PREDICTION OF SURFACE RADIATIVE PROPERTIES

Theoretical frame work

1) electromagnetic theory :
   Maxwell's equation

2) characteristics of radiative wave propagation :
   optical properties (refractive index) from electric and magnetic properties

μ: magnetic permeability, γ: electric permittivity

r_e: electric resistivity

- **dielectrics** $$n = \frac{c_0}{c} = c_0\sqrt{\mu\gamma}$$

n: refractive index

- **conductors**

$$n^2 = \frac{\mu\gamma c_0^2}{2}\left\{1 + \left[1 + \left(\frac{\lambda_0}{2\pi c_0 r_e \gamma}\right)^2\right]^{1/2}\right\}$$

$$\kappa^2 = \frac{\mu\gamma c_0^2}{2}\left\{-1 + \left[1 + \left(\frac{\lambda_0}{2\pi c_0 r_e \gamma}\right)^2\right]^{1/2}\right\}$$

κ: extinction coefficient or absorption index

3) **interaction of the electromagnetic wave with the interface between two media :**

laws of reflection and refraction (Snell's law and Fresnel equation)

4) **prediction of emissivity and reflectivity**

Radiative Wave Propagation within a Medium

Maxwell's equation

$$\nabla \times \underline{H} = \gamma \frac{\partial \underline{E}}{\partial t} + \frac{\underline{E}}{r_e}, \quad \nabla \times \underline{E} = -\mu \frac{\partial \underline{H}}{\partial t},$$

$$\nabla \cdot \underline{E} = 0, \quad \nabla \cdot \underline{H} = 0$$

- H: magnetic intensity,
- E: electric intensity in SI units
- μ: magnetic permeability
- γ: electric permittivity
- r_e: electric resistivity

Within an infinite, homogeneous, isotropic medium

1) Propagation in perfect dielectric media

very large electric resistivity r_e

$$\nabla \times \underline{H} = \gamma \frac{\partial \underline{E}}{\partial t} + \frac{\underline{E}}{r_e}, \quad \nabla \times \underline{E} = -\mu \frac{\partial \underline{H}}{\partial t},$$

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$$\nabla \cdot \underline{E} = 0, \quad \nabla \cdot \underline{H} = 0$$

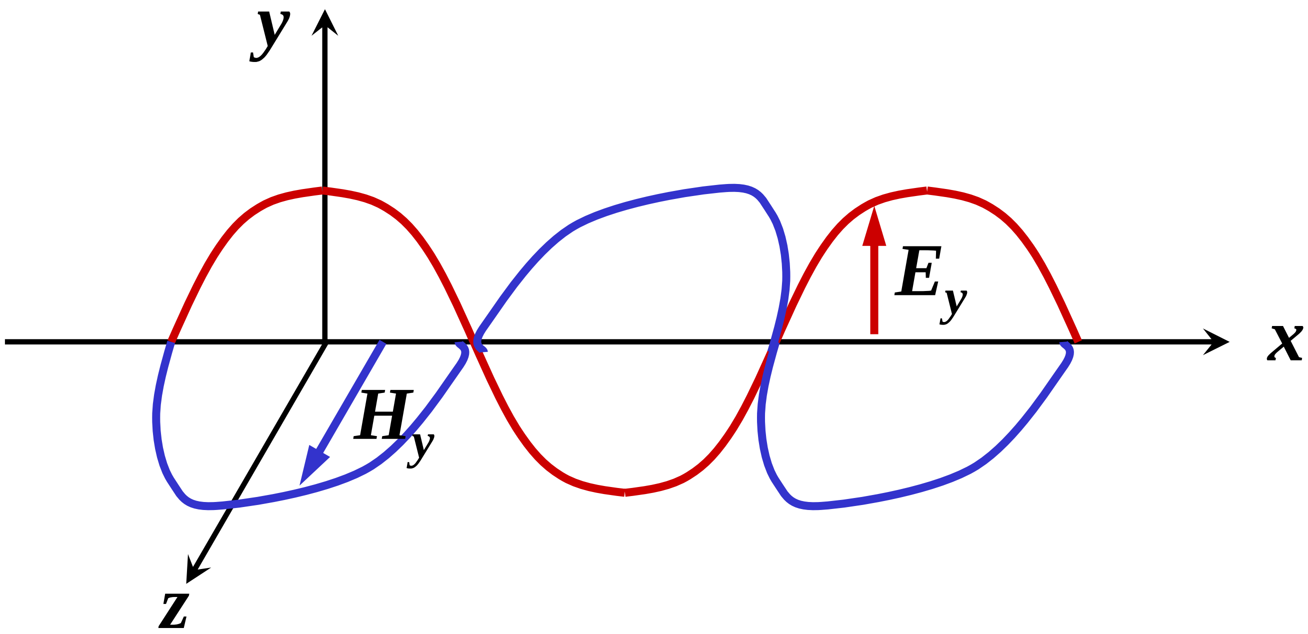
For plane wave: all the quantities concerned with the wave are constant over any *y-z* plane at any time.

Then, $$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

and *H* components can be eliminated.

$$\mu\gamma \frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2}, \quad \mu\gamma = \frac{\partial^2 E_z}{\partial t^2} = \frac{\partial^2 E_z}{\partial x^2}$$

Wave equations governing the propagation of E_y and E_z in the x direction



Electric field wave polarized in *x-y* plane, traveling in *x* direction

For polarized electromagnetic wave: *E* contained in *x-y* plane

$$\mu\gamma \frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2}$$

General solution

$$E_y = f\left(x - \frac{t}{\sqrt{\mu\gamma}}\right) + g\left(x + \frac{t}{\sqrt{\mu\gamma}}\right)$$

propagation in the positive x and negative x directions

Consider a wave propagating in the positive x direction

$$E_y = f\left(x - \frac{t}{\sqrt{\mu\gamma}}\right)$$

propagation speed: $\dfrac{dx}{dt} = \dfrac{1}{\sqrt{\mu\gamma}} = c$ **: speed of light**

For a fixed wavelength (spectral wave)

at the origin, $x = 0$

$$E_y = E_{yM} \exp(i\omega t) = E_{yM} (\cos\omega t + i \sin\omega t)$$

ω **: angular frequency**

$$\omega = 2\pi\nu = 2\pi c / \lambda = 2\pi c_0 / \lambda_0$$

A wave traveling in the positive x **direction**

$$E_y = E_{yM} \exp\left[i\omega\left(t - \frac{x}{c} \right) \right]$$

$$E_y = E_{yM} \exp\left[i\omega\left(t - \sqrt{\mu\gamma}\, x \right) \right]$$

simple refractive index:

$$n = \frac{c_0}{c} = c_0\sqrt{\mu\gamma} = \sqrt{\frac{\mu\gamma}{\mu_0\gamma_0}}$$

$$E_y = E_{yM}\exp\left[i\omega\left(t - \sqrt{\mu\gamma}\,x\right)\right]$$

$$E_y = E_{yM}\exp\left[i\omega\left(t - \frac{n}{c_0}x\right)\right]$$

E_{yM}: **undiminished amplitude through the medium**

Propagation in isotropic media of finite conductivity

$$E_y = E_{yM} \exp\left[i\omega \left(t - \frac{n}{c_0} x \right) \right] \exp\left(-\frac{\omega}{c_0} \kappa x \right)$$

κ: **extinction coefficient** or **absorption index** indicates absorption of the energy of the wave as it travels through the medium.

complex refractive index

$$\bar{n} = n - i\kappa$$

$$\mu\gamma\frac{\partial^2 E_y}{\partial t^2}=\frac{\partial^2 E_y}{\partial x^2}-\frac{\mu}{r_e}\frac{\partial E_y}{\partial t}$$

$$c_0^2\mu\gamma=\left(n-i\kappa\right)^2+\frac{i\mu\lambda_0 c_0}{2\pi r_e},$$

$$n^2-\kappa^2=\mu\gamma c_0^2,\ n\kappa=\frac{\mu\lambda_0 c_0}{4\pi r_e}$$

$$n^2=\frac{\mu\gamma c_0^2}{2}\left\{1+\left[1+\left(\frac{\lambda_0}{2\pi c_0 r_e\gamma}\right)^2\right]^{1/2}\right\}$$

$$\kappa^2=\frac{\mu\gamma c_0^2}{2}\left\{-1+\left[1+\left(\frac{\lambda_0}{2\pi c_0 r_e\gamma}\right)^2\right]^{1/2}\right\}$$

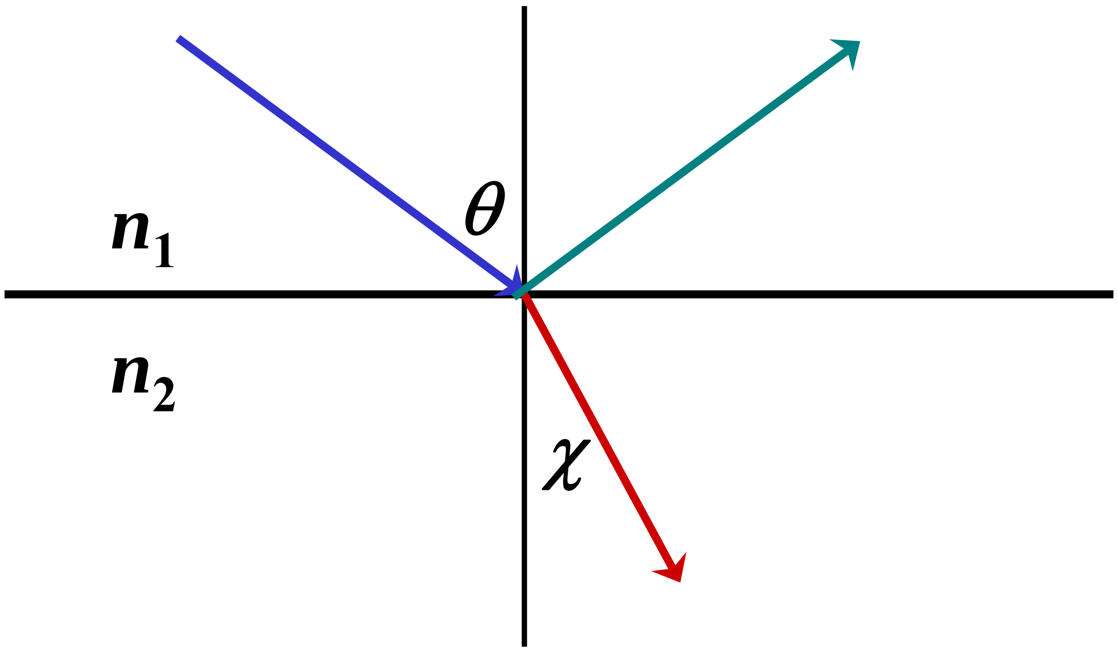
Reflectivity

(directional-hemispherical spectral specular)

Fresnel equation

(M. Born & E. Wolf “Principles of Optics” pp. 36-51)

1) Between two perfect dielectrics



Snell’s law

$$\frac{\sin\chi}{\sin\theta} = \frac{n_1}{n_2}$$

$$\rho'_{\parallel}(\theta)=\left[\frac{\tan(\theta-\chi)}{\tan(\theta+\chi)}\right]^2,\quad \rho'_{\perp}(\theta)=\left[\frac{\sin(\theta-\chi)}{\sin(\theta+\chi)}\right]^2$$

$$\rho'(\theta)=\frac{\rho'_{\parallel}(\theta)+\rho'_{\perp}(\theta)}{2}$$

$$=\frac{1}{2}\left[\frac{\tan^2(\theta-\chi)}{\tan^2(\theta+\chi)}+\frac{\sin^2(\theta-\chi)}{\sin^2(\theta+\chi)}\right]$$

$$=\frac{1}{2}\frac{\sin^2(\theta-\chi)}{\sin^2(\theta+\chi)}\left[1+\frac{\cos^2(\theta+\chi)}{\cos^2(\theta-\chi)}\right]$$

using **Snell's law** $$\frac{\sin\chi}{\sin\theta}=\frac{n_1}{n_2}$$

$$\rho'_{\|}(\theta)=\left\{\frac{(n_2/n_1)^2\cos\theta-\left[(n_2/n_1)^2-\sin^2\theta\right]^{1/2}}{(n_2/n_1)^2\cos\theta+\left[(n_2/n_1)^2-\sin^2\theta\right]^{1/2}}\right\}$$

$$\rho'_{\perp}(\theta)=\left\{\frac{\left[(n_2/n_1)^2-\sin^2\theta\right]^{1/2}-\cos\theta}{\left[(n_2/n_1)^2-\sin^2\theta\right]^{1/2}+\cos\theta}\right\}^2$$

$$\rho'_n=\left(\frac{n_2-n_1}{n_2+n_1}\right)^2$$

2) From vacuum(or air) to absorbing media

$$\rho'_{\parallel}(\theta) = \frac{a^2 + b^2 - 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}{a^2 + b^2 + 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}\rho'_{\perp}(\theta)$$

$$\rho'_{\perp}(\theta) = \frac{a^2 + b^2 - 2a\cos\theta + \cos^2\theta}{a^2 + b^2 + 2a\cos\theta + \cos^2\theta}$$

where

$$2a^2 = \left[\left(n^2 - \kappa^2 - \sin^2\theta\right)^2 + 4n^2\kappa^2\right]^{1/2} + n^2 - \kappa^2 - \sin^2\theta$$

$$2b^2 = \left[\left(n^2 - \kappa^2 - \sin^2\theta\right)^2 + 4n^2\kappa^2\right]^{1/2} - \left(n^2 - \kappa^2 - \sin^2\theta\right)$$

$$\rho'_n = \frac{\left(n_2 - 1\right)^2 + \kappa_2^2}{\left(n_2 + 1\right)^2 + \kappa_2^2}$$

Emissivity

$$\varepsilon'(\theta) = 1 - \rho'(\theta)$$

Dielectrics

$$\varepsilon'_n = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{4n}{(n+1)^2}$$

using $\varepsilon = \frac{1}{\pi}\int_{\cap} \varepsilon' \cos\theta d\omega$

$$\varepsilon = \frac{1}{2} - \frac{(3n+1)(n-1)}{6(n+1)^2} - \frac{n^2(n^2-1)^2}{(n^2+1)^3}\ln\left(\frac{n-1}{n+1}\right)$$

$$+\frac{2n^3(n^2+2n-1)}{(n^2+1)(n^4-1)} - \frac{8n^4(n^4+1)}{(n^2+1)(n^4-1)^2}\ln n$$

Metals : $\cos\chi \sim 1$

$$\varepsilon'_{\parallel}(\theta) = \frac{4n\cos\theta}{\left(n^2+\kappa^2\right)\cos^2\theta + 2n\cos\theta + 1}$$

$$\varepsilon'_{\perp}(\theta) = \frac{4n\cos\theta}{\cos^2\theta + 2n\cos\theta + n^2 + \kappa^2}$$

$$\varepsilon'(\theta) = \frac{\varepsilon'_{\parallel}(\theta) + \varepsilon'_{\perp}(\theta)}{2}$$

$$\varepsilon'_n = \frac{4n}{\left(n+1\right)^2 + \kappa^2}$$

$$\varepsilon = 4n - 4n^2 \ln \frac{1 + 2n + n^2 + \kappa^2}{n^2 + \kappa^2}$$

$$+ \frac{4n\left(n^2 - \kappa^2\right)}{\kappa} \tan^{-1} \frac{\kappa}{n + n^2 + \kappa^2}$$

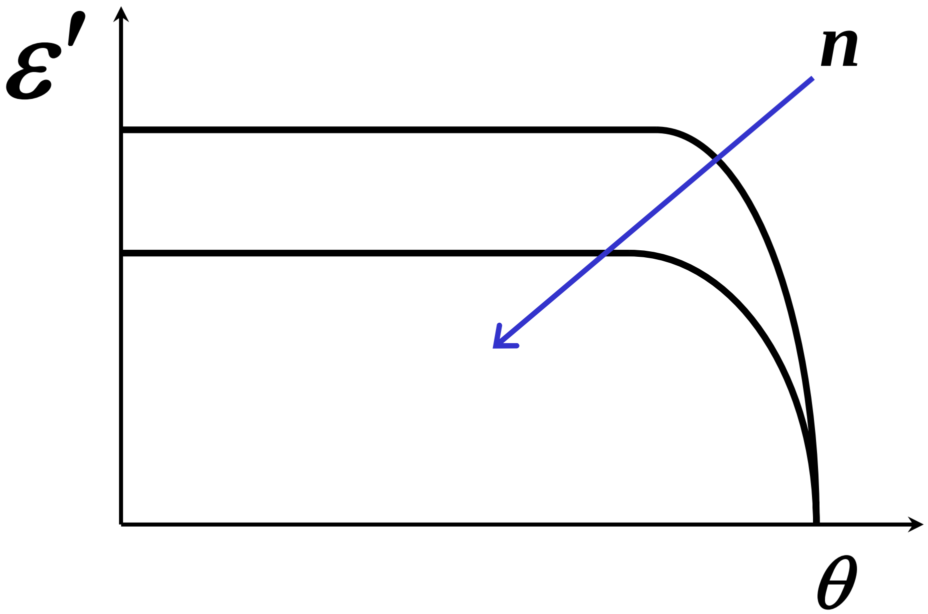
$$+ \frac{4n}{n^2 + \kappa^2} - \frac{4n^2}{\left(n^2 + \kappa^2\right)^2} \ln(1 + 2n + n^2 + \kappa^2)$$

$$- \frac{4n\left(n^2 - \kappa^2\right)}{\kappa\left(n^2 + \kappa^2\right)^2} \tan^{-1} \frac{\kappa}{1 + n}$$

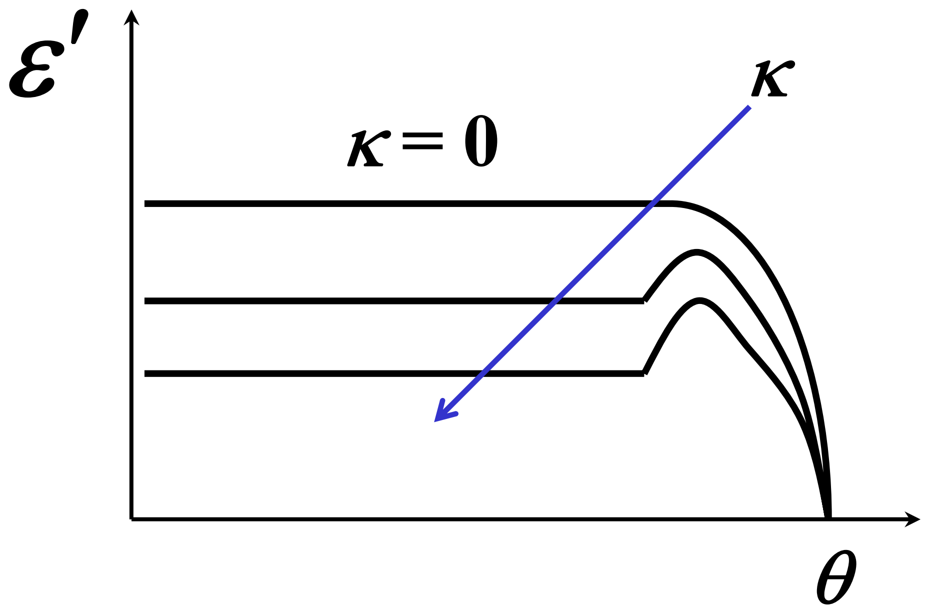
Theoretical trend

- **directional dependence**

1) Dielectrics

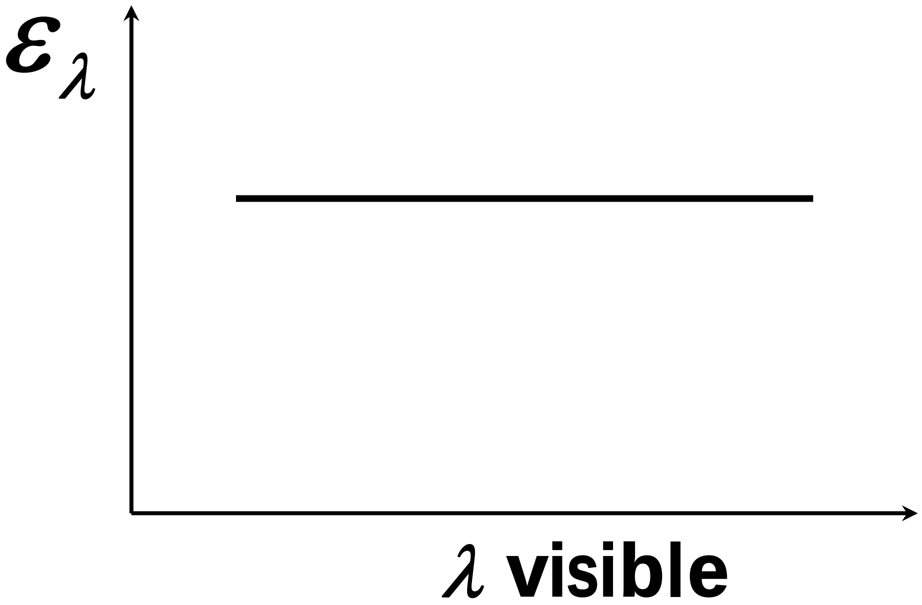


2) Conductors

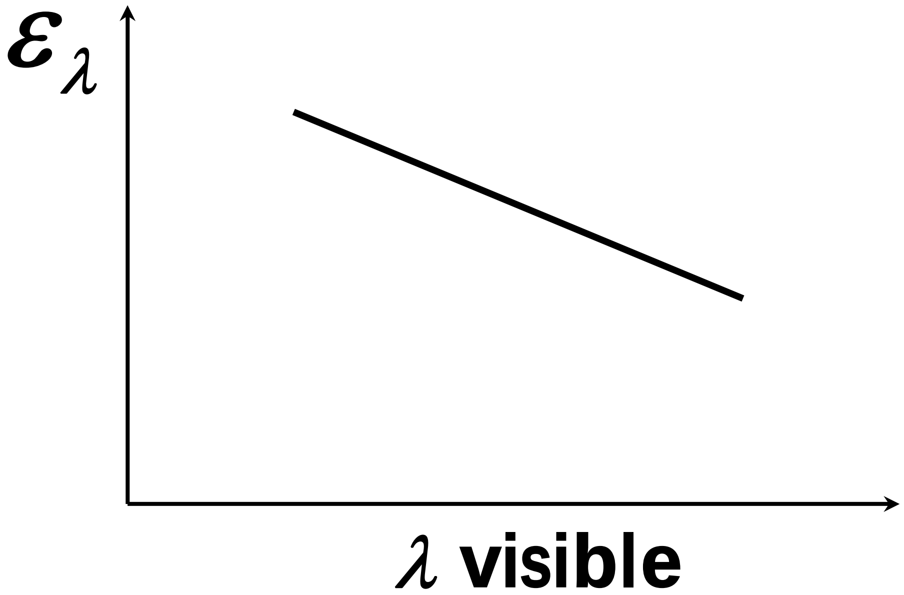


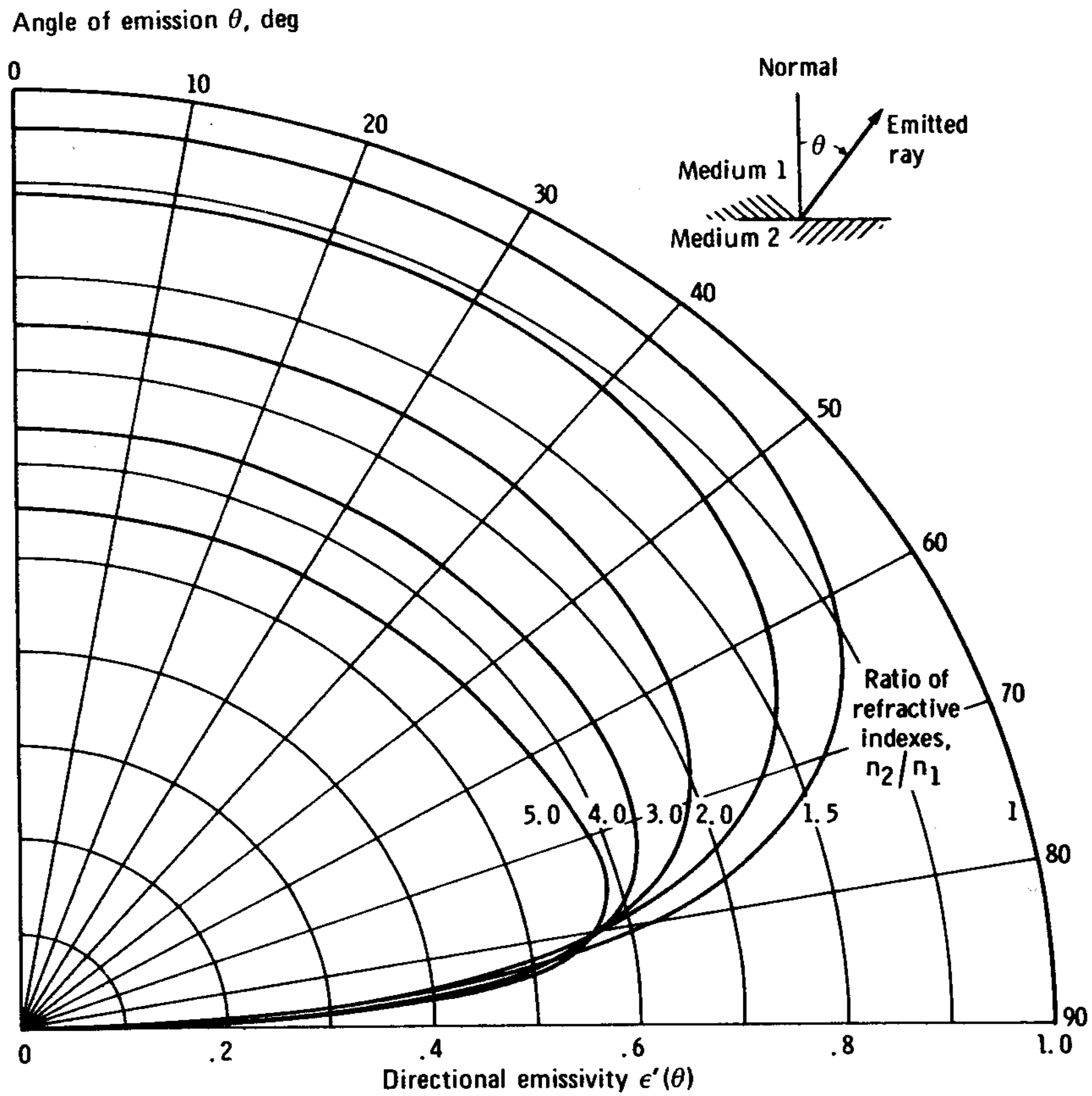
- **spectral dependence**

1) Dielectrics

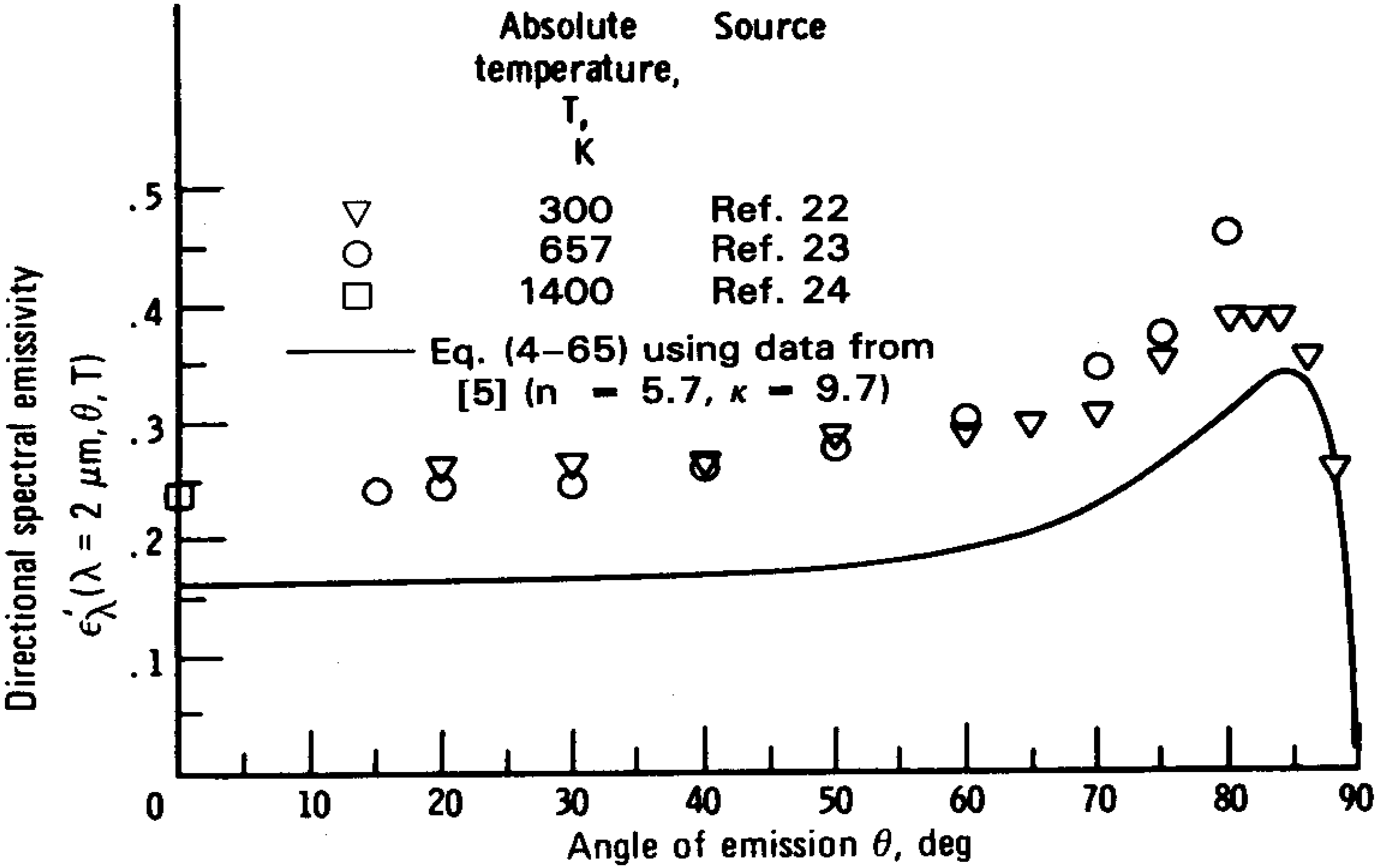


2) Conductors





Directional emissivity of ideal dielectrics predicted by EM theory



Directional spectral emissivity of platinum at wavelength $\lambda = 2\ \mu m$

Hagen – Rubens emissivity relation

for metals with small r_e for long wavelength

$$\lambda_0 > \sim 5\ \mu\text{m}, \qquad \frac{\lambda_0}{2\pi c_0 r_e \gamma} >> 1$$

$$n^2 = \frac{\mu \gamma c_0^2}{2}\left\{1 + \left[1 + \left(\frac{\lambda_0}{2\pi c_0 r_e \gamma}\right)^2\right]^{1/2}\right\}$$

$$\kappa^2 = \frac{\mu \gamma c_0^2}{2}\left\{-1 + \left[1 + \left(\frac{\lambda_0}{2\pi c_0 r_e \gamma}\right)^2\right]^{1/2}\right\}$$

$$n = \kappa = \sqrt{\frac{\mu_0 \lambda_0 c_0}{4\pi r_e}} = \sqrt{\frac{0.003\lambda_0}{r_e}}, \quad \lambda_0[\mu\text{m}], \quad r_e[\Omega - \text{cm}]$$

Directional spectral emissivity

(in the direction normal to the surface)

$$\varepsilon'_{\lambda.n}(\lambda) = 1 - \rho'_{\lambda.n}(\lambda) = \frac{4n}{2n^2 + 2n + 1} \quad \because \varepsilon'_n = \frac{4n}{(n+1)^2 + \kappa^2}$$

$$\varepsilon'_{\lambda.n}(\lambda) = \frac{2}{n} - \frac{2}{n^2} + \frac{1}{n^3} - \frac{1}{2n^5} + \frac{1}{2n^6} - \cdots$$

for $\lambda_0 > \sim 5\ \mu\text{m}$, **large** n $\qquad n = \kappa = \sqrt{\dfrac{0.003\lambda_0}{r_e}}$

$$\varepsilon'_{\lambda.n}(\lambda) = \frac{2}{\sqrt{0.003}}\left(\frac{r_e}{\lambda_0}\right)^{1/2} - \frac{2}{0.003}\frac{r_e}{\lambda_0} + \cdots$$

Note: $\quad r_e \propto T \rightarrow \varepsilon'_{\lambda.n}(\lambda) \propto \sqrt{T}$

$$\varepsilon'_{\lambda.n}(\lambda) = 36.5\left(\frac{r_e}{\lambda_0}\right)^{1/2} - 464\frac{r_e}{\lambda_0}, \quad r_e \approx r_{e,273}\frac{T}{273}$$

Directional total emissivity

$$\varepsilon_n'(T) = \frac{1}{\sigma T^4}\int_0^\infty \varepsilon_{\lambda.n}' e_{\lambda b} d\lambda$$

$$\cong \frac{1}{\sigma T^4}\int_0^\infty 2\left(r_e / 0.003\lambda_0\right)^{1/2} e_{\lambda_0 b} d\lambda_0$$

$$= 0.575\left(r_e T\right)^{1/2} \approx 0.0348\sqrt{r_{e,273}}T$$

with additional terms

$$\varepsilon_n'(T) = 0.575(r_e T)^{1/2} - 0.177(r_e T) + 0.058(r_e T)^{3/2} - \cdots$$

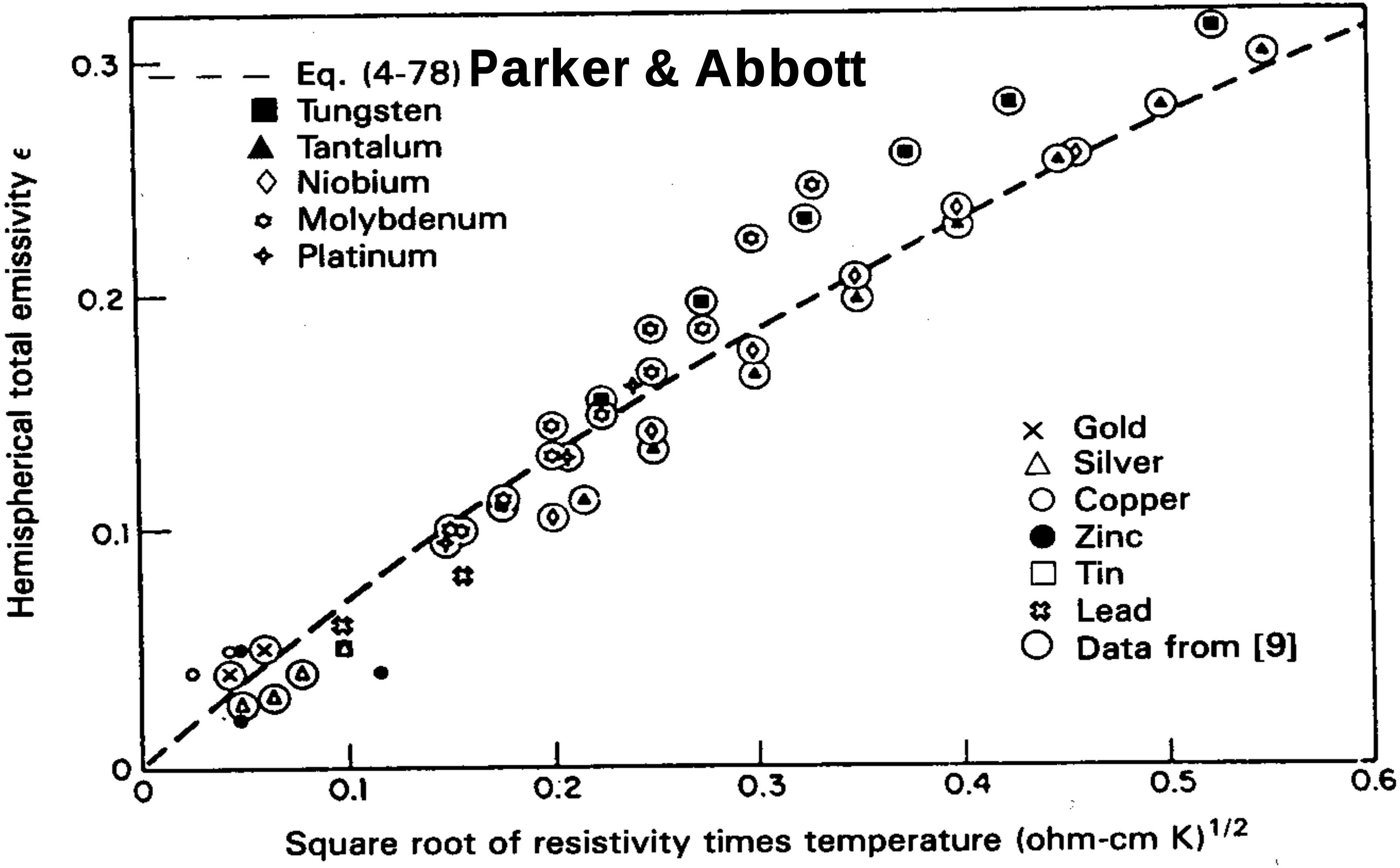
two-term approximation:

$$\varepsilon_n'(T) = 0.576(r_e T)^{1/2} - 0.124(r_e T)$$

three-term approximation:

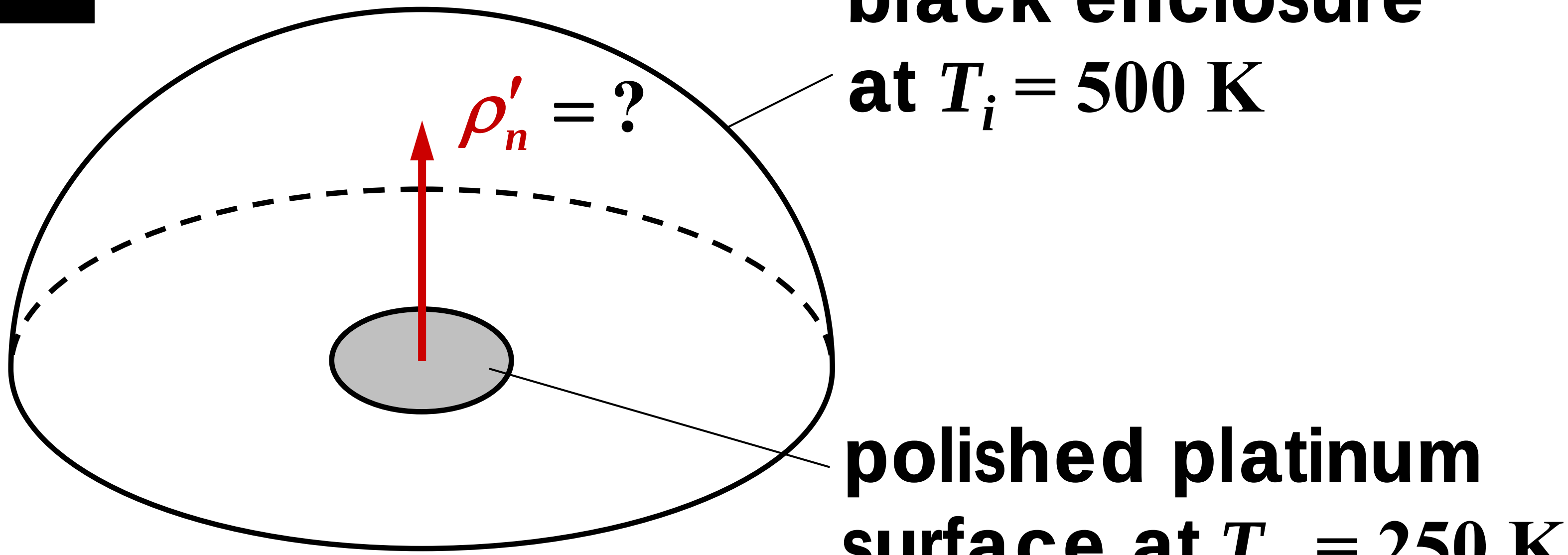
$$\varepsilon_n'(T) = 0.578(r_e T)^{1/2} - 0.178(r_e T) + 0.0584(r_e T)^{3/2}$$

$$\varepsilon(T) = 0.766(r_e T)^{1/2} - \left(0.309 - 0.0889 \ln r_e T\right) r_e T + 0.0175(r_e T)^{3/2}$$



Hemispherical total emissivity of various metals compared with theory

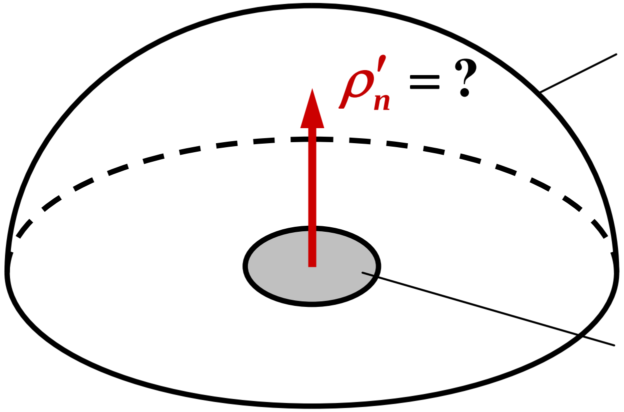
Ex 4-4



Hagen – Rubens emissivity relation

$$\varepsilon_{\lambda,n}' \propto \sqrt{T}, \quad \varepsilon_n'(T) \approx 0.0348\sqrt{r_{e,273}T}$$

platinum : $r_{e,293} = 10^{-5}$



$$\varepsilon'_n(T) \approx 0.0348\sqrt{r_{e,273}}\,T$$

$$\varepsilon'_{\lambda,n} \propto \sqrt{T}$$

$$\rho'_n(250\text{ K}) = 1 - \alpha'_n(250\text{ K}) \stackrel{?}{=} 1 - \varepsilon'_n(250\text{ K})$$

$$\alpha'_n(250\text{ K}) = \frac{\int_0^\infty \alpha'_{\lambda,n}(250\text{ K})\, i_{\lambda b}(500\text{ K})\, d\lambda}{\int_0^\infty i_{\lambda b}(500\text{ K})\, d\lambda}$$

$$= \frac{\int_0^\infty \varepsilon'_{\lambda,n}(250\text{ K})\, i_{\lambda b}(500\text{ K})\, d\lambda}{\int_0^\infty i_{\lambda b}(500\text{ K})\, d\lambda}$$

$$\alpha'_n(250\text{ K}) = \frac{\int_0^\infty \varepsilon'_{\lambda,n}(250\text{ K})\, i_{\lambda b}(500\text{ K})\, d\lambda}{\int_0^\infty i_{\lambda b}\, d\lambda}$$

Since $\varepsilon'_{\lambda,n} \propto \sqrt{T}$

$$\varepsilon'_{\lambda,n}(250\text{ K}) = \sqrt{\frac{250}{500}}\,\varepsilon'_{\lambda,n}(500\text{ K}) = \frac{1}{\sqrt{2}}\,\varepsilon'_{\lambda,n}(500\text{ K})$$

$$\alpha'_n(250\text{ K}) = \frac{\frac{1}{\sqrt{2}}\int_0^\infty \varepsilon'_{\lambda,n}(500\text{ K})\, i_{\lambda b}(500\text{ K})\, d\lambda}{\int_0^\infty i_{\lambda b}(500\text{ K})\, d\lambda}$$

$$= \frac{\varepsilon'_n(500\text{ K})}{\sqrt{2}}$$

$$\rho'_n(250\ \mathrm{K}) = 1 - \alpha'_n(250\ \mathrm{K}) = 1 - \frac{\varepsilon'_n(500\ \mathrm{K})}{\sqrt{2}}$$

$$\varepsilon'_n(500\ \mathrm{K}) = 0.0348\sqrt{r_{e,273}T} = 0.0348\sqrt{r_{e,293}}\sqrt{\frac{273}{293}}T$$

$$= 0.0348\sqrt{10^{-5}}\sqrt{\frac{273}{293}} \times 500 = 0.053$$

$$\rho'_n(250\ \mathrm{K}) = 1 - \frac{0.053}{\sqrt{2}} = 0.963$$

With gray body assumption

$$\rho'_n(250\ \mathrm{K}) = 1 - \alpha'_n(250\ \mathrm{K}) = 1 - \varepsilon'_n(250\ \mathrm{K})$$

$$= 1 - 0.0348\sqrt{r_{e,273}T}$$

$$= 1 - 0.0348\sqrt{10^{-5}}\sqrt{\frac{273}{293}} \times 250 = 1 - 0.027 = 0.973$$