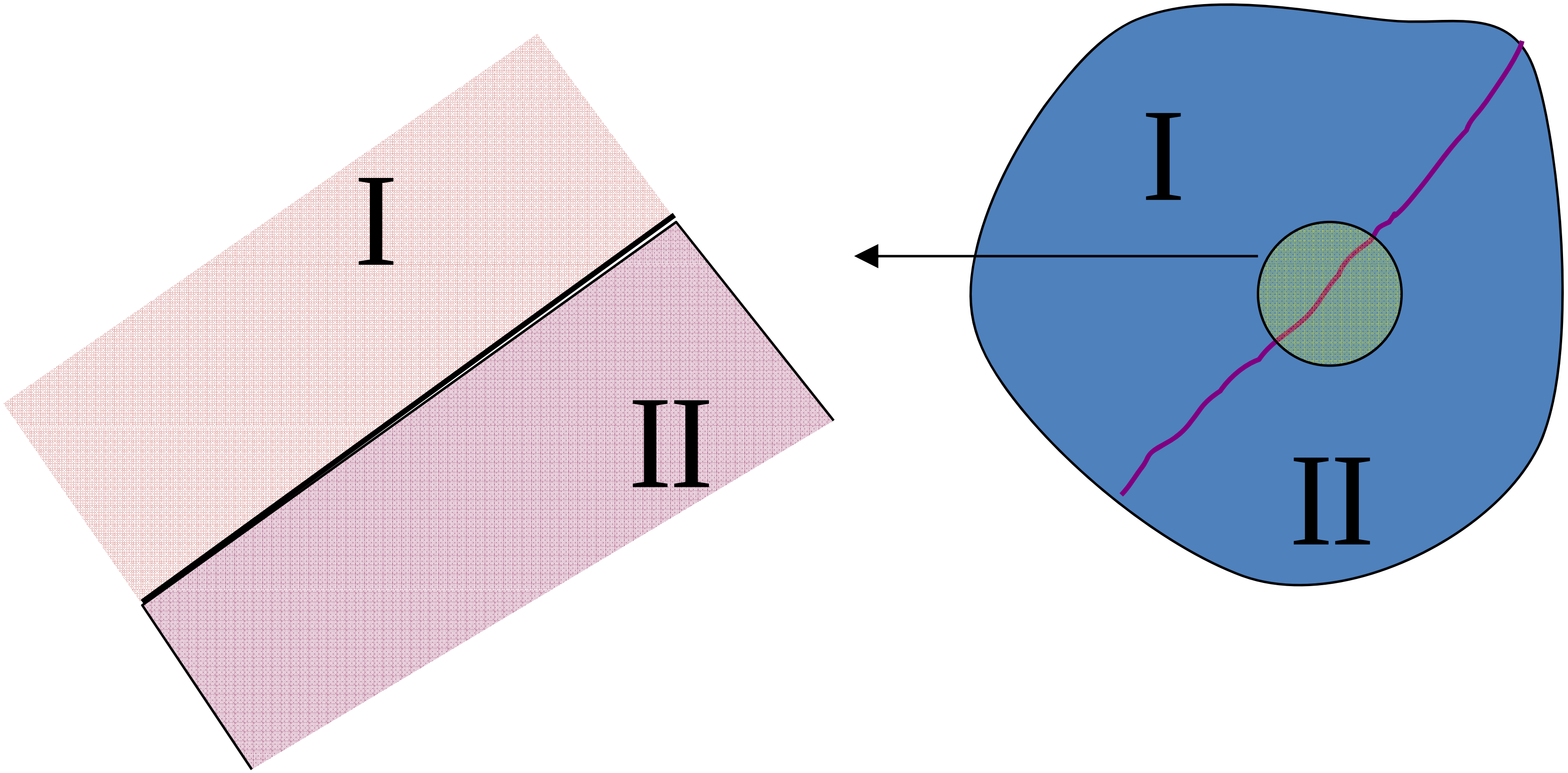
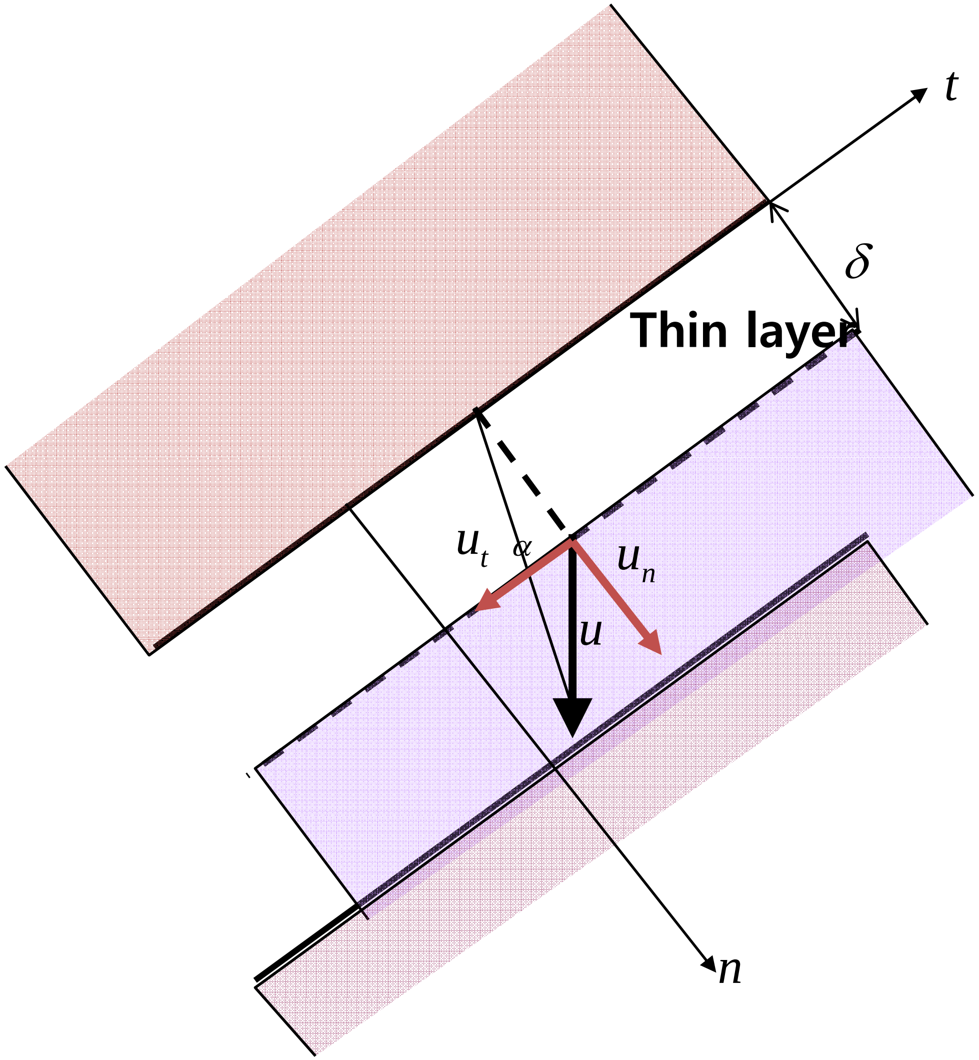
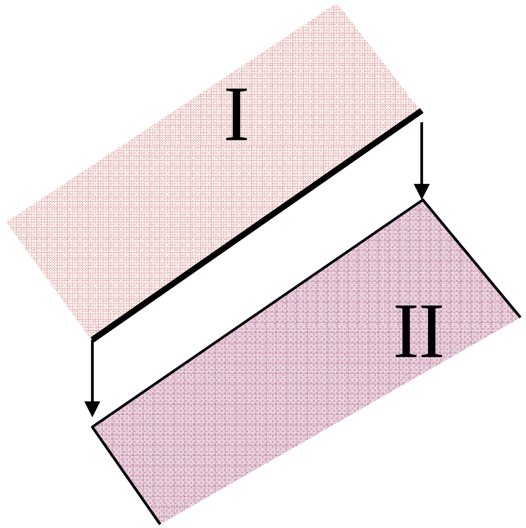
Planes and Lines of Discontinuity



Cracking

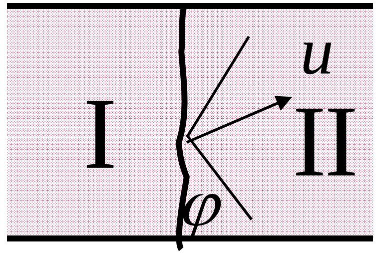




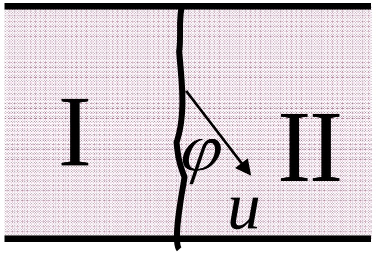
$$\varepsilon_{\max} = \varepsilon^{+} = \frac{1}{2}\frac{u}{\delta}(\sin\alpha + 1)$$

$$\varepsilon_{\min} = \varepsilon^{-} = \frac{1}{2}\frac{u}{\delta}(\sin\alpha - 1)$$

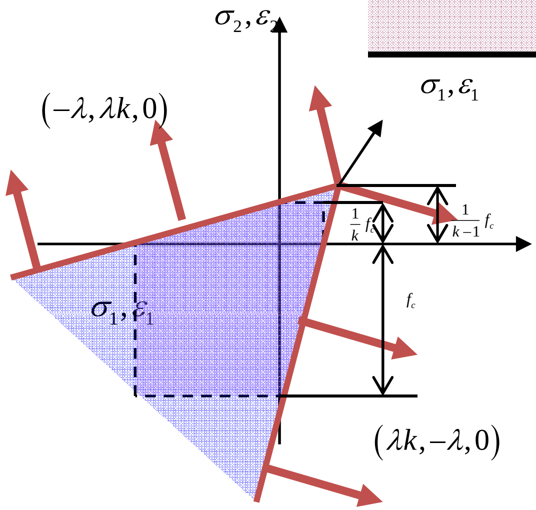
Dissipation work : Plane Strain



Along surfaces

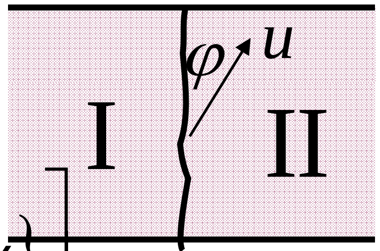


$$W = f_c \frac{1}{2}\frac{u}{\delta}(1-\sin\varphi)$$



$$W_l = ub\frac{1}{2} f_c (1-\sin\varphi)$$

At apex $\varphi \le \alpha \le \pi - \alpha$



$$W_l = ub\frac{1}{2} f_c \left[1-\sin\alpha + \frac{f_t}{f_c}\left\{-(k-1)+(k+1)\sin\alpha\right\}\right]$$

Dissipation work : Plane Stress

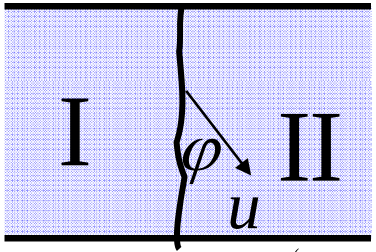
At apex at A $\varphi \le \alpha \le \pi - \alpha$

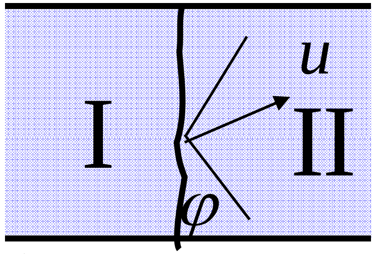
$$W_l = \frac{1}{2}\frac{f_c}{k}ub\left(1+\sin\alpha\right)$$

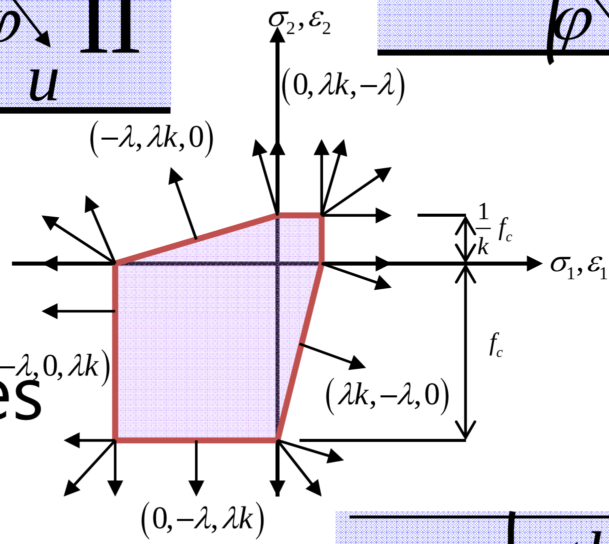
Along other apexes and surfaces

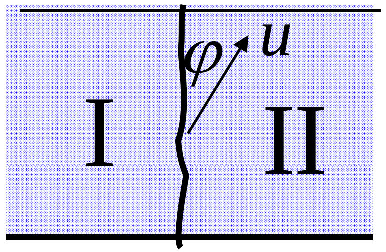
$\alpha \le \varphi$ and $\alpha \ge \pi - \varphi$

$$W_l = \frac{1}{2} f_c ub\left(1-\sin\alpha\right)$$



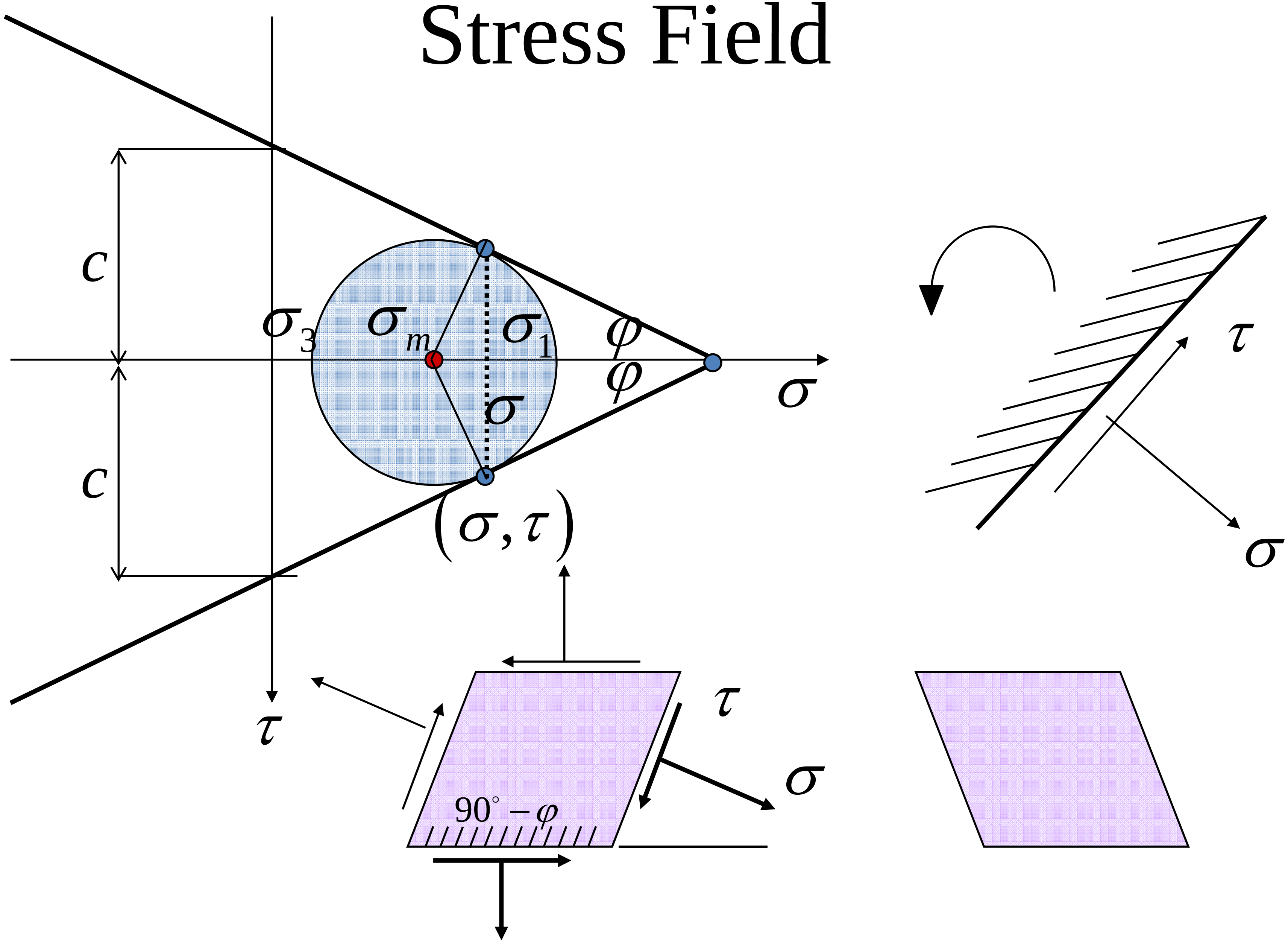




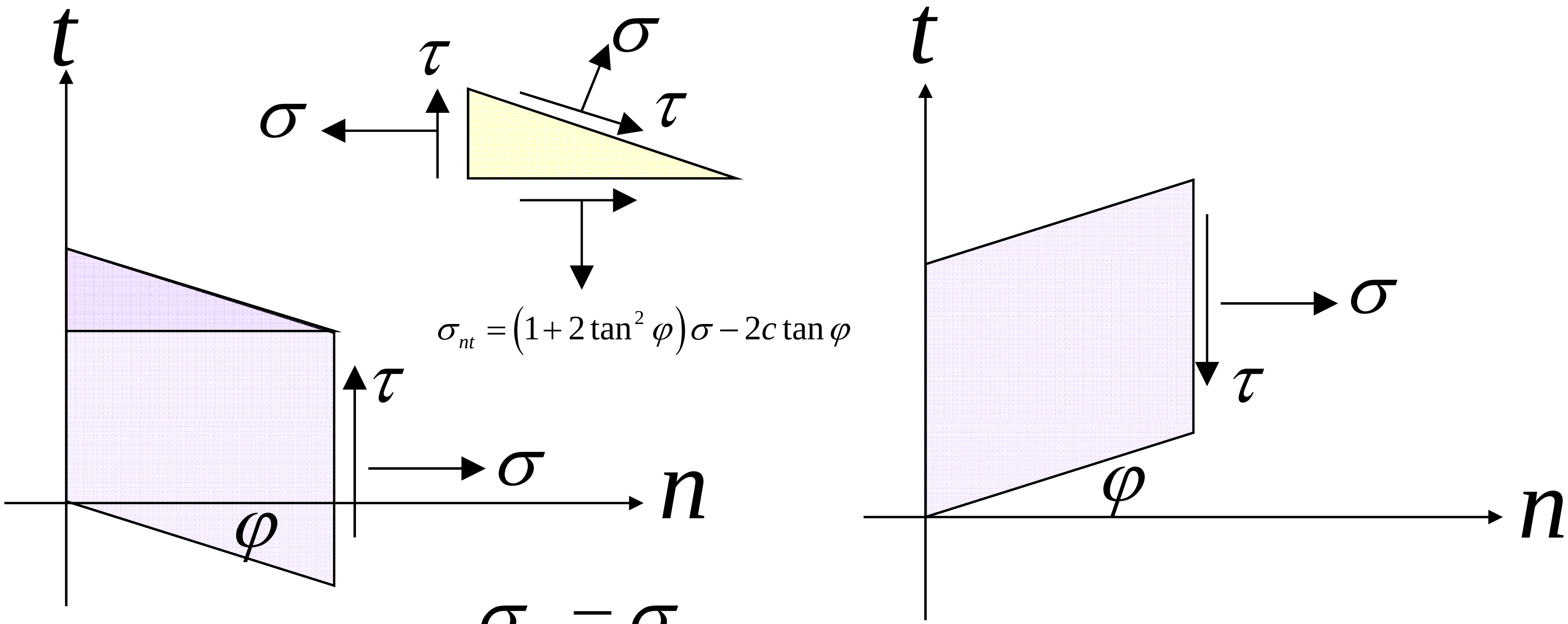


Coulomb Material

Stress Field



Stress on Failure section



$$\sigma_{nt} = \left(1 + 2\tan^2\varphi\right)\sigma - 2c\tan\varphi$$

$$\sigma_n = \sigma$$

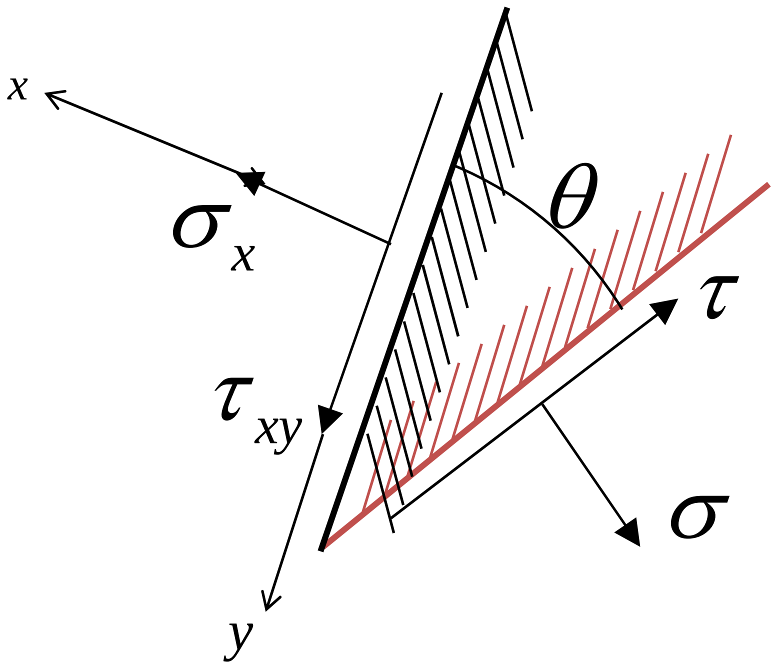
$$\tau_{nt} = \tau = c - \sigma\tan\varphi$$

Stresses from Failure Section

$$\sigma_x = c\cot\varphi + \left(\sigma_m - c\cot\varphi\right)\left[1 - \sin\varphi\sin\left(2\theta+\varphi\right)\right]$$

$$\sigma_y = c\cot\varphi + \left(\sigma_m - c\cot\varphi\right)\left[1 + \sin\varphi\sin\left(2\theta+\varphi\right)\right]$$

$$\tau_{xy} = -\left(\sigma_m - c\cot\varphi\right)\sin\varphi\cos\left(2\theta+\varphi\right)$$



Failure section from a given stresses

For given σ_x and τ_{xy}

$$\tan\beta = -\frac{\tau_{xy}}{\sigma_x - c\cot\varphi}$$

Angle θ is determined.

$$\cos\left(2\theta + \varphi - \beta\right) = \frac{\sin\beta}{\sin\varphi}$$

$$\sigma_m = c\cot\varphi + \frac{\sigma_x - c\cot\varphi}{1 - \sin\varphi\sin\left(2\theta + \varphi\right)}$$

Stress σ and τ are calculated.

$$\sigma = \left(\sigma_m + c\cot\varphi\right)\cos^2\varphi$$

Equilibrium Equations

Rankine Zone

$$\frac{\partial \sigma}{\partial x} \pm \frac{\partial\left(c-\sigma \tan\varphi\right)}{\partial y} = 0$$

$$\frac{\partial\left[\left(1+2\tan^2\varphi\right)\sigma - 2c\tan\varphi\right]}{\partial y} \pm \frac{\partial\left(c-\sigma\tan\varphi\right)}{\partial x} = 0$$

⇒

$$\frac{\partial \sigma}{\partial x} = \frac{\partial \sigma}{\partial y} = 0$$

Prandtl Zone

$$\frac{\partial \sigma}{\partial r} = 0$$

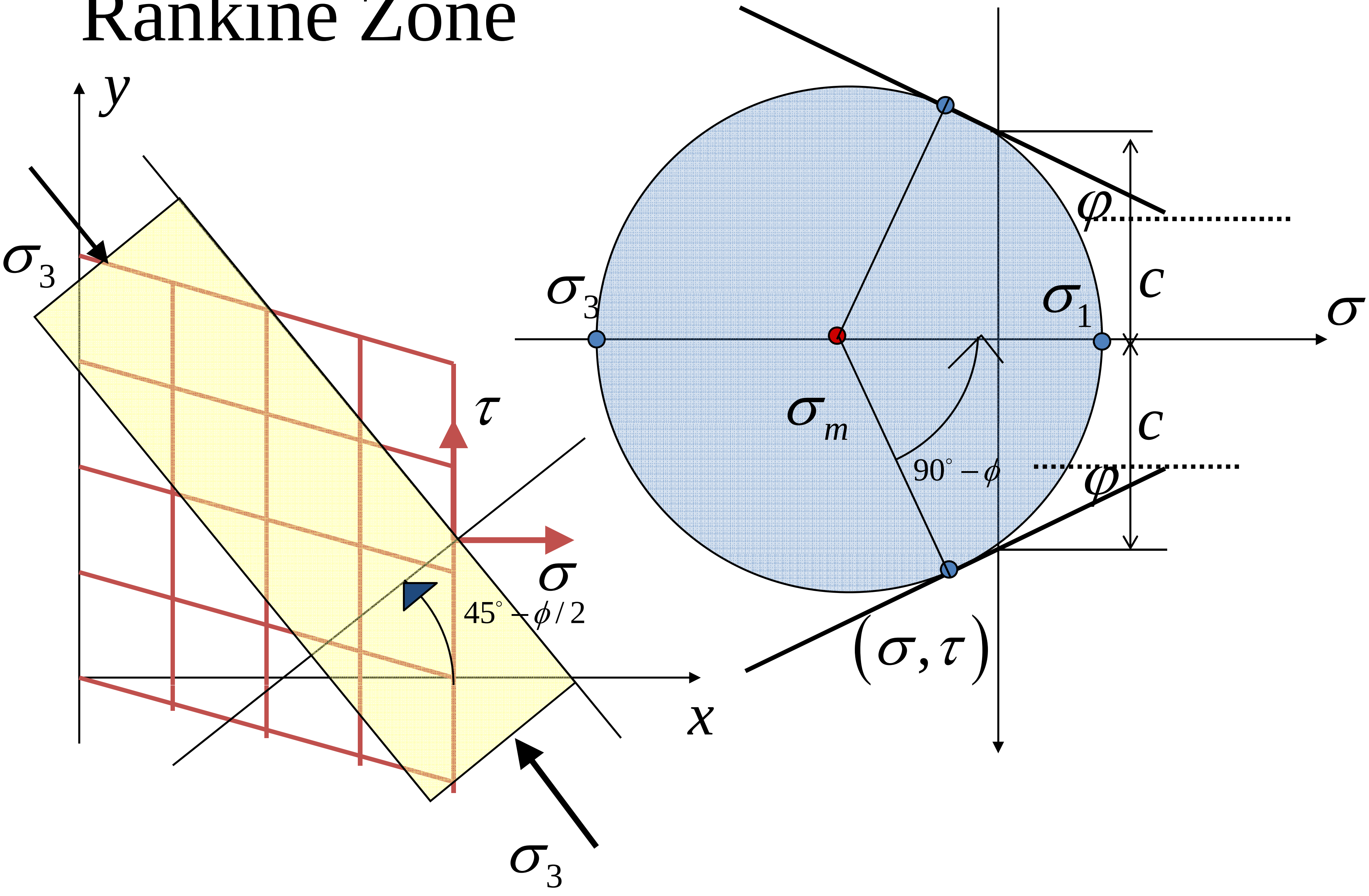
$$\frac{\partial \sigma}{\partial \theta} \pm 2\left(c - \sigma\tan\varphi\right) = 0$$

⇒

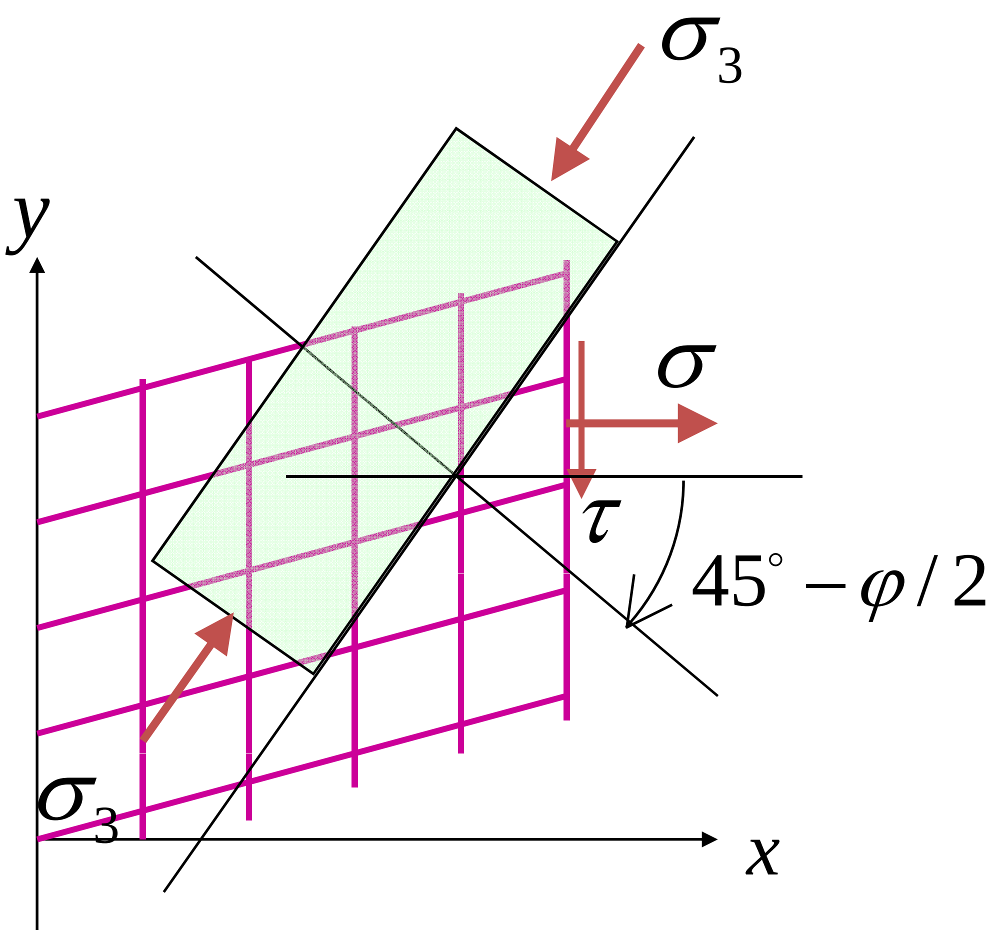
$$\sigma = -Ce^{\pm 2\theta\tan\varphi} + c\cot\varphi$$

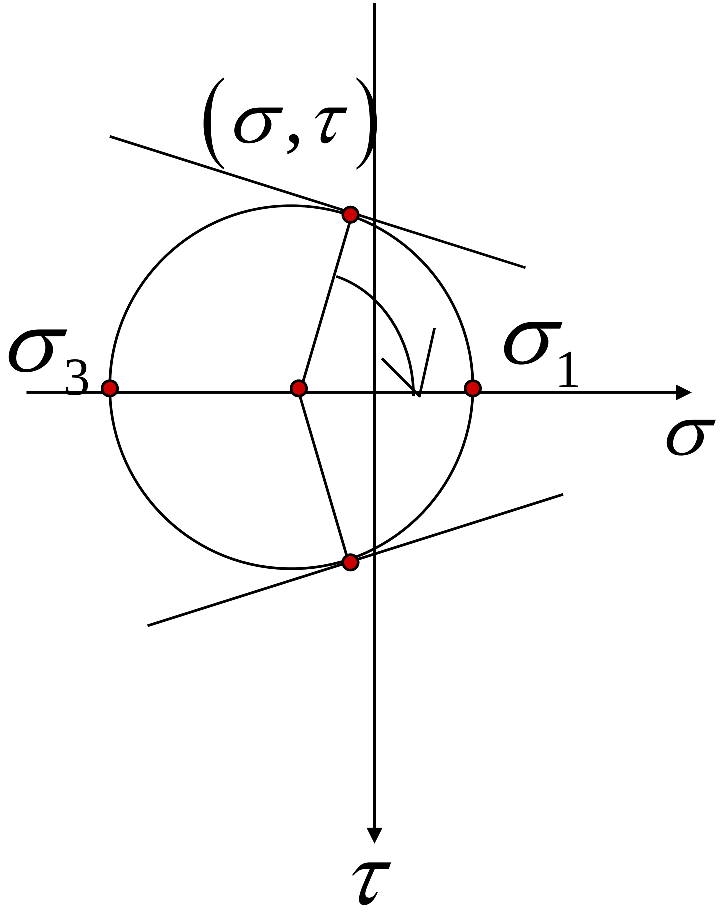
$$\tau_{r\theta} = \pm C\tan\varphi\, e^{\pm 2\theta\tan\varphi}$$

Rankine Zone

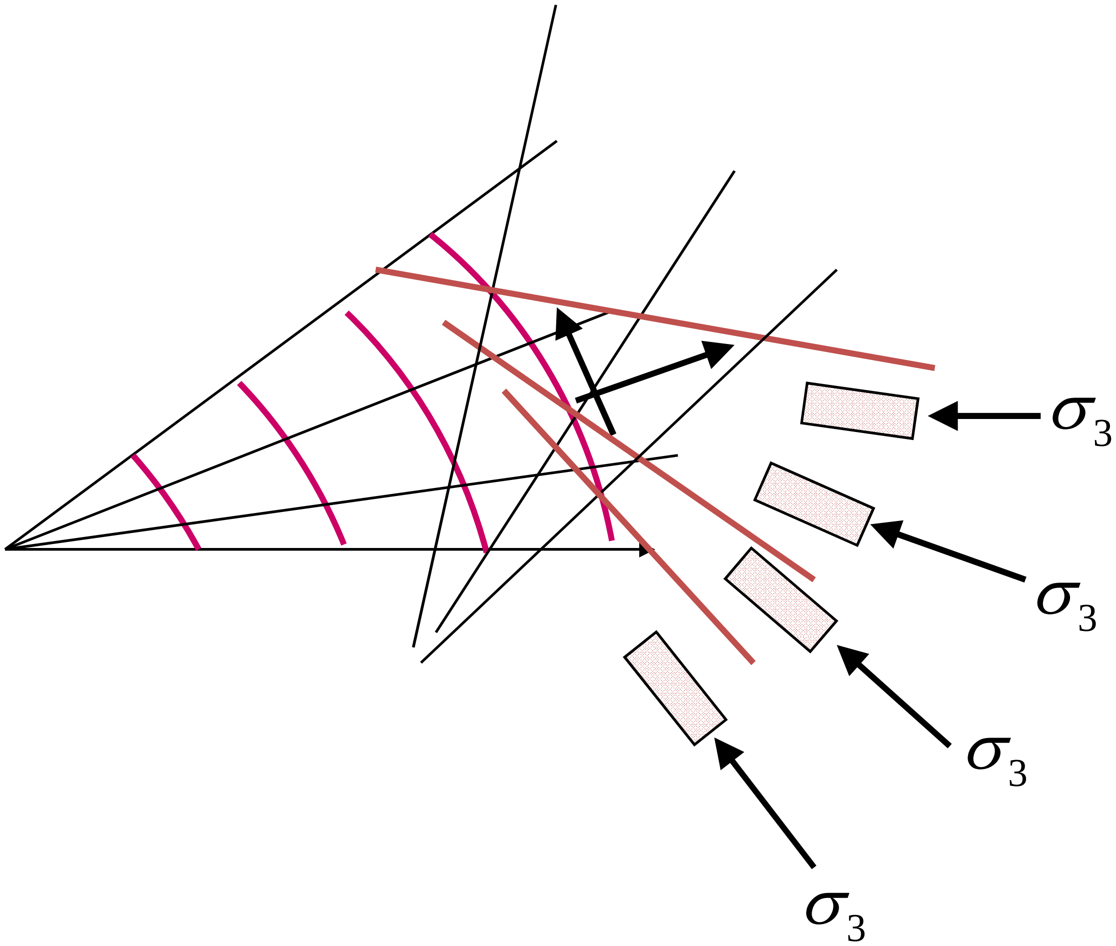


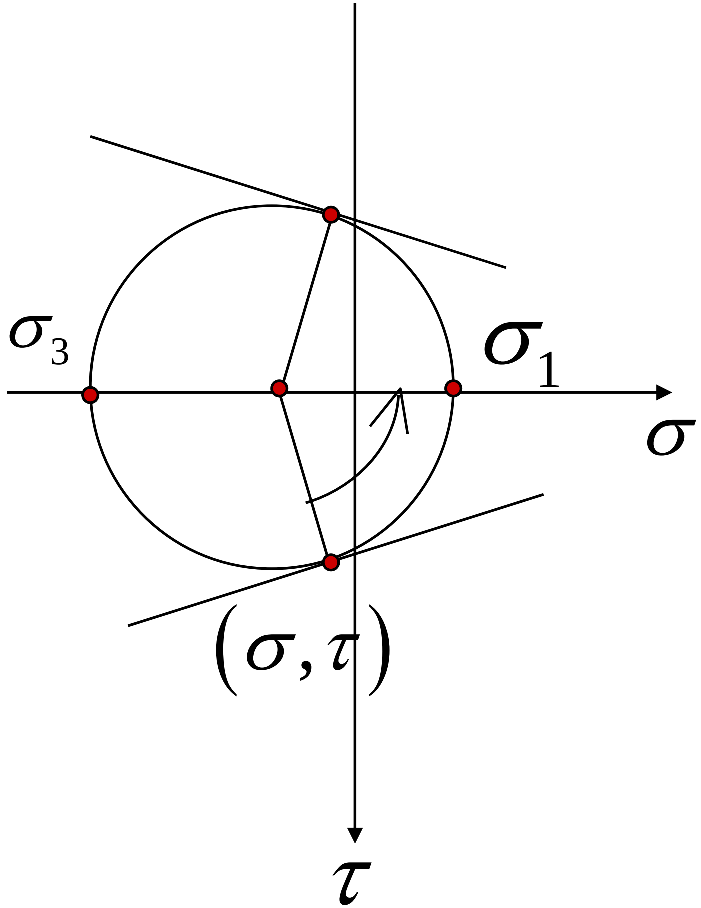
Strut and Rankine Zone



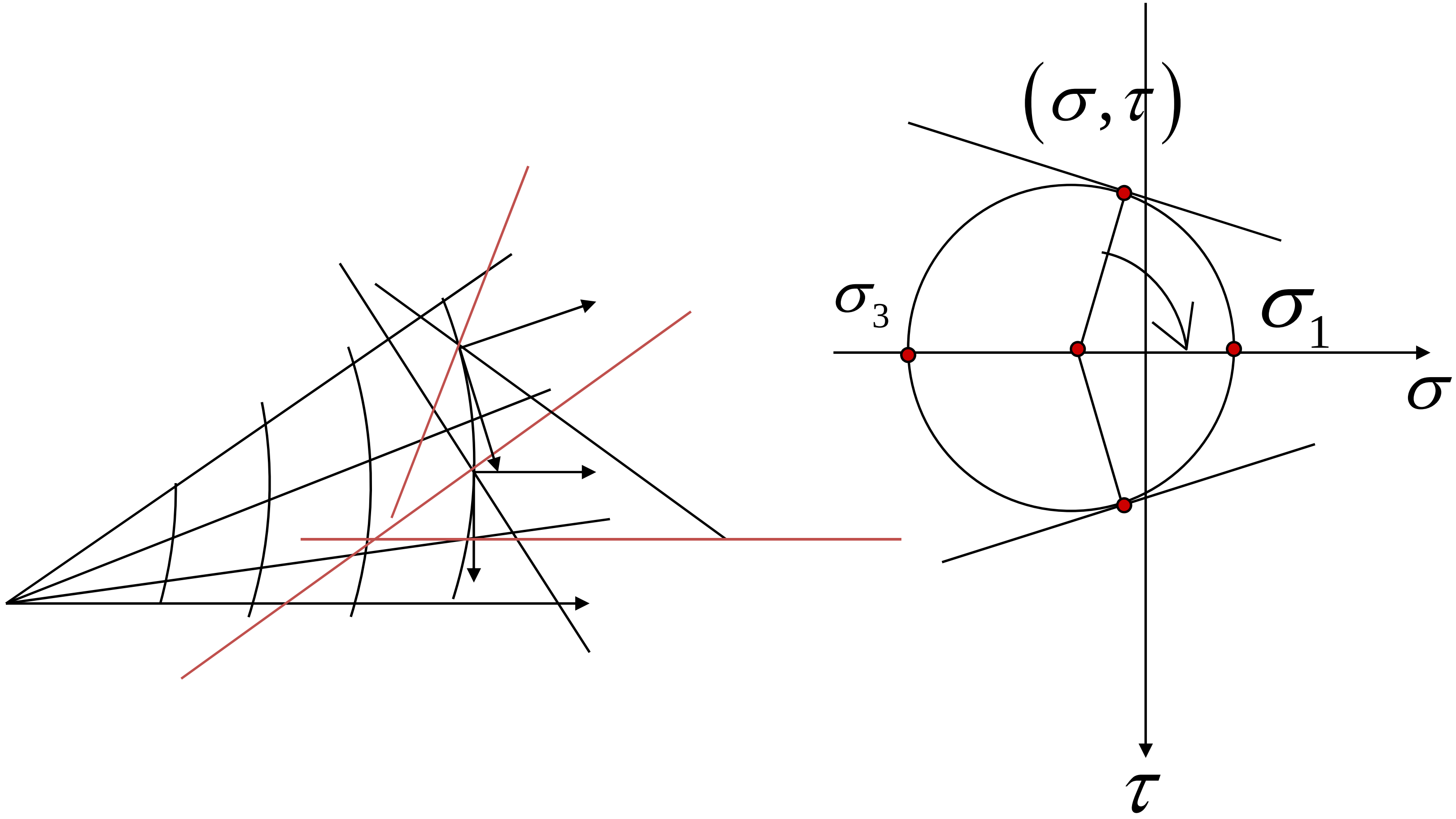


Prandtl Zone



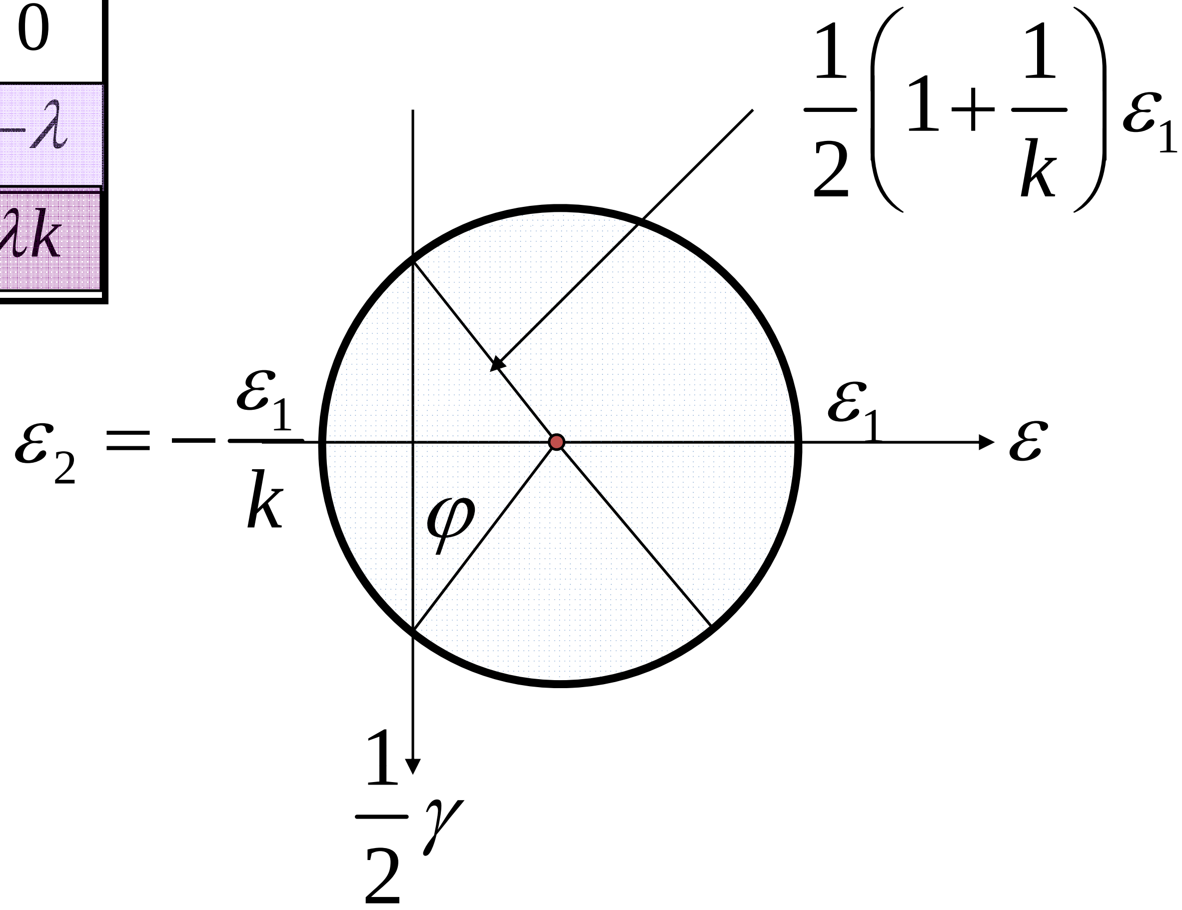


Prandtle Zone



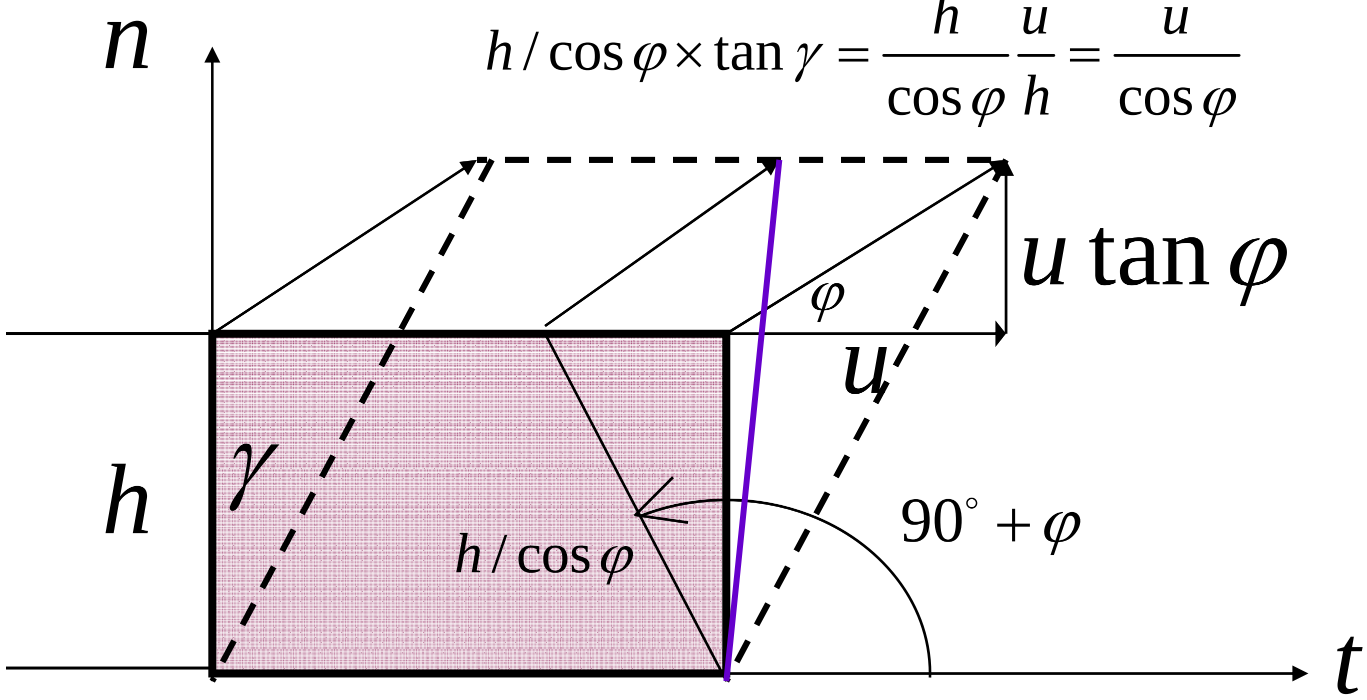
Strain Fields

	ε_1	ε_2	ε_3
1	λk	0	$-\lambda$
2	$-\lambda$	0	λk
3	λk	$-\lambda$	0
4	$-\lambda$	λk	0
5	0	λk	$-\lambda$
6	0	$-\lambda$	λk

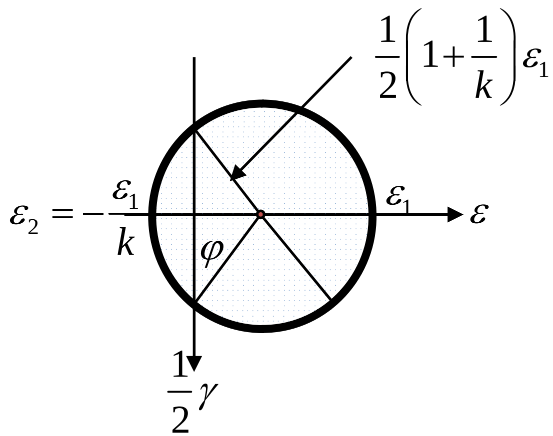


Homogeneous Strain Field

$$h/\cos\varphi \times \tan\gamma = \frac{h}{\cos\varphi}\frac{u}{h} = \frac{u}{\cos\varphi}$$

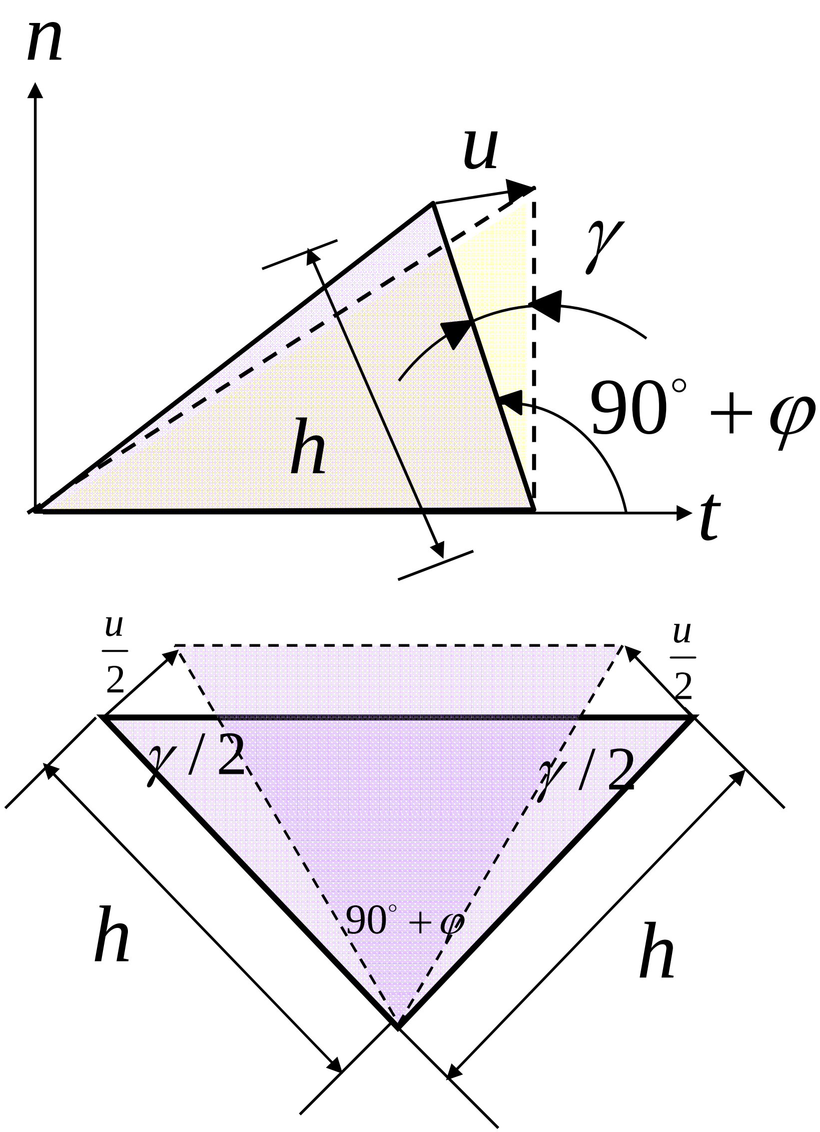


$$\varepsilon_n = \frac{u\tan\varphi}{h} = \gamma\tan\varphi$$

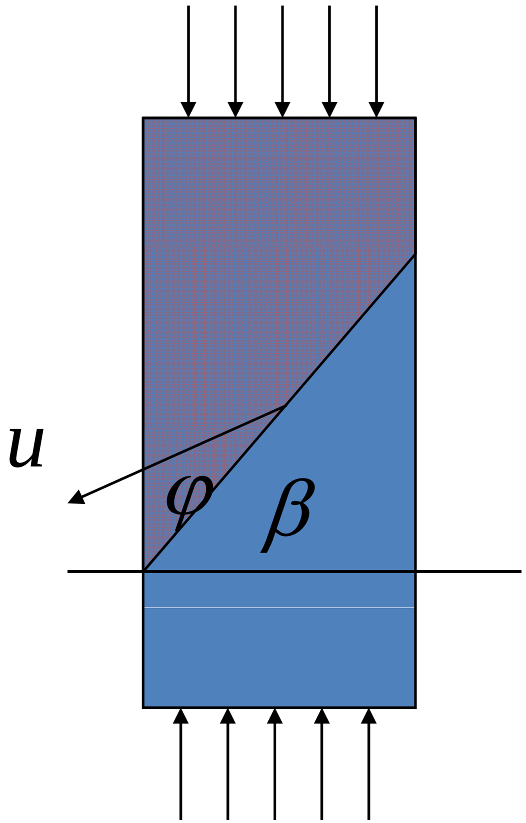


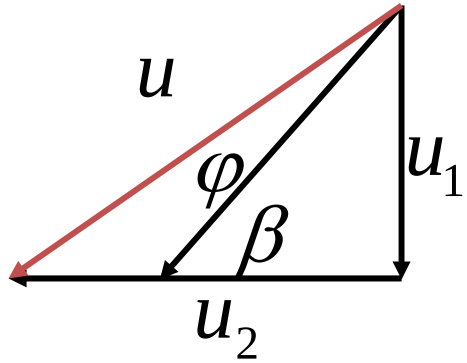
$$\varepsilon_t = 0 \qquad \gamma_{nt} = \gamma = \frac{u}{h}$$

Triangular Zone

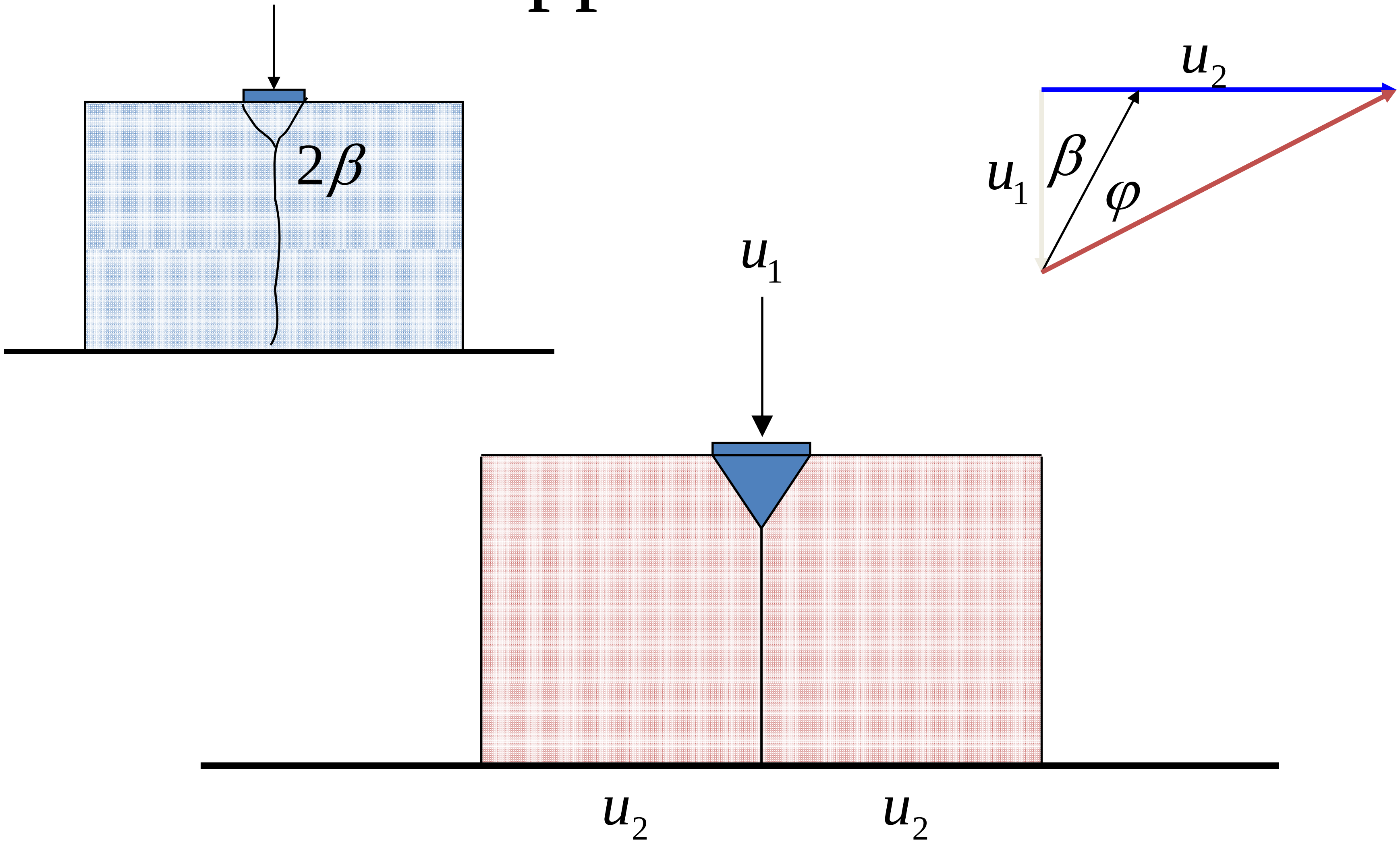


Applications

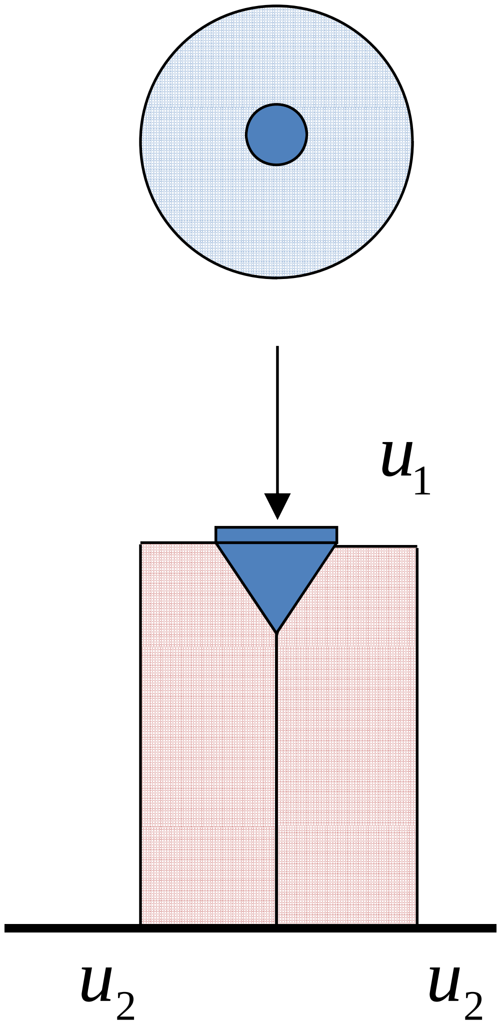


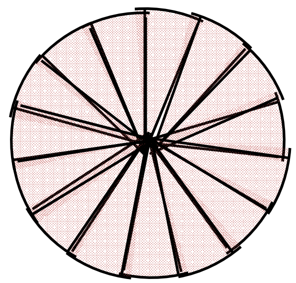


Applications

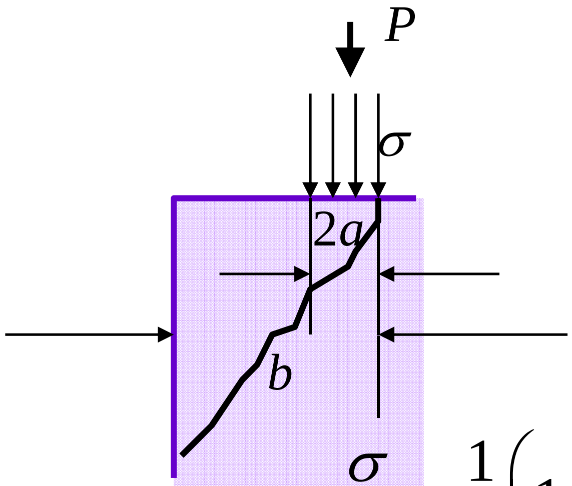


Applications

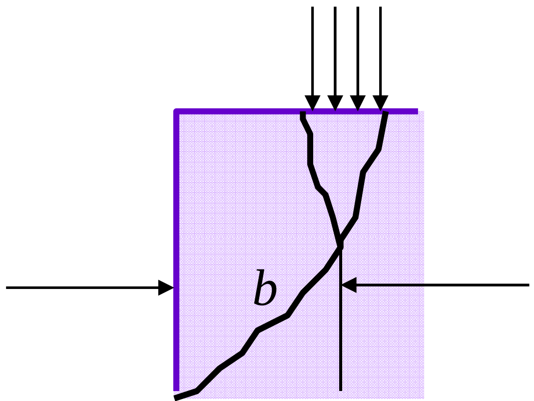




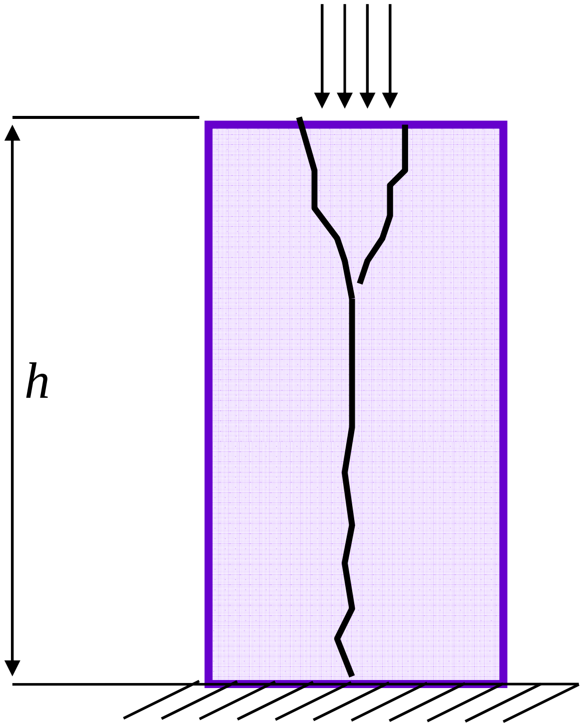
Design for concentrated loading



$$\frac{\sigma}{f_c} = \frac{1}{2}\left(1 + \frac{b}{a}\right)$$



$$\frac{\sigma}{f_c} = 1.5 + K_1\left(\frac{b}{a} - 1\right)\frac{f_t}{f_c}, K_1 \cong 3.6$$

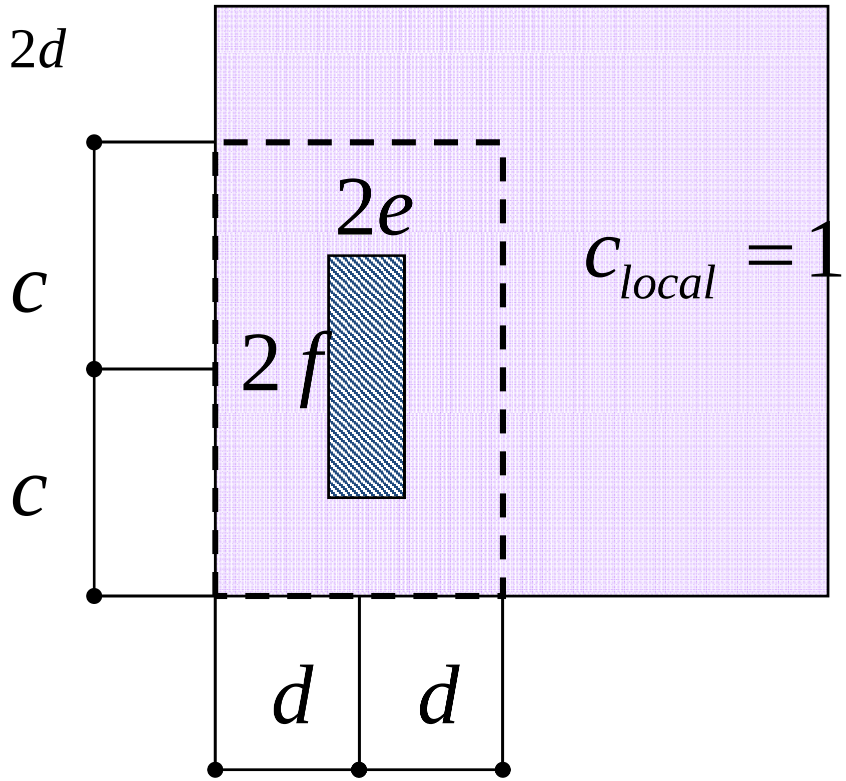


$$\frac{\sigma}{f_c} = 1.0 + K_2\left(\frac{h}{2a} - 1\right)\frac{f_t}{f_c}, K_2 \cong 2.48$$

Semi-empirical formula

$$A_o = 2e \bullet 2f$$

$$A = 2c \bullet 2d$$



$$c_{local} = 1 + K_4 \left(\sqrt{\frac{A}{A_0}} - 1 \right) \frac{f_t}{f_c}$$

$$a = \frac{1}{2}\sqrt{A_0}, b = \frac{1}{2}\sqrt{A}$$