drift

7.3 Data Analysis

7.3.1 Topographic Data

7.3.1.1 Channel

1) Thalweg line

~ a line that connects the lowest points within a river channel

~ a line of fastest flow along a river's course

2) Sinuosity, S_n

~ Sinuosity of river is its tendency to move back and forth across its floodplain, in an S-shaped pattern, over time

~ S_n = thalweg line / straight line = actual path length / shortest path length

7.3 Data Analysis

~ S_n > 1.5 : meandering channel

~ S_n < 1.05 : straight channel

3) Riffle-pool sequence

~ a stream's hydrological flow structure alternates from areas of relatively shallow to deeper water

4) Hydraulic radius, R

~ the ratio of the water area to its wetted perimeter

- $R = A / P$

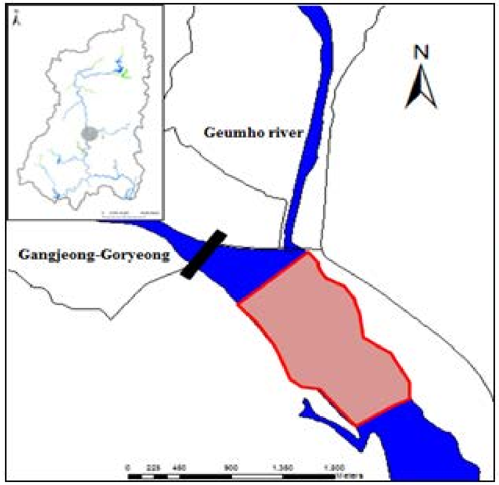
7.3 Data Analysis

1) Manning's roughness coefficient, n

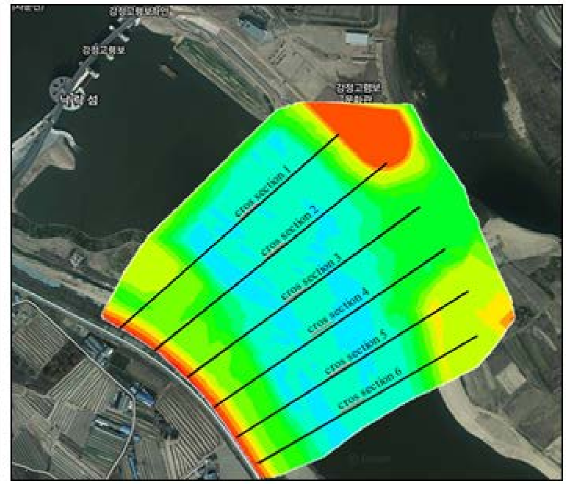
~ approaches in the proper determination of n

- to understand the factors that affect the value of n and thus to acquire a basic knowledge of the problem and narrow the wide range of guesswork
- to consult a table of typical n values for channels of various types
- to examine and became acquainted with the appearance of some typical channels whose roughness coefficients are known
- to determine the value of n by an analytical procedure based on the data of either velocity distribution in the channel cross section and on the data of either velocity or roughness measurement

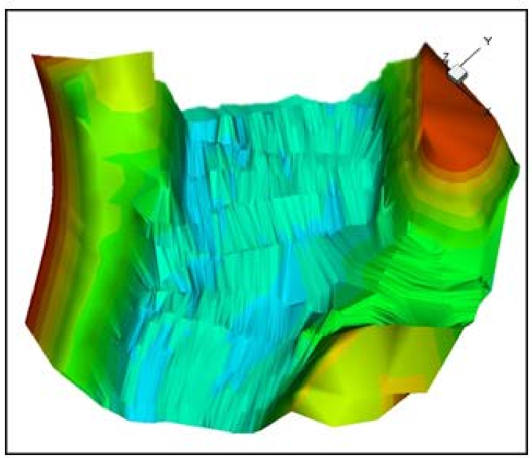
7.3 Data Analysis



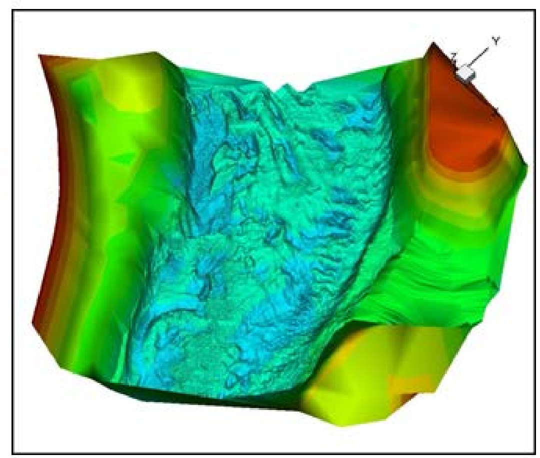
a) Site



b) Using ADCP



c) Using Single-Beam Echo Sounder



d) Using Multi-Beam Echo Sounder

7.3 Data Analysis

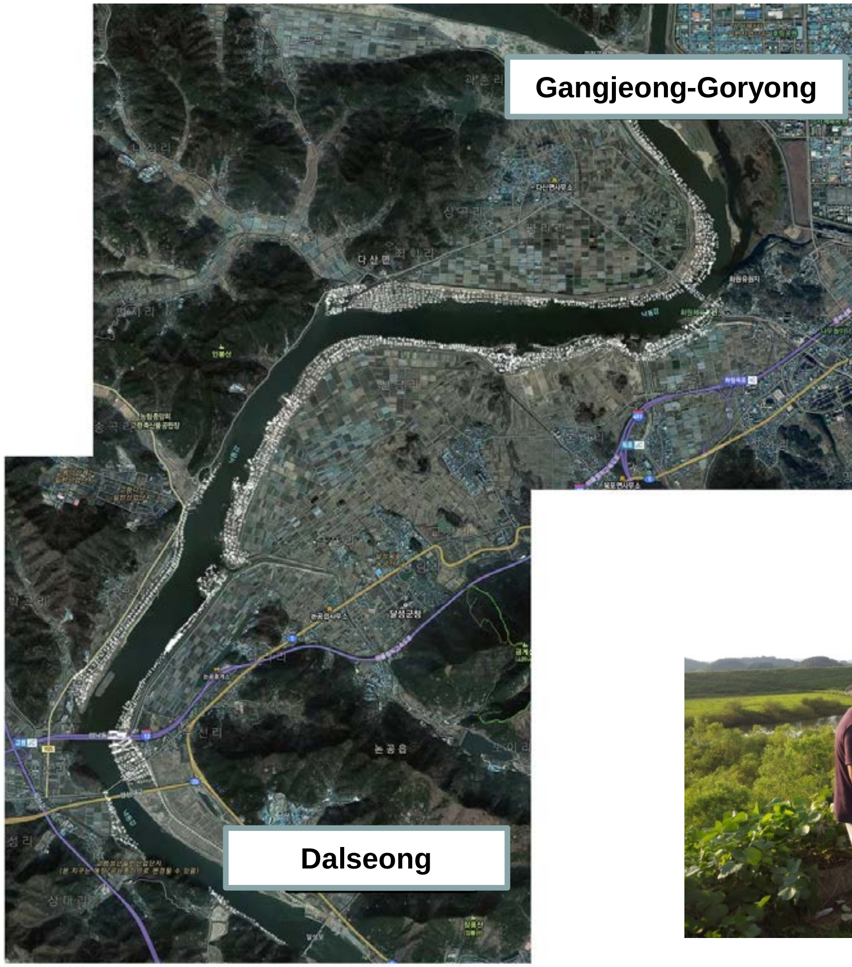
7.3.1.2 Floodplain

1) Merge of Topographic Information for Channel and Floodplain

~ Merge the two data based on a unified reference point to the same coordinate system

~ Adopted Geodetic Reference System 1980 and calculated by converting UTM latitude and longitude coordinates

7.3 Data Analysis



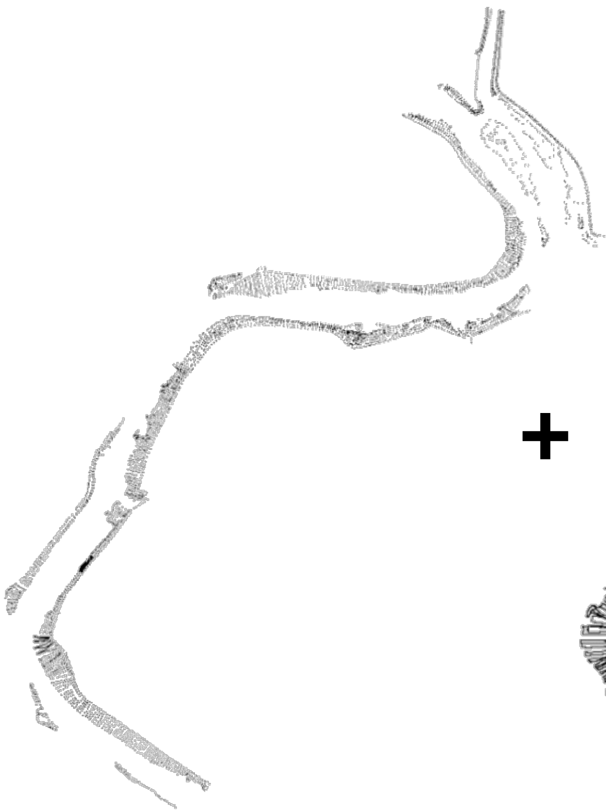
RTK-GPS :

- **GRX1(GPS+GLONASS Receiver and Antena, Digital UHF Modem, Blue Tooth, GCS/GPRS Module and Integrated Receiver with Removable Battery**
- **72 Channels for GPS + GLONASS + SBAS**

Total Station

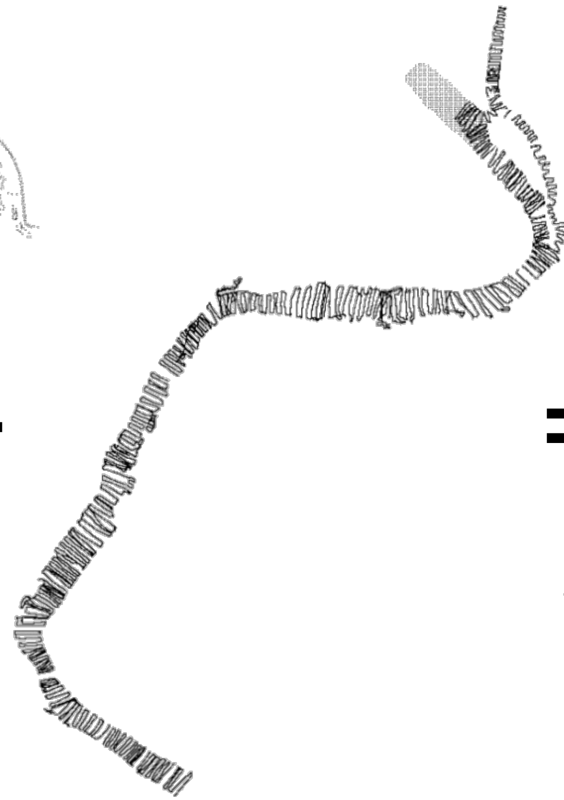
RTK-GPS

7.3 Data Analysis



Floodplain (RTK-GPS)

+



Channel (Single-beam)

=

Merged

7.3 Data Analysis

7.3 Data Analysis

7.3.2 Hydraulic Data

7.3.2.1 Velocity Data

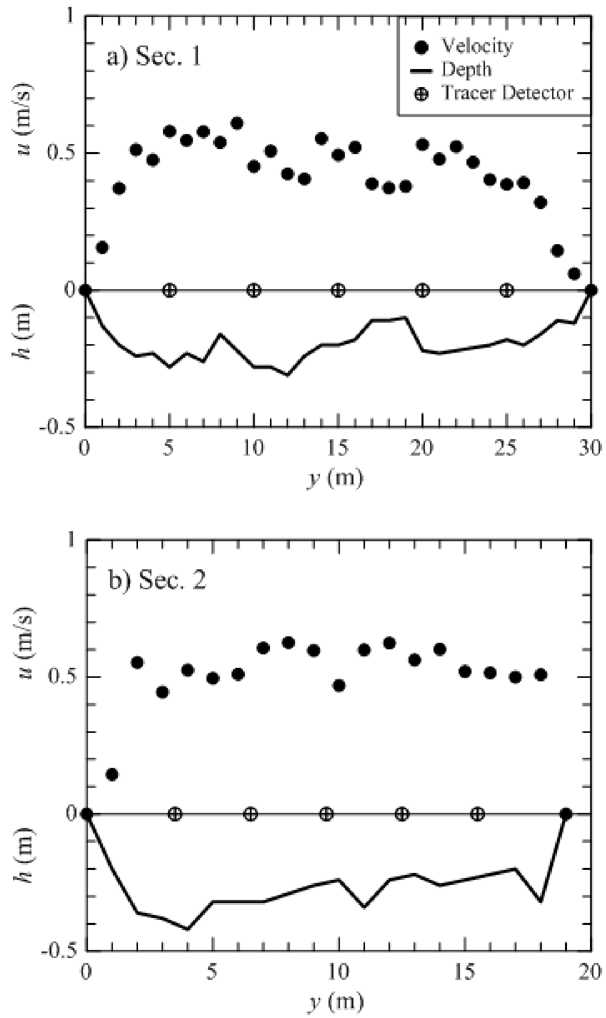
1) Velocity profile

~ Velocity profile is measured by wading with ADV or boat-mounted ADCP

~ Lateral distribution of velocity is obtained

~ Spatial averaged velocity and discharge at each section can be calculated from velocity profiles

7.3 Data Analysis



7.3 Data Analysis

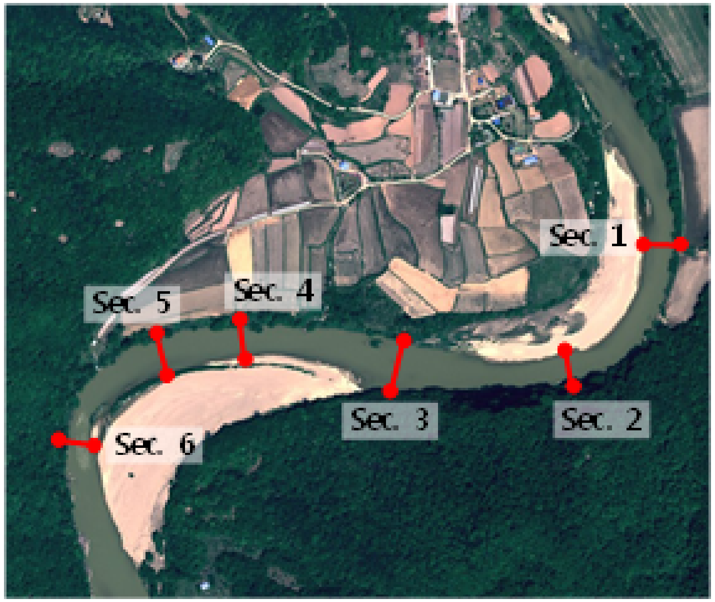
2) Reach-averaged velocity

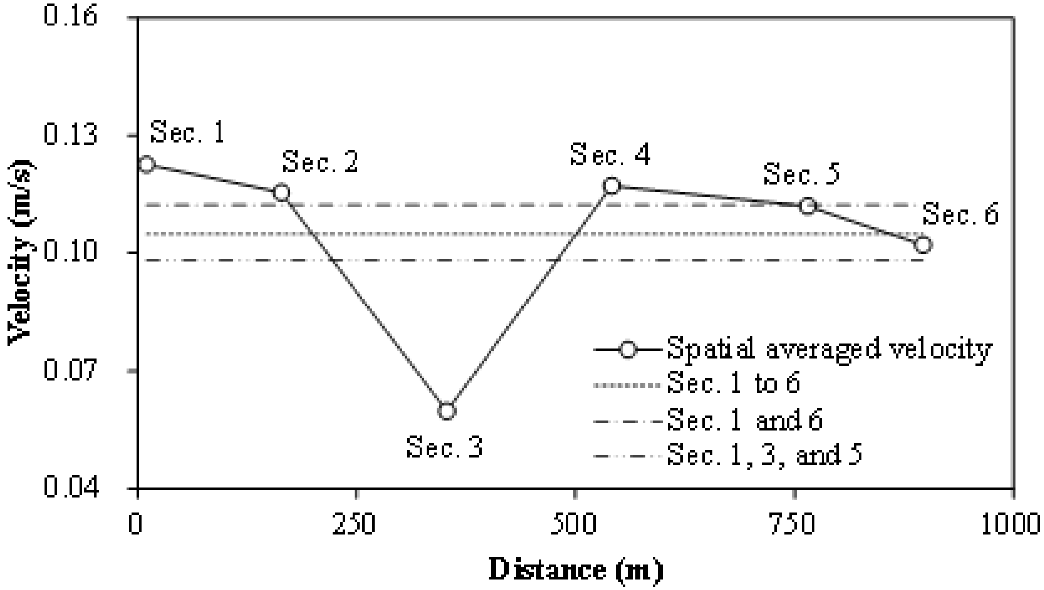
① Velocity measurements at a number of cross sections

~ large number of gauging is needed

~ more accurate reach-averaged velocity would be obtained according to the number of cross sections

7.3 Data Analysis





7.3 Data Analysis

② Tracer method

~ cost effective if tracer is made with an instantaneous injection

$$\bar{v}_x = \frac{x_2 - x_1}{\bar{t}_2 - \bar{t}_1} \tag{7.11}$$

where $\bar{v}_x$ = reach - average velocity; $\bar{t}_1$ and $\bar{t}_2$ = times of passage if the centroid if tracer profile past sites 1 and 2

$$\bar{t}_1 = \frac{\int_0^\infty \tau C\left(x_1, \tau\right) d\tau}{\int_0^\infty C\left(x_1, \tau\right) d\tau} \tag{7.12}$$

where $C(x_1, \tau)$ = cross-sectional averaged concentration at Site 1

7.3 Data Analysis

In the mid-field,

$$C(x,t) = \frac{1}{A}\int_0^W h(x,y)c(x,y,t)dy \tag{7.13}$$

where c = local depth-averaged concentration; h = local depth; A = cross-sectional area.

The observed tracer versus time profile is often uneven because of experimental errors and turbulent eddies in the plume so that some smoothing may be required.

7.3 Data Analysis

7.3.2.2 River depth

1) Use of wading rod

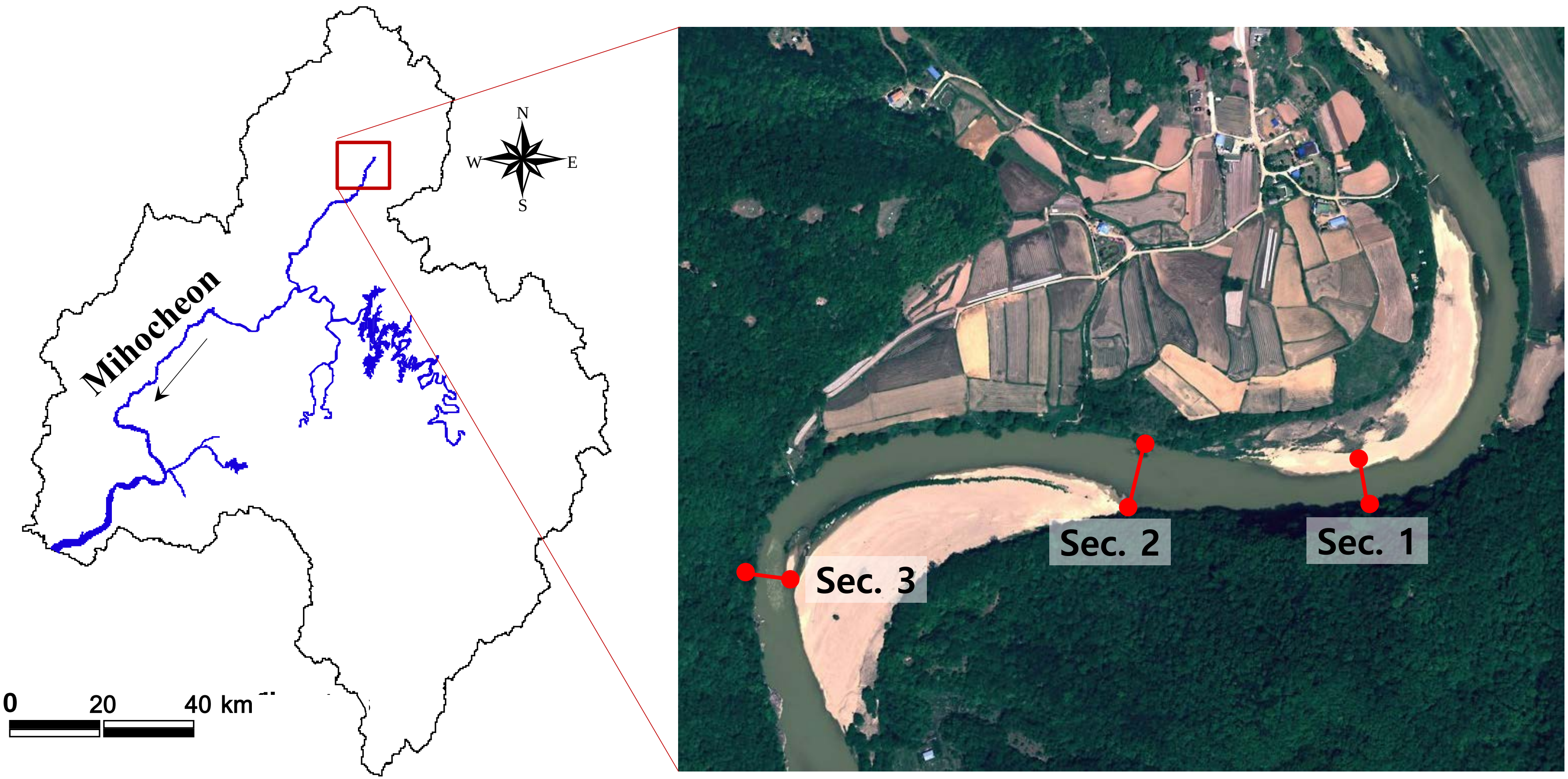
- Depth is shallow enough or measuring from a low footbridge
- Wading rods have a small foot on the bottom to allow the rod to be placed firmly on the streambed.

2) Use of sonic sounder

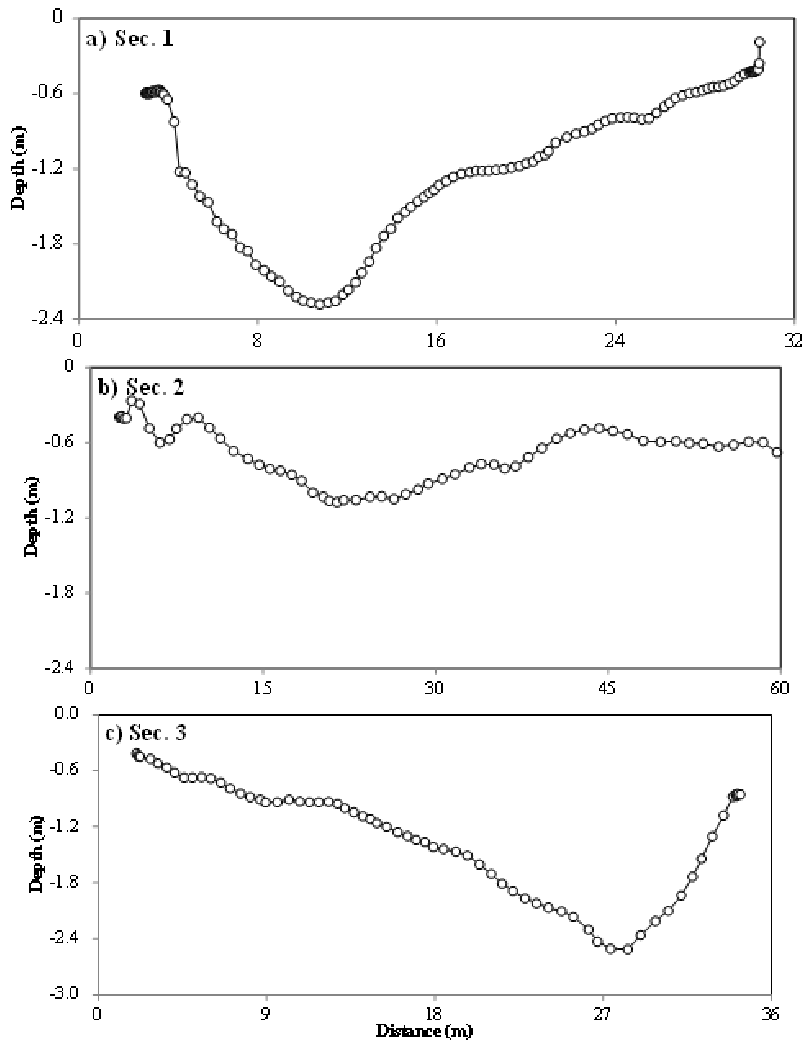
- The sonic sounder has been used primarily for measuring depth when making a moving boat measurement and in ADCP discharge measurements.
- For moving boat measurements, the sonic sounder records a continuous trace of the streambed on a digital or analog chart.

Ex) Depth measurement using sonic sounder in the Mihocheon

7.3 Data Analysis



7.3 Data Analysis



7.3 Data Analysis

7.3.2.3 Discharge

1) Velocity-Area method

- The most practical method of measuring the discharge of a stream.
- The total discharge is the summation of the products of the partial areas of the stream cross section and their respective average velocities.

$$Q = \sum_{i=1}^{n} a_i v_i \qquad (7.14)$$

where a_i is the cross section area, v_i is the corresponding mean velocity

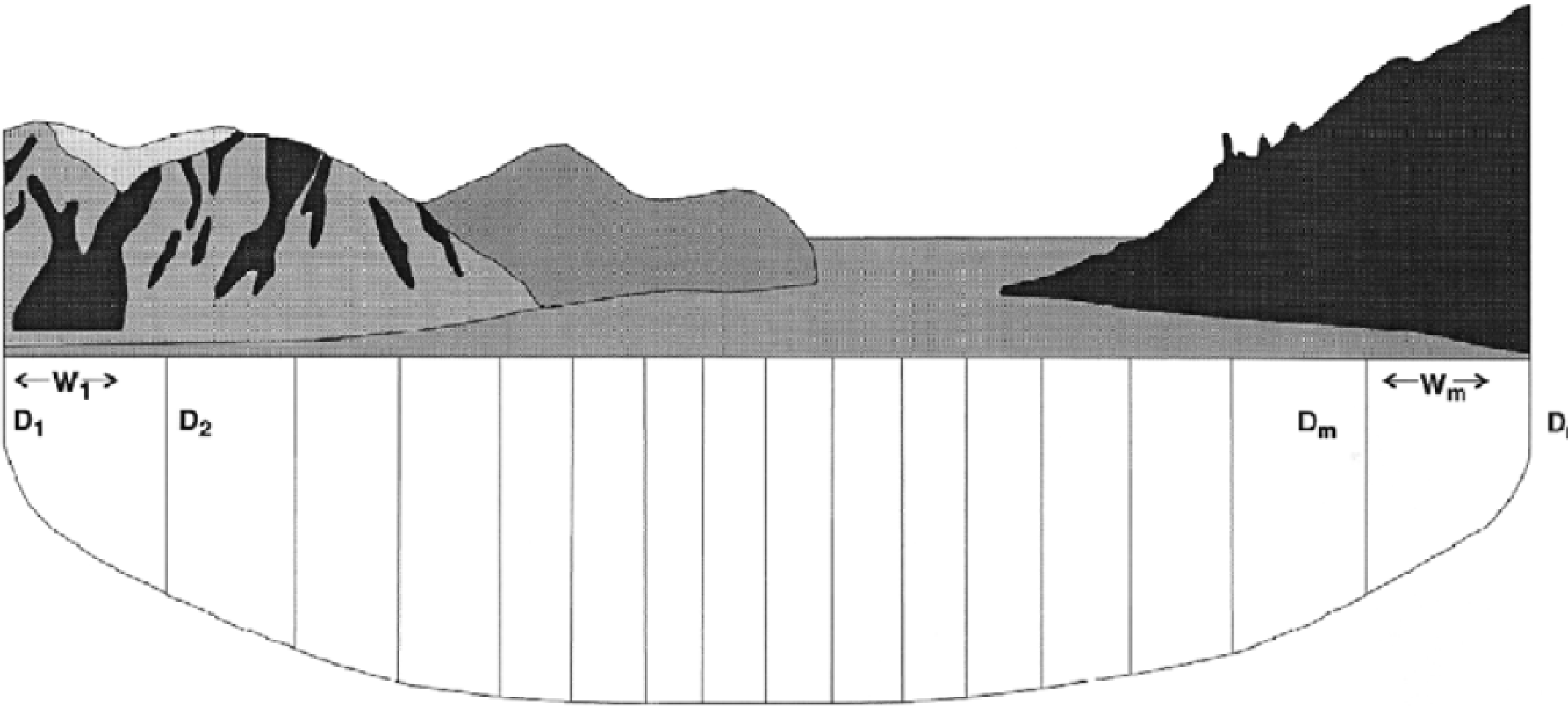
7.3 Data Analysis

2) Mid-section method

- Once the velocity, depth, and distance of the cross section have been determined, the mid-section method can be used for determining the discharge.

$$Q = \sum_{i=1}^{n-1} \left(\frac{D_i + D_{i+1}}{2} \right) \left(\frac{V_i + V_{i+1}}{2} \right) W_i \qquad (7.15)$$

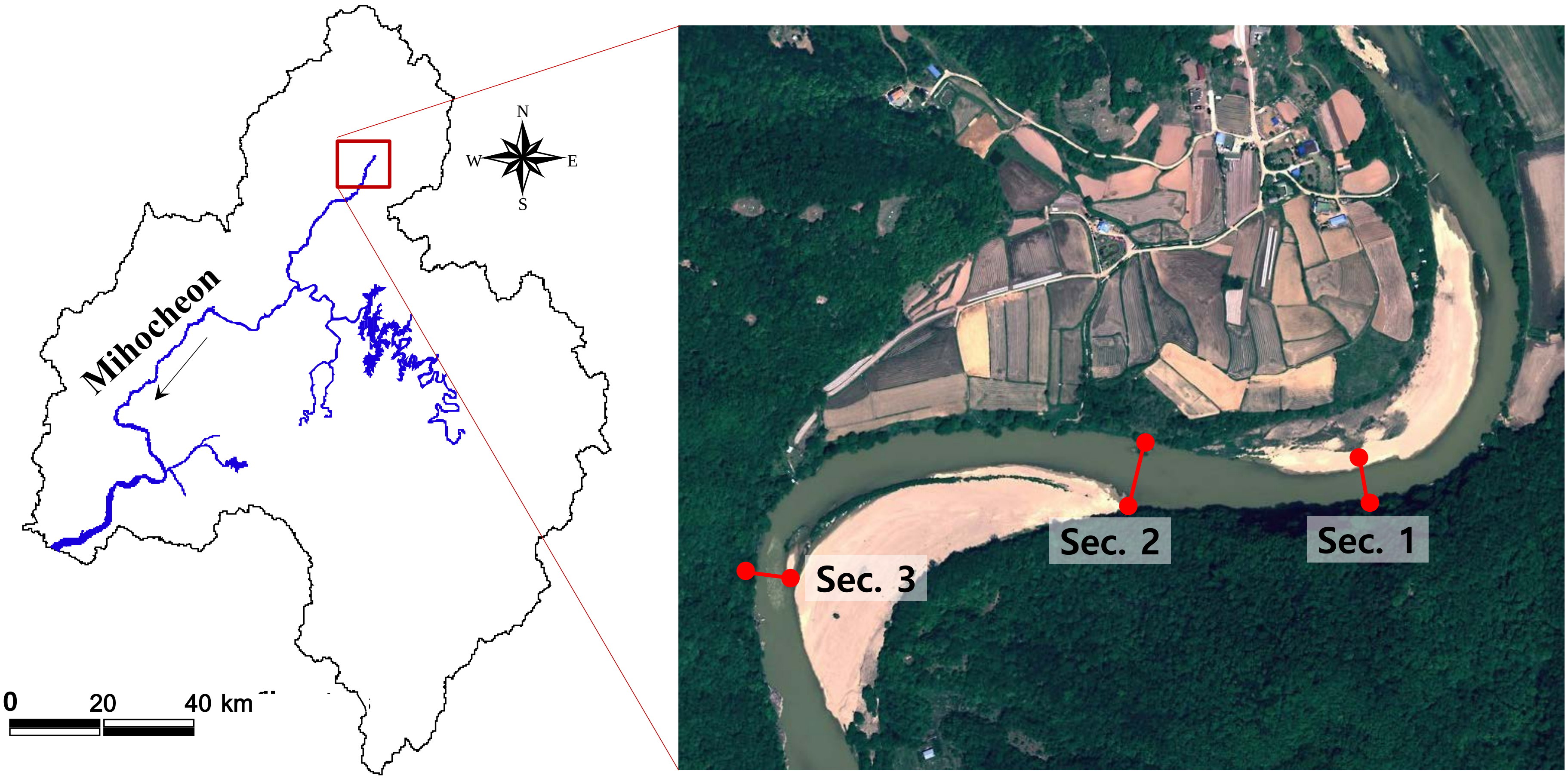
7.3 Data Analysis



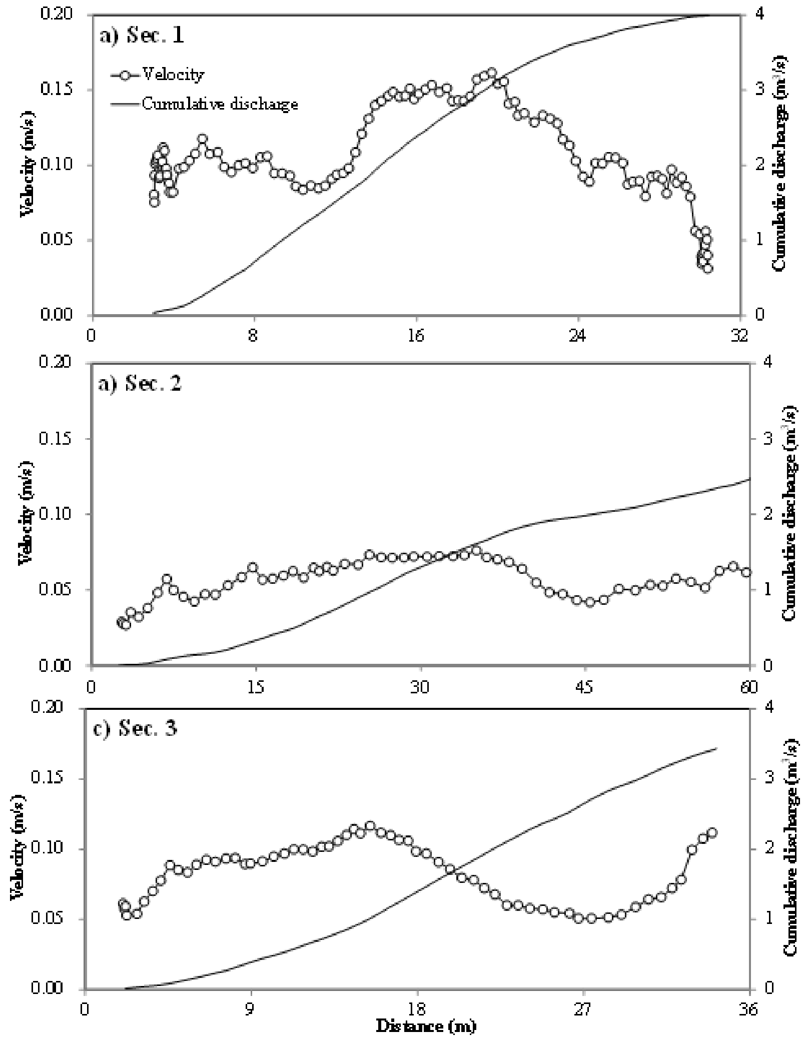
7.3 Data Analysis

Ex) Velocity measurements with ADCP and cumulative discharge in the Mihocheon

7.3 Data Analysis



7.3 Data Analysis



7.3 Data Analysis

7.3.3 Concentration Data

7.3.3.1 C-t profile

1) Central tendency

~ Mean of concentration

~ Median of concentration

2) Dispersion tendency

~ Range of concentration and time

~ Variance of concentration, σ^2

7.3 Data Analysis

3) Symmetry

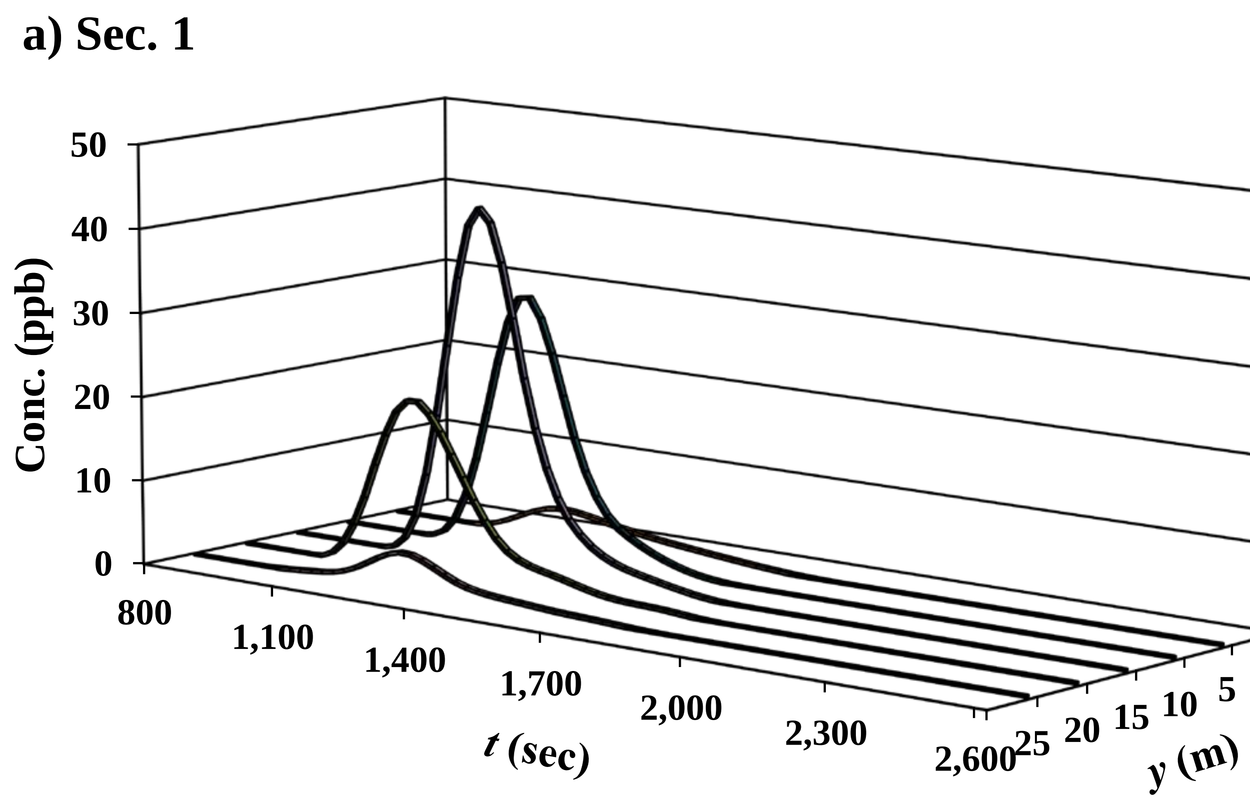
~ Skewness of concentration

4) Peakedness

~ peak of concentration, C_p

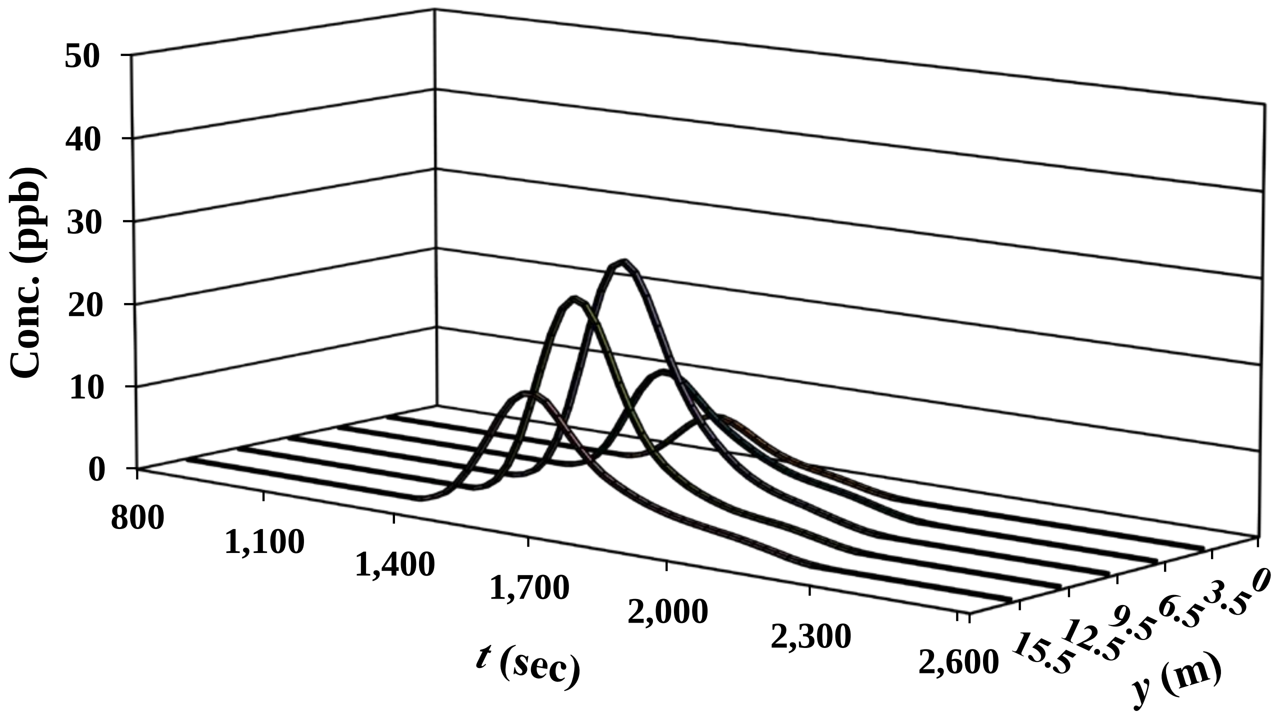
~ time of concentration peak, t_p

7.3 Data Analysis



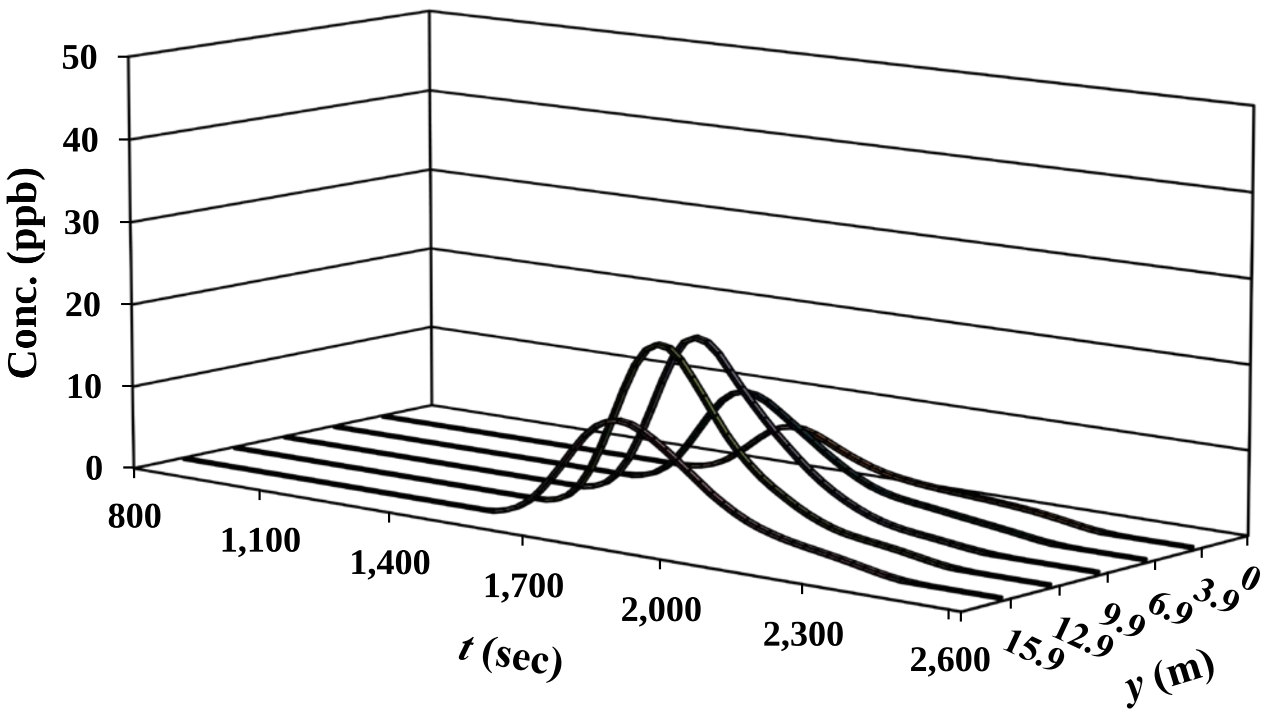
7.3 Data Analysis

b) Sec. 2



7.3 Data Analysis

c) Sec. 3



7.3 Data Analysis

7.3.3.2 Tracer Mass Flow

It is advisable to check for tracer loss due to either photochemical decay, adsorption or poor sampling.

Equations for estimating dispersion coefficients require knowledge of the tracer mass inflow rate.

- In the mid-and far-field the mass flux is

$$m(x) = \int_0^W h(x,y)u(x,y)c(x,y)\,dy \qquad \text{for continuous source} \qquad (7.16a)$$

$$m(x) = \int_0^W \int_0^\infty h(x,y)u(x,y)c(x,y,t)\,dt\,dy \qquad \text{for instantaneous source} \qquad (7.16b)$$

7.3 Data Analysis

[Re]

Mass flux = mass / time = conc · vol / time = $C\,Q = C\,V\,A$

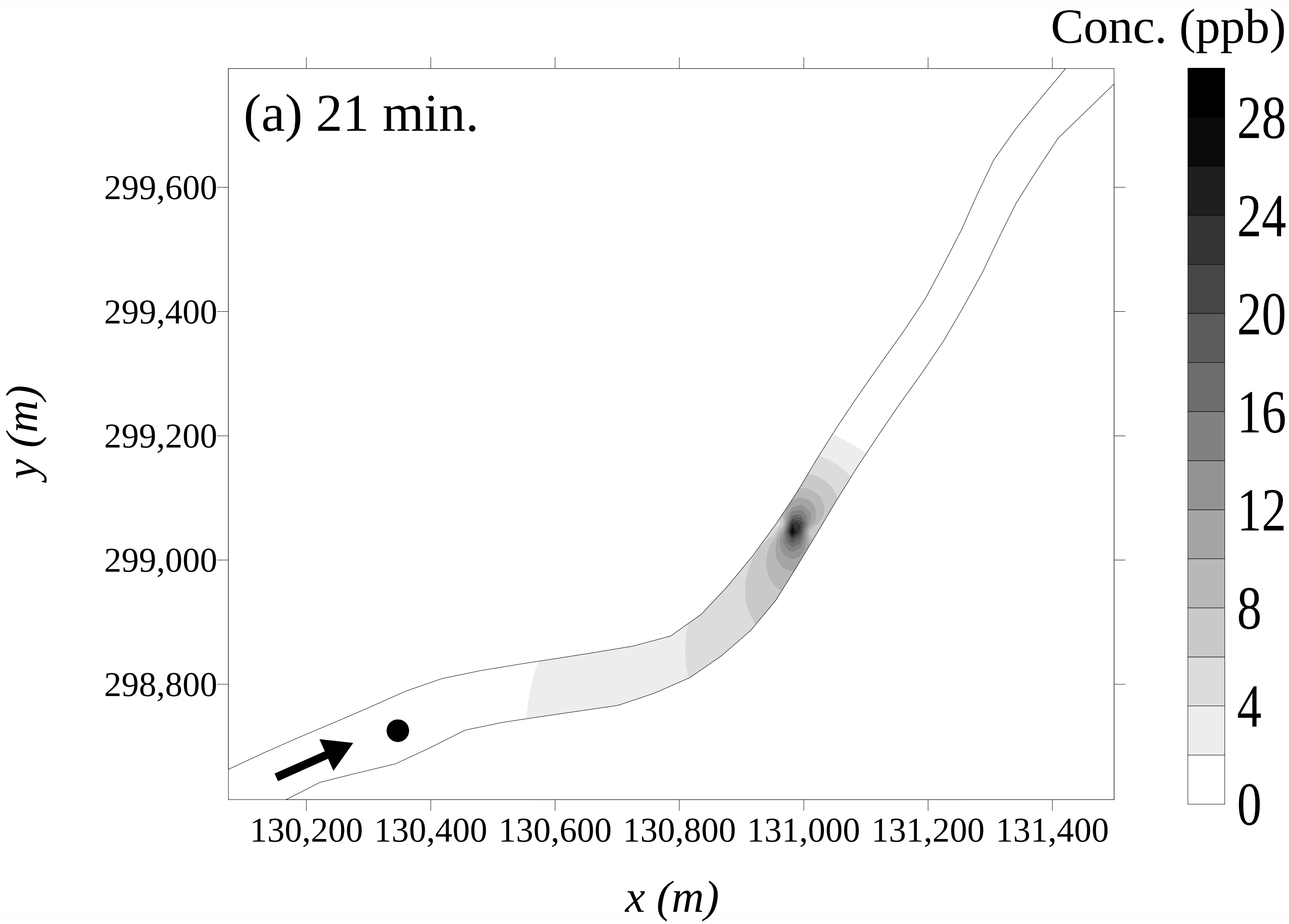
7.3.3.3 Concentration field

1) Compare the behavior of the tracer cloud

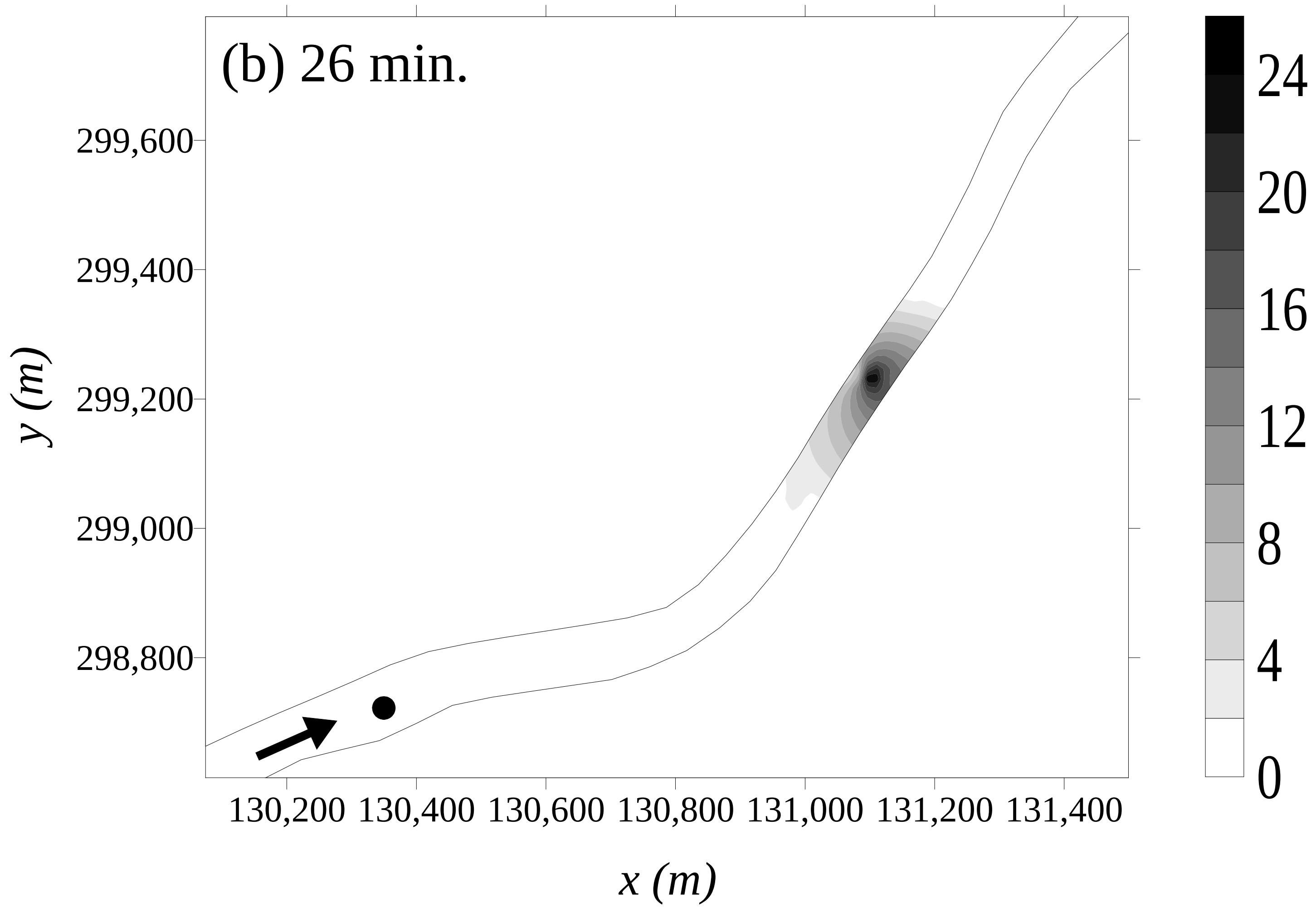
~ plot the contours of the tracer concentration at certain time intervals

~ analysis the spatial distribution of tracer

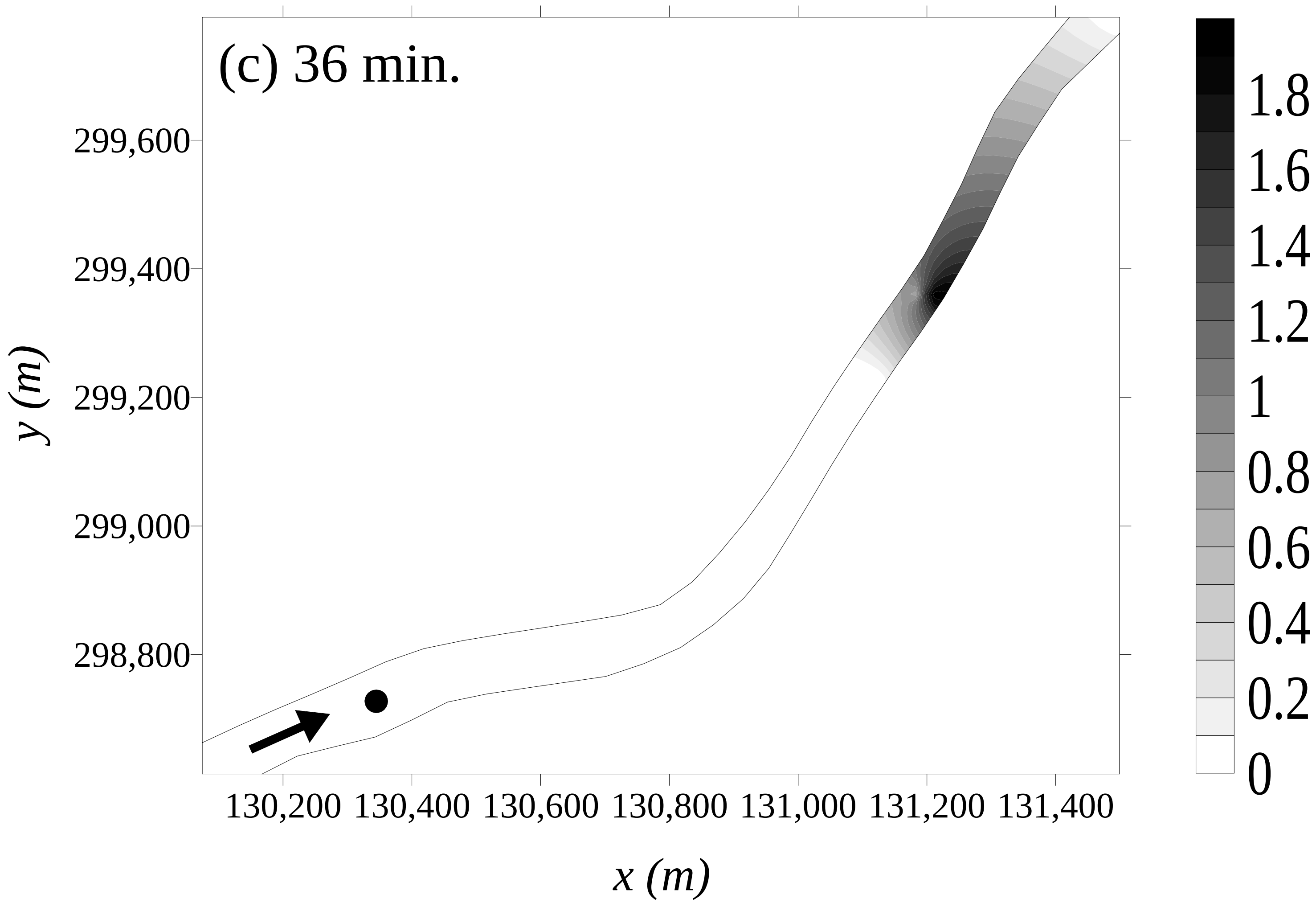
7.3 Data Analysis



7.3 Data Analysis



7.3 Data Analysis

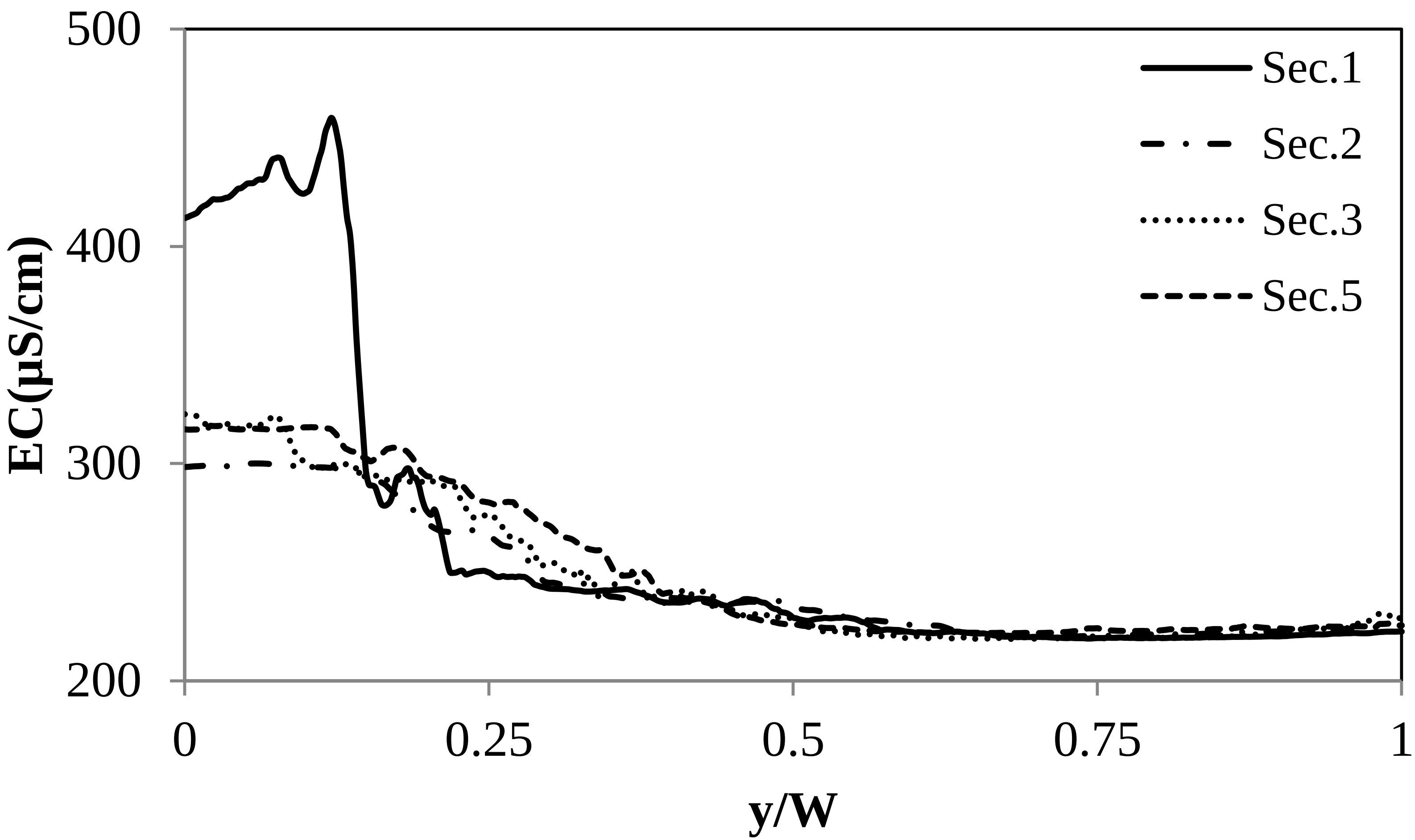


7.3 Data Analysis

7.3.3.4 C-y profile

- At steady state, C-y profile is used to analyze transverse mixing.
- As cloud moving further downstream, the variance of transverse concentration distribution gets larger.

7.3 Data Analysis



7.3 Data Analysis

7.3.4 Uncertainty in Field Measurements

The issue of accuracy in data sampling and measurements is very important for the validity of the model output.

7.3.4.1 Methods of Measurements

The physical property of a compound or an element can be applied as the basis for an analytical measurement.

7.3 Data Analysis

i) Definite methods

These methods are based on the laws that govern the physical and chemical parameters.

~ include volumetric analysis, gravimetry and potentiometry measurements, activation

analysis, isotope dilution mass spectrometry, and voltammetry and polarography

7.3 Data Analysis

ii) Relative methods

These methods consist of comparing the sample with a set of calibration samples of known content.

The sample value is determined by interpolation of the measured quantity with respect to the response curves of standard samples.

iii) Comparative methods

~ use a detection system sensitive to the content of the molecules or elements to be

determined

~ X-ray fluorescence spectrometry

7.3 Data Analysis

iv) On-line monitoring methods

The on-line monitoring consists of the sampling system and sensors. The collected sample from a direct intake of surface water is transferred to the measuring systems.

The sensors:

~ is able to respond rapidly and continuously using the active micro-zone where biological and chemical reactions take place

~ optical sensors / biosensors / chemical sensors are connected to the micro-zone

~ need frequent calibration, growth of biofilm

7.3 Data Analysis

Traceability:

ISOC (1994) defined traceability as “the ability to verify the history, location, or application of an item by means of recorded identification.”

The traceability of measurement depends on proper functioning of measuring equipment, which can be assured by calibration, using reference standards.

The measurement accuracy also depends on the type of sample and the analytical procedure.

7.3 Data Analysis

7.3.4.2 Uncertainty

(1) **Uncertainty of Sampling**

A surface water sample belongs to a population, a statistical term that includes all the possible available measurements of a variable.

The sampling process has to collect data as accurate as possible.

In order to be accurate, the data must be both unbiased and precise.
Random sampling eliminates the sampling bias.

7.3 Data Analysis

New bias can be generated due to errors of the analytical measurement.

In case that a sample is representative of the population, this sample may be used for extracting general conclusions concerning the entire population.

→ Statistical inference

Factors responsible of uncertainty during sampling analysis includes

i) inappropriate methodology of measurement

ii) improper calibration of the instrument

iii) scarce representativity of the collected sample

7.3 Data Analysis

(2) **Uncertainty of Measurement**

Uncertainty of measurement is a non-negative parameter characterizing the dispersion of the values that could be attributed to a variable, based on the used information.

i) Definitional uncertainty

~ a component of measurement uncertainty which result from a finite amount of detail in the definition of the variable

ii) Standard uncertainty

~ a result of a measurement is expressed as a standard deviation

7.3 Data Analysis

[Cf]

• Measurement error

~ the difference between the expected and determined values

• Uncertainty

~ a range into which the expected value may fall within a certain probability

[Cf]

• Bias of measurement

~ gives an estimate of how for the result is from the true value

• Precision

~ gives information on the dispersion of the results

7.3 Data Analysis

- Estimation of total uncertainty

$$u_{total} = (u_{sampling}{}^2 + u_{measure}{}^2 + u_{population}{}^2)^{1/2}$$

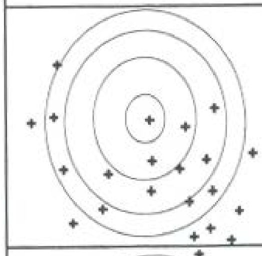
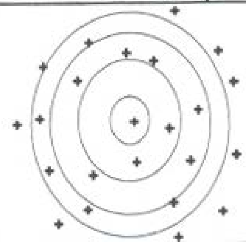
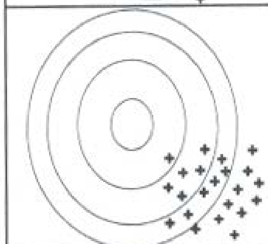
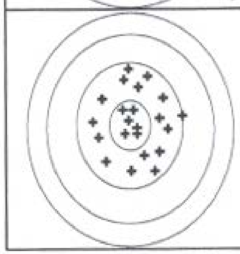
where

$u_{sampling}$ = is determined by repeated collection of at least seven identical samples

$u_{population}$ = is determined by repeated analyses of homogeneous samples

$u_{measure}$ = is related to the sample population

7.3 Data Analysis

	Bias	Precision	Accuracy
	Biased	Imprecise	**Inaccurate**
	Unbiased	Imprecise	**Inaccurate**
	Biased	Precise	**Inaccurate**
	Unbiased	Precise	**Accurate**

7.4 Calculation of Dispersion Coefficients

Observation: calculating the observed values from field concentration data

Prediction: estimating the dispersion coefficient using theoretical or empirical equations

7.4.1 Longitudinal Dispersion Coefficient of 1D ADE

7.4.1.1 Approximate Model for 1D Advection-Dispersion

Approximate model for 1D advection-dispersion in the far-field

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = K\frac{\partial^2 C}{\partial x^2} \qquad (7.17)$$

7.4 Calculation of Dispersion Coefficients

~ neglects the advective zone

~ assume that the mean velocity and longitudinal dispersion coefficient are constant

Taylor solution of Eq. (7.17) for a slug injection is

$$C(x,t) = \frac{M}{A\sqrt{4\pi Kt}} \exp\left[-\frac{(x-Ut)^2}{4Kt}\right] \qquad (7.18)$$

• Concentration versus distance profile at t_n

~ symmetrical about the peak concentration

7.4 Calculation of Dispersion Coefficients

$$C_{\max} = \frac{M}{A\sqrt{4\pi K t_n}} = \frac{M}{A\sqrt{4\pi K \frac{x_{\max}}{U}}} \quad (7.19)$$

$$x_{\max} = U t_n \quad (7.20)$$

$$\overline{x}(t_n) = x_{\max} = U t_n \quad (7.21)$$

- Concentration versus time profile at x_n

~ not symmetrical

The peak concentration occurs at

$$t_{\max} = \sqrt{\frac{K^2}{U^4} + \frac{x_n^2}{U^2}} - \frac{K}{U} \quad (7.22)$$

7.4 Calculation of Dispersion Coefficients

The centroid of the concentration-time curve is

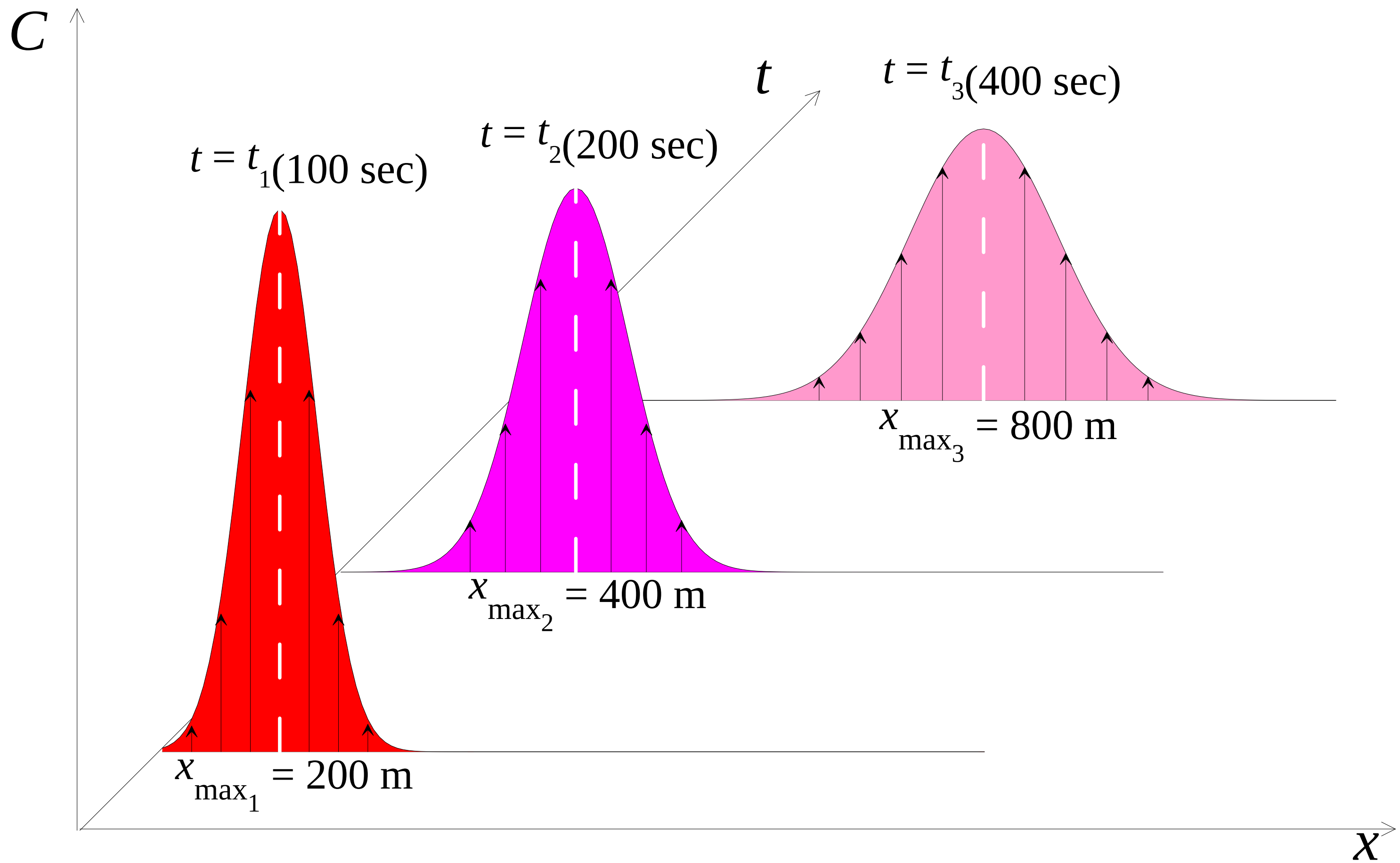
$$\bar{t} = \frac{x_n}{U} + 2\frac{K}{U^2} \tag{7.23}$$

$$C(x,t) = \frac{M}{A\sqrt{4\pi Kt}} \cdot \exp\left[-\frac{(x-Ut)^2}{4Kt}\right] \tag{7.24}$$

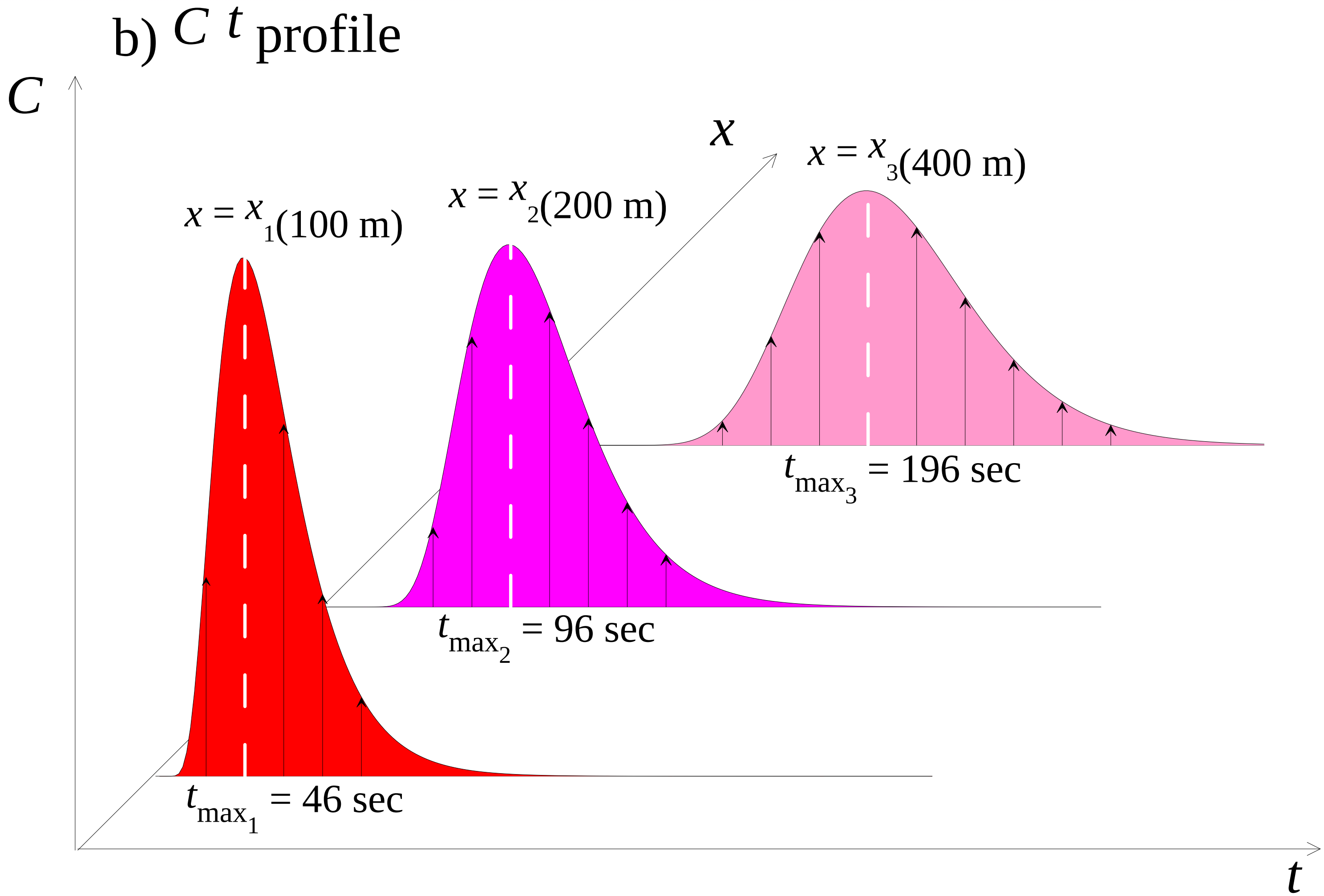
$$(M = 5\ kg, A = 1\ m^2, U = 2\ m/s, K = 20\ m^2/s)$$

7.4 Calculation of Dispersion Coefficients

a) $C\ x$ profile



7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients

The mean travel time is

$$t_t = \frac{x_n}{U} \qquad (7.25)$$

$$t_{\max} < t_t < \overline{t} \qquad (7.26)$$

7.4.1.2 The Frozen Cloud Approximation

In many applications, advection dominates dispersion

$$\frac{x}{U} >> \frac{K}{U^2} \qquad (7.27)$$

$$xU >> K \qquad (7.28)$$

7.4 Calculation of Dispersion Coefficients

Then, the maximum tracer concentration occurs at

$$t_{\max} \approx \bar{t} \approx \frac{x_n}{U} \tag{7.29}$$

The frozen cloud approximation exploits Eq. (7.29) to estimate C-x curves at a fixed time from C-t curves at a fixed location and vice versa.

Frozen cloud concept assumes that <u>no longitudinal dispersion occurs during the time taken for the tracer to pass a sample site</u>.

This concept is not strictly valid in most natural channels because although advection is more rapid than dispersion, the latter does have a discernible effect on tracer concentrations.

7.4 Calculation of Dispersion Coefficients

Under the frozen cloud assumption

$$x_1 = Ut_1 \tag{7.30}$$

$$C(x_1, t) = C\left[x_1 + U(t_1 - t), t_1\right] \tag{7.31}$$

$$C(x_1, t_1) = C\left(x_1, t_1 + \frac{x_1 - x}{U}\right) \tag{7.32}$$

Eq. (7.31) is used to estimate C-t data from C-x data, and Eq. (7.32) to go the other way.

Note that inversion of the axes in Fig. 7.30 concentration measured at negatives values of $t - t_1$ (i.e. in the leading edge) have positive values of $x - x_1$.

7.4 Calculation of Dispersion Coefficients

[Re]

In Eq. (14), $t = t_1 + (t - t_1)$ (A)

Multiplying Eq. (A) by U results in

$$x = Ut = Ut_1 + U(t - t_1)$$

$$= x_1 + U(t - t_1)$$

7.4 Calculation of Dispersion Coefficients

7.4.1.3 Change of Moment Method

For C-x profiles, the change of moment method is written as

$$K = \frac{1}{2}\frac{d\sigma_x^2}{dt} \tag{7.33}$$

$$K = \frac{1}{2}\frac{\sigma_x^2(t_2) - \sigma_x^2(t_1)}{t_2 - t_1} \tag{7.34}$$

where $\sigma_x{}^2$ = longitudinal variance

It is rare for experiments to yield concentration versus distance profiles. It is more common to obtain the concentration data as a function of time at a fixed sampling site downstream.

7.4 Calculation of Dispersion Coefficients

For C-t profiles, Fischer (1966) revised Eq. (7.34) as

$$K = \frac{1}{2}U^2 \frac{\sigma_t^2(x_2) - \sigma_t^2(x_1)}{\overline{t}_2 - \overline{t}_1} \tag{7.35}$$

where

$$\sigma_t^2(x_i) = \frac{\int_{-\infty}^{\infty}(t - \overline{t}_i)C(x_i, t)dt}{\int_{-\infty}^{\infty}C(x_i, t)dt} \tag{7.36}$$

$$\overline{t}_i = \frac{\int_{-\infty}^{\infty}tC(x_i, t)dt}{\int_{-\infty}^{\infty}C(x_i, t)dt} \tag{7.37}$$

7.4 Calculation of Dispersion Coefficients

- Practical difficulties

- Tracer profiles exhibit long tails and sampling must continue for a long time after the bulk if the tracer has passed.
- Small measurement errors in the low concentrations of the leading and trailing edge of the tracer profile are inevitable and these greatly influence the variance and hence the dispersion coefficient.
- Thus, it is difficult to compute a meaningful value of variance when concentration distributions are skewed because of long tails on the observed distributions.

7.4 Calculation of Dispersion Coefficients

7.4.1.4 Routing Method

(1) **Routing a spatial concentration profile**

Use analytical solution, Eq. (7.18) at a specified time $t = t_1$ and apply superposition principle

i) Divide the channel into a number of short segments each Δx long and centered on x_i

7.4 Calculation of Dispersion Coefficients

ii) Then, the mass of tracer in segment *i* is $C(x_i, t_1)\Delta x$ and is assumed to behave like an <u>instantaneous source</u> at $x = x_i$ released $t = t_1$

iii) This source results in concentrations at a later time $t = t_2$ which are predicted using Eq. (7.18), and the separate concentration profiles are summed.

$$C(x, t_2) = \int_{-\infty}^{\infty} \frac{C(\xi, t_1)}{\sqrt{4\pi K(t_2 - t_1)}} \exp\left\{ -\frac{\left[x - \xi - U(t_2 - t_1)\right]^2}{4K(t_2 - t_1)} \right\} d\xi \qquad (7.38)$$

7.4 Calculation of Dispersion Coefficients

where $C(x, t_1)$ = observed concentration as a function of distance at time t_1; $C(x, t_2)$ = predicted concentration at time t_2; ξ = dummy distance variable of integration.

• Restrictions:

Since the analytical solution, Eq. (7.18), is an asymptotic solution which is accurate only a long distance below an instantaneous point source, should be sufficiently large after the injection, t_1 and $(t_2 - t_1)$ should also be large.

7.4 Calculation of Dispersion Coefficients

(2) **Routing a temporal concentration profile**

1) Frozen cloud method

i) Use the frozen cloud assumption to transform the $C-t$ profile at Site 1 (Profile 1) into a $C-x$ profile (Profile 1A) at the mean time of passage using Eq. (7.32).

ii) Profile 1A is then routed downstream using Eq. (7.38) to give a $C-x$ profile (Profile 2A) at the mean time of passage past Site 2.

7.4 Calculation of Dispersion Coefficients

iii) Profile 2A is then transformed into a $C-t$ profile (Profile 2) at Site 2 using Eq. (7.31).

iv) K can be estimated by comparing the $C(x_2,t)$ with the observed $C-t$ profile at Site 2. The best fit value is regarded as the observed dispersion coefficient

$$C(x_2,t)=\int_{-\infty}^{\infty}\frac{C(x_1,\tau)U}{\sqrt{4\pi K(\overline{t_2}-\overline{t_1})}}\exp\left\{-\frac{U^2(\overline{t_2}-\overline{t_1}-t+\tau)^2}{4K(\overline{t_2}-\overline{t_1})}\right\}d\tau \qquad (7.39)$$

7.4 Calculation of Dispersion Coefficients

where $C(x_1, t)$ = observed concentration as a function of time at Site 1; $C(x_2, t)$ = predicted concentration as a function of time at Site 2; $\bar{t}_1$ and $\bar{t}_2$ = mean time of passages at Sites 1 and 2, respectively.

$$\bar{t}_2 = \bar{t}_1 + \frac{x_2 - x_n}{U} \qquad (7.40)$$

• Restrictions:

① Site 1 must be outside the advective zone (x' > 0.4).

② The entire $C - t$ profile must be measured at Site 1.

③ Tracer loss must be negligible so it is necessary to correct for tracer losses caused by adsorption, decay or dilution by tributaries.

7.4 Calculation of Dispersion Coefficients

→ Calculate the mass flux

→ If necessary scale the downstream concentrations so that the mass fluxes are identical.

2) Hayami solution

Barnett (1983) suggested an alternative method for routing a $C-t$ profile using Hayami solution to Eq. (7.17).

$$C(x,t) = \frac{Mx}{AUt\sqrt{4\pi Kt}} \exp\left[-\frac{(x-Ut)^2}{4Kt}\right] \qquad (7.41)$$

7.4 Calculation of Dispersion Coefficients

This solution is simply Eq. (7.18) multiplied by $\frac{x}{Ut}$.

It satisfies the boundary conditions that at $x = 0,\ C = 0$ for all values of t except $t = 0$ when a tracer slug of mass M is introduced over an infinitesimally small time interval.

It can be used to route downstream an observed $C - t$ profile without needing to invoke the frozen cloud approximation.

7.4 Calculation of Dispersion Coefficients

- Comparison between Taylor solution and Hayami solution

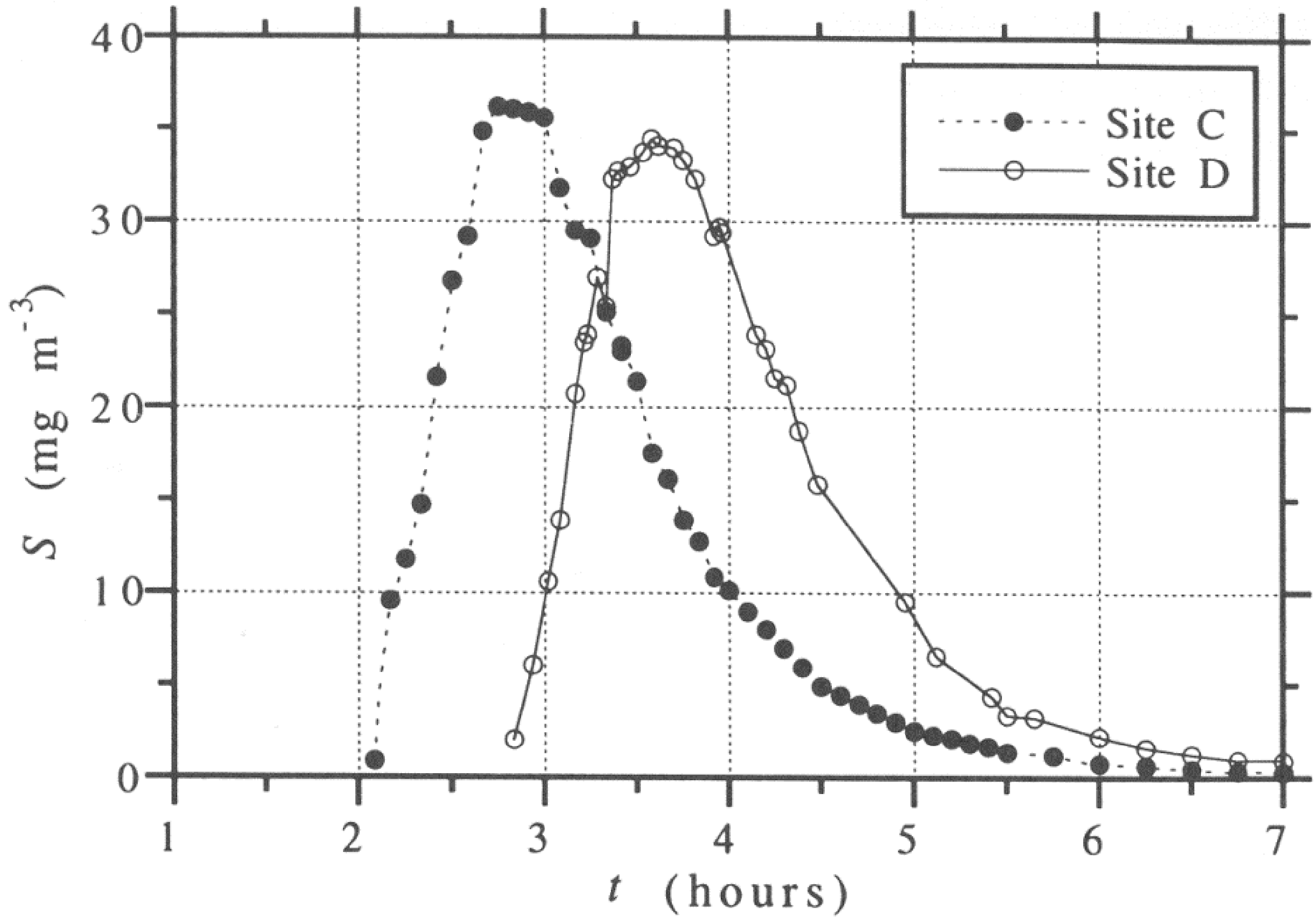
For large x and t, two solutions give similar results.

For small t, the Taylor solution predicts positive concentrations upstream from the source, where the Hayami solution predicts small negative concentrations upstream from the source.

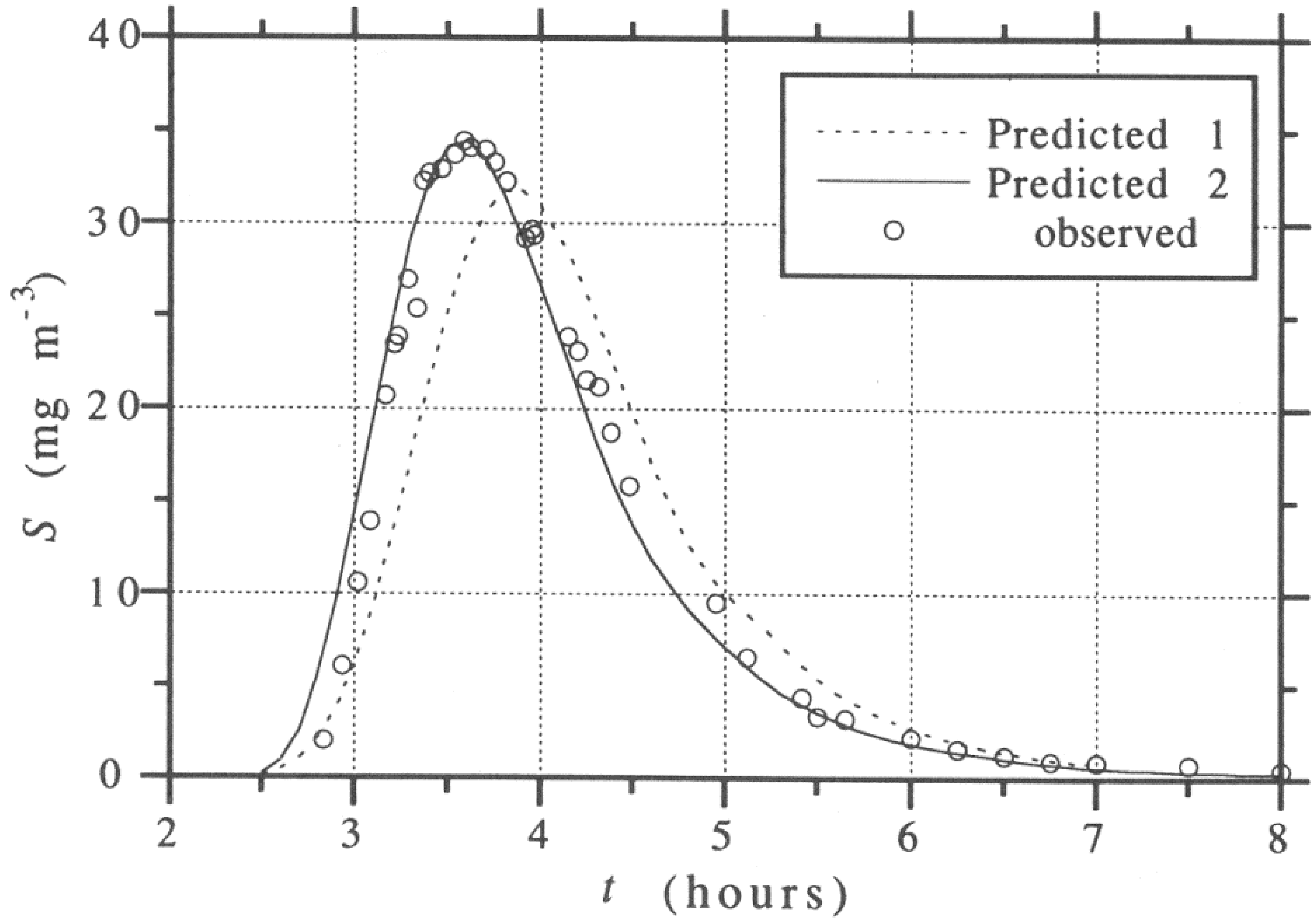
Apply the superposition principle to Eq. (7.41)

$$C(x_2, t) = \int_{-\infty}^{\infty} \frac{C(x_1, \tau)(x_2 - x_1)}{(t-\tau)\sqrt{4\pi K((t-\tau))}} \exp\left[-\frac{[x_2 - x_1 - U(t-\tau)]^2}{4K(t-\tau)}\right] d\tau \qquad (7.42)$$

7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients

7.4.1.5 Graphical Methods

(1) **Chatwin's method**

Rearrange Eq. (7.18) based on Chatwin's transformation (1980)

$$\sqrt{t \log_e \left(\frac{a}{C\sqrt{t}}\right)} = \frac{x}{2\sqrt{K}} - \frac{Vt}{2\sqrt{K}} \tag{7.43}$$

where

$$a = \frac{M}{A\sqrt{4\pi K}} \tag{7.44}$$

7.4 Calculation of Dispersion Coefficients

Plot $\sqrt{t \log_e\left(\frac{a}{C\sqrt{t}}\right)}$ versus t (7.45)

Then, slope is

$$-\frac{V}{2\sqrt{K}} \tag{7.46}$$

And intercept is

$$\frac{x}{2\sqrt{K}} \tag{7.47}$$

This method is only valid in the equilibrium zone (at asymptotically long times).

7.4 Calculation of Dispersion Coefficients

(2) **Thackston's method**

Thackston et al (1967) developed an estimation technique similar to Chatwin's by transforming Eq. (7.18) to give

$$\log_e (C\sqrt{t}) = \log_e a - \frac{(x-Ut)^2}{4Kt} \tag{7.48}$$

Plot $\log_e (C\sqrt{t})$ versus $\dfrac{(x-Ut)^2}{4Kt}$ (7.49)

7.4 Calculation of Dispersion Coefficients

Then, slope is

$$-\frac{1}{4K} \tag{7.50}$$

(3) **Cmax method**

For a conservative tracer, the peak concentration $C_{\max}$ decreases inversely as the square root of distance below the source.

$$C_{\max} = \frac{M}{A\sqrt{4\pi K t_n}} = \frac{M}{A\sqrt{\dfrac{4\pi K x_{\max}}{U}}} \tag{7.51}$$

7.4 Calculation of Dispersion Coefficients

where t_n = time where $c-x$ profile is measured; $x_{\max}$ = location of the maximum concentration.

Plot $C_{\max}$ versus $\frac{1}{\sqrt{x}}$ (7.52)

Then, the slope is

$$\frac{M}{A\sqrt{4\pi K/U}} \tag{7.53}$$

An advantage of this method is that it requires only the peak concentration and not the entire tracer profile.

7.4 Calculation of Dispersion Coefficients

7.4.1.6 Use of Velocity Measurements

In large rivers tracer tests may not be viable because the advective zone is very long and prohibitively large amounts of dye are required.

→ More cost effective to measure the flow distribution at several sites along the channel

→ Calculate LDC from the velocity and depth distribution within the channel.

Flow gauging is made at water level recording stations.

→ Such data can be used to estimate K.

7.4 Calculation of Dispersion Coefficients

Fischer (1967) extended the Taylor and Elder method to natural channels where transverse velocity shear dominates longitudinal dispersion.

$$K = -\frac{1}{A}\int_0^W q'(y)dy\int_0^y \frac{1}{D_T(y)h(y)}dy\int_0^y q'(y)dy \qquad (7.54)$$

where

$$q'(y) = \int_0^{h(y)} u'(y,z)dz \qquad (7.55)$$

$$u'(y,z) = u(y,z) - \overline{u}(y) \qquad (7.56)$$

where $\overline{u}(y)$ = local depth - averaged velocity

7.4 Calculation of Dispersion Coefficients

- Disadvantage:

D_T varies significantly between channels and along a given channel.

The uncertainty in estimating D_T is of the order of $\pm 50\%$ and this leads to a comparable uncertainty in estimates of K.

The reach-averaged dispersion coefficient is

$$\overline{K} = \frac{1}{L}\sum_{i=1}^{n} K_i \Delta x_i \tag{7.57}$$

7.4 Calculation of Dispersion Coefficients

Homework Assignment

Due: Two weeks from today

Concentration-time data listed in Table are obtained from dispersion study by Godfrey and Fredrick (1970).

1) Plot concentration vs. time

2) Calculate time to centroid, variance, skew coefficient.

3) Calculate dispersion coefficient using the change of moment method and routing procedure.

7.4 Calculation of Dispersion Coefficients

4) Compare and discuss the results.

Test reach of the stream is almost straight and necessary data for the calculation of dispersion coefficient are

$$U = 0.52 \text{ m/s}; \qquad W = 18.3 \text{ m};$$

$$H = 0.84 \text{ m}; \qquad U^{*} = 0.10 \text{ m/s}$$

7.4 Calculation of Dispersion Coefficients

Section 1 x = 192 m		Section 2 x =1,009 m		Section 3 x = 1,728 m		Section 4 x = 2,399 m		Section 5 x = 3,353 m		Section 6 x = 4,130 m	
T (hr)	C/C_0	T (hr)	C/C_0	T (hr)	C/C_0	T (hr)	C/C_0	T (hr)	C/C_0	T (hr)	C/C_0
1111.5	0.00	1125.0	0.00	1138.0	0.00	1149.0	0.00	1210.0	0.00	1226.0	0.00
1112.5	2.00	1126.0	0.15	1139.0	0.12	1152.0	0.26	1215.0	0.05	1231.0	0.07
1112.5	16.50	1127.0	1.13	1140.0	0.30	1155.0	0.67	1220.0	0.25	1236.0	0.22
1113.0	13.45	1128.0	2.30	1143.0	1.21	1158.0	0.95	1225.0	0.52	1241.0	0.40
1113.5	7.26	1128.5	2.74	1145.0	1.61	1200.0	1.09	1228.0	0.64	1245.0	0.50
1114.0	5.29	1129.0	2.91	1147.0	1.64	1202.0	1.13	1231.0	0.70	1249.0	0.58
1115.0	3.37	1129.5	2.91	1149.0	1.56	1204.0	1.10	1234.0	0.72	1251.0	0.59
1116.0	2.29	1130.0	2.80	1153.0	1.26	1206.0	1.04	1237.0	0.71	1253.0	0.59

7.4 Calculation of Dispersion Coefficients

1117.0	1.54	1131.0	2.59	1158.0	0.86	1208.0	0.95	1240.0	0.65	1257.0	0.54
1118.0	1.03	1133.0	2.18	1203.0	0.53	1213.0	0.72	1244.0	0.55	1304.0	0.44
1120.0	0.40	1137.0	1.34	1208.0	0.30	1218.0	0.50	1248.0	0.45	1313.0	0.27
1124.0	0.10	1143.0	0.60	1213.0	0.17	1223.0	0.31	1258.0	0.24	1323.0	0.14
1128.0	0.04	1149.0	0.23	1218.0	0.10	1228.0	0.21	1308.0	0.12	1333.0	0.06
1133.0	0.02	1158.0	0.08	1228.0	0.04	1238.0	0.08	1318.0	0.06	1343.0	0.03
1138.0	0.00	1208.0	0.03	1238.0	0.01	1248.0	0.02	1333.0	0.03	1403.0	0.02
-	-	1218.0	0.00	1248.0	0.00	1300.0	0.00	1353.0	0.00	1423.0	0.00

7.4 Calculation of Dispersion Coefficients

7.4.2 Longitudinal and Transverse Dispersion Coefficients for 2D ADE

7.4.2.1 Introduction

For mixing predictions in the intermediate field, the depth-averaged 2D advection-dispersion equation can be used.

$$\frac{\partial c}{\partial t}+\frac{\partial}{\partial x}(huc)+\frac{\partial}{\partial y}(hvc)=\frac{\partial}{\partial x}(hD_L\frac{\partial c}{\partial x})+\frac{\partial}{\partial y}(hD_T\frac{\partial c}{\partial y}) \qquad (7.58)$$

where over-bars for depth-averaged concentrations are eliminated.

7.4 Calculation of Dispersion Coefficients

Field study for 2D model is conducted to obtain bathymetric and hydraulic data at each section and depth averaged concentration data $(c(x, y, t))$.

Then, longitudinal and transverse dispersion coefficients are calculated from the field data.

Two types of tracer injection are used:

i) Continuous injection: steady source

ii) Instantaneous injection: non-steady source

7.4 Calculation of Dispersion Coefficients

A single instantaneous tracer injection gives information about only one of the ensemble of possible outcomes.

→ In practice, results from a single instantaneous injection are usually assumed to describe the typical plume behavior.

→ A steady source is preferred when studying transverse dispersion because the plume can be traversed several times and the results time averaged.

7.4 Calculation of Dispersion Coefficients

For steady source, only transverse dispersion coefficient can be obtained by

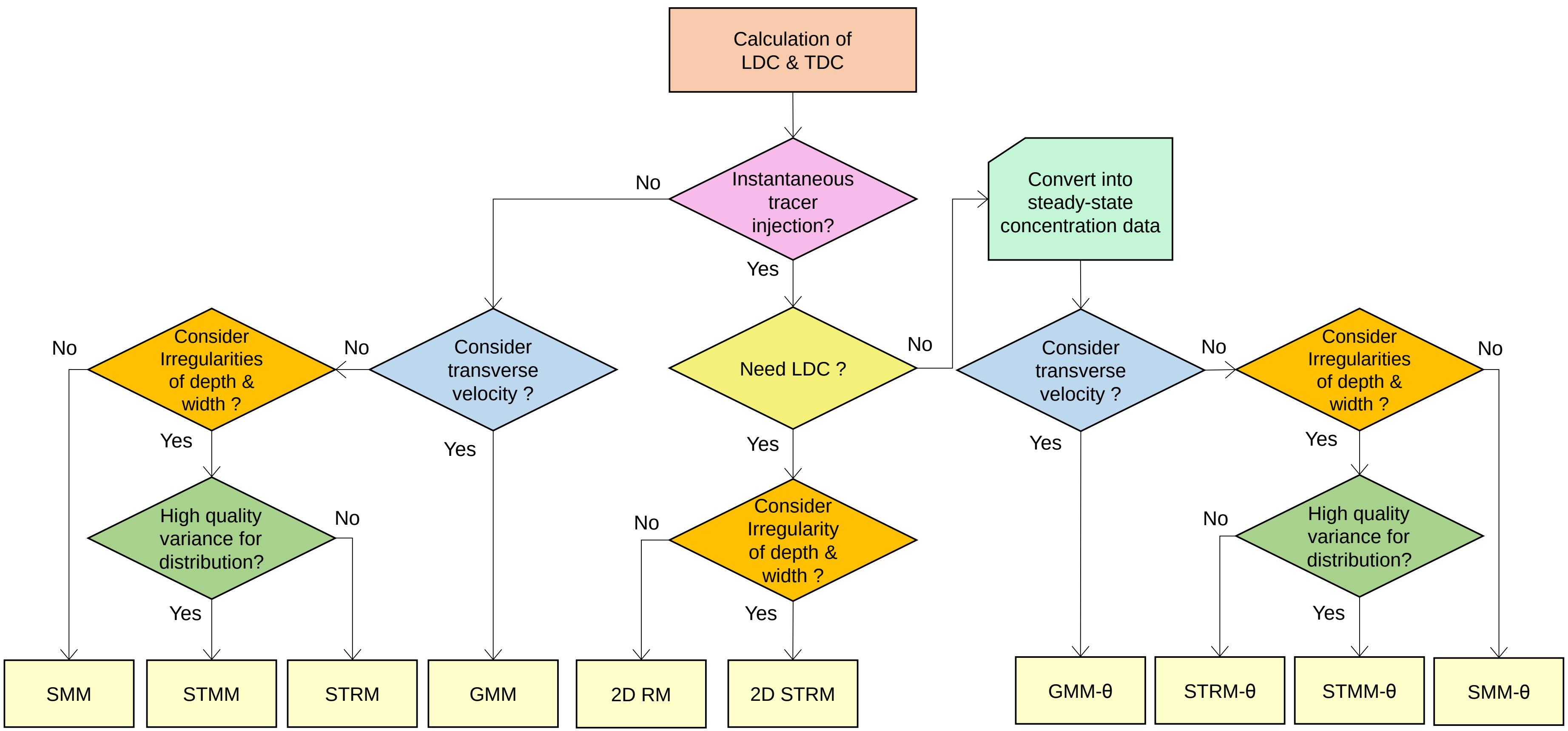
i) Moment method

ii) Routing method

iii) Graphical method

For non-steady source, both longitudinal and transverse dispersion coefficients can be obtained by the 2D routing methods whereas only the transverse dispersion coefficient can be obtained by the moment methods.

7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients

7.4.2.2 Moment-based Methods for Steady Source

For steady injection of tracer, only transverse dispersion coefficient (TDC) is calculated by the moment method.

(1) Simple Moment Method (SMM)

TDC is determined by the rate of change of spatial variance of the tracer profile.

$$D_T = \frac{1}{2}\frac{d\sigma_y{}^2}{dt} \tag{7.59}$$

7.4 Calculation of Dispersion Coefficients

where t = time of travel; and

$$\sigma_y^{\ 2}(x) = \frac{\int_0^w (y-\overline{y})^2 c(x,y)dy}{\int_0^w c(x,y)dy} \tag{7.60}$$

$$\overline{y}(x) = \frac{\int_0^w yc(x,y)dy}{\int_0^w c(x,y)dy} \tag{7.61}$$

- Finite difference form is

$$D_T = \frac{1}{2}U\frac{\sigma_y^{\ 2}(x_2) - \sigma_y^{\ 2}(x_1)}{x_2 - x_1} \tag{7.62}$$

7.4 Calculation of Dispersion Coefficients

- Graphically, the slope of plot of $\sigma_y{}^2(x)$ versus x is

$$\frac{2D_T}{U} \tag{7.63}$$

- Restrictions:

Tracer dispersion must obey Fick’s law with a constant dispersion coefficient.

Tracer must mot impinge on either bank.

The channel must be uniform so that the plume does not expand and contract.

The upstream transect must be placed far enough downstream from the source for tracer to be well mixed vertically.

7.4 Calculation of Dispersion Coefficients

- Experimental errors (uncertainties):

The sampling error is ±10% plus random error by sensor limit.

The entire transverse tracer profile must be measured because the variance estimates are greatly affected by experimental errors in concentration measurements at the edges of the tracer profile where s is small but $(y-\bar{y})^2$ is large.

→ It is advisable to plot concentration profile and either drop or smooth out any suspicious concentration data at the edges of the plume.

7.4 Calculation of Dispersion Coefficients

(2) **Generalized Moment Method (GMM)**

When the channel bathymetry varies longitudinally then the plume undergoes cyclical expansions and contractions, SMM cannot give accurate results.

Hollley et al (1972) developed the GMM for a steady tracer input which takes into account transverse and longitudinal variations of depth and velocity, non-zero transverse velocities, and allows for tracer impinging on the bank.

7.4 Calculation of Dispersion Coefficients

The depth-averaged 2D advection-dispersion equation for channels with irregular depth is

$$\frac{\partial c}{\partial t}+\frac{\partial}{\partial x}(huc)+\frac{\partial}{\partial y}(hvc)=\frac{\partial}{\partial x}(hD_L\frac{\partial c}{\partial x})+\frac{\partial}{\partial y}(hD_T\frac{\partial c}{\partial y}) \qquad (7.64)$$

For steady-state, Eq. (6) becomes

$$\frac{\partial}{\partial x}(huc)+\frac{\partial}{\partial y}(hvc)=\frac{\partial}{\partial y}(hD_T\frac{\partial c}{\partial y}) \qquad (7.65)$$

where the longitudinal dispersion term is neglected

7.4 Calculation of Dispersion Coefficients

Multiply each term in Eq. (7) by $(y-\bar{y})^2$

Here $\bar{y}$ is given as

$$\bar{y} = \frac{\int_0^w yhucdy}{\int_0^w hucdy} \tag{7.66}$$

Then, integrate each term in Eq. (7.65) across the channel to have

$$\frac{d}{dx}\int_0^w (y-\bar{y})^2 hucdy - 2\int_0^w (y-\bar{y})hvcdy + 2\int_0^w (y-\bar{y})D_T \frac{\partial c}{\partial y}dy = 0 \tag{7.67}$$

7.4 Calculation of Dispersion Coefficients

Normalize each term in Eq. (7.67) by dividing the total longitudinal tracer flux

$$\int_0^w hucdy \tag{7.68}$$

Then, in case of constant h, u, and D_T , Eq. (7.67) and Eq. (7.59) are identical.

$$\frac{d\sigma_{y1}^{\ 2}(x)}{dx} - g(x) + 2D_y f(x) = 0 \tag{7.69}$$

7.4 Calculation of Dispersion Coefficients

where

$$\sigma_{y1}{}^{2}(x) = \frac{\int_0^w (y-\overline{y})^2 hucdy}{\int_0^w hucdy} \qquad (7.70)$$

$$g(x) = \frac{2\int_0^w (y-\overline{y})hvcdy}{\int_0^w hucdy} \qquad (7.71)$$

$$f(x) = \frac{\int_0^w (y-\overline{y})h\varphi \dfrac{\partial c}{\partial y} dy}{\int_0^w hucdy} \qquad (7.72)$$

$$D_T = D_y \varphi(y) \qquad (7.73)$$

7.4 Calculation of Dispersion Coefficients

where

$$\int_0^w \varphi(y)dy = 1 \tag{7.74}$$

• Practical difficulties

① The transverse velocity is rarely measured in the field.

② In an irregular channel, the variance of the tracer flux $\sigma_3{}^2(x)$ does not vary linearly with distance.

In a non-uniform channel both $g(x)$ and $f(x)$ also vary non-linearly.

→ Holley suggests that this problem can be overcome by integrating Eq. (7.69) along the channel to give

7.4 Calculation of Dispersion Coefficients

$$\sigma_{y1}{}^{2}(x_2) - \sigma_{y1}{}^{2}(x_1) - \int_{x_1}^{x_2} g(x)dx + 2D_y \int_{x_1}^{x_2} f(x)dx = 0 \qquad (7.75)$$

Eqs. (7.69) and (7.75) require accurate estimation of the variance of the tracer flux and measurement errors in concentrations at the edges of the tracer cloud affect the variance.

→ GMM does not produce robust estimates of the dispersion coefficient for field measurements in irregular natural channels (Rutherford, 1994).

→ The stream-tube method is much to be preferred in natural streams.

7.4 Calculation of Dispersion Coefficients

(3) **Stream-tube Moment Method (STMM)**

Yotsukura and Sayre (1976) developed a more popular tool than the GMM in a non-uniform channel.

This transformation is of merit because a fixed value of q is attached to a fixed streamline so that the coordinate system shifts back and forth within the cross section along with the flow.

Assuming that the factor of diffusivity is constant, the solution for a steady vertical line source is

7.4 Calculation of Dispersion Coefficients

$$c(x,q) = \frac{m}{\sqrt{4\pi Dx}} \exp\left[-\frac{(q-q_0)^2}{4Dx} \right] \qquad (7.76)$$

where m = tracer injection rate; q_0 = cumulative discharge at the source, and

$$q = \int_0^y hudy \qquad (7.77)$$

$$D = \psi H^2 U D_T \qquad (7.78)$$

where H = mean depth; U = mean velocity; and

$$\psi = \frac{1}{W} \int_0^W \left(\frac{h}{H}\right)^2 \frac{u}{U} dy \qquad (7.79)$$

7.4 Calculation of Dispersion Coefficients

where ψ = a dimensionless shape factor.

Rewrite Eq. (7.76) as

$$c(x,q) = c_{\max}(x)\exp\left[-\frac{(q-q_0)^2}{4Dx}\right] \qquad (7.80)$$

If tracer does not impinge on either bank,

Plot $\sqrt{\log_e \dfrac{c_{\max}(x)}{c(x,q)}}$ versus q

Then the slope is

$$\frac{1}{\sqrt{4Dx}} \qquad (7.81)$$

7.4 Calculation of Dispersion Coefficients

[Re] Stream-tube GMM

$$\frac{d\sigma_q^{\ 2}}{dx} = -2D_q f(x) \tag{7.82}$$

where

$$\sigma_q^{\ 2}(x) = \frac{\int_0^Q (q-\overline{q})^2 c(x,q)dq}{\int_0^Q c(x,q)dq} \tag{7.83}$$

$$\overline{q}(x) = \frac{\int_0^Q qc(x,q)dq}{\int_0^Q c(x,q)dq} \tag{7.84}$$

7.4 Calculation of Dispersion Coefficients

$$f(x) = \frac{\int_0^Q q\psi(q) \frac{\partial c(x,q)}{\partial q} dq}{\int_0^Q c(x,q) dq} \quad (7.85)$$

$$D(q) = D_q \psi(q) \quad (7.86)$$

$$D_q = \frac{1}{2} \frac{d}{dx} (\sigma_q)^2 \quad (7.87)$$

7.4 Calculation of Dispersion Coefficients

(4) **Graphical estimation method**

1) Nokes' method (1986)

This method assumes that the tracer concentration distributions are Gaussian.

Use a normalized cumulative concentration

$$c_c(y) = \frac{\int_o^y c(y)dy}{\int_o^w c(y)dy} \tag{7.88}$$

7.4 Calculation of Dispersion Coefficients

Plot $c_c(y)$ versus y on arithmetic probability paper.

Then, a straight line can be fitted to a plot of experimental data

→ Gaussian distributions plot as a straight line

A robust estimate of the variance of the tracer distribution is

$$\sigma_y^{\ 2} = \frac{1}{4}(y_{0.84} - y_{0.16})^2 \qquad (7.89)$$

where $y_{0.84}$ and $y_{0.16}$ = transverse locations at which $c_c(y)$ = 0.84 and 0.16 respectively.

7.4 Calculation of Dispersion Coefficients

2) Constant - coefficient model

$$c(x,y)=\frac{m}{h\sqrt{4\pi D_T xu}}\exp\left[-\frac{u(y-y_0)^2}{4D_T x}\right] \tag{7.90}$$

$$c_{\max}(x)=\frac{m}{h\sqrt{4\pi D_T xu}} \tag{7.91}$$

Plot $\sqrt{\log_e\left[\frac{c_{\max}(x)}{c(x,y)}\right]}$ versus y

Then, the slope of this plot is

$$\sqrt{\frac{u}{4D_T x}} \tag{7.92}$$

7.4 Calculation of Dispersion Coefficients

7.4.2.3 Moment-based Methods for Non-Steady Source

For non-steady injection of tracer, the transverse dispersion coefficient can be calculated by the moment method after converting into steady-state concentration data.

Beltaos (1975) presented a procedure for converting the governing equation of the transient condition to the steady-state condition.

Integrate Eq. (7.2) with respect to time assuming that u, v, h, D_L, and D_T are independent of time to yield

7.4 Calculation of Dispersion Coefficients

$$\frac{\partial}{\partial x}(hu\theta)+\frac{\partial}{\partial y}(hv\theta)=\frac{\partial}{\partial x}\left(hD_L\frac{\partial\theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(hD_T\frac{\partial\theta}{\partial y}\right) \tag{7.93}$$

where θ is the dosage of the tracer, which is defined as,

$$\theta(x,y)\equiv\int_0^\infty c(x,y,t)dt \tag{7.94}$$

Eq. (7.93) is the same differential equation as Eq. (7.2), with the dosage θ used instead of the concentration c in a continuous injection tracer test, without the time variation term.

7.4 Calculation of Dispersion Coefficients

(1) Generalized Moment Method for θ (GMM-θ)

Simplify Eq. (7.2) by dropping the longitudinal dispersion term under a steady concentration condition, and assuming that the depth and the transverse dispersion coefficient are constant in the transverse direction

$$\frac{\partial}{\partial x}(u\theta)+\frac{\partial}{\partial y}(v\theta)=D_T\frac{\partial^2\theta}{\partial y^2} \tag{7.95}$$

Multiplying each side of Eq. (7.95) by y^2 and integrating it with respect to y using the Leibnitz's rule and integral by parts to yield the generalized moment equation as

7.4 Calculation of Dispersion Coefficients

$$D_T = \frac{1}{2}\frac{\partial \sigma_{\theta 2}^2}{\partial x}\frac{1}{f} - \frac{g}{f} \tag{7.96}$$

where

$$\sigma_{\theta 2}^2 = \frac{\int_0^W u\theta y^2 dy}{\int_0^W u\theta dy} \tag{7.97a}$$

$$f = -\frac{\int_0^W \theta dy - [\theta y]_0^W}{\int_0^W u\theta dy} \tag{7.97b}$$

$$g = \frac{\int_0^W v\theta y dy}{\int_0^W u\theta dy} \tag{7.97c}$$

7.4 Calculation of Dispersion Coefficients

Plot $\sigma_{\theta 2}^{2}$ against x

Then, the slope of the straight line is

$$2f\left(D_T + \frac{g}{f}\right) \qquad (7.98)$$

(2) **Simple Moment Method for θ (SMM- θ)**

The simple moment method (Sayre and Chang, 1968) can be deduced from the generalized moment method.

Assuming that the longitudinal velocity is constant, and the tracer impinging on banks and transverse velocity are negligible, Eq. (7.97) becomes

7.4 Calculation of Dispersion Coefficients

$$f = \frac{1}{U}, \qquad g = 0 \tag{7.99}$$

where U is the reach-averaged velocity.

Incorporating Eq. (7.99) into Eq. (7.96) yields

$$D_T = \frac{U}{2} \frac{\partial \sigma_\theta^2}{\partial x} \tag{7.100}$$

where σ_θ^2 = the variance of the transverse distribution of the tracer dosage

$$= \frac{\int_0^W \theta y^2 dy}{\int_0^W \theta dy} \tag{7.101}$$

7.4 Calculation of Dispersion Coefficients

(3) **Stream-tube Moment Method for θ (STMM- θ)**

Beltaos (1980) derived the stream-tube moment method which can be applied to meandering streams that have a skewed concentration distribution due to irregular depths and velocities.

Apply the stream-tube concept neglecting the transverse velocity into Eq. (7.95) to get

$$\frac{\partial \theta}{\partial x} = \frac{\partial}{\partial q}\left(E_T \frac{\partial \theta}{\partial q} \right) \qquad (7.102)$$

7.4 Calculation of Dispersion Coefficients

where E_T = the diffusion factor defined as

$$E_T = \Psi D_T U H^2 \tag{7.103}$$

where Ψ is a dimensionless shape-velocity factor that has a range of 1.0 - 3.6 (Sayre, 1979; Beltaos, 1980) and is defined by

$$\Psi = \frac{1}{Q}\int_0^Q h^2 u\,dq \tag{7.104}$$

Normalizing Eq. (7.102) using η and S

$$\frac{\partial S}{\partial x} = \frac{E_T}{Q^2}\frac{\partial^2 S}{\partial \eta^2} \tag{7.105}$$

7.4 Calculation of Dispersion Coefficients

where

$$\eta \equiv q / Q$$

$$S \equiv \theta / \Theta$$

$$\Theta = \int_0^1 \theta d\eta.$$

Conducting a similar procedure as the simple moment method for Eq. (7.105) gives

$$D_T = \frac{Q^2}{2\Psi U H^2} \frac{d\sigma_\eta^2}{dx} \frac{1}{\left(1 - (1 - \eta_0) S_1 - \eta_0 S_0\right)} \qquad (7.106)$$

7.4 Calculation of Dispersion Coefficients

where σ_η^2 and η_0 are the variance and the centroid of the $S-\eta$ distribution, respectively, and S_0 and S_1 are the normalized dosages at the left and right banks, respectively.

Ex) Case study of streamtube moment method

Field tracer experiment in Nakdong River

Pollutant inflow from left side of tributaries

Using conductivity as tracer

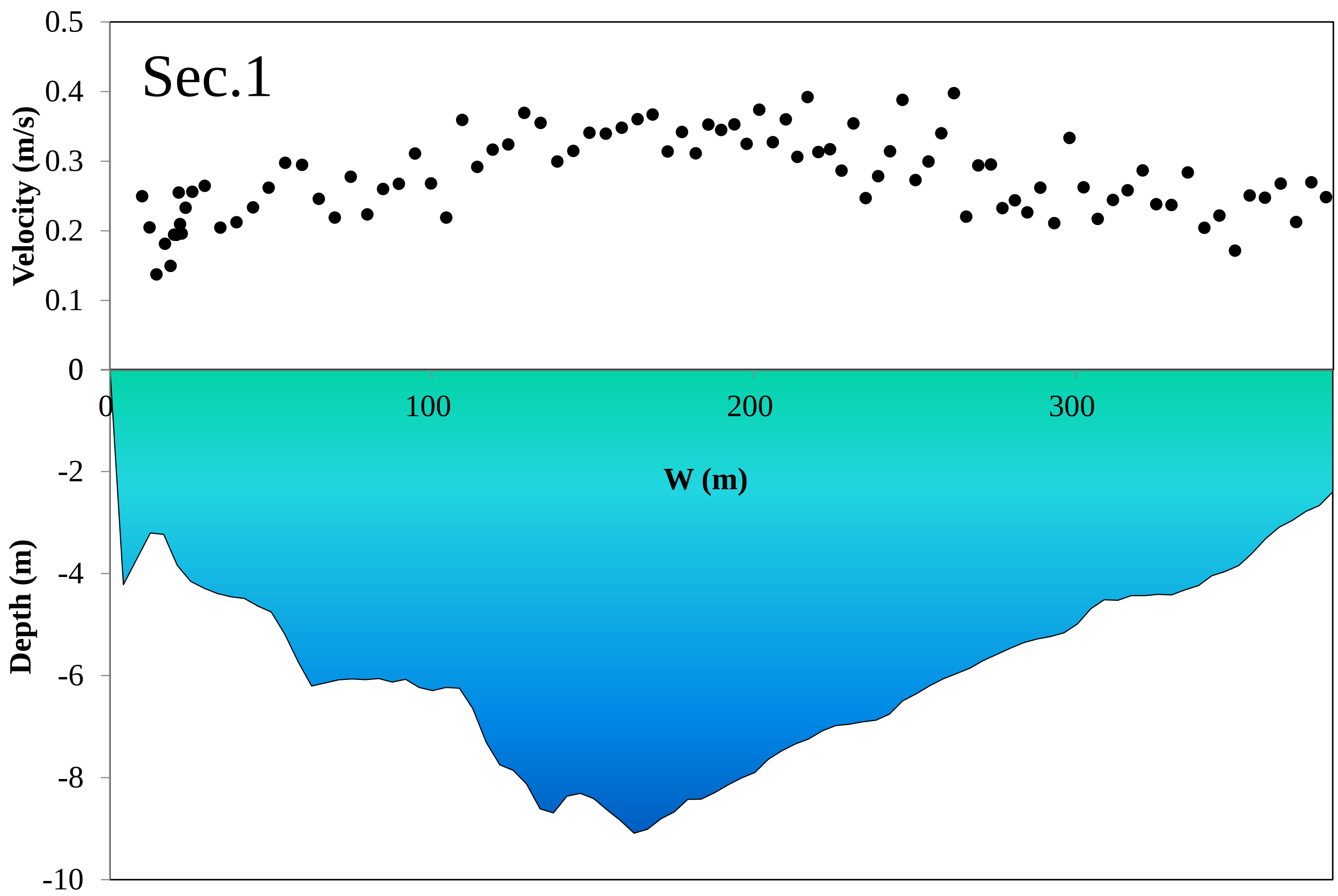
Using Beltaos(1980)'s streamtube moment method

7.4 Calculation of Dispersion Coefficients

i) Hydraulic data measurement

Discharge, velocity, water depth

7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients

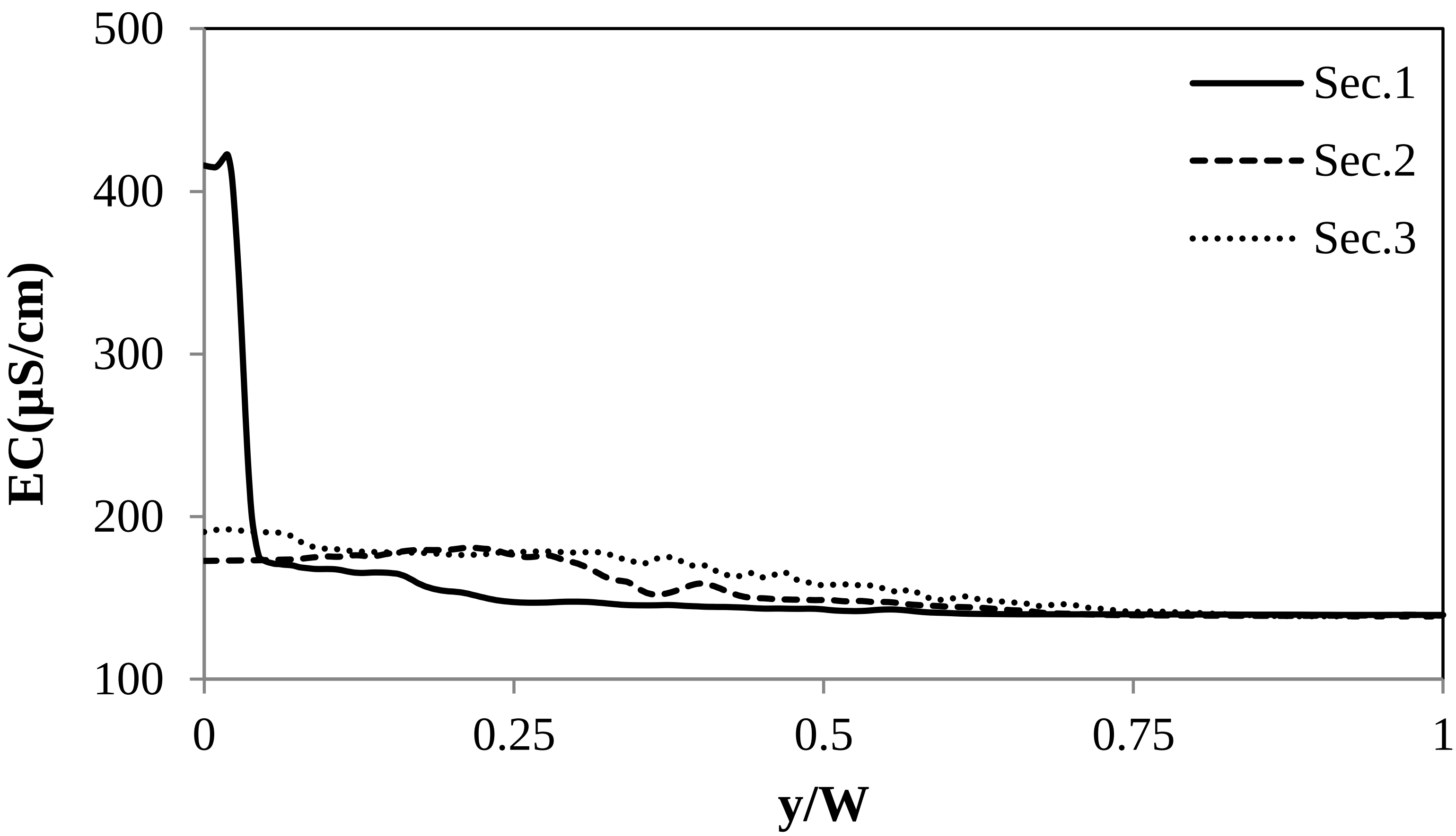
Date	Section	Q (m^3/s)	H (m)	U (m/s)	u^* (m/s)	W (m)	W/H	U/u^*
2014-08-28	1	445.7	5.93	0.27	0.024	376	63.4	11.3
	2	476.8	4.98	0.22	0.020	432.5	86.9	11.0
	3	467.4	5.17	0.22	0.020	410.3	79.4	11.0

7.4 Calculation of Dispersion Coefficients

ii) Concentration data measurement

Conductivity, temperature

7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients

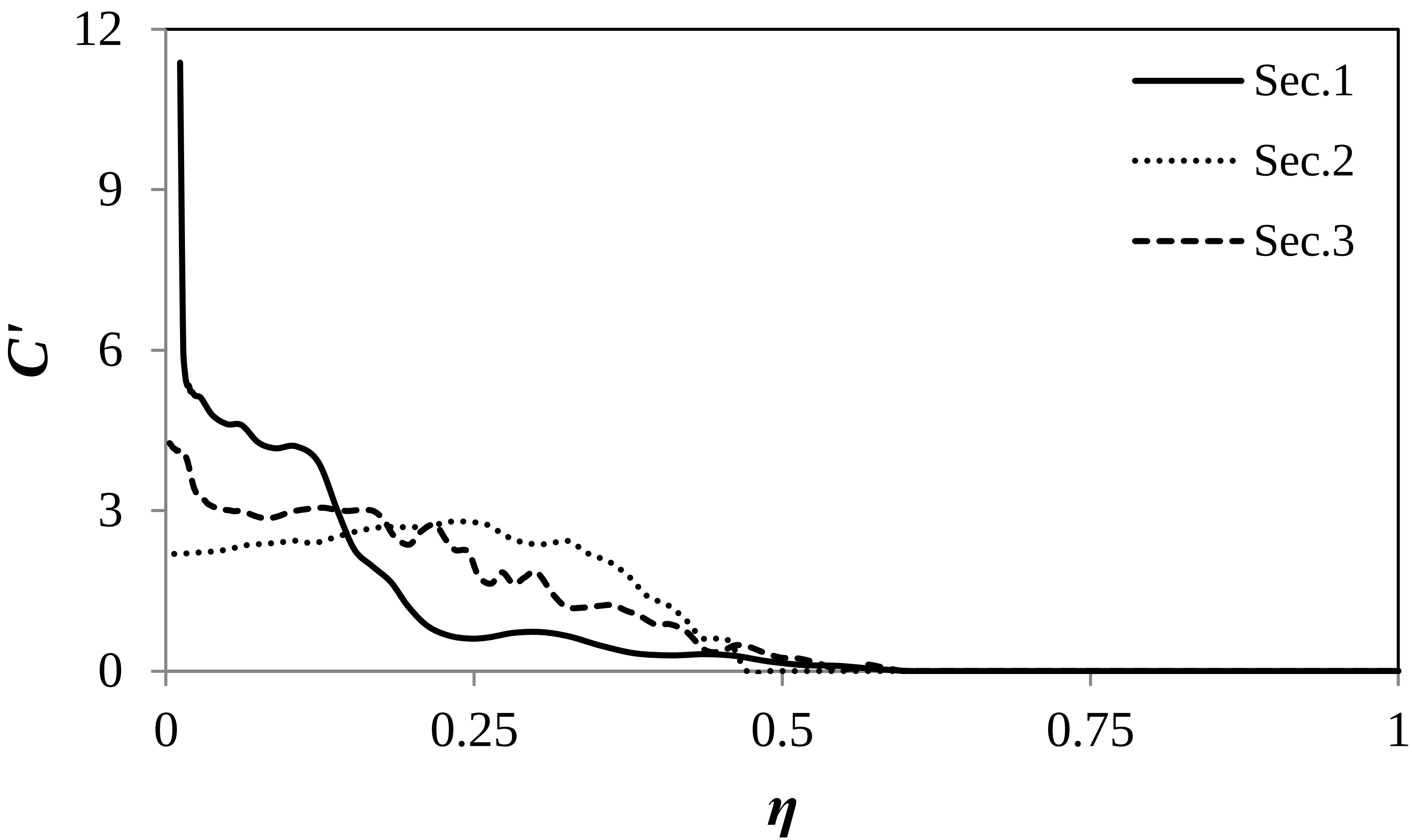
iii) calculation of ψ

$$\psi = \frac{1}{Q}\int_0^Q h^2 u dq$$

iv) plot $C' - \eta$ graph

$$\eta \equiv \frac{q}{Q}, \; C' \equiv \frac{C}{C_\infty}, \; C_\infty = \int_0^1 C d\eta$$

7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients

v) plot $\sigma^2 - F(x)$ graph

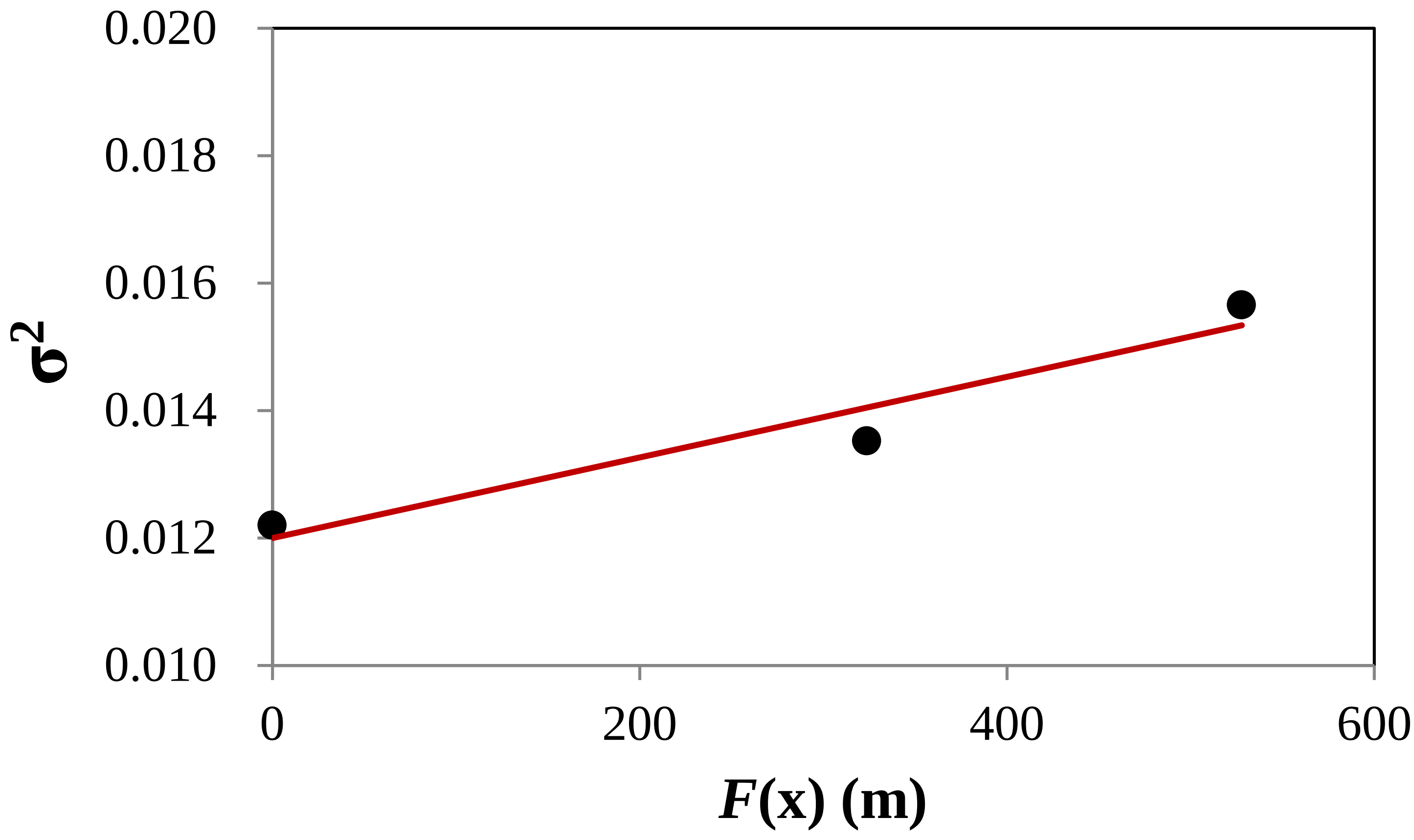
$$f(x) = 1 - (1 - \eta_0) C_1' - \eta_0 C_0'$$

$$F(x) = \int_0^x f(x)\, dx$$

Slope = $\dfrac{2E_y}{Q^2}$

from diffusion factor $E_y = \psi D_T U H^2$, transverse dispersion coefficient can be calculated.

7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients

7.4.2.4 Routing Method for Non-Steady Source

(1) **Stream-tube Moment Method for θ (STRM- θ)**

The stream-tube routing method can be derived from the analytical solution of the stream-tube model of 2D ADE for tracer dosage transport in irregular channels (Seo et al., 2006).

The analytical solution to Eq. (7.102) for a mass being instantaneously injected at $x = 0$ and $\eta = \omega$ can be obtained as

7.4 Calculation of Dispersion Coefficients

$$S(x,\eta) = \frac{\Theta}{\sqrt{4\pi B_C x}} \exp\left(-\frac{(\eta-\omega)^2}{4B_C x} \right) \qquad (7.107)$$

where $B_C = E_T / Q^2.$

Using the frozen cloud assumption in which no dispersion occurs during the passage of the tracer cloud past the measuring station (Fischer, 1968; Rutherford, 1994), the routing equation of the tracer dosage can be written as

$$S(x_2,\eta) = \int_0^1 \frac{S(x_1,\omega)}{\sqrt{4\pi B_C (x_2 - x_1)}} \exp\left(\frac{-(\eta-\omega)^2}{4B_C (x_2 - x_1)} \right) d\omega \qquad (7.108)$$

7.4 Calculation of Dispersion Coefficients

where $S(x_1, \omega)$ = the observed dosage distribution as a function of the dimensionless discharge at an upstream site; $S(x_2, \eta)$ = the predicted dosage distribution as a function of dimensionless discharge at a downstream site; and ω = a normalized dummy transverse distance variable of integration.

Eq. (19) cannot obtain the longitudinal dispersion coefficient because the concentration profile is converted into the dosage profile.

7.4 Calculation of Dispersion Coefficients

(2) **Two-dimensional Routing Method (2D RM)**

The two-dimensional routing method can be derived from the analytical solution of the 2D advection-dispersion equation for the initial distribution of the mass which is injected instantaneously at ξ and ψ (Baek et al., 2006).

$$C(x,y,t)=\int_0^W\int_{-\infty}^{\infty}\frac{f(\xi,\psi)}{4\pi t\sqrt{D_L D_T}}\exp\left(-\frac{(x-\xi)^2}{4D_L t}\right)\exp\left(-\frac{(y-\psi)^2}{4D_T t}\right)d\xi d\psi \qquad (7.109)$$

7.4 Calculation of Dispersion Coefficients

where $f(\cdot)$ is a dummy function.

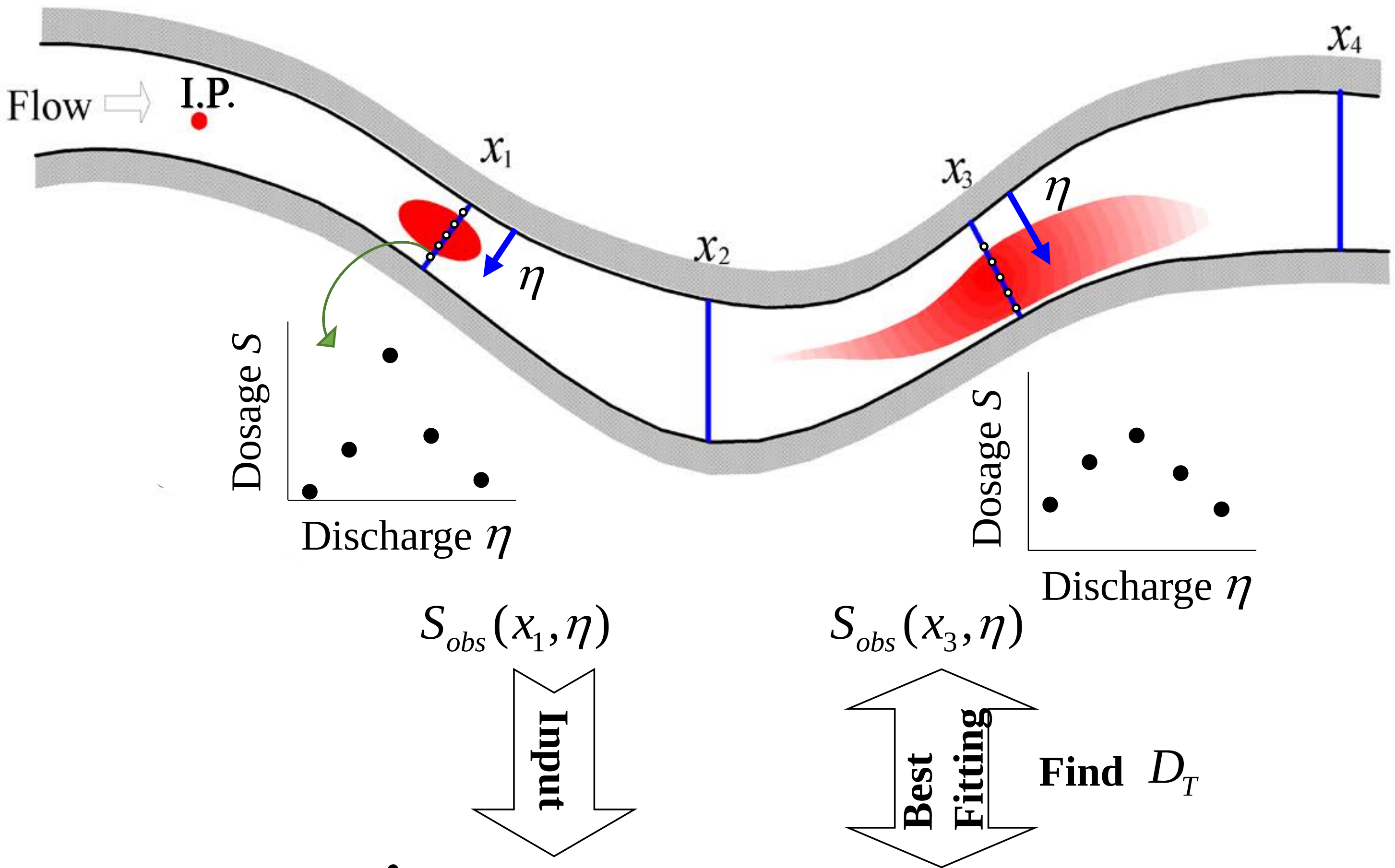
Using the frozen cloud assumption in which no dispersion occurs during the passage of the tracer cloud past the measuring station (Fischer, 1968; Rutherford, 1994), the routing equation of the temporal tracer concentration can be written as

$$C(x_2, y, t)$$
$$= \int_0^W \int_{-\infty}^{\infty} \frac{C(x_1, \psi, \tau) U}{4\pi(\bar{t}_2 - \bar{t}_1)\sqrt{D_L D_T}} \exp\left(-\frac{U^2(\bar{t}_2 - \bar{t}_1 - t + \tau)^2}{4D_L(\bar{t}_2 - \bar{t}_1)}\right) \exp\left(-\frac{(y-\psi)^2}{4D_T(\bar{t}_2 - \bar{t}_1)}\right) d\tau d\psi \qquad (7.110)$$

7.4 Calculation of Dispersion Coefficients

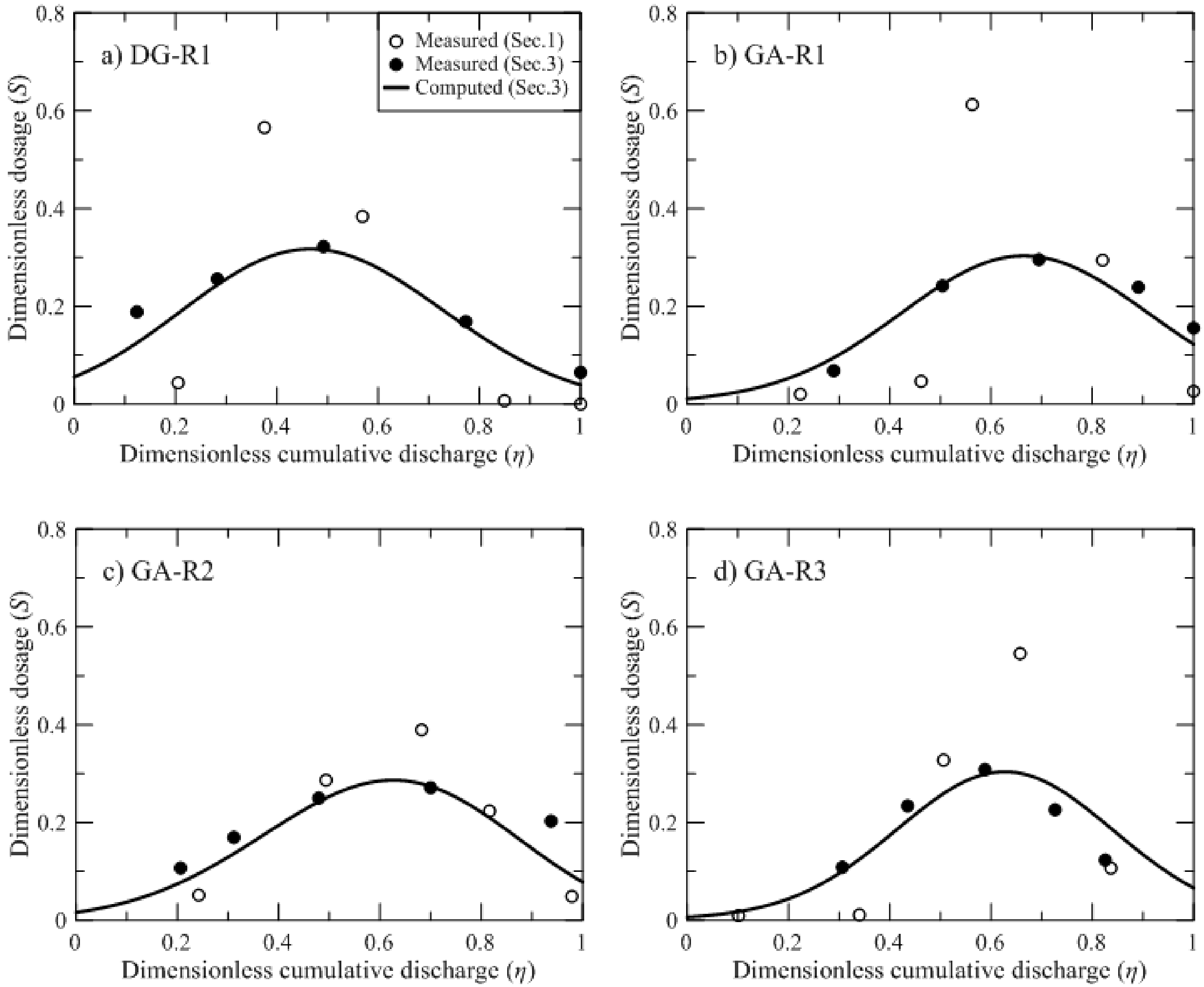
where $C(x_2, y, t)$ = the predicted depth-averaged concentration as a function of time and lateral distance, y, at x_2; $C(x_1, \psi, \tau)$ = the observed depth-averaged concentration as a function of time and lateral distance at x_1.

7.4 Calculation of Dispersion Coefficients



$$\int_\eta S_{obs}(x_1,\eta) f(D_T)\, d\eta = S_{pre}(x_3,\eta)$$

7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients

(3) **Two-dimensional Stream-tube Routing Method (2D STRM)**

The two-dimensional stream-tube routing method can be derived from the solution of the transport equation combined with the stream-tube concept (Baek and Seo, 2010).

$$C(x_2,\eta,t) = \int_0^1 \frac{\left[\int_{-\infty}^{\infty} \frac{C(x_1,\omega,\tau)U}{\sqrt{4\pi D_L(\bar{t}_2-\bar{t}_1)}} \cdot \exp\left(-\frac{U^2(\bar{t}_2-\bar{t}_1-t+\tau)^2}{4D_L(\bar{t}_2-\bar{t}_1)}\right)d\tau\right]}{\sqrt{4\pi B_C(x_2-x_1)}} \cdot \exp\left(-\frac{(\eta-\omega)^2}{4B_C(x_2-x_1)}\right)d\omega \qquad (7.111)$$

7.4 Calculation of Dispersion Coefficients

where $C(x_2, \eta, t)$ = the temporal profile of the predicted concentration at a downstream section, x_2; $C(x_1, \omega, \tau)$ = the temporal profile of the measured concentration at an upstream section, x_1.

Eq. (7.111) is the combination of the 1D routing equation (Eq. 17) and the stream-tube routing equation (Eq. 7.108).

Merits of 2D STRM:

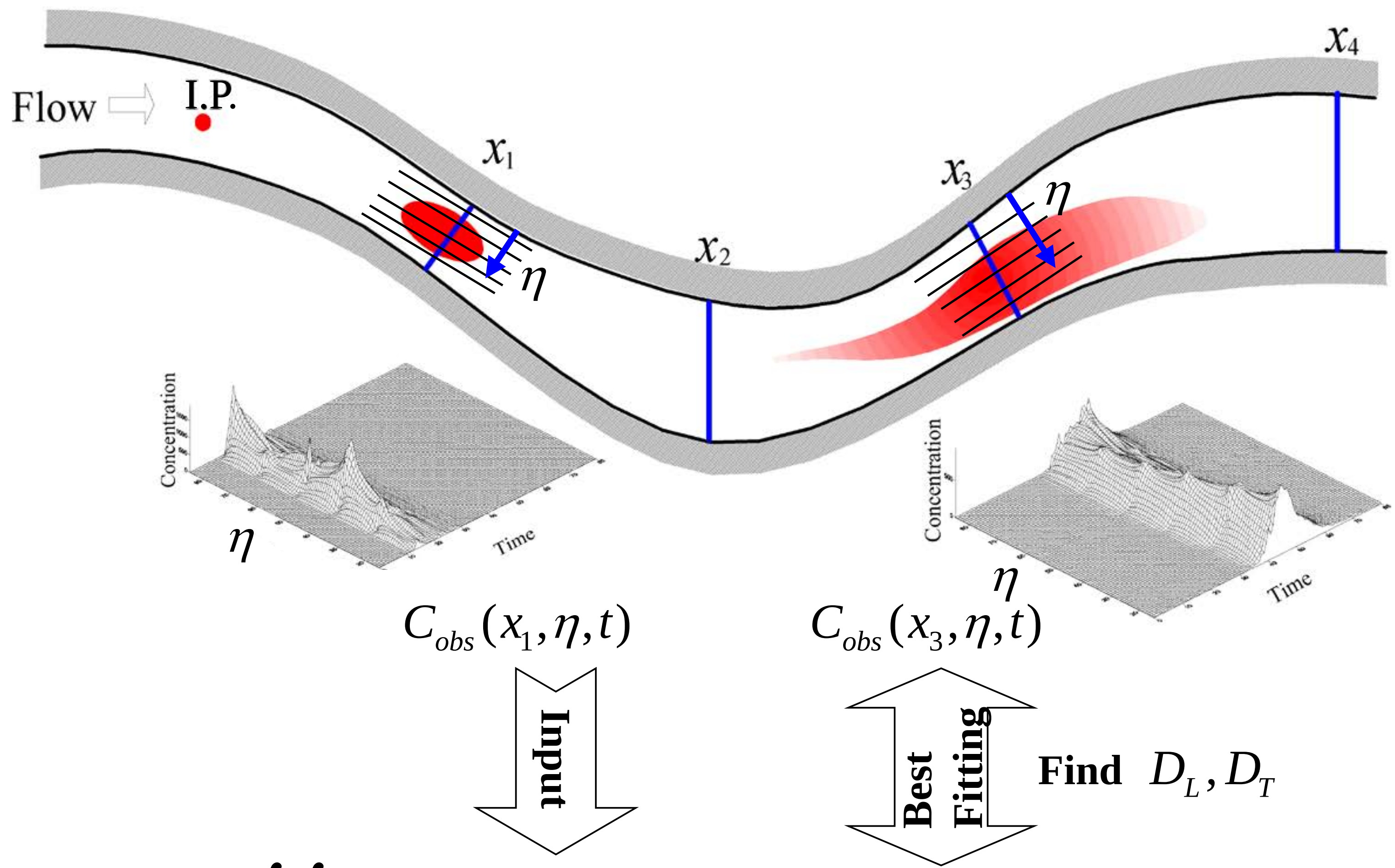
1) The dispersion coefficients can be obtained under the transient concentration condition without the conversion of the governing equation of unsteady condition into that of steady condition.

7.4 Calculation of Dispersion Coefficients

2) Both longitudinal and transverse dispersion coefficients can be obtained simultaneously, whereas only the transverse dispersion coefficient can be obtained by the moment methods.

3) The dispersion coefficients can be calculated by directly fitting the analytical solution of the upstream concentration distribution to the observed concentration profile at the downstream section without using variances of the concentration profiles which are usually skewed due to irregular geometry and the tracer impinging on banks at several sections.

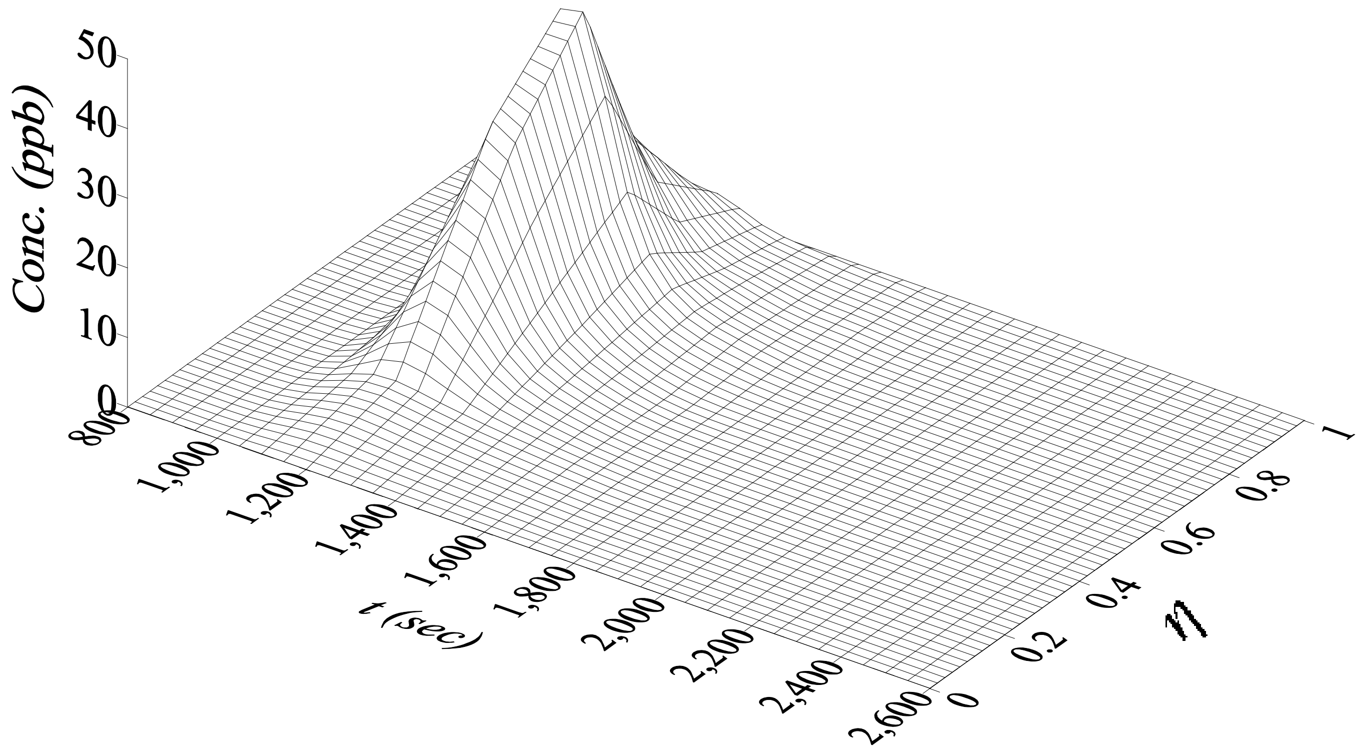
7.4 Calculation of Dispersion Coefficients



$$\int_\eta \int_t C_{obs}(x_1,\eta,t)\, f(D_L, D_T)\, dt\, d\eta = C_{pre}(x_3,\eta,t)$$

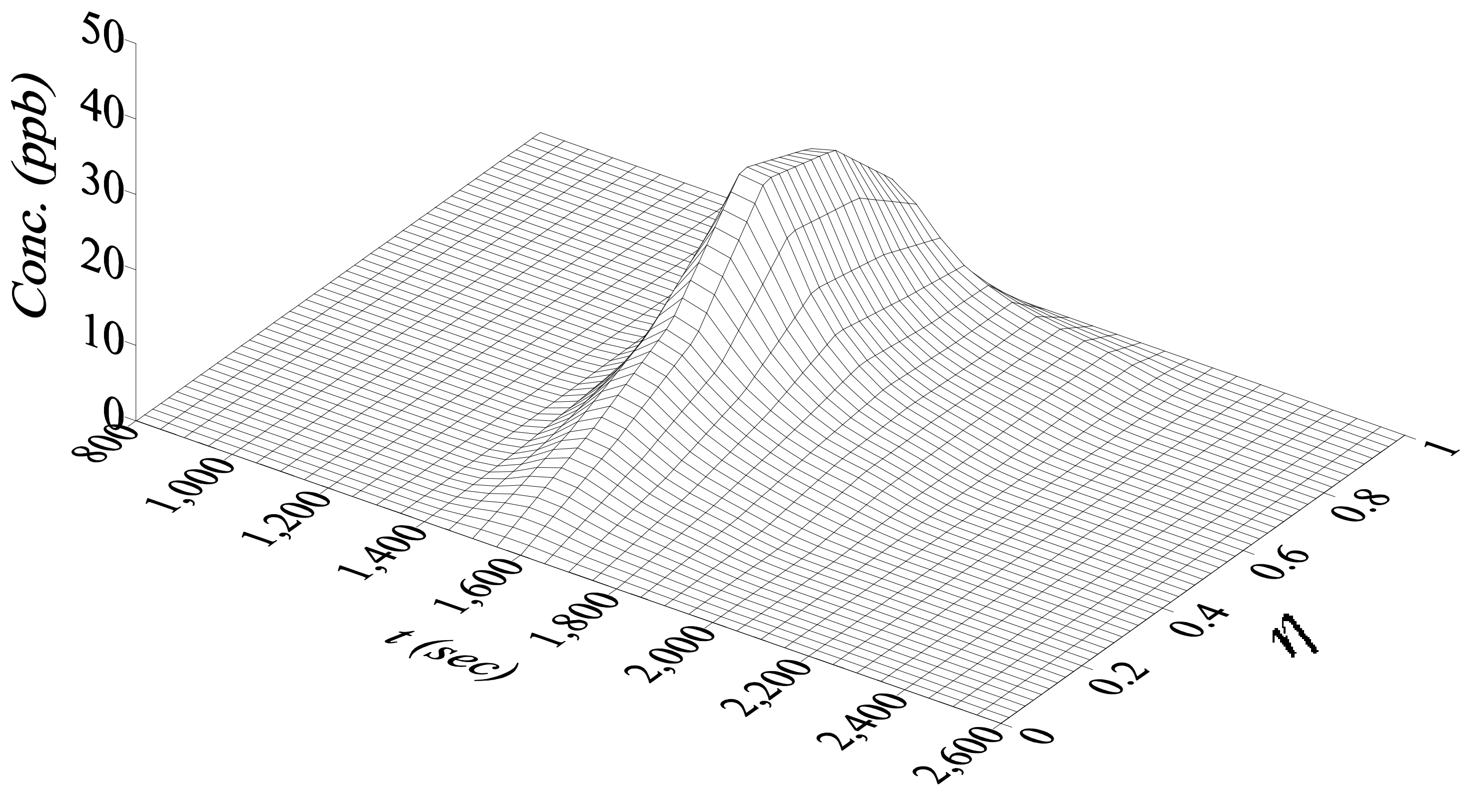
7.4 Calculation of Dispersion Coefficients

a) Sec. 1



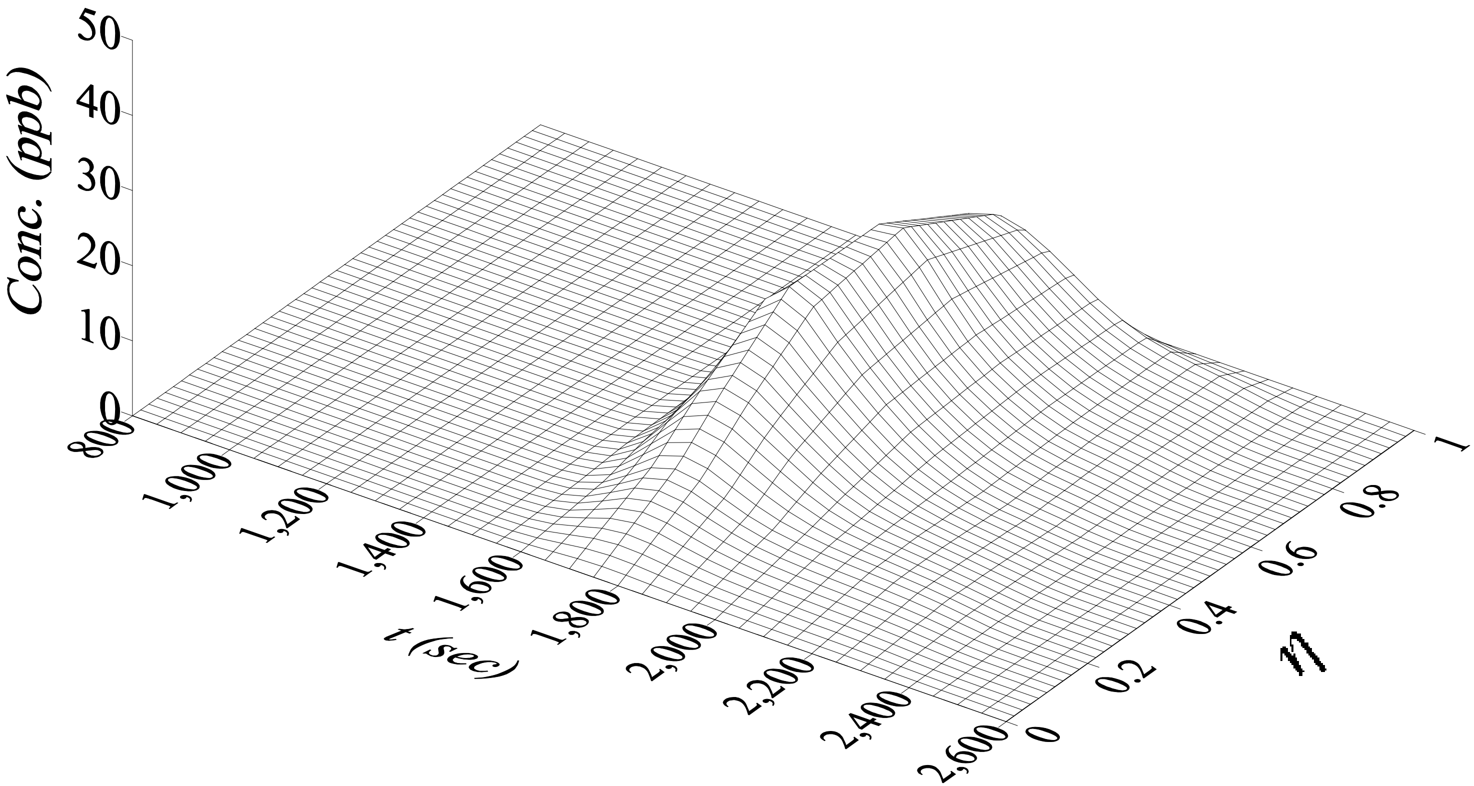
7.4 Calculation of Dispersion Coefficients

b) Sec. 2

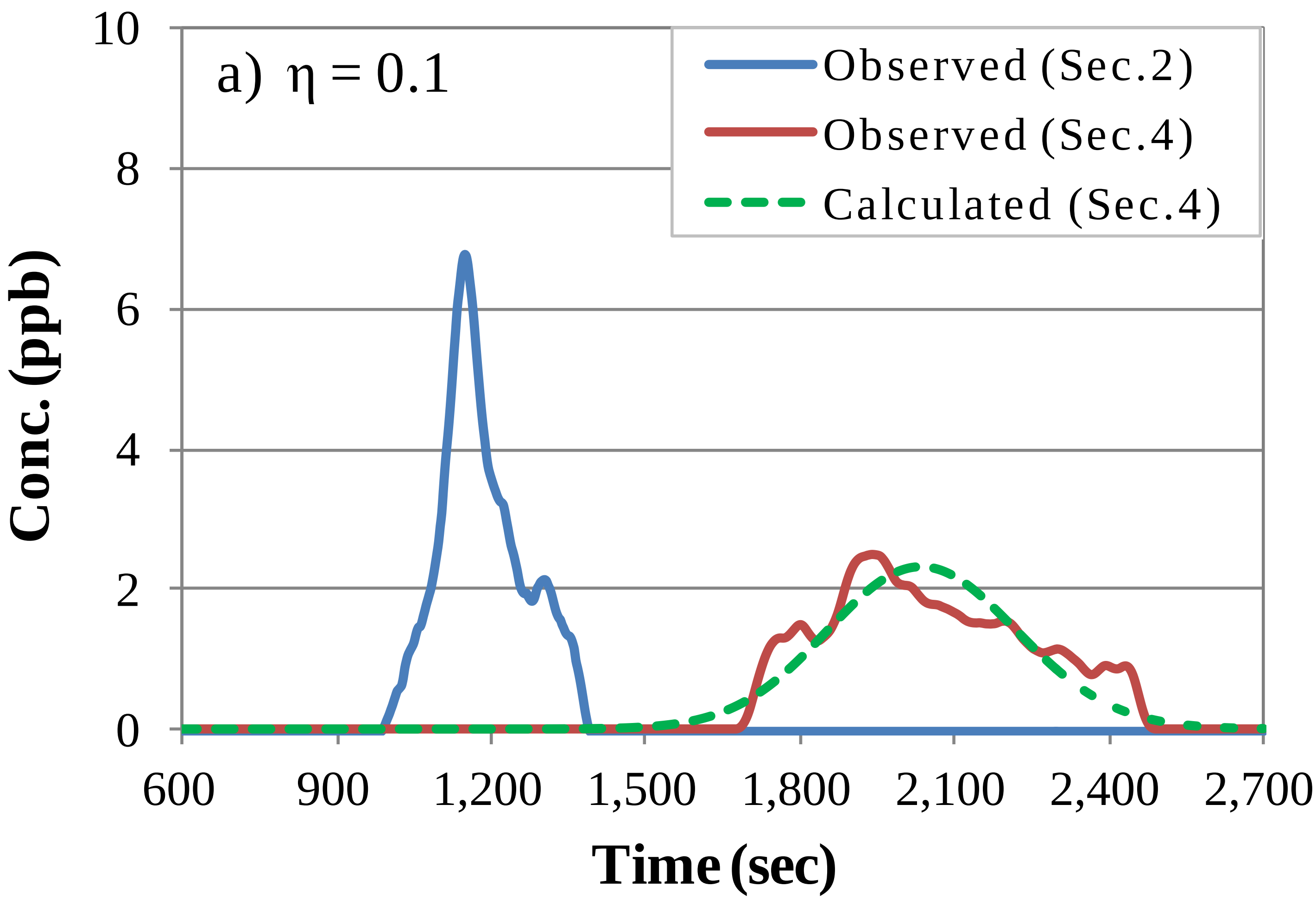


7.4 Calculation of Dispersion Coefficients

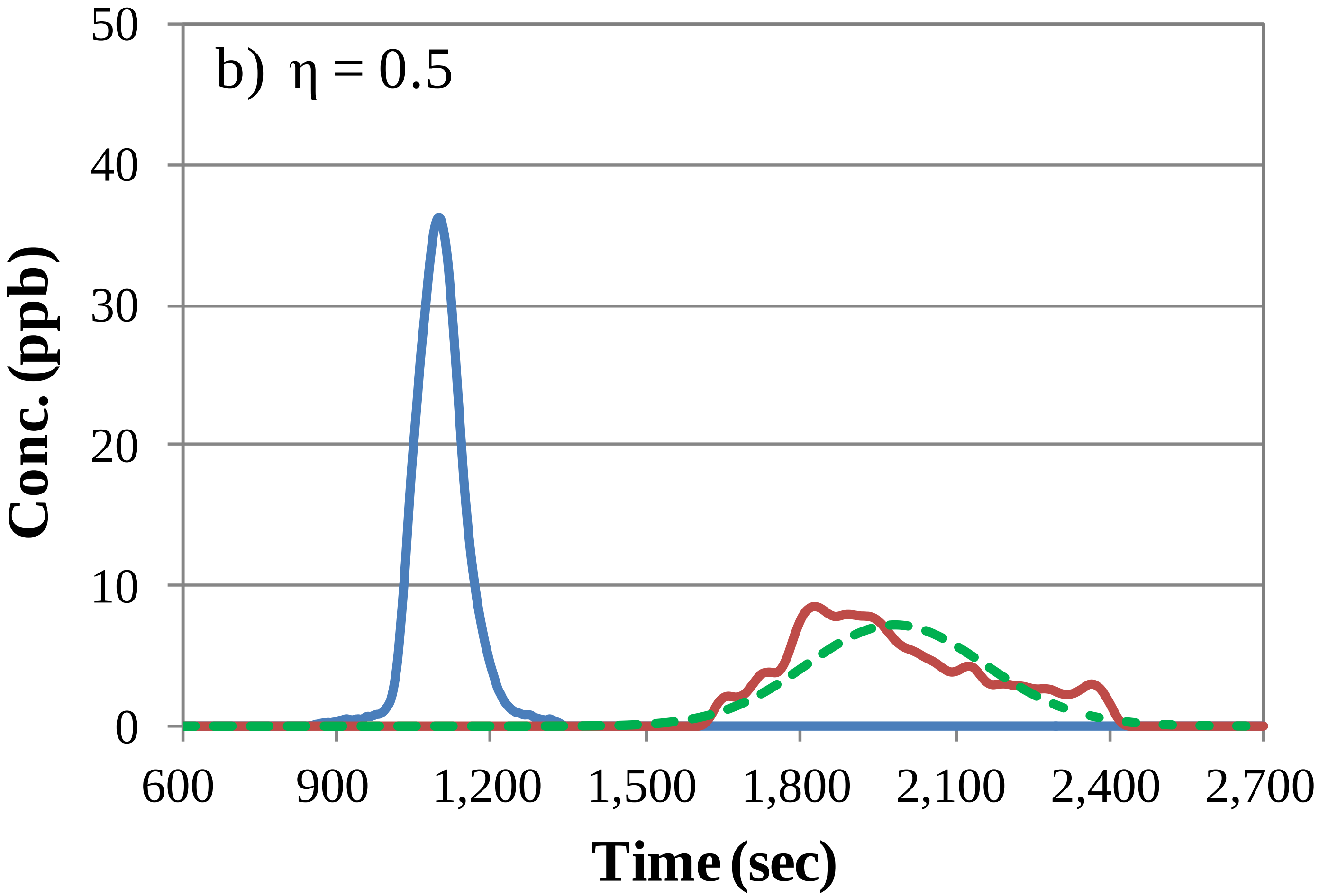
c) Sec. 3



7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients



7.4 Calculation of Dispersion Coefficients

[Re] 2D numerical model

For non-steady source, data are best analyzed using a two-dimensional numerical model.

→ Model calibration involves trial and error adjustments of the dispersion coefficients D_L and D_T until there is a satisfactory match between observed and predicted tracer concentrations.

7.4 Calculation of Dispersion Coefficients

7.4.2.5 Drogues for Measuring Transverse Dispersion

Surface floats:

~ offer a convenient tool for making low cost estimates of U and D_T

~ Floats with GPS sensors can be used.

i) Velocity measurements

Record the transverse location and time of travel of drogues at each section where the tag line is installed across the river

→ Velocity can be estimated from the time taken for drogues to reach the section using a small number of drogues (five to10).

7.4 Calculation of Dispersion Coefficients

ii) measurement D_T

Release drogues (30 -50) sequentially from the same point so that they do not interfere with each other

After measuring transect is divided transversely into m segments of equal width, total number of drogues passing through each section is counted.

The transverse variance is

$$\sigma_y{}^2 = \frac{1}{N}\sum_{i=1}^{m} n(y_i)(y_i - \overline{y})^2 \qquad (7.112)$$

where N = total number of drogues; m = total number of segments;

$n(y_i)$ =number of drogues in the segment centered on y_i and

7.4 Calculation of Dispersion Coefficients

$$\overline{y} = \frac{1}{N}\sum_{x=1}^{m} y_i n(y_i) \tag{7.113}$$

$$D_T = \frac{1}{2}\frac{d\sigma_y^{\ 2}}{dt} = \frac{1}{2}U\frac{d\sigma_y^{\ 2}}{dx} \tag{7.114}$$

A more robust estimate of the variance can be made by calculating the cumulative number of drogues to the left of each segment.

$$n_c(y_j) = \sum_{i=1}^{j} n(y_i) \tag{7.115}$$

Then, plot $n_c(y_j)/N$ versus y on arithmetic probability paper, a straight line can be fitted.

7.4 Calculation of Dispersion Coefficients

$$\sigma_y^{\;2} = \frac{1}{4}(y_{0.84} - y_{0.16})^2 \qquad (7.116)$$

where $y_{0.84}$ and $y_{0.16}$ = transverse locations at which the cumulative proportion of drogues equals 0.84 and 0.16, respectively.

- Restrictions:

- D_T is higher near the surface than near the bed.
- Surface floats tend to over-estimate the depth-averaged D_T (Okoye 1970; Nokes 1986).
- Movement of drogues is easily affected by snags, wind, and surface eddies.

7.4 Calculation of Dispersion Coefficients

7.4.2.6 Use of Transverse Mixing Distance

Use the transverse mixing distance (L_y) that can be measured by an aerial photograph or an infra-red photographs

[Re] visible tracer: humic compounds, suspended soilds

Heat tracer: cooling water plume

For tracer originating from a point source

$$D_T = \beta \frac{UW^2}{L_y} \tag{7.117}$$

7.4 Calculation of Dispersion Coefficients

where β = 0.536 for bankside source

= 0.134 for mid-channel source

For the simplified stream tube model

$$D = \beta \frac{Q^2}{L_y} \tag{7.118}$$

where D = factor of diffusion; Q = discharge

$$D = \psi H^2 U D_T \tag{7.119}$$

7.4 Calculation of Dispersion Coefficients

where ψ = shape factor

Substitute Eq. (7.119) into Eq. (7.118)

$$D_T = \beta \frac{UW^2}{\psi L_y} \tag{7.120}$$