# 457.562 Special Issue on River Mechanics (Sediment Transport)
# .12 Bed forms, flow resistance, and sediment transport



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# 1. Form Drag and Skin Friction

- Bed forms can have a profound influence on flow resistance, and thus on sediment transport in an alluvial channel.
- Consider, the case of normal flow in a wide rectangular channel. In the presence of bed forms must be

$$\overline{\tau}_b = \rho g H S$$

- Where overbar tau is an effective boundary shear stress, where the overbar denotes averaging over the bed forms, and can be defined as the streamwise drag force per unit area.
- H represents the depth averaged over the bed forms.

# 1. Form Drag and Skin Friction

- The effective boundary shear stress consists of two major sources of "skin friction (local shear stress)", and "form drag (pressure)".

$$\bar{\tau}_b \equiv \tau_{bs} + \tau_{bf}$$

$\tau_{bs}$ : Skin friction

$\tau_{bf}$ : Form drag

- The form drag results from the net pressure distribution over an entire bed form.
- Drag is not directly on the particle itself (since normal to surface).
- Drag appears when flow separates in the lee of the crest, and this one is substantial (so, skin friction dominant)

# 1. Form Drag and Skin Friction

- The part of the effective shear stress that governs sediment transport is thus seen to be the skin friction.
- Any equation of the bed-load is to be applied, it is necessary to replace the Shields stress by the Shields stress associated with skin friction only:

$$\tau_s^* = \frac{\tau_{bs}}{\rho g R D}$$

# 2. Shear Stress Partitions

- Einstein Partition
  - Einstein was the first to recognize the necessity of distingui shing between skin friction and form drag.

$$\bar{\tau}_b = \rho C_f U^2$$

  - Where $C_f$ represents a resistance coefficient that includes both skin friction and form drag.

$$\tau_{bs} = \rho C_{fs} U^2$$

  - Where $C_{fs}$ is the frictional resistance coefficient that would result if bed forms were absent. In rough turbulent flow

$$C_{fs} = \left[ \frac{1}{\kappa} \ln\left( 11 \frac{H_s}{k_s} \right) \right]^{-2}$$

  - $H_s$ denotes the depth that would result in the absence of b edforms

# 2. Shear Stress Partitions

- The purpose of the calculation is to get $H_s$. In normal flo w (steady, uniform)

$$\bar{\tau}_b = \rho C_f U^2 = \rho g H S$$

$$\tau_{bs} = \rho C_{fs} U^2 = \rho g H_s S$$

- Now,

$$H_s = \frac{U^2}{gS}\left[\frac{1}{\kappa}\ln\left(11\frac{H_s}{k_s}\right)\right]^{-2}$$

- If form drag is

$$\tau_{bf} = \rho C_{ff} U^2 = \rho g H_f S$$

$$\tau_{bf} = \bar{\tau}_b - \tau_{bs}$$

# 2. Shear Stress Partitions

- Then the coefficients are

$$C_f = C_{fs} + C_{ff}$$

$$H = H_s + H_f$$

# Example Calculations

- Consider a sand-bed stream at a given cross section with a slope of 0.0004, a mean depth of 2.9m, a median bed sediment size of 0.35mm and a discharge per unit width $q = U \times H = 4.4 m^2 / s$. Assume that the flow is under near -normal condition. Compute values of $\tau_{bs}, \tau_{bf}, C_{fs}, C_{ff}, H_s$ and $H_f$
- Sol) The mean flow velocity is given by U=4.4/2.9=1.52m/s
  An appropriate esimate of $k_s$ for sand-bed steam is $k_s$=2.5$D_{50}$. First, $H_s$=1.047, then $H_f$. From the each depth, you may can each shear stress. After this, you can get friction coefficietns of each.

# 2. Shear Stress Partitions

- Finally the form-induced Shield stress is

$$\tau_f^* = \frac{\tau_{bf}}{\rho g R D}$$

- The previous case (Example) only 30% of total Shield stress is skin Shield stress (which contributes to the transport of sediment).
- Einstein partition needs the ***velocity information***.

# 2. Shear Stress Partitions (Nelson-Smith Partition)

- Nelson and Smith (1989) consider flow over a dune; the f low is taken to separate in the lee of the dune.

$$D_{ffs} = \frac{1}{2}\rho C_D B \Delta U_r^2$$

$D_{ffs}$ denotes that potion of the streamwise drag force $D_{fs}$ that is due to form drag, $B$ is the width, $U_r$ is a reference velocity. $C_D$ is site specific but in the Columbia River 0.21.

$$\tau_{bf} = \frac{1}{2}\rho C_D \frac{\Delta}{\lambda} U_r^2 = \frac{D_{ffs}}{B\lambda}$$

# 2. Shear Stress Partitions (Nelson-Smith Partition)

- The reference velocity $U_r$ is defined to be the mean veloc ity that would prevail between $z=k_s$ and $z=\Delta$ if the bed for ms were not there.
- From the logarithmic profile

$$\frac{U_r}{\sqrt{\tau_{bs}/\rho}} = \frac{1}{\kappa}\left[\ln\left(30\frac{\Delta}{k_s}\right)-1\right]$$

- It is assumed that a rough logarithmic law with roughnes s $k_s$ *prevail from* $z=k_s$ and $z=\Delta$ and from $z=\Delta$ to $z=H$, $k_c$ r epresents an another roughness, presenting a composit e roughness length, including the effects of both skin or grain friction and form drag.

# 2. Shear Stress Partitions (Nelson-Smith Partition)

- Now there are two velocity distributions exist

$$\frac{\bar{u}(z)}{\sqrt{\tau_{bs}/\rho}} = \frac{1}{\kappa}\ln\left(30\frac{z}{k_s}\right), \qquad k_s < z < \Delta$$

$$\frac{\bar{u}(z)}{\sqrt{\left(\tau_{bs}+\tau_{bf}\right)/\rho}} = \frac{1}{\kappa}\ln\left(30\frac{z}{k_c}\right), \quad \Delta < z < H$$

- Nelson and Smith match the above two laws at the level *z=Δ* (the top of the dune).

$$\frac{\tau_{bs}+\tau_{bf}}{\tau_{bs}} = \left[\frac{\ln\left(30\Delta/k_s\right)}{\ln\left(30\Delta/k_c\right)}\right]^2$$

# 2. Shear Stress Partitions (Nelson-Smith Partition)

- The partition requires a prior knowledge of total boundar y shear stress as well as roughness height and dune hei ght, and wave length
- From the partition

$$\tau_{bf} = \overline{\tau}_b - \tau_{bs} = \frac{1}{2} C_D \frac{\Delta}{\lambda \kappa^2} \left[ \ln\left( 30 \frac{\Delta}{k_s} \right) - 1 \right]^2 \tau_{bs}$$

- This equation will give skin shear stress and so from fricti on. With this information using the previous equation of

$$\frac{\tau_{bs} + \tau_{bf}}{\tau_{bs}} = \left[ \frac{\ln\left(30\Delta / k_s\right)}{\ln\left(30\Delta / k_c\right)} \right]^2$$

- You can get the composite roughness.

# 2. Example

- Chosen to rather similar to the previous one, let

  $H = 2.9m;\ S = 0.0004;\ \ k_s = 2.5D_{50};\ \ \mathrm{D}_{50} = 0.35mm,$

  $\Delta = 0.4m;\ \lambda = 15m$

- Then $\bar{\tau}_b = \rho g H S,$ from

$$\bar{\tau}_b - \tau_{bs} = \frac{1}{2} C_D \frac{\Delta}{\lambda \kappa^2} \left[ \ln\left( 30 \frac{\Delta}{k_s} \right) - 1 \right]^2 \tau_{bs}$$

- $\tau_{bf} = \bar{\tau}_b - \tau_{bs}$, now you can get $k_c$ from,

$$\frac{\tau_{bs} + \tau_{bf}}{\tau_{bs}} = \left[ \frac{\ln(30\Delta / k_s)}{\ln(30\Delta / k_c)} \right]^2$$

- And you can get,

$$C_f = C_{fs} + C_{ff}$$

$$H = H_s + H_f$$

# 2. Example

- In computing friction coefficients, the flowing relationship was used for the depth-averaged velocity

$$\frac{U}{\sqrt{\left(\tau_{bs}+\tau_{bf}\right)/\rho}}=\frac{1}{\kappa}\ln\left(11\frac{H}{k_c}\right)$$

- Now you can calculate the depth-average flow velocity.
- In another way, if you know the mean depth averaged ve locity, you can get friction coefficient for form drag as

$$k_c=\frac{11H}{e^{\kappa U/u_*}}\qquad\left(u_*=\sqrt{\left(\tau_{bs}+\tau_{bf}\right)/\rho}\right)$$

- ***The Nelson-Smith does not require the assumption o f quasi-normal flow (Einstein method)***

# 3. Empirical Formulas for Stage-Discharge Relations

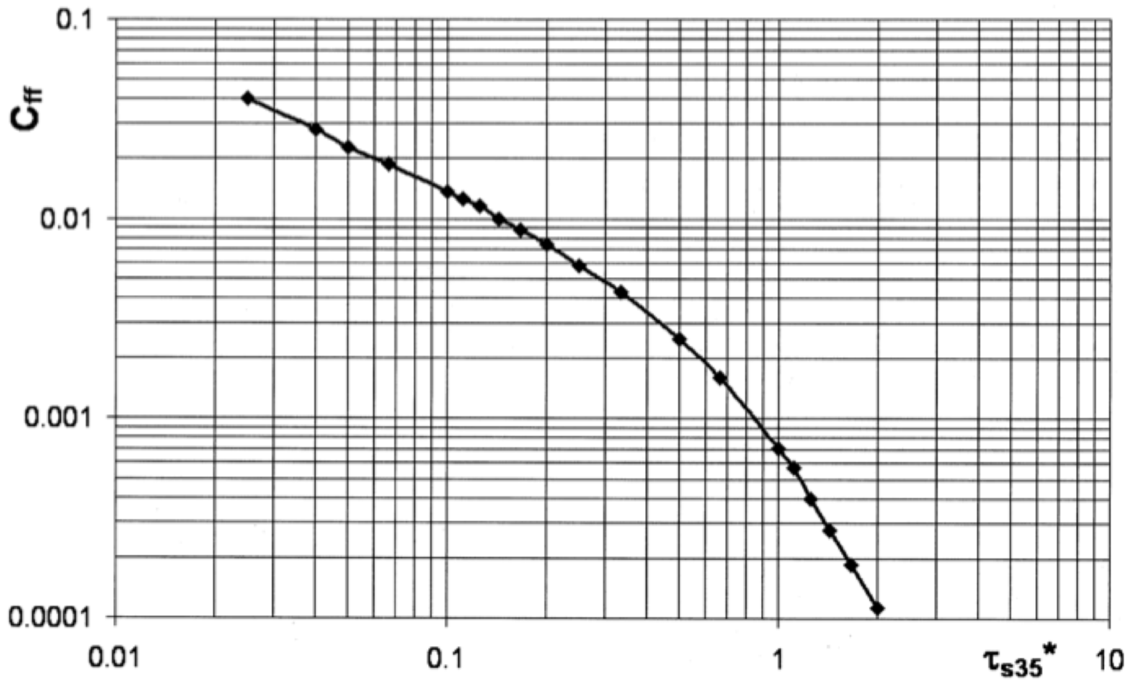
- To use the partition method, it is necessary to know in ad vance the total effective boundary shear stress.
- Therefore, partition itself cannot say anything.
- The Einstein-Barbarossa Method
  - For the case of dune resistance in sand-bed streams.
  - Empirical relation of the following form:

$$C_{ff} = f\left(\tau^*_{s35}\right)$$

$$\tau^*_{s35} = \frac{\tau_{bs}}{\rho g R D_{35}}$$

# 3. Empirical Techniques for Stage-Discharge Relations

- The friction coefficient for the bed forms declines for incr easing shear.
- Increased intensity of flow causes a decrease in form dra g (transition from dunes to flat bed).



# 3. Empirical Techniques for Stage-Discharge Relations

- We need to find a relation between H and water discharg e Q.
- It is assumed that the river slope S and the sizes $D_{50}$ and $D_{35}$ are known. The river is taken to be sufficiently wide s o that the hydraulic radius $R_h=H$: (Hydraulic radius can b e sued for the depth).
- The channel width is

$$B = B(H)$$

# 3. Empirical Techniques for Stage-Discharge Relations

$$H_s \rightarrow C_{fs} \qquad \left( C_{fs} = \left[ \frac{1}{\kappa} \ln\left( 11 \frac{H_s}{k_s} \right) \right]^{-2} \right)$$

$$C_{fs}, H_s \rightarrow U \qquad \left( U^2 = \frac{g H_s S}{C_{fs}} \right)$$

$$H_s \rightarrow \tau_{bs} \rightarrow \tau^*_{s35} \qquad \left( \tau_{bs} = \rho g H_s S,\ \tau^*_{s35} = \frac{\tau_{bs}}{\rho g R D_{35}} \right)$$

$$\tau^*_{s35} \rightarrow C_{ff}$$

Use diagram

$$C_{ff}, U \rightarrow H_f \qquad \left( \tau_{bf} = \rho C_{ff} U^2 = \rho g H_f S \right)$$

$$H = H_s + H_f$$

$$Q = UHB$$

- So result may be plotted in terms of H versus Q for the desired depth-discharge relation.

# 3. Empirical Techniques for Stage-Discharge Relations

- With the previous value, we may have the bed-load tran sport rate.

$$\tau_s^* = \frac{\tau_{bs}}{\rho g R D_{50}}$$

- The volumetric bed load transport rate $Q_b$ is

$$Q_b = q_b B$$

- This is for the depth-discharge prediction in the sand-be d streams.

# 3. Stage-Discharge Relations (Engelund-Hansen Method)

- Sand-bed streams.
- More accurate than the method of Einstein and Barbaro ssa.
- The method assumes quasi-uniform material; it is neces sary to know only a single grain size *D*. Roughness heig ht $k_s$ is $k_s = 2.5 D_{50}$
- The method uses the Einstein partition. Skin friction is c omputed

$$C_{fs} = \left[ \frac{1}{\kappa} \ln\left( 11 \frac{H_s}{k_s} \right) \right]^{-2}$$

$$\tau^* = \frac{\bar{\tau}_b}{\rho RgD}; \qquad \tau_s^* = \frac{\tau_{bs}}{\rho RgD}$$

# 3. Stage-Discharge Relations (Engelund-Hansen Method)

- The previous equation have two branches.
- The lower-regime branch is given by

$$\tau_s^* = 0.064 + 0.4\left(\tau^*\right)^2$$

- Upper branch satisfies the relation

$$\tau_s^* = \tau^*$$

over a range; this implies on upper-regime plane bed. For higher values of Shields stress again exceeds $\tau_s^*$ implying form drag due to the development of antidunes.

- The procedure rather closely parallels that of the Einstein Barbarossa method. It is assumed that values of S and D are know, as well as cross-sectional geometry.

# 3. Stage-Discharge Relations (Engelund-Hansen Method)

- Values of $H_s$ are selected, ranging from a low value to near b ank-full. The calculation then proceeds as follows

$$H_s \to C_{fs} \to U \qquad \left( C_{fs} = \left[ \frac{1}{\kappa} \ln \left( 11 \frac{H_s}{k_s} \right) \right]^{-2}, U^2 = \frac{gH_s S}{C_{fs}} \right)$$

$$H_s \to \tau_{bs} \to \tau_s^* \qquad \left( \tau_{bs} = \rho g H_s S,\ \tau_s^* = \frac{\tau_{bs}}{\rho g R D} \right)$$

$$\tau_s^* \to \tau^* \qquad \left( \tau_s^* = f\left(\tau^*\right), \tau_s^* = 0.006 + 0.4\left(\tau^*\right)^2 \right)$$

$$\tau^* \to \bar{\tau}_b \to H \qquad \left( \tau_s^* = \frac{\tau_{bs}}{\rho R g D}, \bar{\tau}_b = \rho g H S \right)$$

$$Q = UHB$$

- The value of $\tau_s^*$ may then be used to calculate bedload trans port rates, in a fashion completely analogous to the procedur e outlined for the Einstein-Barbarossa method.